Legendre Polynomials

Legendre Polynomial on [-1,1]

- Bonnet's recursion formula

$$(n+1)L_{n+1}(t) = (2n+1)tL_n(t) - nL_{n-1}(t).$$

$$\begin{aligned}
&L_0(t) = 1 \\
&L_1(t) = t \\
&L_2(t) = 2^{-1}(3t^2 - 1) \\
&L_3(t) = 2^{-1}(5t^3 - 3t) \\
&L_4(t) = 8^{-1}(35t^4 - 30t^2 + 3) \\
&L_5(t) = 8^{-1}(63t^5 - 70t^3 + 15t) \\
&L_6(t) = 16^{-1}(231t^6 - 315t^4 + 105t^2 - 5) \\
&L_7(t) = 16^{-1}(429t^7 - 693t^5 + 315t^3 - 35t) \\
&L_8(t) = 128^{-1}(6435t^8 - 12012t^6 + 6930t^4 - 1260t^2 + 35) \\
&L_9(t) = 128^{-1}(12155t^9 - 25740t^7 + 18018t^5 - 4620t^3 + 315t) \\
&L_{10}(t) = 256^{-1}(46189t^{10} - 109395t^8 + 90090t^6 - 30030t^4 + 3465t^2 - 63)
\end{aligned}$$

- Othonormal basis

$$P_n(t) = \left(\frac{2n+1}{2}\right)^{1/2} L_n(t)$$

$$\begin{aligned}
&P_0(t) = (1/2)^{1/2}2^0 \\
&P_1(t) = (3/2)^{1/2}2^0 t \\
&P_2(t) = (5/2)^{1/2}2^{-1}(3t^2 - 1) \\
&P_3(t) = (7/2)^{1/2}2^{-1}(5t^3 - 3t) \\
&P_4(t) = (9/2)^{1/2}2^{-3}(35t^4 - 30t^2 + 3) \\
&P_5(t) = (11/2)^{1/2}2^{-3}(63t^5 - 70t^3 + 15t) \\
&P_6(t) = (13/2)^{1/2}2^{-4}(231t^6 - 315t^4 + 105t^2 - 5) \\
&P_7(t) = (15/2)^{1/2}2^{-4}(429t^7 - 693t^5 + 315t^3 - 35t) \\
&P_8(t) = (17/2)^{1/2}2^{-7}(6435t^8 - 12012t^6 + 6930t^5 - 1260t^2 + 35) \\
&P_9(t) = (19/2)^{1/2}2^{-7}(12155t^9 - 25740t^7 + 18018t^5 - 4620t^3 + 315t) \\
&P_{10}(t) = (21/2)^{1/2}2^{-8}(46189t^{10} - 109395t^8 + 90090t^6 - 30030t^4 + 3465t^2 - 63)
\end{aligned}$$

Legendre Polynomial on [0,1]

$$P_{n+1}(t) = (2n+3)^{1/2}\left[\frac{(2n+1)^{1/2}}{n+1}(2t-1)P_n(t) - \frac{n}{(n+1)(2n-1)^{1/2}}P_{n-1}(t)\right],$$
$$\frac{P_n(t)}{(2n+1)^{1/2}} = \frac{2(n-1)+1}{n}(2t-1)\frac{P_{n-1}(t)}{(2(n-1)+1)^{1/2}} - \frac{(n-2)+1}{n}\frac{P_{n-2}(t)}{(2(n-2)+1)^{1/2}}$$

$P_0(t) = 1$

$P_1(t) = 3^{1/2}(2t-1)$

$P_2(t) = 5^{1/2}(6t^2 - 6t + 1)$

$P_3(t) = 7^{1/2}(20t^3 - 30t^2 + 12t - 1)$

$P_4(t) = 9^{1/2}(70t^4 - 140t^3 + 90t^2 - 20t + 1)$

$P_5(t) = 11^{1/2}(252t^5 - 630t^4 + 560t^3 - 210t^2 + 30t - 1)$

$P_6(t) = 13^{1/2}(924t^6 - 2772t^5 + 3150t^4 - 1680t^3 + 420t^2 - 42t + 1)$

$P_7(t) = 15^{1/2}(3432t^7 - 12012t^6 + 16632t^5 - 11550t^4 + 4200t^3 - 756t^2 + 56t - 1)$

$P_8(t) = 17^{1/2}(12870t^8 - 51480t^7 + 84084t^6 - 72072t^5 + 34650t^4 - 9240t^3 + 1260t^2 - 72t + 1)$

$P_9(t) = 19^{1/2}(48620t^9 - 218790t^8 + 411840t^7 - 420420t^6 + 252252t^5 - 90090t^4 + 18480t^3$
$\quad - 1980t^2 + 90t - 1)$

$P_{10}(t) = 21^{1/2}(184756t^{10} - 923780t^9 + 1969110t^8 - 2333760t^7 + 1681680t^6 - 756756t^5 + 210210t^4$
$\quad - 34320t^3 + 2970t^2 - 110t + 1)$