Something from Nothing

Giorgio Sbardolini
Nihil + Meaning Language and Cognition, ILLC, March 2024

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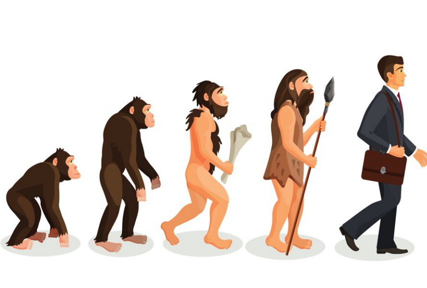
First theme: **logic and change**

- Many people think that **logic** matters for the study of **what meaning is**.



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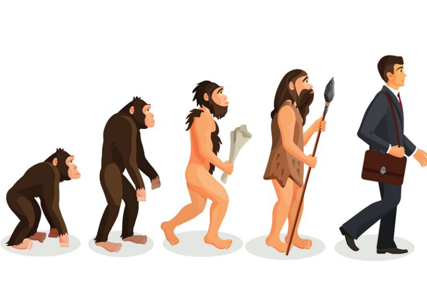

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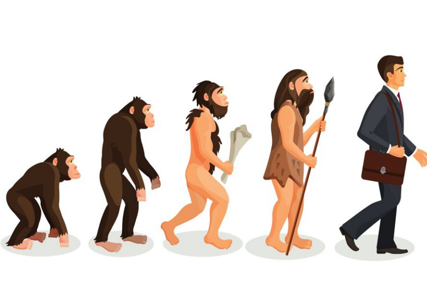
- Many people think that **logic** matters for the study of **what meaning is**.
- Not many people think that **logic** matters for the study of **how meaning changes**.
- (Of course, evolution has its own "logic", but not in the sense of "valid arguments".)



Second theme: **fossils**

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- At lexical level we find a convergence to universal negatives, regardless of morphological history.

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Third theme: **negation**

- Negation is not **just** Boolean negation.

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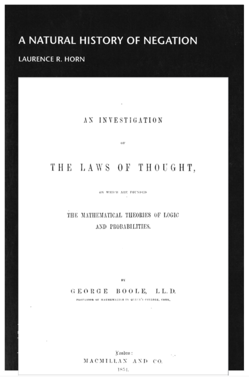
$$\models \varphi \vee \neg\varphi$$



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- **Russell's Thesis**: "all forms of negation are reducible to a suitably placed *it is not the case that*" (Arthur Prior [16, 459]).
- Today's "Russellians": Boolean negation + Scalar Implicatures (*exh*)

Aim

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- Homogeneity, NPIs, Lexical gaps: not separate phenomena but stages of a common history.
- Different "grades" of cognitive involvement:

BSML		Classical Logic
BSML$+$	Aloni (2022)	Free Choice
BSML$\times$		Homogeneity
BSML$+\times$	w/ T. Roberts	NPIs (*even*, *any*)
BSML$\circledast$	w/ L. Incurvati w/ F. Carcassi	Lexical gaps

Overview

Background

Neglect Zero

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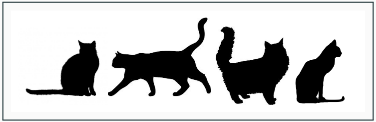

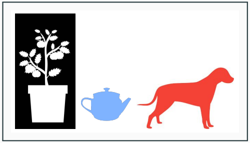

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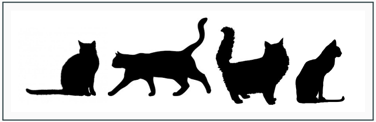

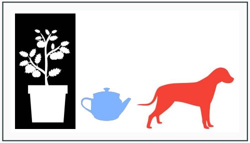

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- O&D-Sampling is the hypothesis that speakers may be sloppy in verification tasks that require a sampling method.

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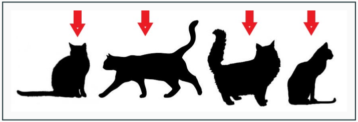

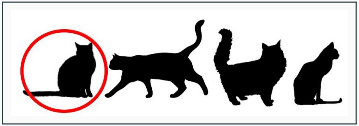

Homogeneity

- Homogeneity inferences arise with definite plurals, conjunctions, and other constructions [3, 13, 14, 19, 21].

(2) a. Our questions were answered. ⇒ Every question was answered.
b. Our questions were not answered. ⇒ No question was answered.

(3) a. They saw Delta and Omicron coming. ⇒ Delta was expected.
b. They didn't see Delta and Omicron coming. ⇒ Delta was not expected.

- Homogeneity effects are "all or nothing" readings.
- Not for today: Non-maximality.

$\Rightarrow$ Definite plurals have $\forall$ semantics.

Homogeneity effects arise whenever negation expresses contrariety ($\neg\exists$), instead of contradiction ($\neg\forall$): if so, negation is not Boolean.

Hom is modeled in BSML$\times$.

NPIs

(4) a. Even John left. $\models \begin{cases} \text{John left.} & (i) \\ \text{John was the least likely to leave.} & (ii) \\ \text{Someone else left.} & (iii) \\ \text{Everyone else left.} & (iv) \end{cases}$

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- (*iv*) is not predicted but see [6, 8, 12], a.o.
- None of the inferences in (3b) are predicted.

$\Rightarrow$ *Not even* is the (non-Boolean) negation of *even*.

- *Even* has $\forall$ "homogeneous" semantics, modeled in BSML$+\times$.
- *Not even* is "on the way" to lexicalization (and it is lexicalized in other languages, such as Greek and Italian).
- The biases modeled in BSML$+\times$ can be suspended, blocking universal and additive inferences. (Rullman's data)

(6) a. I didn't see any cake.
b. *I saw any cake.

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(7) a. Any student can do it. $\Rightarrow$ *a* can do it and *b* can do it ...
b. Any worker who arrived late was fired.

Any

$\Rightarrow$ *any* $\approx$ EVEN ONE but **obligatorily** in BSML+$\times$ (cf. Lahiri [15], also: Dayal 1998, van Rooij 2008)

- Both *any* $\models \forall$ and $\neg$*any*$\models \neg\exists$.

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- Disharmony is avoided by preventing *any* from being asserted, unless restricted (e.g. by a relative clause): $\textit{any } x \textit{ is } F \not\models \forall x F x$.

Lexical gaps

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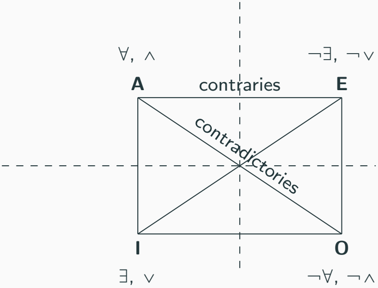
all
always
both
and

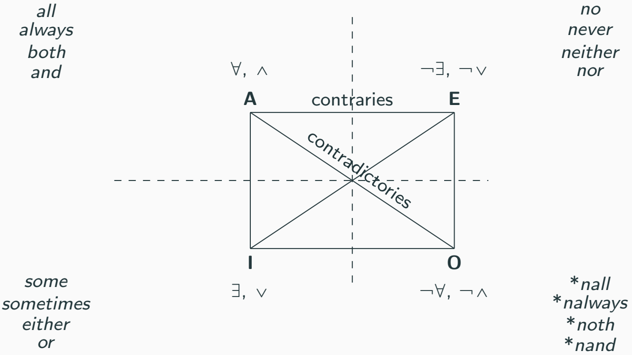
no
never
neither
nor



some
sometimes
either
or

**nall*
**nalways*
**noth*
**nand*

Lexical gaps



- The Boolean negation of *some* can lexicalize as *no*, but the Boolean negation of *all* does not lexicalize as **nall*. Same for *or/nor* but not *and/*nand*. (Horn, a.o.)

$\Rightarrow$ Lexical negation *n-* is not Boolean.

- In BSML$\ast$, the Boolean negations of *every* ($\forall$) and *and* ($\wedge$) cannot be expressed (but $\neg\exists$ and $\neg\vee$ can).
- Lexical gaps in the domain of connectives and quantifiers are explained by **undefinability** results in the (non-classical) logic of mental representations: the logic of the lexicon. [4, 10, 18]

A fossil

An English fossil: **ever**.

I. **Always, at all times; in all cases. Now mostly replaced by forever *adv.***

I.1.a. Throughout all time, eternally; throughout all past or all future time; forever; perpetually (often hyperbolically or in relative sense: throughout one's life, etc.). Also emphatically **ever and ever**, †**ever ay**, †**ever and o**. Now *archaic* and *literary*. **Old English–**

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1549 That wee may euer liue with thee in the worlde to come.
Booke of Common Prayer (STC 16267) Firste Daie of Lente f. xxxiiii*

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PROSPERO Spirits, which by mine art
I have from their confines called to enact
My present fancies.
FERDINAND Let me live here ever.
So rare a wondered father and a wise
Makes this place paradise.

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Varieties of negation

- Developed in Maria Aloni's [1] account of Free Choice.
- Background: team logic [23], inquisitive semantics [5], alternative semantics [2], situation semantics [11], truth-maker semantics [7], bilateral logic [20].
- A classical modal logic as background, on top of which we may define different kinds of cognitive biases (here, two).
- A weak Neglect Zero bias (NE) accounts for Free Choice Disjunction.

Semantics

$$M, s \vDash Pt \text{ iff } \forall i \in s : I_i(t) \in I_i(P, w_i)$$

$$M, s \dashv Pt \text{ iff } \forall i \in s : I_i(t) \notin I_i(P, w_i)$$

$$M, s \vDash \neg\varphi \text{ iff } M, s \dashv \varphi$$

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$M, s \models \neg\varphi$ iff $M, s \dashv \varphi$

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BSML+ (Aloni)

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- BSML translates into Classical Modal Logic over the language of ML.
- BSML+ is obtained as follows (Aloni 2022):

$$M, s \models \mathrm{NE} \text{ iff } s \neq \varnothing$$

$$M, s \dashv \mathrm{NE} \text{ iff } s = \varnothing$$

$$M, s \models p^{+} \text{ iff } M, s \models p \wedge \mathrm{NE}$$

$$M, s \dashv p^{+} \text{ iff } M, s \dashv p \wedge \mathrm{NE}$$

$$M, s \models [O^{n}(\varphi_1, \ldots, \varphi_n)]^{+} \text{ iff } M, s \models [O^{n}(\varphi_1^{+}, \ldots, \varphi_n^{+})] \wedge \mathrm{NE}$$

$$M, s \dashv [O^{n}(\varphi_1, \ldots, \varphi_n)]^{+} \text{ iff } M, s \dashv [O^{n}(\varphi_1^{+}, \ldots, \varphi_n^{+})] \wedge \mathrm{NE}$$

with $O \in \{\neg, \vee, \wedge, \forall, \exists\}$

- BSML$\times$ is obtained as follows:

$$M, s \models \text{NS} \text{ iff } s = \varnothing \ \& \ \forall t \subseteq s : M, t \dashv \text{NS} \rightarrow t = s$$

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$$M, s \models p^{\times} \text{ iff } M, s \models p \vee \text{NS}$$

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- Neglect Zero

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  with $O \in \{\neg, \vee, \wedge, \forall, \exists\}$
- Neglect Zero
- One-and-Done Sampling

BSML$\times$ implements NZ+O&D: if a state rejects something, it is equivalent to any of its (nonempty) substates that reject anything.

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$$M, s \models \varphi \wedge \psi \text{ iff } M, s \models \varphi \ \& \ M, s \models \psi$$

$$M, s \mathrel{=\!\!|} \varphi \wedge \psi \text{ iff } \exists t, t' : s = t \cup t' \ \& \ M, t \mathrel{=\!\!|} \varphi \ \& \ M, t' \mathrel{=\!\!|} \psi$$

BSML×

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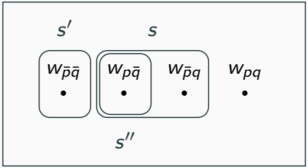
$$M, s \Dashv \varphi \wedge \psi \text{ iff } \exists t, t' : s = t \cup t' \;\&\; M, t \Dashv \varphi \;\&\; M, t' \Dashv \psi$$



$$s \models \neg(p \wedge q)$$
$$s' \models \neg(p \wedge q)$$
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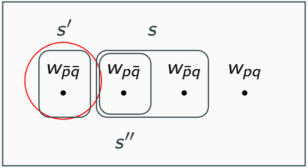

$$s \models \neg(p \wedge q) \text{ and } s \not\models [\neg(p \wedge q)]^{\times}$$

$$s' \models \neg(p \wedge q) \text{ and } s' \models [\neg(p \wedge q)]^{\times}$$

$$s'' \models \neg(p \wedge q) \text{ and } s'' \not\models [\neg(p \wedge q)]^{\times}$$

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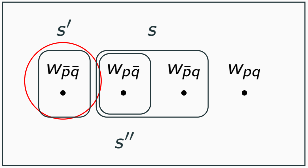
$$M, s \dashv \varphi \wedge \psi \text{ iff } \exists t, t' : s = t \cup t' \;\&\; M, t \dashv \varphi \;\&\; M, t' \dashv \psi$$



$s \vDash \neg(p \wedge q)$ and $s \nvDash [\neg(p \wedge q)]^{\times}$
$s' \vDash \neg(p \wedge q)$ and $s' \vDash [\neg(p \wedge q)]^{\times}$
$s'' \vDash \neg(p \wedge q)$ and $s'' \nvDash [\neg(p \wedge q)]^{\times}$

$$[\neg(p \wedge q)]^{\times} \vDash \neg p \wedge \neg q$$

$$M, s \models \forall x \varphi \text{ iff } \forall d : M, s^x_d \models \varphi$$

$$M, s \mathrel{=\!\!|} \forall x \varphi \text{ iff } \exists S{:}s = \cup S \ \& \ \forall t_{\in S}{:}\exists d : M, t^x_d \mathrel{=\!\!|} \varphi \ \& \ \forall d' \exists t'_{\in S}{:}t'^x_{d'} \mathrel{=\!\!|} \varphi$$

$M, s \models \forall x\varphi$ iff $\forall d : M, s^x_d \models \varphi$

$M, s \mathrel{=\!\!|} \forall x\varphi$ iff $\exists S{:}s = \cup S$ & $\forall t_{\in S}{:}\exists d : M, t^x_d \mathrel{=\!\!|} \varphi$ & $\forall d'\exists t'_{\in S}{:}t'^x_{d'} \mathrel{=\!\!|} \varphi$

$$[\neg\forall x F : G]^\times \models \neg\exists x F : G$$

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$$[\neg\forall xF : G]^{\times} \models \neg\exists xF : G$$

-
- BSML× accounts for Homogeneity assuming definite plurals have $\forall$ semantics.

- BSML$\times$ does not predict lexical gaps (**nand* and **nall* are definable);
- BSML$\ast$ is obtained as follows:

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- BSML$\ast$ is obtained as follows:

$$M, s \models \textsc{ln} \text{ iff } s \neq \varnothing \ \&\ \forall t \subseteq s : M, t \mathrel{=\!\!|} \textsc{ln} \rightarrow t = s$$

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$$M, s \models p^{*} \text{ iff } M, s \models p \ \&\ M, s \models \textsc{ln}$$

$$M, s \mathrel{=\!\!|} p^{*} \text{ iff } M, s \mathrel{=\!\!|} p \ \&\ M, s \mathrel{=\!\!|} \textsc{ln}$$

$$M, s \models [O^n(\varphi_1, \ldots, \varphi_n)]^{*} \text{ iff } M, s \models O^n(\varphi_1^{*}, \ldots, \varphi_n^{*}) \ \&\ M, s \models \textsc{ln}$$

$$M, s \mathrel{=\!\!|} [O^n(\varphi_1, \ldots, \varphi_n)]^{*} \text{ iff } M, s \mathrel{=\!\!|} O^n(\varphi_1^{*}, \ldots, \varphi_n^{*}) \ \&\ M, s \mathrel{=\!\!|} \textsc{ln}$$

with $O \in \{\neg, \vee, \wedge, \forall, \exists\}$

The operators and relations highlighted in orange are definable in BSML$\ast$. The others are not definable.

Lexicalizability

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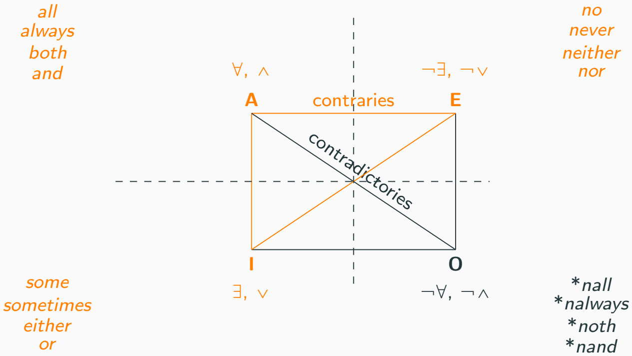

$$[\neg p \wedge \neg q]^{\circledast} \vDash [\neg(p \wedge q)]^{\circledast} \qquad [\neg(p \wedge q)]^{\circledast} \vDash [\neg p \wedge \neg q]^{\circledast}$$

$$[\neg\exists x\varphi]^{\circledast} \vDash [\neg\forall x\varphi]^{\circledast} \qquad [\neg\forall x\varphi]^{\circledast} \vDash [\neg\exists x\varphi]^{\circledast}$$

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Negation in natural language is many things (**contrary to Russell's Thesis**).

- Lexical negatives are defined in BSML$\ast$.
- Lexicalization goes by BSML$\ast$ rather than BSML because BSML$\ast$ is cognitively simpler.

BSML+

$s \models \text{NE}$ iff $s \neq \varnothing$ $\qquad$ $s \models p^+$ iff $s \models p \wedge \text{NE}$

$s \dashv \text{NE}$ iff $s = \varnothing$ $\qquad$ $s \dashv p^+$ iff $s \dashv p \wedge \text{NE}$

$$s \models [O^n(\varphi_1, \ldots, \varphi_n)]^+ \text{ iff } s \models [O^n(\varphi_1^+, \ldots, \varphi_n^+)] \wedge \text{NE}$$

$$s \dashv [O^n(\varphi_1, \ldots, \varphi_n)]^+ \text{ iff } s \dashv [O^n(\varphi_1^+, \ldots, \varphi_n^+)] \wedge \text{NE}$$

BSML×

$s \models \text{NS}$ iff $s = \varnothing$ & $\forall t \subseteq s : t \dashv \text{NS} \rightarrow t = s$ $\qquad$ $s \models p^\times$ iff $s \models p \vee \text{NS}$

$s \dashv \text{NS}$ iff $s \neq \varnothing$ & $\forall t \subseteq s : t \dashv \text{NS} \rightarrow t = s$ $\qquad$ $s \dashv p^\times$ iff $s \dashv p \vee \text{NS}$

$$s \models [O^n(\varphi_1, \ldots, \varphi_n)]^\times \text{ iff } s \models [O^n(\varphi_1^\times, \ldots, \varphi_n^\times)] \vee \text{NS}$$

$$s \dashv [O^n(\varphi_1, \ldots, \varphi_n)]^\times \text{ iff } s \dashv [O^n(\varphi_1^\times, \ldots, \varphi_n^\times)] \vee \text{NS}$$

BSML⁎

$s \models \text{LN}$ iff $s \neq \varnothing$ & $\forall t \subseteq s : t \dashv \text{LN} \rightarrow t = s$ $\qquad$ $s \models p^*$ iff $s \models p$ & $s \models \text{LN}$

$s \dashv \text{LN}$ iff $s \neq \varnothing$ & $\forall t \subseteq s : t \dashv \text{LN} \rightarrow t = s$ $\qquad$ $s \dashv p^*$ iff $s \dashv p$ & $s \dashv \text{LN}$

$$s \models [O^n(\varphi_1, \ldots, \varphi_n)]^* \text{ iff } s \models O^n(\varphi_1^*, \ldots, \varphi_n^*) \;\&\; s \models \text{LN}$$

$$s \dashv [O^n(\varphi_1, \ldots, \varphi_n)]^* \text{ iff } s \dashv O^n(\varphi_1^*, \ldots, \varphi_n^*) \;\&\; s \dashv \text{LN}$$

Even with T. Roberts (UU)

- In BSML$+\times$, which is exactly what you think it is.

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- In BSML+$\times$, which is exactly what you think it is.
- Add to BSML+$\times$ an operator: $\textsc{even}\ a\varphi$ asserts that φa and that φa is at the bottom of the $\leqslant$-ordering of focus alternatives.

$$\textsc{even}\ a\varphi := \varphi a \wedge \forall x \in Alt(a) : \varphi a \leqslant \varphi x$$

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- Assume MEP: If $A, B \in Alt$, A is true, and $A \leqslant B$, then B is true.

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$$[\textsc{even}\ a\varphi]^{+\times} \models \exists x \in Alt(a) : \varphi x \qquad \text{(by } +\text{)} \tag{iii}$$

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$$
\begin{array}{lll}
[\textsc{even}\ a\varphi]^{+\times} \models \varphi a & & (i) \\
[\textsc{even}\ a\varphi]^{+\times} \models \forall x \in Alt(a) : \varphi a \leqslant \varphi x & & (ii) \\
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\end{array}
$$

(2) a. Even John left. $\models \begin{cases} \text{John left.} & (i) \\ \text{John was the least likely to leave.} & (ii) \\ \text{Someone else left.} & (iii) \\ \text{Everyone else left.} & (iv) \end{cases}$

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$$\text{(2) a. Even John left.} \models \begin{cases} \text{John left.} & (i) \\ \text{John was the least likely to leave.} & (ii) \\ \text{Someone else left.} & (iii) \\ \text{Everyone else left.} & (iv) \end{cases}$$

- $+\times$ is a bias: if absent, (*iii*) and (*iv*) do not follow (cf. Rullmann [17], a.o.)

- Moreover, in BSML$+\times$ we obtain $[\neg\textsc{even}\ a\varphi]^{+\times} \equiv [\textsc{even}\ a\neg\varphi]^{+\times}$.

- Moreover, in BSML$+\times$ we obtain $[\neg\text{EVEN } a\varphi]^{+\times} \equiv [\text{EVEN } a\neg\varphi]^{+\times}$.
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- Moreover, in BSML$+\times$ we obtain $[\neg\textsc{even}\ a\varphi]^{+\times} \equiv [\textsc{even}\ a\neg\varphi]^{+\times}$.
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(2) b. Not even John left. $\models \begin{cases} \text{John did not leave.} & (i) \\ \text{John was the most likely to leave.} & (ii) \\ \text{Someone else did not leave.} & (iii) \\ \text{No one else left.} & (iv) \end{cases}$

- In BSML+$\times$ *even* may be under NZ+O&D but doesn't have to. What happens if an operator **lexicalizes** NZ+O&D?

$$M, s \models \text{ANY } x\varphi \text{ iff } \forall d : M, s^x_d \models \varphi^{+\times}$$

$$M, s \dashv \text{ANY } x\varphi \text{ iff } \exists S{:}s = \cup S \;\&\; \forall t_{\in S}{:}\exists d : M, t^x_d \dashv \varphi^{+\times} \;\&\; \forall d'\exists t'_{\in S}{:}t'^x_{d'} \dashv \varphi^{+\times}$$

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- Consequently, both $\textsc{any}\ x\varphi \models \forall x\varphi$ and $\neg\textsc{any}\ x\varphi \models \neg\exists x\varphi$

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- But the speaker is not biased! The system is still classical, hence $\textsc{contraposition}$ holds ($\neg\varphi \models \neg\psi \Longrightarrow \psi \models \varphi$). Therefore:

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- *any* violates **harmony** (Prior 1960, Dummett 1991).

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- *any* violates **harmony** (Prior 1960, Dummett 1991).
- Harmony is kept by preventing *any* from occurring unrestricted in an assertion (upward monotonic contexts).

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- Consequently, both $\text{ANY } x\varphi \models \forall x\varphi$ and $\neg\text{ANY } x\varphi \models \neg\exists x\varphi$
- But the speaker is not biased! The system is still classical, hence CONTRAPOSITION holds ($\neg\varphi \models \neg\psi \implies \psi \models \varphi$). Therefore:

$$\exists x\varphi \models \text{ANY } x\varphi \models \forall x\varphi$$

- *any* violates **harmony** (Prior 1960, Dummett 1991).
- Harmony is kept by preventing *any* from occurring unrestricted in an assertion (upward monotonic contexts).
- *any* in assertion is only possible if $\text{ANY } x\varphi \not\models \forall x\varphi$ (restricted "universal" *any*)

Conclusions and outlook

First theme: **Logic and change**.

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Second theme: **Linguistic fossils**.

Conclusion

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- and eventually, the same constraint explains lexical gaps.

BSML+

$s \models \mathrm{NE}$ iff $s \neq \varnothing$ $\qquad$ $s \models p^+$ iff $s \models p \wedge \mathrm{NE}$

$s \dashv \mathrm{NE}$ iff $s = \varnothing$ $\qquad$ $s \dashv p^+$ iff $s \dashv p \wedge \mathrm{NE}$

$s \models [O^n(\varphi_1, \ldots, \varphi_n)]^+$ iff $s \models [O^n(\varphi_1^+, \ldots, \varphi_n^+)] \wedge \mathrm{NE}$

$s \dashv [O^n(\varphi_1, \ldots, \varphi_n)]^+$ iff $s \dashv [O^n(\varphi_1^+, \ldots, \varphi_n^+)] \wedge \mathrm{NE}$

BSML$\times$

$s \models \mathrm{NS}$ iff $s = \varnothing$ & $\forall t \subseteq s : t \dashv \mathrm{NS} \rightarrow t = s$ $\qquad$ $s \models p^\times$ iff $s \models p \vee \mathrm{NS}$

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$s \models [O^n(\varphi_1, \ldots, \varphi_n)]^\times$ iff $s \models [O^n(\varphi_1^\times, \ldots, \varphi_n^\times)] \vee \mathrm{NS}$

$s \dashv [O^n(\varphi_1, \ldots, \varphi_n)]^\times$ iff $s \dashv [O^n(\varphi_1^\times, \ldots, \varphi_n^\times)] \vee \mathrm{NS}$

BSML$\ast$

$s \models \mathrm{LN}$ iff $s \neq \varnothing$ & $\forall t \subseteq s : t \dashv \mathrm{LN} \rightarrow t = s$ $\qquad$ $s \models p^\ast$ iff $s \models p$ & $s \models \mathrm{LN}$

$s \dashv \mathrm{LN}$ iff $s \neq \varnothing$ & $\forall t \subseteq s : t \dashv \mathrm{LN} \rightarrow t = s$ $\qquad$ $s \dashv p^\ast$ iff $s \dashv p$ & $s \dashv \mathrm{LN}$

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- Cancellation effects with *all* and *both*:

(9) a. Bianca likes the girls. $\approx$ Bianca likes every girl.
    b. Bianca does not like all the girls. $\not\approx$ Bianca likes no girl.

(10) a. Mia bought a and b. $\approx$ Mia bought both a and b.
     b. Mia did not buy a and b. $\not\approx$ Mia did not buy both a and b.

$$\begin{array}{rl} \mathit{both}(\varphi \wedge \psi) \equiv \varphi \wedge \psi & [\mathit{both}(\varphi \wedge \psi)]^{\times} \equiv \varphi^{\times} \wedge \psi^{\times} \\ \mathit{all}(\forall x \varphi) \equiv \forall x \varphi & [\mathit{all}(\forall x \varphi)]^{\times} \equiv \forall x \varphi^{\times} \\ \neg \mathit{both}(\varphi \wedge \psi) \not\models \neg \varphi & [\neg \mathit{both}(\varphi \wedge \psi)]^{\times} \not\models \neg \varphi \\ \neg \mathit{all}(\forall x \varphi) \not\models \neg \varphi a & [\neg \mathit{all}(\forall x \varphi)]^{\times} \not\models \neg \varphi a \end{array}$$

There is a "homogeneous" universal $\forall$ and then there is EVERY. The latter always expresses *not every* under negation even under cognitive bias.

$$M, s \models \forall x\varphi \text{ iff } \forall d : M, s^x_d \models \varphi$$

$$M, s \dashv \forall x\varphi \text{ iff } \exists S{:}s = \cup S \ \& \ \forall t_{\in S}{:}\exists d : M, t^x_d \dashv \varphi \ \& \ \forall d' \exists t'_{\in S}{:}t'^x_{d'} \dashv \varphi$$

$$M, s \models \textsc{every}x\varphi \text{ iff } \forall d : M, s^x_d \models \varphi$$

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