Lab07: Image Formation and Epipolar Constraint

Oct 2022

Problem 1. Image Formation

$$\boldsymbol{point} = [\boldsymbol{x}; \boldsymbol{y}; \boldsymbol{z}; \mathbf{1}]\boldsymbol{inthepointcloud}$$

$$tform = [R1, T1; 0001]$$

$$\boldsymbol{pointincameracordinate} \rightarrow [\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}, \mathbf{1}]^{\boldsymbol{T}} \cdot \boldsymbol{tform}$$

select $1-3$row of matrix above as the input$[\boldsymbol{x}', \boldsymbol{y}', \boldsymbol{z}']$

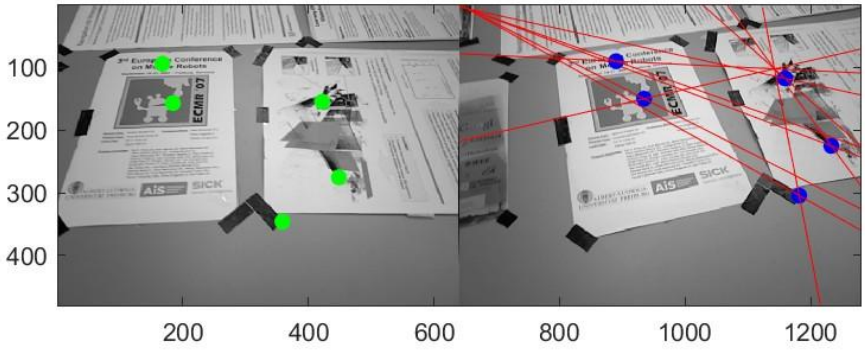
$$\boldsymbol{pointtmp} = \boldsymbol{intrinsics} * [\boldsymbol{x}', \boldsymbol{y}', \boldsymbol{z}']^{\boldsymbol{T}}$$

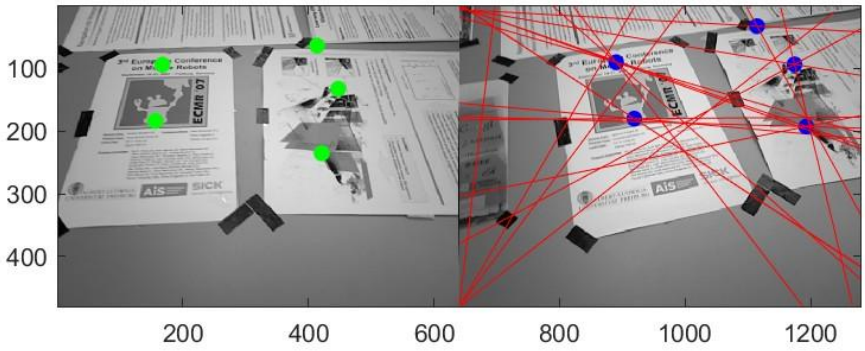
$$\boldsymbol{point\ in\ camera\ frame} = \boldsymbol{pointtmp}(\mathbf{1}{:}\,\mathbf{2})/\boldsymbol{pointtmp}(\mathbf{3}) = (\boldsymbol{x\_camera}, \boldsymbol{y\_camera})$$

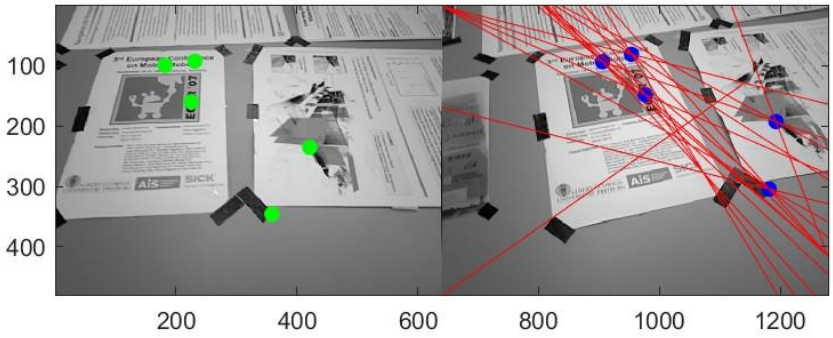
The output images are as below:

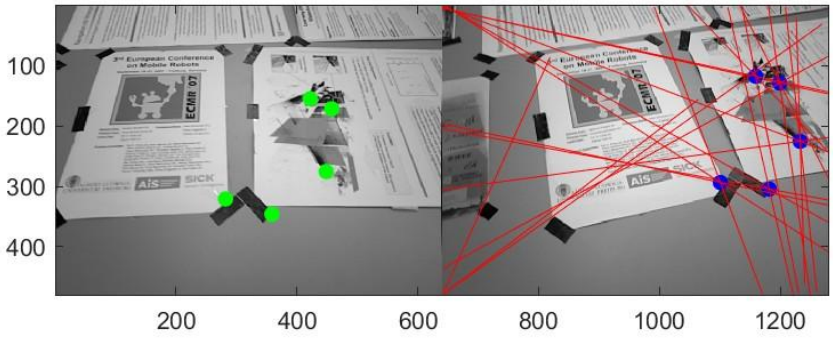
Problem 2. Essential Matrix Constraint

Result:









These are just some of the results, since the points are selected randomly, the number of possible matrixes varies, it can be 2,4,6, etc. Thus, the image can be different.

We can tell from the image that some of the lines of different essential matrixes all cross the point which are exactly the corresponding points of sift feature points in the first image.

Problem 3:

Question1: It is known that $\det(\mathrm{AB}) = \det(\mathrm{A})\det(\mathrm{B})$for every A and B.

$$E = [T]_x R$$

$$since\ RR^T = R^T R = I$$

$$EE^T = [T]_x R R^T [T]_x^T = [T]_x [T]_x^T$$

$$since\ \det([T]_x) = \det\left(\begin{bmatrix} 0 & -t3 & t2 \\ t3 & 0 & -t1 \\ -t2 & t1 & 0 \end{bmatrix}\right) = 0, \det(R[T]_x) = \det\left([T]_x^T\right), \det(E) = \det(E^T)$$

$$thus\ det(E) = 0.$$

Question2:

Firstly, the distance which the line based upon the candidate Essential Matrix to the candidate points in the second points can be used as an evaluation criterion, we may sum up all 5 distances to be a comparing distance.

Extra Points:

Given R1, R2, T1, T2, the transformation matrix of Camera 1 and 2 can be presented as:

$$tform1 = [R1, T1; 0001];\ \ tform2 = [R2, T2; 0001];$$

Thus the relative pose of camera 2 with respect to camera 1 is

$$tform1^{-1} \cdot tform2$$