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Black-Scholes Stock Price Analysis and Delta Hedging Report

Data Collection Summary

Data Collection Date	2/16/2022						
	DKNG	AMD	TWTR				
Stock Price at Data Collection	22.94	115.34	35.8				
Closing price on 04/14/2022	16.49	93.06	45.08				
Option A-E Information	Strike Price	Call Price	Put Price	Call volume	Put volume	Maturity	Dividends Info (ex-dividend dates, amounts)
ITM at maturity A (TWTR)	36	2.54	2.66	232	18	4/14/2022	N/A
ATM at maturity B (DKNG)	17.5	6.6	0.95	1	10	4/14/2022	N/A
OTM at maturity C (AMD)	120	8.4	13.35	828	514	4/14/2022	N/A
ITM on 2/16/2022 D (TWTR)	35	4.5	3.75	1	5	6/17/2022	N/A
OTM on 2/16/2022 E (AMD)	120	12.85	16.9	237	63	6/17/2022	N/A

Option Data Summary:

	Option A	Option B	Option C	Option D	Option E
Option ticker	TWTR	DKNG	AMD	TWTR	AMD
Expiration date	4/16/2022	4/16/2022	4/16/2022	6/17/2022	6/17/2022
Call price no dividends (computed)	3.21	6.44	12.09	4.74	16.99
Call price (market)	2.54	6.60	13.35	4.5	12.85
Percentage difference**	26.5%	-2.4%	-9.4%	5.4%	32.2%
Put price no dividends (computed)	3.40	0.99	16.71	3.87	21.39
Put price (market)	2.66	0.95	13.35	3.75	16.90
Percentage difference**	27.9%	4.5%	25.2%	3.2%	26.6%

Analysis of Prices

Option A Analysis- Option A was ITM at maturity on 4/16/22 with a stock price of $45.08 and a strike price of $36. Our predicted call price was $3.21, which was 26.5 percent higher than the market value of $2.54. Our predicted put price was $3.40, which was 27.9% higher than the market price of $2.66.

Option B Analysis- Option B was the closest to ATM at maturity on 4/16/22 with a stock price of $16.49 and a strike price of $17.50. Our predicted call price was $6.44, which was 2.4% percent lower than the market value of $6.60. Our predicted put price was $0.99, which was 4.5% higher than the market price of $0.95.

Option C Analysis- Option C was OTM at maturity on 4/16/22 with a stock price of $93.06 and a strike price of $120. Our predicted call price was $12.09, which was 9.4 percent lower than the market value of $13.35. Our predicted put price was $16.71, which was 25.2% higher than the market price of $13.35.

The predicted prices for the short-term options compared to the actual market prices were a mixed bag. Both the call and put options for option B were decently accurate and the call option for option C was pretty accurate. However, the predicted call and put for option A and the put for option C were decently far off the market price.

Option D Analysis – Option D was ITM at the date of data collection and will expire on 6/17/22 with a strike price of $35. Our predicted call price was $4.74 which was 5.4 percent higher than the market value of $4.50. Our predicted put price was $3.87, which was 3.2% higher than the market price of $3.75.

Option E Analysis – Option E was OTM at the date of data collection and will expire on 6/17/22 with a strike price of $120. Our predicted call price was $16.99 which was 32.2 percent higher than the market value of $12.85. Our predicted put price was $21.39, which was 26.6% higher than the market price of $3.75.

For our long-term options, option D's predicted prices were far more accurate than option E's predicted prices when compared to the market prices.

The major reasons why our predicted prices using the Black-Scholes model do not align with the market prices well is because of the assumptions that we make along with the Black-Scholes model. The major assumption that is violated is that we assumed there is a constant volatility for the underlying stock throughout the options lifetime. A stock prices volatility will never be constant day-to-day. For example, the AMD options (C and E) had pretty high volatility and their stock prices fluctuated a lot during the past 2 months. Due to this rampant change in volatility, the predicted AMD prices were in general the worst compared to DKNG and TWTR. An assumption that is made by the Black-Scholes but was not violated is that there are no dividends paid during the option's lifetime. None of the 3 companies chosen pay dividends so this assumption is relaxed. Lastly, the Black-Scholes model disregards transaction costs, but in reality, there will always be transaction costs, so our predicted prices were affected by this too.

Call option A Greeks - We found a Delta of 0.54, meaning that when the underlying stock price increases by $1.00, the price of the option will increase by $0.54. The calculated Gamma was 0.0479, meaning that when the underlying stock price increases by $1.00, the delta of the option will increase by .0479. The options Vega was calculated to be 6.70 meaning when the volatility of underlying stock price increases by 1%, the corresponding option price will increase by 0.067. The calculated Theta was -7.43 meaning when one day passes, the option price will decrease by $0.020 = (-7.43/365). The Rho of the option was calculated to be 3.56 meaning when the risk-free rate increases by 1%, the value of the option will increase by $0.0356.

Analysis of Delta Hedging Strategy

Delta hedging is a trading strategy for stock options. The purpose of using delta hedging is to reduce/minimize the risk that is associated with changes in the underlying stock. The aim is to keep a delta-neutral portfolio so there is no bias towards either side of the hedge.

There are some common patterns seen within the dynamic of the delta value. When rebalancing daily, like in our case, the delta of the stock generally increases when the stock price goes up (In some cases the delta goes down when the stock price increase is very minimal). When the price of the underlying stock decreases, the delta of the stock will decrease when doing daily rebalancing. As the definition states, the delta of the option is the probability that the underlying stock will expire in-the-money. So, when the price of the stock is well-above the strike price, the delta will be closer to one. When the price of the stock is far below the strike price, the delta will be closer to zero. To further this point, as we get closer and closer to maturity, if the option is going to expire in the money, the delta will be one, and if the option is going to expire out of the money, the delta will be zero. This theory is shown perfectly in our delta hedging. For option A, ten days before maturity, the stock price ($50.98) was far above the strike price ($36), so the delta was almost one. As we got closer to maturity, we got even closer to one because it became clear the option was going to expire ITM. Similarly for option C (OTM), as we got closer to maturity, the options delta began to approach zero, as it became clear the option was going to expire ITM. For option B, which expired slightly out of the money, it was not as clear as the other two options. Since we were so close to the strike price nearing maturity, the delta was not too close to one or zero because there was no obvious indicator that the option would expire OTM or ITM. On the day of maturity, the delta became zero because the option expired OTM.

For option A we lost 74.18% of our initial investment ($18,848.11) as a result from hedging this option. If we had not hedged option A our losses would have been much more significant at 257.36% of our initial investment ($65,391.64) since option A expired far ITM for buyer. Although we would have lost money either way, the hedging was decently successful as we would have limited our losses had we not hedged the option.

For option B we lost 10.44% of our initial investment ($6,892.30) as a result of the hedge. We would have had a profit of $66,021.76 with no hedge. This hedge was a complete failure as we would have lost money if we hedged the option as opposed to making a lot of profit if we did not hedge the option.

For option C we lost 25.92% of our initial investment ($21,781.25) as a result of the hedge. If we had not hedged this option, we would have made a profit of $84,027.70. Just like option B, this hedge was a failure as we would have lost money if we hedged the option but would have gained a huge profit if we didn’t hedge the option.