1 Structural Path Analysis

An Input-Output model is a network with sectors as nodes and inter-industry flows as the edges. It is possible to decompose this network by power series expansion of $(\overline{I}-\overline{A})^{-1}$, defined as Leontief Inverse and explore the intricate chain of production processes through Structural Path Analysis technique (SPA). Overline represents activity in economy scale. Environmentally Extended Input Output (EEIO) analysis gives the total environmental impact as

$$\overline{E} = \overline{B}(\overline{I} - \overline{A})^{-1}\overline{F} \tag{1}$$

where $\overline{B}$ is the vector of direct environmental interventions from all economic sectors and $\overline{F}$ is vector of final demands from the sectors. $\overline{A}$ is the Direct Requirements matrix that contains information about contribution of different sectors to the input flow of every other sector. Applying power series expansion to the Leontief Inverse, we obtain

$$(\overline{I} - \overline{A})^{-1} = \overline{I} \ + \overline{A} \ + \ \overline{A}^2 \ + \ \overline{A}^3 \ + \ldots.. \tag{2}$$

The detailed mathematical proof of this equation can be obtained from relevant studies.[?,?] Expressed using power series expansion, the total environmental emission instigated by final demand $\overline{F}$ is

$$\overline{E} = \overline{B}\ \overline{A^0}\ \overline{F} + \overline{B}\ \overline{A^1}\ \overline{F} + \overline{B}\ \overline{A^2}\ \overline{F} + \ldots... \tag{3}$$

Assuming that the final demand is subjected only to i-th sector and $F = [0\ \ldots.\ 0\ f_i\ 0\ \ldots.\ 0]^T$, Eq. 3 can be rewritten as,

$$\overline{E}_i \ = \ b_i f_i \ + \ \sum_{j=1}^{n} b_j a_{ji} f_i \ + \ \sum_{j=1}^{n}\sum_{k=1}^{n} b_k a_{kj} a_{ji} f_i \ + \sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} b_l a_{lk} a_{kj} a_{ji} f_i + \ldots... \tag{4}$$

where i, j, k, l are sector indices, n is the total number of sectors and f_i is the final demand for i-th sector. In this hierarchical disintegration, each term of Eq. 4 corresponds to a path order level determined by the number of linkages to the i-th sector final demand. Thus the term, $\sum_{j=1}^{n} b_j a_{ji} f_i$ contains all the n first order paths as they are linked to the final demand directly. The i-th sector itself is the zeroth order. Similarly, $\sum_{j=1}^{n}\sum_{k=1}^{n} b_k a_{kj} a_{ji} f_i$ represents the sum of all n^2 second order paths. These operations are explained through a two sector IO model as shown in Fig. 1. It consists of two economic sectors, 1 and 2 with final demand vector $[f_1\ 0]^T$ and environmental interventions matrix as $[b_1, b_2]$. The direct requirements for the two-sector economy is

$$\overline{A} \ = \ \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \tag{5}$$

Using Eq. 1, total environmental impact due to the given final demand can be calculated. Alternatively, $\overline{E}$ can be disintegrated to calculate the individual terms in Eq. 4, and represent them as separate nodes in Fig. 1. Using Eq. 4, we get

$$\begin{aligned} \overline{E}_1 \ = \ & b_1 f_1 \ + \ b_1 a_{11} f_1 \ + \ b_2 a_{21} f_1 \ + \ b_1 a_{11} a_{11} f_1 \ + \ b_2 a_{21} a_{11} f_1 \\ & + \ b_1 a_{12} a_{21} f_1 \ + \ b_2 a_{22} a_{21} f_1 \ + \ b_1 a_{11} a_{11} a_{11} f_1 \ + \ b_2 a_{21} a_{11} a_{11} f_1 \ + \ \ldots.. \end{aligned} \tag{6}$$

For clarification, the term $b_2 a_{21} a_{11} a_{11} f_1$ is the direct environmental impact of sector 2 due to economic flow from itself to sector 1 along the path $2 \rightarrow 1 \rightarrow 1 \rightarrow 1$ for satisfying the final demand f_1.

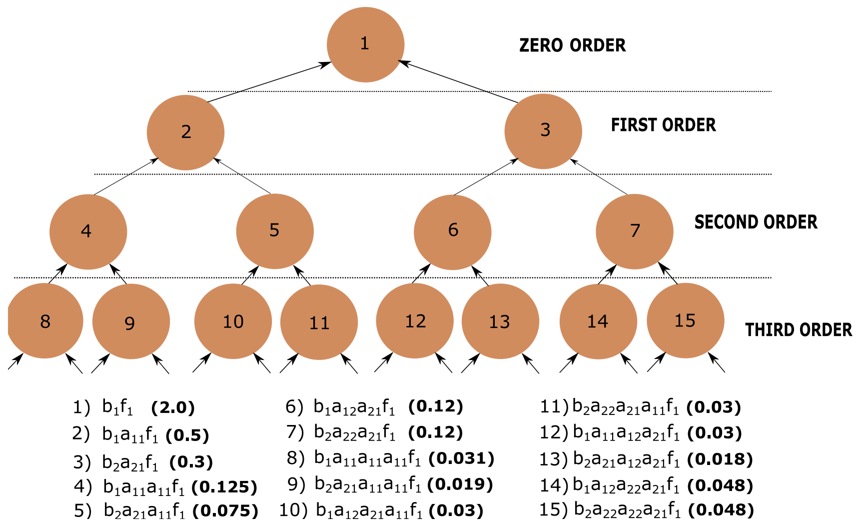

Figure 1: Tree Structure network of 2×2 sector IO model obtained using SPA. The labels of the different nodes are given at the bottom with the expression and impact value (calculated from Eq. 6 in brackets

After the tree is generated, we extract nodes along with their paths and rank them based on their percentage contribution to the total environmental impact. The contribution of a certain sector x to the total environmental intervention of sector a due to the path linkage $x \to y \to z \to a$ is calculated by

$$E_{\%} = \frac{b_x a_{xy} a_{yz} a_{za} f_a}{E_a} \times 100 \tag{7}$$

As observed from the tree, the number of nodes grow exponentially with addition of each order level. A quick calculation for a 389 sector economy reveals a tree network with $384^3 = 358863869$ nodes if built only up till the third order. Thus, adding another level to the tree will result in additional 21 billion nodes! This makes the problem computationally intensive and intractable. Fig. 1 shows a small initial part of the infinite tree structure. So, to build the tree, a pruning technique is employed to skip the "unimportant" nodes. In this technique, if the value of a node is larger than a certain threshold, the branches from that node for the next rounds are calculated and stored, otherwise further branches starting from that node are "pruned". However, it might be that the children nodes within branches occurring from this particular node may have values higher than the threshold. To decrease the possibility of this occurrence, the threshold is compared with economic activity of that node multiplied with the maximum intervention factor possible in the interventions matrix. The SPA algorithm stops when none of the sectors have higher emissions values than the threshold value. The results are sorted, the paths ranked and stored. Using this approach, efficient use of memory is possible since only important nodes and paths are explored.