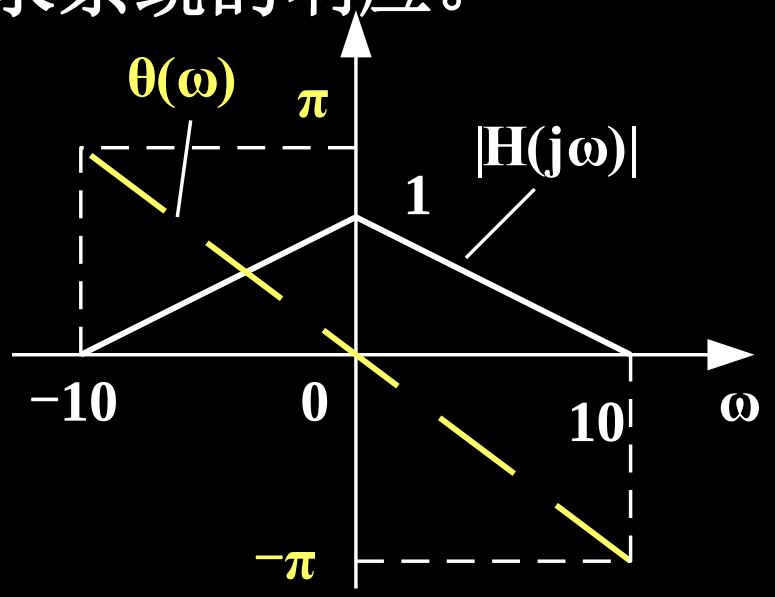
# 频域分析例

例：某LTI系统的 $|H(j\omega)|$ 和 $\theta(\omega)$ 如图，
若 $f(t)= 2 + 4\cos(5t) + 4\cos(10t)$，求系统的响应。




解法一：用傅里叶变换

$F(j\omega) = 4\pi\delta(\omega) + 4\pi[\delta(\omega-5) + \delta(\omega+5)] + 4\pi[\delta(\omega-10) + \delta(\omega+10)]$

$H(j\omega)= |H(j\omega)|\ e^{j\theta(\omega)}$

$Y(j\omega) = F(j\omega)H(j\omega) =$

$4\pi\delta(\omega)\ H(0) + 4\pi[\delta(\omega-5)\ H(j5) + \delta(\omega+5)\ H(-j5)] + 4\pi[\delta(\omega-10)\ H(j10) + \delta(\omega+10)\ H(-j10)]$

$= 4\pi\delta(\omega) + 4\pi[-j0.5\delta(\omega-5) + j0.5\delta(\omega+5)]$

$y(t) = F^{-1}[Y(j\omega)]= 2 +2\sin(5t)$

# 解法二：用三角傅里叶级数

$f(t)$的基波角频率$\Omega=5\text{rad/s}$

$$f(t)= 2 + 4\cos(\Omega t) + 4\cos(2\Omega t)$$

$$H(0) =1,\quad H(j\Omega) = 0.5e^{-j0.5\pi},\quad H(j2\Omega) = 0$$

$$\begin{aligned} y(t) &= 2 + 4\times 0.5\cos(\Omega t - 0.5\pi) \\ &= 2 + 2\sin(5t) \end{aligned}$$