

Laguerre Polynomials

$$(n+1)L_{n+1}(t) = (2n+1-t)L_n(t) - nL_{n-1}(t).$$

$$L_0(t) = 1$$

$$L_1(t) = -t + 1$$

$$L_2(t) = 2^{-1}(t^2 - 4t + 2)$$

$$L_3(t) = 6^{-1}(-t^3 + 9t^2 - 18t + 6)$$

$$L_4(t) = 24^{-1}(t^4 - 16t^3 + 72t^2 - 96t + 24)$$

$$L_5(t) = 120^{-1}(-t^5 + 25t^4 - 200t^3 + 600t^2 - 600t + 120)$$

- Othonormal basis

$$\varphi_n(t) = \sqrt{2}L_n(2t)e^{-t}$$

$$(n+1)\varphi_{n+1}(t) = (2n+1-2t)\varphi_n(t) - n\varphi_{n-1}(t).$$

$$\varphi_0(t) = \sqrt{2}e^{-t}$$

$$\varphi_1(t) = \sqrt{2}(-2t+1)e^{-t}$$

$$\varphi_2(t) = \sqrt{2}2^{-1}(4t^2 - 8t + 2)e^{-t}$$

$$\varphi_3(t) = \sqrt{2}6^{-1}(-8t^3 + 36t^2 - 36t + 6)e^{-t}$$

$$\varphi_4(t) = \sqrt{2}24^{-1}(16t^4 - 128t^3 + 288t^2 - 192t + 24)e^{-t}$$

$$\varphi_5(t) = \sqrt{2}120^{-1}(-32t^5 + 400t^4 - 1600t^3 + 2400t^2 - 1200t + 120)e^{-t}$$

Orthonormal Laguerre Basis on the positive real axis

