TU Dortmund University
Department of Statistics

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Time Series Analysis

Exercise 5

Submission: Please submit until 12:00pm on 10th of May in the Moodle room. It is mandatory to submit in groups of 2-3 people. Please submit only **one** PDF file with the name LASTNAMES_SHEET5, where LASTNAMES has to be substituted with the last names of the group members (It is sufficient if one group member submits the file).
For `R` code you can use `R`-Markdown (`https://rmarkdown.rstudio.com/lesson-1.html`). Please write your full names and student ID number on the submitted pdf file.

Problem 14 (Linear Filters)

Examine your understanding of the lecture material by answering the following questions:

1. In what sense is a linear filter linear?
2. Is $\Delta = 1 - B$ a causal filter?
3. What is the advantage of a causal filter over a non-causal filter?
4. What is the difference between the z-transform and the characteristic polynomial of a linear filter?
5. Does it also hold for the characteristic polynomial that the corresponding linear filter can be derived unambiguously from it?
6. What is the interpretation of the kernel of a linear filter?
7. What is the interpretation of the space of invariants of a linear filter?

Problem 15 (Combining Linear Filters)

Consider the two filters,

$$\begin{aligned} V &= 0.2B^{-1} + 0.6 + 0.2B \quad \text{and} \\ W &= \Delta_5 = 1 - B^5. \end{aligned}$$

a) What could be the use of applying each of the two filters to a time series $(y_t : t \in \mathbb{Z})$?

b) In what order would you apply the two filters to a time series?

c) Is there a filter U that leads to the same result as applying V and W one after the other, i.e. $U = W \circ V$? Specify it if there is one.

d) Find a linear filter $\tilde{W}$ such that $W = \Delta \circ \tilde{W}$.

Problem 16 (Inverse of linear filters)

Consider the two linear filters,

$$\begin{aligned} U &= 1 - 0.8B + 0.15B^2 \\ W &= \Delta_5 = 1 - B^5. \end{aligned}$$

a) Does a causal inverse filter with absolute summable weights exist for U and W? Explain your answer.

b) Determine the inverse U^{-1} of the linear filter U.
Hint: It could be helpful to factorize the filter U using its roots. The filter U can then be represented as $U = U_1 \circ U_2$ with appropriate linear filters U_1 and U_2. Then Remark I from the lecture can be used to find U^{-1}.

Problem 17 (Difference filter)

Eliminate trend and seasonal effects from the `Transport.txt` (see Moodle) time series using appropriate difference filters Δ^r and Δ_l, $r, l \in \mathbb{N}$. The values in the time series are monthly values of a public transport price index from spain starting in January 1993 and ending in December 2001. When eliminating the trend you may start by differencing once and checking wether significant trends are still present.