Travelling Salesperson Problem - a single and multi-objective implementation

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Code available at: https://github.com/miguelince/tsp-optimization

Introduction

For CiFO's final project we were challenged to solve an optimization problem using a Genetic Algorithm (GA). Therefore, this report aims to detail how our group built a GA to solve an instance of a travelling salesperson problem (TSP).

TSP is a combinatorial optimization problem where, given a list of locations (usually cities), the objective is to define a route that, usually, minimizes a certain metric, i.e., distance, having visited all the locations once and returning to the original point of departure. Using data from Boston's school buses pick-up locations we have implemented two algorithms: the first one minimizing the duration of the route and the second, and more complex, a multi-objective optimization minimizing duration and distance. In both cases, to be able to select the best algorithm we ran 30 trials with several combinations of selection, crossover and mutation methods and compared the average results of the best individual after 100 generations of those trials.

Problem definition

TSP is defined by a set of locations, i.e., cities, and weights, connecting those locations such as distance. The objective is to find a route that visits all locations only once and returns to the starting point. Figure 1 shows graph representation of single-objective TSP.

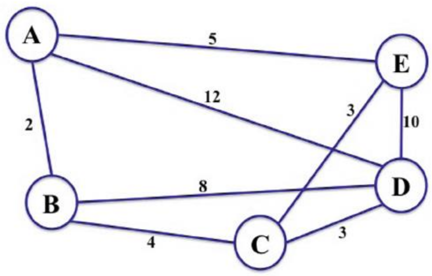

Figure 1 - TSP graph representation for single objective with cities as nodes and distance as weight of the connections

However, when defining the best route, one may desire to, in addition to distance, minimize additional metrics such as duration. To do so, one needs to associate distance and duration to each one of the edges connecting the different locations – figure 2.

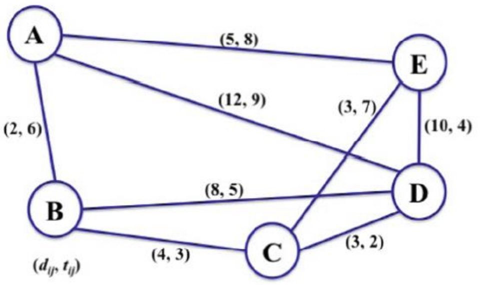

Figure 2 - Multi-objective TSP with distance and duration as weights

The main difference between single-objective and multi-objective optimization is that instead of having a single solution, there is a set of solutions that outperform the rest in respect to at least one of the metrics, also known as non-dominated or Pareto-set. To choose the "best solution" from that set we decided to calculate the minimal distance between our candidate points and an optimum theoretical point in respect to fitness. This theoretical point defined as [0,0]. Even though the optimum point is not a feasible solution, we will always want the individual that is closer to that point.

Data

Boston Public Schools hosted a competition to solve routing and bell times. The objective was to assign students to school bus stops while operating within their fleet of buses. The dataset was composed of fictitious students and schools' details. Using this dataset, we selected specific fields (Table 1) to perform the optimization. Mainly we needed a set of locations, given by GPS coordinates.

Feature	Description
Latitude	Latitude of the stop
Longitude	Longitude of the stop
Assigned School	Name of the school
School Latitude	Schools' latitude
School Longitude	School's longitude

Table 1 - Selected columns from the original dataset

With the coordinates of the school and the stops, we have retrieved the driving distances and durations between all the points using openrouteservice API for python.

Experiments

Duration optimization

Single-objective optimization refers to a problem where the function to be optimized takes into consideration one metric. Most of the TSP uses cases, lay on minimization of the total distances travelled. Having access to both distances and durations we decided to solve TSP using durations instead. So, the representation of possible solutions (Individuals) for the problem is a list of locations indexes from the 48 locations and the fitness is the sum of the duration between all of them.

Possible representation of an individual in the population:

$$[\ 1\ 23\ 14\ 43\ 14\ 6\ 18 \ldots 15]$$

Mathematical model for singe-objective was defined as:

$$\min(fitness) = min \sum_{i=1}^{48} duration_i$$

We have experiment with 3 different selection methods - Fitness proportion (also known as roulette wheel), ranking and tournament selection (with tournament size equal to 5) – combined with 2 different crossovers – Single point and Cycle crossovers. Swap mutation was the mutation method used in all the experiments. Furthermore, we defined the population size equal to 20 individuals and crossover and mutation probability equal to 90% and 10%, respectively.

In annex 1 is shown the fitness of the best individual in the population, for each of the 30^{th} trial runs, after 100 generations. Comparing the results of each of the experiments we can conclude that using ranking selection with single point crossover was the experiment with the minimum average fitness equal to 2.64 hours with a standard deviation of 0.33 hours (or around 20 minutes). Even though ranking selection and single point crossover presented the lowest average fitness, there is not a significant difference in the expected fitness, between all experiments using single point crossover (1,2,3). On the other hand, experiments using cycle crossover (4,5) were significantly worse. A summary of the results is shown in figure 3.

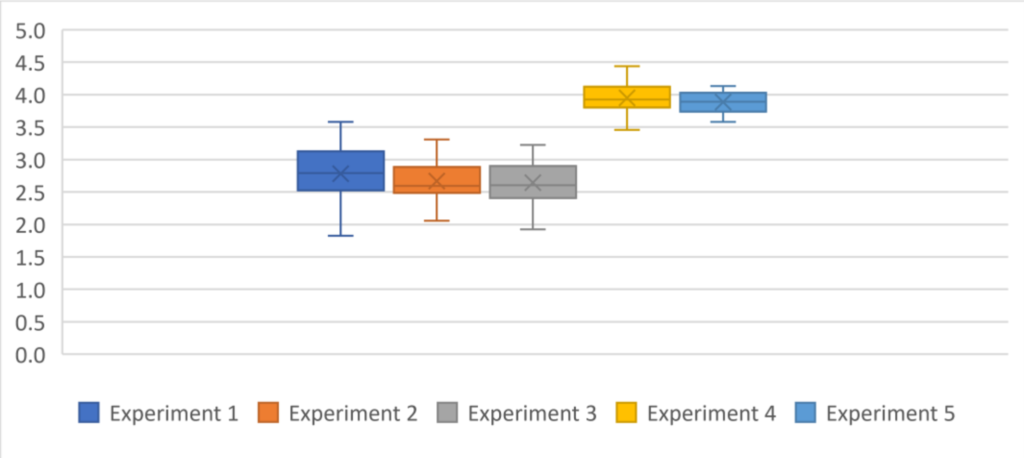

Figure 3 - Boxplots of fitness by experiments

Curious enough, the individual with the best fitness comparing all trials and all experiments was generated using fitness proportion as selection and single pojnt crossover with 1.8 hours. Was also with this combination that the standard deviation was the highest among all experiments. Experiments using cycle crossover, even though showed the worse fitness, presented the highest consistency with small deviation among the best individuals in the trials.

Theoretically, we were expecting cycle crossover to perform better than single point, and not the opposite. One possible reason for observed results, is the fact that, cycle crossover results in a higher change in the offspring representation, on one side, inducing higher diversity, but on the other, if the parents are good solutions could also have a destructive action. We also expected that with tournament selection we would select better individuals but possibly due to a small range of values and a uniform distribution of feasible fitness values the impact of the selection method was diminished.

Multi-objective optimization

To model the problem as multi-objective, the objective function had to be adapted from single-objective implementation, to consider distance and duration.

$$min \begin{cases} \sum_{i,j \in 48} distance_{ij} \\ \sum_{i,j \in 48} duration_{ij} \end{cases}$$

Also, for the selection of the parents our group used the concept of Pareto optimality, to identify a set of non-dominated points which have the minimum fitness regarding at least one of the metrics. Projecting the possible solutions in a 2-dimensional graph, those points would lay under an optimal frontier that divides dominated from non-dominated points – the pareto frontier. Pareto selection works by iterating all possible solutions and removing the non-dominated points in each of those iterations, marking them with the number of the iteration (flag). The probability of an individual to be selected is inverse to its flag, attributing higher chance of selection to better individuals.

For multi-objective, we ran once again, 30 trials of 2 different experiments, each using a different crossover method – single point and cycle. The results were consistent with what we observed in single-objective, where single point crossover, on average, (93.8km and 2.8h) outperformed cycle crossover (133.6km and 4h) - figure 4.

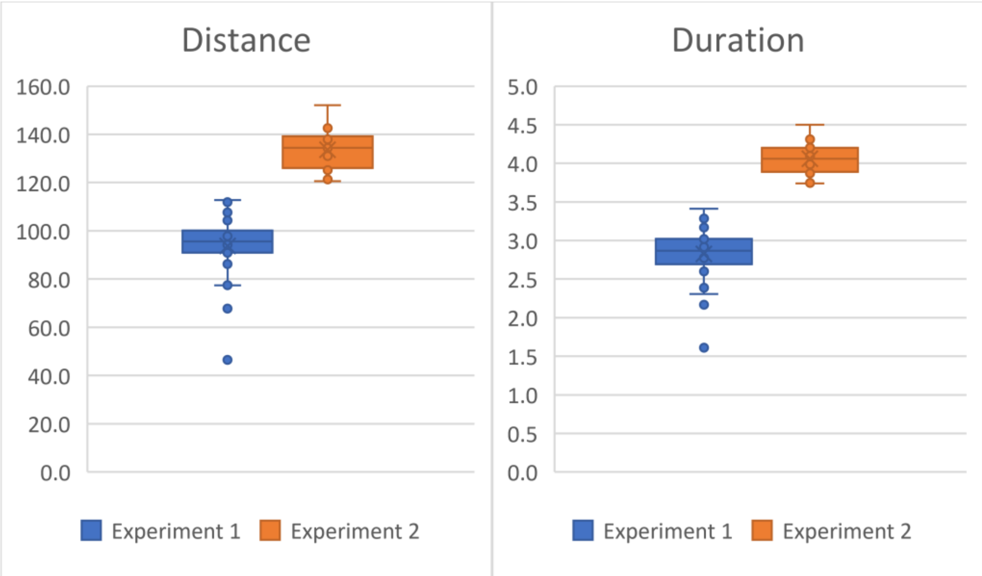

Figure 4 - Boxplots for fitness of best individual on each trial; left distance, right duration

Cycle crossover, again, showed a highest consistency in the results obtained with lower standard deviation. In one of the trials using single point the best individual after 100 generations showed an impressive fitness of 46.5km and 1.6h, even lower fitness comparing with was obtain in the single-objective (best individual had fitness equal to 1.8h).

Conclusion

To solve the TSP optimization our group implemented two approaches: one to minimize the total duration of the route and the second one, a multi-objective, to minimize both duration and distance. In single-objective we combined different selection and crossover methods and concluded that, even though ranking selection and single point crossover, presented the lowest average fitness, there were not significant differences in the expected fitness of the best individuals after 100 generations using single point crossover among experiments, but presented significant better results compared with cycle crossover. One possible reason is the fact that cycle crossover could destroy good representation due to a high level of change in the representation of the offspring.

For multi-objective, we have implemented a Pareto selection and experiment with single point and cycle crossover. Once again, single point presented the best individuals, on average. Was using multi-objective that the best individual was generated with a total distance of 46.5km and 1.6h, less 0.2h (12 minutes) than the best individual generated in single-objective.

References

Vanneschi, L. (n.d.). Computation Intelligence for Optimization. Springer

Hameed, I.A. (2020). Multi-objective Solution of Traveling Salesman Problem with Time. Advances in Intelligent Systems and Computing, vol 921. Springer, Cham. https://doi.org/10.1007/978-3-030-14118-9_13

Annex

Annex 1 – Experiments for single-objective

Selection	fps	tournament	ranking	ranking	tournament
Crossover	single point	single point	single point	cycle	cycle
Trials	**Experiment 1**	**Experiment 2**	**Experiment 3**	**Experiment 4**	**Experiment 5**
1	3.3	2.7	2.4	3.8	3.9
2	3.2	2.9	2.9	3.6	3.9
3	3.3	2.6	1.9	4.0	3.8
4	**1.8**	2.9	2.9	3.9	3.8
5	3.3	2.1	2.9	3.8	4.1
6	2.9	2.7	2.4	3.9	4.1
7	2.5	2.9	2.7	4.0	4.1
8	2.7	3.1	2.4	3.9	4.0
9	2.5	2.6	2.5	4.2	4.0
10	2.1	2.6	3.1	4.4	3.7
11	3.0	3.3	2.8	4.1	3.7
12	2.6	3.1	2.4	4.0	4.1
13	2.9	2.2	3.1	3.8	3.7
14	2.5	2.5	3.1	4.2	3.7
15	3.6	2.2	2.6	3.7	4.0
16	3.5	3.3	2.4	3.8	3.6
17	2.7	2.6	2.5	4.0	3.8
18	3.0	2.4	2.4	4.1	3.9

19	2.6	2.2	2.7	3.9	3.9
20	1.9	3.2	2.3	3.9	4.1
21	2.8	2.8	3.2	4.3	3.9
22	2.5	2.5	3.0	3.7	4.0
23	2.2	2.5	2.5	4.0	4.0
24	2.8	2.9	2.7	4.1	3.9
25	3.1	2.2	2.7	3.7	4.1
26	2.9	2.5	2.1	4.2	3.7
27	2.3	2.5	3.0	3.6	3.8
28	3.1	2.8	2.5	3.5	4.1
29	2.9	3.0	2.3	3.9	3.6
30	2.8	2.5	2.6	4.4	3.6
Mean	2.78	2.67	2.64	3.95	3.88
Std Deviation	0.43	0.33	0.32	0.22	0.16

Annex 2 – Experiments for multi-objective

Crossover	**single point**	**cycle**	**single point**	**cycle**
Trials	**Experiment 1**	**Experiment 2**	**Experiment 1**	**Experiment 2**
	Distance	**Distance**	**Duration**	**Duration**
1	95.9	121.3	2.8	3.7
2	107.0	139.4	3.2	4.2
3	90.9	144.4	2.8	4.3
4	97.8	134.9	3.0	4.1
5	112.8	131.1	3.3	4.0
6	**46.5**	132.9	**1.6**	4.0
7	94.3	127.4	2.9	3.9
8	92.1	152.1	2.6	4.5
9	67.8	123.6	2.2	3.8
10	104.3	143.0	3.3	4.3
11	111.9	132.0	3.4	4.1
12	95.3	142.5	2.8	4.3
13	77.4	125.7	2.4	4.0
14	79.8	136.9	2.4	4.1
15	96.5	138.7	2.9	4.1
16	91.1	123.0	2.8	3.9
17	99.5	135.6	3.0	4.1
18	96.6	131.7	2.9	4.0
19	98.3	126.0	3.0	3.8
20	94.3	140.1	2.9	4.2
21	98.2	120.6	3.0	3.8
22	88.4	134.2	2.7	4.1
23	95.6	134.5	2.8	4.1

24	86.2	134.4	2.6	4.1
25	77.5	138.0	2.3	4.2
26	107.6	139.2	3.3	4.2
27	100.1	141.1	2.9	4.3
28	107.3	134.6	3.1	4.0
29	94.4	123.4	3.0	3.8
30	108.5	125.2	3.0	3.7
Mean	93.79	133.59	2.83	4.06
Std Deviation	13.50	7.60	0.37	0.19