Escola Politécnica - USP

MAP3121 - Método Numéricos e Aplicações - 2021

Matheus Silva Coutinho - 10410156
Taiki Hashizume - 10791812

Exercício Programa 2

Autovalores e Autovetores de Matrizes Reais Simétricas
–
O Algoritmo QR
–
Aplicação em Treliças Planas

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1. Introdução

O EP2 da disciplina foi escrito em linguagem Python 3. Para este EP foi desenvolvido o algoritmo de Householder para usá-lo em conjunto com o algoritmo QR desenvolvido no EP1. As tarefas do enunciado foram separadas em três items: (a) e (b) para verificação dos resultados dos algoritmos desenvolvidos e (c) para aplicação do algoritmo em um problema de treliças planas.

2. Algoritmo para Transformações de Householder e redução de uma matriz simétrica a uma matriz tridiagonal simétrica semelhante

O algoritmo foi desenvolvido dentro do corpo de uma função chamada HH(), que recebe como uma das entradas a matriz simétrica a ser processada. Cria-se primeiro uma matriz identidade de mesma ordem da matriz A em que irão ser aplicadas multiplicações a direita das matrizes H_ω que representam as transformações de Householder.

Começamos os processos dentro de um laço “for” que percorre desde a linha 1 da matriz até sua última linha. Defini-se primeiro o ãT1 que corresponde a primeira coluna da matriz com seu primeiro elemento substituído por zero. Defini-se sigma (δ) como sendo o sinal do segundo elemento de ãT1. Depois disso calcula-se o ω_1:

$$\omega_1 = \tilde{a}_1 + \delta \| \tilde{a}_1 \| e_2$$

O vetor “e” possui seu primeiro elemento igual a zero, o segundo igual a 1 e o restante igual a zero e possui mesma dimensão que o vetor ãT1. Definimos $\bar{\omega}$ como sendo o vetor ω_1 sem o primeiro elemento. Agora é feita uma multiplicação a esquerda de todas as colunas sendo analisadas com:

$$\bar{a} = \bar{a} - 2\frac{\bar{\omega}.\bar{a}}{\bar{\omega}.\bar{\omega}}\bar{\omega}$$

A seguir é feita a multiplicação a direita de todas as linhas sendo analisadas a partir da coluna subsequente a ãT1. Iguala-se a primeira linha com os valores calculados da primeira coluna e a segunda linha, da primeira coluna em diante, com valores calculados da segunda coluna, da segunda linha em diante. Aplica-se os mesmo método á matriz Identidade, porém apenas com uma multiplicação a direita. Isto define um ciclo do laço, e é iniciado um novo ciclo a partir de uma submatriz menor começando na terceira linha e terceira coluna. O laço se repete até atingir a última linha da matriz e o resultado final é uma matriz semelhante a inicial só que tridiagonalizada e ainda simétrica.

3. Item a

Aplica-se o método de Householder e em seguida o algoritmo QR a matriz de entrada dada obtendo-se os autovalores e autovetores da matriz. É comparado com uma operação lógica se Av=λv é satisfeita para todos os autovetores e autovalores e é verificado se os autovetores são ortogonais por $V * V^T = I$. Também é comparado se as soluções analíticas são iguai ás soluções calculadas numericamente. As saídas deste item estão a seguir:

```
[[2. 4. 1. 1.]
 [4. 2. 1. 1.]
 [1. 1. 1. 2.]
 [1. 1. 2. 1.]]

Matriz Tridiagonalizada Simétrica:
[[ 2.      -4.24264 -0.      -0.     ]
 [-4.24264  3.       1.41421  0.     ]
 [-0.       1.41421  2.       0.     ]
 [-0.       0.       0.      -1.     ]]

Matriz HT:
[[ 1.       0.       0.       0.     ]
 [ 0.      -0.94281  0.33333  0.     ]
 [ 0.      -0.2357  -0.66667  0.70711]
 [ 0.      -0.2357  -0.66667 -0.70711]]

Matriz A Final:
[[ 7.  0. -0.  0.]
 [ 0. -2.  0. -0.]
 [-0. -0.  2. -0.]
 [ 0. -0. -0. -1.]]

Matriz V Final:
[[ 0.63246 -0.70711  0.31623 -0.     ]
 [ 0.63246  0.70711  0.31623 -0.     ]
 [ 0.31623 -0.      -0.63246 -0.70711]
 [ 0.31623 -0.      -0.63246  0.70711]]

iterações k com deslocamento: 4

A*v1= [4.42719 4.42719 2.21359 2.21359]  λ1*v1= [4.42719 4.42719 2.21359
2.21359]  Iguais: True

A*v2= [ 1.41421 -1.41421 -0.      -0.     ]  λ2*v2= [ 1.41421 -1.41421  0
.       0.     ]  Iguais: True

A*v3= [ 0.63246  0.63246 -1.26491 -1.26491]  λ3*v3= [ 0.63246  0.63246 -1
.26491 -1.26491]  Iguais: True

A*v4= [-0.      -0.       0.70711 -0.70711]  λ4*v4= [ 0.       0.       0
.70711 -0.70711]  Iguais: True

V*V^T:
 [[ 1. -0.  0.  0.]
 [-0.  1.  0.  0.]
 [ 0.  0.  1.  0.]
 [ 0.  0.  0.  1.]]
```

```
V*V^T = I: True

Soluções Calculadas: [-2. -1.  2.  7.]

Soluções analíticas: [-2. -1.  2.  7.]

Autovalores dados iguais a Autovetores Calculados: True
```

4. Item b

São repetidas as mesmas tarefas do item (a) porém agora para uma matriz de ordem 20. Os resultados estão a seguir:

```
[[20. 19. 18. 17. 16. 15. 14. 13. 12. 11. 10.  9.  8.  7.  6.  5.  4.  3.
   2.  1.]
 [19. 19. 18. 17. 16. 15. 14. 13. 12. 11. 10.  9.  8.  7.  6.  5.  4.  3.
   2.  1.]
 [18. 18. 18. 17. 16. 15. 14. 13. 12. 11. 10.  9.  8.  7.  6.  5.  4.  3.
   2.  1.]
 [17. 17. 17. 17. 16. 15. 14. 13. 12. 11. 10.  9.  8.  7.  6.  5.  4.  3.
   2.  1.]
 [16. 16. 16. 16. 16. 15. 14. 13. 12. 11. 10.  9.  8.  7.  6.  5.  4.  3.
   2.  1.]
 [15. 15. 15. 15. 15. 15. 14. 13. 12. 11. 10.  9.  8.  7.  6.  5.  4.  3.
   2.  1.]
 [14. 14. 14. 14. 14. 14. 14. 13. 12. 11. 10.  9.  8.  7.  6.  5.  4.  3.
   2.  1.]
 [13. 13. 13. 13. 13. 13. 13. 13. 12. 11. 10.  9.  8.  7.  6.  5.  4.  3.
   2.  1.]
 [12. 12. 12. 12. 12. 12. 12. 12. 12. 11. 10.  9.  8.  7.  6.  5.  4.  3.
   2.  1.]
 [11. 11. 11. 11. 11. 11. 11. 11. 11. 11. 10.  9.  8.  7.  6.  5.  4.  3.
   2.  1.]
 [10. 10. 10. 10. 10. 10. 10. 10. 10. 10. 10.  9.  8.  7.  6.  5.  4.  3.
   2.  1.]
 [ 9.  9.  9.  9.  9.  9.  9.  9.  9.  9.  9.  9.  8.  7.  6.  5.  4.  3.
   2.  1.]
 [ 8.  8.  8.  8.  8.  8.  8.  8.  8.  8.  8.  8.  8.  7.  6.  5.  4.  3.
   2.  1.]
 [ 7.  7.  7.  7.  7.  7.  7.  7.  7.  7.  7.  7.  7.  7.  6.  5.  4.  3.
   2.  1.]
 [ 6.  6.  6.  6.  6.  6.  6.  6.  6.  6.  6.  6.  6.  6.  6.  5.  4.  3.
   2.  1.]
 [ 5.  5.  5.  5.  5.  5.  5.  5.  5.  5.  5.  5.  5.  5.  5.  5.  4.  3.
   2.  1.]
 [ 4.  4.  4.  4.  4.  4.  4.  4.  4.  4.  4.  4.  4.  4.  4.  4.  4.  3.
   2.  1.]
 [ 3.  3.  3.  3.  3.  3.  3.  3.  3.  3.  3.  3.  3.  3.  3.  3.  3.  3.
   2.  1.]
 [ 2.  2.  2.  2.  2.  2.  2.  2.  2.  2.  2.  2.  2.  2.  2.  2.  2.  2.
   2.  1.]
 [ 1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.
   1.  1.]]
```

Matriz Tridiagonalizada Simétrica:

```
[[ 20.      -49.69909   0.        0.        0.        0.        0.
  0.        0.        0.        0.        0.        0.        0.        0.
          0.        0.        0.        0.        0.      ]
 [-49.69909 152.2     -16.52453   0.        0.        0.        0.
  0.        0.        0.        0.        0.        0.        0.        0.
          0.        0.        0.        0.        0.      ]
 [  0.      -16.52453  17.02222   4.74992  -0.       -0.       -0.
 -0.       -0.       -0.       -0.        0.        0.        0.        0.
          0.        0.        0.        0.        0.      ]
 [  0.        0.        4.74992   6.62393  -2.21346  -0.        0.
  0.        0.        0.        0.       -0.       -0.       -0.       -0.
         -0.       -0.       -0.       -0.       -0.      ]
 [  0.        0.       -0.       -2.21346   3.56561   1.26177  -0.
 -0.        0.        0.        0.        0.        0.        0.        0.
          0.       -0.       -0.       -0.       -0.      ]
 [  0.        0.       -0.       -0.        1.26177   2.2549    0.80323
 -0.       -0.       -0.       -0.        0.        0.        0.       -0.
         -0.       -0.       -0.       -0.       -0.      ]
 [  0.        0.       -0.        0.       -0.        0.80323   1.57333
 -0.54753  -0.       -0.       -0.        0.        0.        0.        0.
          0.       -0.       -0.       -0.       -0.      ]
 [  0.        0.       -0.        0.       -0.       -0.       -0.54753
  1.17379   0.39047   0.        0.        0.        0.        0.       -0.
         -0.       -0.       -0.        0.        0.      ]
 [  0.        0.       -0.        0.        0.       -0.       -0.
  0.39047   0.91954   0.28712  -0.       -0.        0.        0.        0.
          0.        0.        0.        0.        0.      ]
 [  0.        0.       -0.        0.        0.       -0.       -0.
  0.        0.28712   0.74775   0.2155    0.        0.       -0.       -0.
         -0.       -0.       -0.        0.        0.      ]
 [  0.        0.       -0.        0.        0.       -0.       -0.
  0.       -0.        0.2155    0.62624   0.16383   0.        0.        0.
          0.        0.        0.        0.        0.      ]
 [  0.        0.        0.       -0.        0.        0.        0.
  0.       -0.        0.        0.16383   0.53713   0.12534   0.        0.
          0.        0.        0.        0.        0.      ]
 [  0.        0.        0.       -0.        0.        0.        0.
  0.        0.        0.        0.        0.12534   0.46984   0.09589   0.
          0.        0.        0.        0.        0.      ]
 [  0.        0.        0.       -0.        0.        0.        0.
  0.        0.       -0.        0.        0.        0.09589   0.41779  -0.
 07286   0.       -0.       -0.       -0.        0.      ]
 [  0.        0.        0.       -0.        0.       -0.        0.
 -0.        0.       -0.        0.        0.        0.       -0.07286   0.
 3767    0.05451   0.       -0.       -0.       -0.      ]
 [  0.        0.        0.       -0.        0.       -0.        0.
 -0.        0.       -0.        0.        0.        0.        0.        0.
 05451   0.34369  -0.03964   0.        0.        0.      ]
 [  0.        0.        0.       -0.       -0.       -0.       -0.
 -0.        0.       -0.        0.        0.        0.       -0.        0.
         -0.03964   0.31677   0.02743   0.        0.      ]
 [  0.        0.        0.       -0.       -0.       -0.       -0.
 -0.        0.       -0.        0.        0.        0.       -0.       -0.
          0.        0.02743   0.29454   0.01727   0.      ]
 [  0.        0.        0.       -0.       -0.       -0.       -0.
  0.        0.        0.        0.        0.        0.       -0.       -0.
          0.        0.        0.01727   0.27596  -0.00868]
```

```
 [  0.         0.         0.        -0.        -0.        -0.        -0.
  0.         0.         0.         0.         0.         0.         0.        -0.
         0.         0.         0.        -0.00868   0.26027]]

Matriz HT:
[[ 1.        0.        0.        0.        0.        0.        0.        0.
   0.        0.        0.        0.        0.        0.        0.        0.
  0.        0.        0.        0.     ]
 [ 0.       -0.3823  -0.5136   0.51061  0.42588 -0.30774  0.19493  0.10866
 -0.05331  0.02296 -0.00864  0.00282 -0.00079  0.00019  0.00004 -0.00001
-0.        0.       -0.       -0.     ]
 [ 0.       -0.36218 -0.35141  0.1075  -0.20173  0.42112 -0.48219 -0.41176
  0.28339 -0.16193  0.07775 -0.03146  0.01069 -0.00303 -0.0007   0.00013
  0.00002 -0.        0.        0.     ]
 [ 0.       -0.34206 -0.21114 -0.14309 -0.37136  0.28323  0.03022  0.33371
 -0.46281  0.41272 -0.27713  0.14734 -0.06317  0.02191  0.0061  -0.00134
-0.00023  0.00003 -0.       -0.     ]
 [ 0.       -0.32194 -0.09157 -0.27504 -0.29503 -0.04728  0.34438  0.28214
  0.05437 -0.36291  0.4548  -0.36044  0.20909 -0.09286 -0.03194  0.00845
  0.00168 -0.00024  0.00002  0.     ]
 [ 0.       -0.30182  0.00852 -0.31789 -0.11813 -0.27185  0.26203 -0.12443
  0.36013 -0.16803 -0.21503  0.43729 -0.40529  0.24973  0.11119 -0.0365
-0.00873  0.00146 -0.00016 -0.00001]
 [ 0.       -0.2817   0.09035 -0.29708  0.06646 -0.31287 -0.00925 -0.32603
  0.10351  0.29491 -0.286   -0.09416  0.40475 -0.42299 -0.2654   0.11326
  0.03365 -0.00679  0.00086  0.00005]
 [ 0.       -0.26157  0.15513 -0.23421  0.20615 -0.20587 -0.236   -0.19944
 -0.24228  0.23662  0.20195 -0.33075 -0.02816  0.38803  0.42462 -0.25583
-0.09856  0.0246  -0.00373 -0.00028]
 [ 0.       -0.24145  0.20408 -0.14726  0.27809 -0.02858 -0.29736  0.0745
 -0.28835 -0.13815  0.29067  0.14229 -0.34143 -0.02106 -0.39854  0.41154
  0.22106 -0.07087  0.01319  0.00118]
 [ 0.       -0.22133  0.23842 -0.05091  0.28027  0.14192 -0.19451  0.26726
 -0.05091 -0.30619 -0.07458  0.30448  0.13557 -0.33536  0.07286 -0.42938
-0.37449  0.16353 -0.03854 -0.00418]
 [ 0.       -0.20121  0.25936  0.04324  0.22415  0.25123 -0.00495  0.26416
  0.21244 -0.11352 -0.30511 -0.06755  0.29654  0.1833   0.30183  0.18178
  0.45438 -0.30073  0.09381  0.01266]
 [ 0.       -0.18109  0.26813  0.12642  0.12861  0.27503  0.17379  0.09549
  0.28181  0.18564 -0.11994 -0.30631 -0.11804  0.25534 -0.27184  0.20545
-0.32822  0.43054 -0.19055 -0.03303]
 [ 0.       -0.16097  0.26593  0.19243  0.01527  0.21707  0.2676  -0.11975
  0.12754  0.28289  0.20238 -0.07165 -0.29684 -0.2157  -0.14475 -0.35259
-0.00797 -0.44933  0.32123  0.07474]
 [ 0.       -0.14085  0.254    0.23737 -0.09511  0.10168  0.24776 -0.25709
 -0.11359  0.10816  0.27143  0.25359  0.03892 -0.23075  0.31381  0.0637
  0.32086  0.27026 -0.4415  -0.14715]
 [ 0.       -0.12073  0.23355  0.25937 -0.18516 -0.03601  0.13198 -0.25031
 -0.26408 -0.15725  0.03246  0.21874  0.29721  0.19857  0.05148  0.29879
-0.30563  0.06291  0.47318  0.25226]
 [ 0.       -0.10061  0.20578  0.25839 -0.2425  -0.15972 -0.02961 -0.1136
 -0.22832 -0.27608 -0.23356 -0.10365  0.0779   0.2444  -0.31305 -0.21689
-0.0444  -0.339   -0.34696 -0.37538]
 [ 0.       -0.08048  0.17193  0.2359  -0.26052 -0.24006 -0.17585  0.07674
 -0.04135 -0.15694 -0.24565 -0.28406 -0.25476 -0.15232 -0.01005 -0.1941
  0.33192  0.3268   0.07003  0.48049]
```

```
 [ 0.      -0.06036  0.13321  0.19467 -0.23851 -0.25994 -0.25583  0.22499
  0.16853  0.09011 -0.00382 -0.10393 -0.19805 -0.27137  0.30701  0.28707
-0.19468 -0.01826  0.23817 -0.51803]
 [ 0.      -0.04024  0.09084  0.13849 -0.18113 -0.21696 -0.24424  0.26138
  0.26689  0.25947  0.23801  0.2017   0.15006  0.08312 -0.00161  0.09273
-0.19646 -0.30301 -0.39878  0.44693]
 [ 0.      -0.02012  0.04603  0.07191 -0.09753 -0.12277 -0.14748  0.17153
  0.19476  0.21699  0.23801  0.25757  0.27535  0.29093 -0.30375 -0.31298
 0.31736  0.31466  0.30028 -0.26071]]

Matriz A Final:
[[170.40427   0.        0.       -0.        0.        0.       -0.
 0.        0.        0.        0.        0.        0.       -0.       -0.
       -0.        0.       -0.       -0.       -0.     ]
 [  0.       19.0081   -0.       -0.        0.        0.       -0.
 0.        0.        0.        0.        0.        0.       -0.       -0.
       -0.        0.       -0.       -0.       -0.     ]
 [  0.        0.        6.89678   0.       -0.        0.       -0.
-0.        0.       -0.       -0.       -0.       -0.       -0.        0.
        0.       -0.        0.       -0.       -0.     ]
 [ -0.       -0.       -0.        3.56048   0.       -0.        0.
 0.       -0.        0.        0.       -0.        0.        0.       -0.
       -0.        0.       -0.        0.        0.     ]
 [  0.        0.       -0.        0.        2.18808  -0.       -0.
 0.        0.        0.        0.        0.       -0.       -0.       -0.
        0.       -0.        0.       -0.       -0.     ]
 [  0.        0.        0.       -0.        0.        1.49399   0.
 0.       -0.        0.        0.        0.        0.        0.        0.
       -0.        0.       -0.        0.        0.     ]
 [ -0.       -0.       -0.        0.       -0.        0.        1.09545
-0.        0.       -0.        0.       -0.        0.        0.        0.
       -0.        0.       -0.        0.        0.     ]
 [  0.        0.       -0.        0.        0.        0.        0.
 0.84612   0.       -0.        0.       -0.       -0.       -0.        0.
        0.       -0.        0.       -0.       -0.     ]
 [  0.        0.        0.       -0.        0.       -0.       -0.
 0.        0.68025   0.       -0.        0.       -0.        0.        0.
       -0.       -0.        0.       -0.       -0.     ]
 [  0.        0.       -0.        0.        0.        0.        0.
-0.       -0.        0.56477   0.       -0.        0.       -0.       -0.
        0.        0.        0.       -0.        0.     ]
 [  0.        0.       -0.        0.        0.        0.        0.
-0.       -0.       -0.        0.48156   0.       -0.        0.        0.
       -0.       -0.        0.       -0.       -0.     ]
 [  0.        0.       -0.       -0.        0.       -0.       -0.
-0.        0.        0.        0.        0.42003   0.       -0.       -0.
        0.        0.        0.       -0.        0.     ]
 [  0.       -0.       -0.        0.        0.        0.        0.
-0.       -0.       -0.       -0.       -0.       -0.        0.37369   0.        0.
       -0.       -0.        0.        0.       -0.     ]
 [ -0.       -0.       -0.        0.       -0.        0.        0.
-0.       -0.       -0.       -0.       -0.       -0.        0.33836  -0.
        0.        0.        0.       -0.        0.     ]
 [ -0.       -0.        0.       -0.       -0.        0.        0.
 0.       -0.       -0.        0.       -0.       -0.       -0.        0.
31129   0.        0.       -0.       -0.        0.     ]
```

```
 [ -0.       -0.        0.        0.       -0.       -0.       -0.
 0.       -0.       -0.        0.        0.       -0.        0.       -0.
        0.29061  -0.       -0.        0.       -0.     ]
 [  0.       -0.       -0.        0.        0.        0.        0.
-0.       -0.       -0.       -0.       -0.        0.       -0.        0.
        0.        0.27504  -0.        0.       -0.     ]
 [ -0.        0.        0.       -0.        0.       -0.       -0.
 0.        0.        0.        0.        0.        0.        0.       -0.
       -0.        0.        0.26369   0.       -0.     ]
 [ -0.       -0.       -0.        0.       -0.        0.        0.
-0.       -0.       -0.       -0.       -0.       -0.       -0.        0.
        0.       -0.        0.        0.25147  -0.     ]
 [ -0.       -0.       -0.        0.       -0.        0.        0.
-0.       -0.       -0.       -0.       -0.       -0.       -0.        0.
        0.       -0.        0.        0.        0.25596]]

Matriz V Final:
[[ 0.31212  0.31029  0.30663 -0.30118  0.29396 -0.28502 -0.2744   0.26217
  0.24841  0.23318  0.21659  0.19873  0.1797   0.15962 -0.13859 -0.11676
 0.09424 -0.07117  0.02391  0.04768]
 [ 0.31029  0.29396  0.26217 -0.21659  0.15962 -0.09424 -0.02391 -0.04768
 -0.11676 -0.1797  -0.23318 -0.2744  -0.30118 -0.31212  0.30663  0.28502
-0.24841  0.19873 -0.07117 -0.1386 ]
 [ 0.30663  0.26217  0.1797  -0.07117 -0.04768  0.15962  0.24841 -0.30118
 -0.31029 -0.2744  -0.19873 -0.09424  0.02391  0.13859 -0.23318 -0.29396
 0.31212 -0.28502  0.11676  0.21659]
 [ 0.30118  0.21659  0.07117  0.09424 -0.23318  0.30663  0.29396 -0.19873
 -0.04768  0.11676  0.24841  0.31029  0.28502  0.1797  -0.02391  0.13859
-0.26217  0.31212 -0.15961 -0.2744 ]
 [ 0.29396  0.15962 -0.04768  0.23318 -0.31212  0.24841  0.07117  0.13859
  0.28502  0.30118  0.1797  -0.02391 -0.21659 -0.31029  0.26217  0.09424
 0.11676 -0.2744   0.19872  0.30664]
 [ 0.28502  0.09424 -0.15962  0.30663 -0.24841  0.02391 -0.21659  0.31212
  0.19873 -0.04768 -0.26217 -0.30118 -0.13859  0.11676 -0.29396 -0.2744
 0.07117  0.1797  -0.23318 -0.31029]
 [ 0.2744   0.02391 -0.24841  0.29396 -0.07117 -0.21659 -0.30663  0.11676
 -0.1797  -0.31212 -0.15962  0.13859  0.31029  0.19873  0.09424  0.30118
-0.23319 -0.04768  0.26217  0.28502]
 [ 0.26217 -0.04768 -0.30118  0.19873  0.13859 -0.31212 -0.11676 -0.21659
 -0.29396 -0.02391  0.2744   0.24841 -0.07117 -0.30663  0.1797  -0.15961
 0.31029 -0.09424 -0.28502 -0.23319]
 [ 0.24841 -0.11676 -0.31029  0.04768  0.28502 -0.19873  0.1797  -0.29396
  0.02391  0.30663  0.13859 -0.23318 -0.26217  0.09424 -0.31212 -0.07117
-0.2744   0.21659  0.30118  0.15962]
 [ 0.23318 -0.1797  -0.2744  -0.11676  0.30118  0.04768  0.31212 -0.02391
  0.30663  0.09424 -0.28502 -0.15962  0.24841  0.21659  0.19873  0.26218
 0.13859 -0.29396 -0.31029 -0.07117]
 [ 0.21659 -0.23318 -0.19873 -0.24841  0.1797   0.26217  0.15962  0.2744
  0.13859 -0.28502 -0.11676  0.29396  0.09424 -0.30118  0.07117 -0.30663
 0.04768  0.31029  0.31212 -0.02391]
 [ 0.19873 -0.2744  -0.09424 -0.31029 -0.02391  0.30118 -0.13859  0.24841
 -0.23318 -0.15962  0.29396  0.04768 -0.31212  0.07117 -0.28502  0.1797
-0.21659 -0.26217 -0.30664  0.11676]
 [ 0.1797  -0.30118  0.02391 -0.28502 -0.21659  0.13859 -0.31029 -0.07117
 -0.26217  0.24841  0.09424 -0.31212  0.11676  0.23319  0.2744   0.04768
 0.30663  0.15961  0.29397 -0.19873]
```

```
 [ 0.15962 -0.31212  0.13859 -0.1797  -0.31029 -0.11676 -0.19873 -0.30663
  0.09424  0.21659 -0.30118  0.07117  0.23318 -0.29396 -0.04768 -0.24841
-0.28502 -0.02391 -0.2744   0.26217]
 [ 0.13859 -0.30663  0.23318 -0.02391 -0.26217 -0.29396  0.09424 -0.1797
  0.31212 -0.19873 -0.07117  0.28502 -0.2744   0.04768 -0.21659  0.31029
 0.15961 -0.11676  0.24841 -0.30118]
 [ 0.11676 -0.28502  0.29396  0.13859 -0.09424 -0.2744   0.30118  0.15962
  0.07117 -0.26217  0.30663 -0.1797  -0.04768  0.24841  0.31029 -0.19873
 0.02391  0.23319 -0.21659  0.31211]
 [ 0.09424 -0.24841  0.31212  0.26217  0.11676 -0.07117  0.23318  0.31029
 -0.2744   0.13859  0.04768 -0.21659  0.30663 -0.28502 -0.15961 -0.02391
-0.19873 -0.30118  0.1797  -0.29396]
 [ 0.07117 -0.19873  0.28502  0.31212  0.2744   0.1797  -0.04768  0.09424
 -0.21659  0.29396 -0.31029  0.26217 -0.15962  0.02391 -0.11676  0.23318
 0.30118  0.30664 -0.1386   0.2484 ]
 [ 0.04768 -0.13859  0.21659  0.2744   0.30663  0.31029 -0.28502 -0.23318
  0.15962 -0.07117 -0.02391  0.11676 -0.19873  0.26217  0.30118 -0.31212
-0.29396 -0.24841  0.09424 -0.1797 ]
 [ 0.02391 -0.07117  0.11676  0.15962  0.19873  0.23318 -0.26217 -0.28502
  0.30118 -0.31029  0.31212 -0.30663  0.29396 -0.2744  -0.24841  0.21659
 0.1797   0.1386  -0.04768  0.09424]]

iterações k com deslocamento: 23

A*v1= [53.18629 52.87417 52.25177 51.32273 50.09251 48.56833 46.75912 44.
67552 42.32974 39.73556 36.90819 33.86423 30.62154 27.19915 23.61714 19.8
9654 16.05918 12.12758  8.12481  4.07436]  λ1*v1= [53.18629 52.87417 52.2
5177 51.32273 50.09251 48.56833 46.75912 44.67552 42.32974 39.73556 36.90
819 33.86423 30.62154 27.19915 23.61714 19.89654 16.05918 12.12758  8.124
81  4.07436]  Iguais: True

A*v2= [ 5.89796  5.58767  4.98342  4.117    3.03399  1.79136  0.45448 -0.
9063  -2.2194  -3.41574 -4.43239 -5.21584 -5.7249  -5.93278 -5.82853 -5.4
1766 -4.72176 -3.77746 -2.63442 -1.3528 ]  λ2*v2= [ 5.89796  5.58767  4.9
8342  4.117    3.03399  1.79136  0.45448 -0.9063  -2.2194  -3.41574 -4.43
239 -5.21584 -5.7249  -5.93278 -5.82853 -5.41766 -4.72176 -3.77746 -2.634
42 -1.3528 ]  Iguais: True

A*v3= [ 2.11479  1.80816  1.23935  0.49084 -0.32884 -1.10083 -1.71321 -2.
07719 -2.13998 -1.89249 -1.37059 -0.64996  0.1649   0.95586  1.60822  2.0
274   2.15261  1.96571  1.49379  0.80527]  λ3*v3= [ 2.11479  1.80816  1.2
3935  0.49084 -0.32884 -1.10083 -1.71321 -2.07719 -2.13998 -1.89249 -1.37
059 -0.64996  0.1649   0.95586  1.60822  2.0274   2.15261  1.96571  1.493
79  0.80527]  Iguais: True

A*v4= [-1.07235 -0.77117 -0.2534   0.33555  0.83025  1.09177  1.04665  0.
70757  0.16976 -0.41572 -0.88445 -1.10477 -1.0148  -0.63982 -0.08513  0.4
9346  0.93346  1.11129  0.977    0.56831]  λ4*v4= [-1.07235 -0.77117 -0.2
534   0.33555  0.83025  1.09177  1.04665  0.70757  0.16976 -0.41572 -0.88
445 -1.10477 -1.0148  -0.63982 -0.08513  0.49346  0.93346  1.11129  0.977
    0.56831]  Iguais: True

A*v5= [ 0.64321  0.34925 -0.10433 -0.51023 -0.68294 -0.54354 -0.15572  0.
30326  0.62364  0.65901  0.3932  -0.05232 -0.47392 -0.67893 -0.57366 -0.2
0621  0.25548  0.60041  0.67094  0.43483]  λ5*v5= [ 0.64321  0.34925 -0.1
0433 -0.51023 -0.68294 -0.54354 -0.15572  0.30326  0.62364  0.65901  0.39
32  -0.05232 -0.47392 -0.67893 -0.57366 -0.20621  0.25548  0.60041  0.670
94  0.43483]  Iguais: True
```

A*v6= [-0.42581 -0.1408   0.23846 0.45811 0.37112 0.03572 -0.32359 -0.
4663 -0.2969   0.07123 0.39168 0.44996 0.20706 -0.17444 -0.43918 -0.4
0995 -0.10633 0.26847 0.46357 0.34837] λ6*v6= [-0.42581 -0.1408   0.2
3846 0.45811 0.37112 0.03572 -0.32359 -0.4663 -0.2969   0.07123 0.39
168 0.44996 0.20706 -0.17444 -0.43918 -0.40995 -0.10633 0.26847 0.463
57 0.34837] Iguais: True

A*v7= [-0.30059 -0.02619 0.27212 0.32202 0.07796 -0.23727 -0.3359 -0.
12791 0.19685 0.34191 0.17485 -0.15182 -0.3399 -0.2177   0.10324 0.3
2993 0.25544 -0.05223 -0.31222 -0.2872] λ7*v7= [-0.30059 -0.02619 0.2
7212 0.32202 0.07796 -0.23727 -0.3359 -0.12791 0.19685 0.34191 0.17
485 -0.15182 -0.3399 -0.2177   0.10324 0.32993 0.25544 -0.05223 -0.312
22 -0.2872] Iguais: True

A*v8= [0.22183 -0.04034 -0.25484 -0.16815 0.11727 0.26409 0.09879 -0.
18326 -0.24873 -0.02023 0.23218 0.21018 -0.06022 -0.25945 -0.15205 0.1
3505 0.26254 0.07974 -0.1973 -0.24116] λ8*v8= [0.22183 -0.04034 -0.2
5484 -0.16815 0.11727 0.26409 0.09879 -0.18326 -0.24873 -0.02023 0.23
218 0.21018 -0.06022 -0.25945 -0.15205 0.13505 0.26254 0.07974 -0.197
3 -0.24116] Iguais: True

A*v9= [0.16898 -0.07943 -0.21107 -0.03243 0.19389 0.13519 -0.12224 -0.
19997 0.01626 0.20859 0.09428 -0.15862 -0.17834 0.06411 0.21232 0.0
4841 -0.18666 -0.14734 0.10858 0.20488] λ9*v9= [0.16898 -0.07943 -0.2
1107 -0.03243 0.19389 0.13519 -0.12224 -0.19997 0.01626 0.20859 0.09
428 -0.15862 -0.17834 0.06411 0.21232 0.04841 -0.18666 -0.14734 0.108
58 0.20488] Iguais: True

A*v10= [0.1317 -0.10149 -0.15497 0.06594 0.1701 -0.02693 -0.17627 -0
.0135   0.17318 0.05322 -0.16097 -0.09015 0.14029 0.12232 -0.11224 -0.
14807 0.07827 0.16602 -0.04019 -0.17524] λ10*v10= [0.1317 -0.10149 -
0.15497 0.06594 0.1701 -0.02693 -0.17627 -0.0135   0.17318 0.05322 -0
.16097 -0.09015 0.14029 0.12232 -0.11224 -0.14807 0.07827 0.16602 -0.
04019 -0.17524] Iguais: True

A*v11= [0.1043 -0.11229 -0.0957   0.11962 0.08654 -0.12625 -0.07686 0
.13214 0.06674 -0.13725 -0.05623 0.14156 0.04538 -0.14504 -0.03427 0.
14766 0.02296 -0.14942 -0.01151 0.1503] λ11*v11= [0.1043 -0.11229 -
0.0957   0.11962 0.08654 -0.12625 -0.07686 0.13214 0.06674 -0.13725 -0
.05623 0.14156 0.04538 -0.14504 -0.03427 0.14766 0.02296 -0.14942 -0.
01151 0.1503] Iguais: True

A*v12= [0.08347 -0.11526 -0.03958 0.13033 -0.01004 -0.12651 0.05821 0
.10434 -0.09794 -0.06704 0.12347 0.02003 -0.1311   0.02989 0.11972 -0.
07548 -0.09098 0.11012 0.04904 -0.1288] λ12*v12= [0.08347 -0.11526 -
0.03958 0.13033 -0.01004 -0.12651 0.05821 0.10434 -0.09794 -0.06704 0
.12347 0.02003 -0.1311   0.02989 0.11972 -0.07548 -0.09098 0.11012 0.
04904 -0.1288] Iguais: True

A*v13= [0.06715 -0.11255 0.00893 0.10651 -0.08094 -0.05179 0.11595 -0
.0266 -0.09797 0.09283 0.03522 -0.11663 0.04363 0.08714 -0.10254 -0.
01782 0.11459 -0.05965 -0.07426 0.10985] λ13*v13= [0.06715 -0.11255
0.00893 0.10651 -0.08094 -0.05179 0.11595 -0.02659 -0.09797 0.09283 0
.03522 -0.11663 0.04363 0.08714 -0.10254 -0.01782 0.11459 -0.05965 -0.
07426 0.10985] Iguais: True

```
A*v14= [ 0.05401 -0.10561  0.04689  0.0608  -0.10499  0.03951  0.06724 -0
.10375  0.03189  0.07329 -0.10191  0.02408  0.0789  -0.09946  0.01613  0.
08405 -0.09644  0.00809  0.08871 -0.09285]  λ14*v14= [ 0.05401 -0.10561
0.04689  0.0608  -0.10499  0.03951  0.06724 -0.10375  0.03189  0.07329 -0
.10191  0.02408  0.0789  -0.09946  0.01613  0.08405 -0.09644  0.00809  0.
08871 -0.09285]  Iguais: True

A*v15= [-0.04314  0.09545 -0.07259 -0.00744  0.08161 -0.09151  0.02934  0
.05594 -0.09716  0.06186  0.02216 -0.08872  0.08542 -0.01484 -0.06742  0.
09659 -0.04969 -0.03635  0.09375 -0.07733]  λ15*v15= [-0.04314  0.09545 -
0.07259 -0.00744  0.08161 -0.09151  0.02934  0.05594 -0.09716  0.06186  0
.02216 -0.08872  0.08542 -0.01484 -0.06742  0.09659 -0.04969 -0.03635  0.
09375 -0.07733]  Iguais: True

A*v16= [-0.03393  0.08283 -0.08543  0.04028  0.02739 -0.07974  0.08753 -0
.04639 -0.02068  0.07619 -0.08911  0.05222  0.01386 -0.07219  0.09017 -0.
05775 -0.00695  0.06777 -0.0907   0.06294]  λ16*v16= [-0.03393  0.08283 -
0.08543  0.04028  0.02739 -0.07974  0.08753 -0.04639 -0.02068  0.07619 -0
.08911  0.05222  0.01386 -0.07219  0.09017 -0.05775 -0.00695  0.06777 -0.
0907   0.06294]  Iguais: True

A*v17= [ 0.02592 -0.06832  0.08584 -0.07211  0.03211  0.01957 -0.06413  0
.08534 -0.07547  0.03812  0.01311 -0.05957  0.08434 -0.07839  0.0439   0.
00658 -0.05466  0.08284 -0.08085  0.04942]  λ17*v17= [ 0.02592 -0.06832
0.08584 -0.07211  0.03211  0.01957 -0.06413  0.08534 -0.07547  0.03812  0
.01311 -0.05957  0.08434 -0.07839  0.0439   0.00658 -0.05466  0.08284 -0.
08085  0.04942]  Iguais: True

A*v18= [-0.01877  0.0524  -0.07516  0.0823  -0.07236  0.04738 -0.01257 -0
.02485  0.05711 -0.07751  0.08182 -0.06913  0.04209 -0.0063  -0.03079  0.
06149 -0.07942  0.08086 -0.0655   0.03655]  λ18*v18= [-0.01877  0.0524  -
0.07516  0.0823  -0.07236  0.04738 -0.01257 -0.02485  0.05711 -0.07751  0
.08182 -0.06913  0.04209 -0.0063  -0.03079  0.06149 -0.07942  0.08086 -0.
0655   0.03655]  Iguais: True

A*v19= [ 0.00601 -0.0179   0.02936 -0.04014  0.04997 -0.05864  0.06593 -0
.07167  0.07574 -0.07803  0.07849 -0.07711  0.07392 -0.06901  0.06247 -0.
05447  0.04519 -0.03485  0.0237  -0.01199]  λ19*v19= [ 0.00601 -0.0179
0.02936 -0.04014  0.04997 -0.05864  0.06593 -0.07167  0.07574 -0.07803  0
.07849 -0.07711  0.07392 -0.06901  0.06247 -0.05447  0.04519 -0.03485  0.
0237  -0.01199]  Iguais: True

A*v20= [ 0.0122  -0.03548  0.05544 -0.07024  0.07849 -0.07942  0.07296 -0
.05969  0.04086 -0.01822 -0.00612  0.02989 -0.05087  0.06711 -0.07709  0.
07989 -0.07524  0.06358 -0.046    0.02412]  λ20*v20= [ 0.0122  -0.03548
0.05544 -0.07024  0.07849 -0.07942  0.07296 -0.05969  0.04086 -0.01822 -0
.00612  0.02989 -0.05087  0.06711 -0.07709  0.07989 -0.07524  0.06358 -0.
046    0.02412]  Iguais: True

V*V^T:
 [[ 1.  0. -0.  0. -0.  0.  0. -0. -0. -0.  0.  0.  0.  0. -0.  0. -0.  0
.  0.  0.]
 [ 0.  1.  0. -0. -0. -0. -0. -0.  0.  0. -0.  0.  0. -0.  0. -0.  0.  0.
  0.  0.]
 [-0.  0.  1.  0.  0.  0. -0. -0.  0.  0.  0. -0. -0.  0.  0.  0.  0.  0.
  0.  0.]
 [ 0. -0.  0.  1. -0.  0.  0.  0. -0.  0. -0.  0. -0. -0.  0.  0.  0. -0.
 -0.  0.]
```

```
 [-0. -0.  0. -0.  1.  0.  0. -0.  0. -0.  0.  0.  0. -0. -0.  0. -0.  0.
  0. -0.]
 [ 0. -0.  0.  0.  0.  1.  0.  0.  0.  0.  0.  0.  0. -0. -0. -0. -0. -0.
  0.  0.]
 [ 0. -0. -0.  0.  0.  0.  1.  0.  0.  0. -0. -0. -0.  0. -0.  0.  0. -0.
 -0. -0.]
 [-0. -0. -0.  0. -0.  0.  0.  1.  0. -0.  0. -0. -0. -0.  0. -0. -0. -0.
 -0. -0.]
 [-0.  0.  0. -0.  0.  0.  0.  0.  1. -0. -0.  0. -0. -0. -0. -0. -0. -0.
 -0. -0.]
 [-0.  0.  0.  0. -0.  0.  0. -0. -0.  1.  0. -0.  0. -0. -0. -0. -0. -0.
 -0.  0.]
 [ 0. -0.  0. -0.  0.  0. -0.  0. -0.  0.  1. -0. -0.  0.  0. -0. -0. -0.
  0.  0.]
 [ 0.  0. -0.  0.  0.  0. -0. -0.  0. -0. -0.  1. -0. -0. -0.  0. -0.  0.
 -0. -0.]
 [ 0.  0. -0. -0.  0.  0. -0. -0. -0.  0. -0. -0.  1.  0. -0.  0.  0. -0.
 -0. -0.]
 [ 0. -0.  0. -0. -0. -0.  0. -0. -0. -0.  0. -0.  0.  1. -0.  0.  0.  0.
  0.  0.]
 [-0.  0.  0.  0. -0. -0. -0.  0. -0. -0.  0. -0. -0. -0.  1. -0. -0.  0.
  0.  0.]
 [ 0. -0.  0.  0.  0. -0.  0. -0. -0. -0. -0.  0.  0.  0. -0.  1.  0. -0.
  0.  0.]
 [-0.  0.  0.  0. -0. -0.  0. -0. -0. -0. -0. -0.  0.  0. -0.  0.  1.  0.
 -0. -0.]
 [ 0.  0.  0. -0.  0. -0. -0. -0. -0. -0. -0.  0. -0.  0.  0. -0.  0.  1.
 -0. -0.]
 [ 0.  0.  0. -0.  0.  0. -0. -0. -0. -0.  0. -0. -0.  0.  0.  0. -0. -0.
  1. -0.]
 [ 0.  0.  0.  0. -0.  0. -0. -0. -0.  0.  0. -0. -0.  0.  0.  0. -0. -0.
 -0.  1.]]

V*V^T = I: True

Soluções Calculadas: [  0.25147   0.25596   0.26369   0.27504   0.29061
 0.31129   0.33836   0.37369   0.42003   0.48156   0.56477   0.68025   0.
84612   1.09545   1.49399   2.18808   3.56048   6.89678  19.0081  170.404
27]

Soluções analíticas: [  0.25147   0.25596   0.26369   0.27504   0.29061
 0.31129   0.33836   0.37369   0.42003   0.48156   0.56477   0.68025   0.
84612   1.09545   1.49399   2.18808   3.56048   6.89678  19.0081  170.404
27]

Autovalores analíticos iguais a autovalores Calculados: True
```

5. Item c – Treliças Planas

Para este item foi necessário, primeiro, converter o arquivo de input em informações usáveis. O arquivo de input é aberto em modo “r” para sua leitura e transformado em uma lista de strings de cada linha. Posteriormente tranforma-se esta lista em um dicionário em que cada chave é um tuple de dois nós representando as barras e seus valores guardam as informações de ângulo e comprimento, nesta ordem. As outras informações de aréa de secção transversal,

módulo E e densidade do material são atribuídas a variáveis normais. São também guardadas informações do número de barras e número de nós não fixos para construção das matrizes de massa e de rigidez da treliça inteira. O próximo passo que o código executa é calcular a matriz de rigidez 4x4 de cada barra e adicionar suas contribuições à matriz de rigidez total em um laço que circula pelo dicionário criado anteriormente. Neste mesmo laço são adicionadas as contribuições de massas na matriz de massa total da treliça. O método utilizado para se calcular a matriz K total é iniciar com uma matriz de zeros de ordem 28 e, após a adição de todas as contribuições, extrair apenas as primeiras 24 linhas e colunas.

Depois disso, com a matriz de rigidez total e matriz de massa total, faz-se a multiplicação à esquerda e depois à direita de K pela matriz $M^{-\frac{1}{2}}$ e assim obtemos a matriz $\widetilde{K}$ simétrica. Calcula-se os autovalores e autovetores desta matriz através do algoritmo de Householder e QR para obtermos as 5 menores frequências e seus respectivos modos de vibração. Os resultados estão a seguir:

```
Matriz Tridiagonalizada Simétrica:
[[ 203305.95354 -167930.55062       0.            0.            0.
     0.            0.            0.            0.            0.
  0.            0.            0.            0.            0.            0
.           0.            0.            0.            0.            0.
          0.            0.            0.     ]
 [-167930.55062  213926.81412  115650.915          0.            0.
     0.            0.            0.            0.            0.
  0.            0.            0.            0.            0.            0
.           0.            0.            0.            0.            0.
          0.            0.            0.     ]
 [      0.       115650.915    278200.56854  -91760.99646       0.
     0.            0.           -0.            0.            0.
  0.            0.           -0.            0.           -0.           -0
.           0.            0.            0.            0.           -0.
          0.            0.            0.     ]
 [      0.            0.       -91760.99646  219567.32267  -96828.47991
     0.           -0.           -0.           -0.           -0.
 -0.           -0.            0.           -0.           -0.           -0
.           0.           -0.           -0.           -0.           -0.
          0.            0.            0.     ]
 [      0.            0.            0.       -96828.47991  209047.2959  -
169876.88037      -0.           -0.           -0.            0.
  0.           -0.           -0.            0.            0.            0
.          -0.            0.            0.            0.            0.
         -0.           -0.           -0.     ]
 [      0.            0.            0.            0.      -169876.88037
250635.06082   92360.73808       0.            0.            0.
  0.            0.            0.            0.            0.            0
.           0.            0.            0.            0.            0.
          0.            0.            0.     ]
 [      0.            0.            0.           -0.           -0.
 92360.73808  266051.83453  -80439.77107      -0.           -0.
  0.            0.            0.           -0.            0.            0
.           0.           -0.            0.           -0.            0.
          0.           -0.           -0.     ]
 [      0.            0.           -0.           -0.           -0.
     0.       -80439.77107  188938.96882  -97716.21621       0.
  0.            0.            0.           -0.           -0.            0
.           0.           -0.           -0.            0.            0.
          0.            0.           -0.     ]
```

```
[      0.            0.            0.           -0.           -0.
     0.           -0.       -97716.21621  201181.53049  102991.30698
  0.            0.            0.            0.            0.            0
.            0.            0.            0.            0.            0.
          0.            0.            0.     ]
 [      0.            0.            0.           -0.            0.
     0.           -0.            0.       102991.30698  221057.94381   55
907.4658        0.           -0.           -0.            0.            0
.           -0.            0.            0.            0.           -0.
          0.           -0.           -0.     ]
 [      0.            0.            0.            0.           -0.            0.
     0.            0.            0.            0.        55907.4658   249
323.89639 -134365.0285       -0.           -0.           -0.           -0
.            0.            0.           -0.            0.           -0.
         -0.            0.           -0.     ]
 [      0.            0.            0.           -0.           -0.
     0.            0.            0.            0.            0.      -134
365.0285   172091.46586  109306.604         0.            0.            0
.            0.            0.            0.           -0.           -0.
          0.            0.           -0.     ]
 [      0.            0.           -0.            0.           -0.
     0.            0.            0.            0.           -0.
 -0.        109306.604    180752.37187   62602.6076        0.            0
.            0.            0.            0.            0.            0.
          0.            0.            0.     ]
 [      0.            0.            0.           -0.            0.
     0.           -0.           -0.            0.           -0.
 -0.            0.        62602.6076   305758.0128   152941.51111      -0
.            0.            0.            0.           -0.            0.
         -0.           -0.            0.     ]
 [      0.            0.           -0.           -0.            0.
     0.            0.           -0.            0.            0.
 -0.            0.            0.       152941.51111  223758.80049  -55690
.06214       0.            0.           -0.           -0.           -0.
         -0.           -0.            0.     ]
 [      0.            0.           -0.           -0.            0.
     0.            0.            0.            0.            0.
 -0.            0.            0.           -0.       -55690.06214  254687
.65977   78293.76478      -0.           -0.           -0.           -0.
         -0.           -0.           -0.     ]
 [      0.            0.            0.            0.           -0.
     0.            0.            0.            0.           -0.
  0.            0.            0.            0.            0.        78293
.76478  159122.34433  -21230.39543       0.            0.            0.
          0.            0.            0.     ]
 [      0.            0.            0.           -0.            0.
     0.           -0.           -0.            0.            0.
  0.            0.            0.            0.            0.           -0
.       -21230.39543  169130.08282 -127022.44278       0.            0.
         -0.           -0.            0.     ]
 [      0.            0.            0.           -0.            0.
     0.            0.           -0.            0.            0.
 -0.            0.            0.            0.           -0.           -0
.            0.      -127022.44278  177312.72877  -74785.82852      -0.
          0.           -0.           -0.     ]
 [      0.            0.            0.           -0.            0.
     0.           -0.            0.            0.            0.
  0.           -0.            0.           -0.           -0.           -0
```

```
.            0.            0.       -74785.82852  181947.14215  -74065.19
041       0.            0.            0.     ]
 [      0.            0.           -0.           -0.            0.
     0.            0.            0.            0.            0.           -0.
 -0.          -0.            0.            0.           -0.           -0
.            0.            0.           -0.       -74065.19041  263804.61
508  -21905.03006       0.            0.     ]
 [      0.            0.            0.            0.            0.          -0.
     0.            0.            0.            0.            0.
 -0.            0.            0.           -0.           -0.           -0
.            0.          -0.            0.            0.       -21905.03
006  183156.23927   59507.23155       0.     ]
 [      0.            0.            0.            0.            0.          -0.
     0.          -0.            0.            0.           -0.
  0.            0.            0.           -0.           -0.           -0
.            0.          -0.           -0.            0.            0.
      59507.23155  107386.95883  -29727.60509]
 [      0.            0.            0.            0.            0.          -0.
     0.          -0.           -0.            0.           -0.
 -0.          -0.            0.            0.            0.           -0
.            0.            0.           -0.            0.            0.
          0.       -29727.60509   19744.43044]]

Matriz HT:
[[ 1.       0.       0.       0.       0.       0.       0.       0.
  0.       0.       0.       0.       0.       0.       0.       0.
 0.       0.       0.       0.       0.       0.       0.       0.     ]
 [ 0.      -0.31623 -0.11616  0.24612 -0.06533 -0.24775  0.09855 -0.12776
 -0.1486  -0.24607  0.33741 -0.00956  0.18158  0.01267  0.04437  0.21747
 0.24103  0.22591 -0.01724  0.08755  0.29536 -0.13648 -0.30047  0.37689]
 [ 0.       0.89443 -0.08214  0.28778  0.03055 -0.12275  0.08437 -0.06044
 -0.05854 -0.06856  0.10973  0.00467  0.12391  0.05038 -0.02085  0.12179
 0.12436 -0.03115 -0.01908  0.10753  0.02687 -0.03426 -0.0074  -0.00197]
 [ 0.       0.      -0.61605 -0.03934  0.24949  0.09796  0.10487  0.11861
 -0.11355  0.03611  0.0028  -0.11099 -0.00518 -0.05464 -0.0373  -0.42361
 0.15862 -0.01396  0.23137  0.12451 -0.18681  0.28833 -0.00929  0.31487]
 [ 0.       0.      -0.      -0.5042  -0.04673 -0.09088  0.09496  0.25402
 -0.20504 -0.09836  0.30798 -0.0056   0.04386  0.07509  0.30923  0.23901
 0.17696 -0.03822  0.06989  0.38403  0.03084  0.09807  0.35833 -0.17543]
 [ 0.       0.       0.      -0.       0.42924 -0.04883 -0.33899 -0.01191
 -0.15196 -0.13083  0.00244  0.46095 -0.01804  0.30949  0.2544  -0.02421
-0.15253 -0.28401 -0.22473 -0.11107  0.03822 -0.04966  0.12871  0.31136]
 [ 0.       0.      -0.43561 -0.10464 -0.2632  -0.18266  0.01364 -0.26343
  0.29762  0.01222 -0.06194  0.44137  0.10955 -0.30006  0.07509 -0.05392
-0.30907 -0.05959  0.07762  0.2198   0.16799 -0.17557  0.09734 -0.08471]
 [ 0.       0.       0.58082  0.08631 -0.09248 -0.12951  0.1305  -0.10096
  0.10961  0.03369 -0.04227  0.24856  0.12537 -0.11824  0.03556 -0.19613
 0.04047 -0.05675  0.30924  0.31973 -0.24502  0.22317  0.05037  0.37606]
 [ 0.       0.22361  0.08214 -0.29288 -0.24629 -0.16617 -0.05728  0.09731
 -0.14356 -0.20305  0.26069 -0.07429 -0.31663 -0.20359  0.25802 -0.23742
-0.34628  0.12558 -0.00332 -0.27489 -0.07507  0.04649 -0.33384  0.1612 ]
 [ 0.       0.22361  0.08214 -0.51017  0.03171  0.30678 -0.14082 -0.03622
  0.16758  0.12928 -0.22246  0.0421   0.07779  0.01999 -0.11186  0.05783
 0.18971  0.3185   0.05525 -0.03141  0.3853  -0.10247 -0.0615   0.37967]
 [ 0.       0.      -0.20535 -0.02331 -0.26488 -0.06064  0.14888  0.13022
  0.16415  0.20125 -0.18095  0.22131 -0.0322   0.18642  0.13533  0.51745
 0.04449  0.0257  -0.10265 -0.21669 -0.29248  0.41018 -0.17386  0.17074]
```

```
 [ 0.       0.       0.      -0.16807  0.00114 -0.46913 -0.07141  0.21554
 -0.02236 -0.05431 -0.22432 -0.24992  0.12724 -0.36655 -0.33747  0.10643
 0.03762 -0.28135 -0.32317 -0.04908 -0.05975 -0.07088  0.18249  0.2807 ]
 [ 0.       0.       0.       0.       0.      -0.21817 -0.12363  0.01563
  0.26921  0.35558  0.24658  0.10875 -0.11346  0.04858  0.08526 -0.30647
 0.33953  0.34142 -0.4641   0.08285 -0.2507  -0.16334  0.05696 -0.04518]
 [ 0.       0.       0.      -0.      -0.       0.49588  0.13477 -0.24152
  0.03191  0.07753  0.44948  0.01212 -0.26571 -0.28571 -0.19617  0.22686
-0.07387 -0.23488 -0.26505  0.08955 -0.1167   0.00024  0.12112  0.24297]
 [ 0.       0.       0.       0.      -0.29344  0.08043  0.17894  0.13789
 -0.16014 -0.22914 -0.02787  0.43311 -0.18615 -0.06598 -0.17191 -0.17706
 0.50889 -0.17374  0.10178 -0.37296  0.03494 -0.16505  0.13352 -0.07125]
 [ 0.       0.      -0.       0.43267 -0.094    0.04827 -0.07893  0.4063
  0.00907  0.15304 -0.05165 -0.02309 -0.273   -0.20989  0.20221  0.04616
-0.12795  0.24909  0.05926 -0.02288  0.34289  0.08718  0.44627  0.18773]
 [ 0.       0.      -0.      -0.06992  0.05544 -0.33773  0.03219  0.09439
  0.21382  0.25204  0.2684   0.00944 -0.40522  0.37466 -0.41234  0.02103
-0.09267 -0.21231  0.29925  0.03942  0.24844  0.00735 -0.06277  0.03369]
 [ 0.       0.      -0.13216  0.04949 -0.38521  0.05191 -0.45379 -0.14277
 -0.04312 -0.16017 -0.05453 -0.20345 -0.12095  0.27788 -0.06284  0.10791
 0.00159  0.06893  0.2435   0.10279 -0.38875 -0.31539  0.23695  0.20498]
 [ 0.       0.       0.      -0.      -0.09121  0.06665  0.29825  0.16282
 -0.23594 -0.23426 -0.28288  0.16975 -0.2277   0.24599 -0.29557 -0.08634
-0.24607  0.25789 -0.34718  0.43457 -0.01913 -0.04334 -0.07599  0.02752]
 [ 0.       0.       0.      -0.      -0.34063  0.09629  0.37632 -0.01856
  0.13591  0.09789  0.00217 -0.28702  0.22996  0.38018  0.25355 -0.29379
-0.09288 -0.31004 -0.20613 -0.10345  0.1821  -0.12407  0.14806  0.21417]
 [ 0.       0.       0.       0.03614 -0.02065  0.1997  -0.09714  0.53418
  0.30355 -0.20327  0.34732  0.14854  0.4708   0.059   -0.26906 -0.03424
-0.2203   0.09634  0.03749 -0.08752 -0.13876 -0.04661 -0.02005 -0.00594]
 [ 0.       0.       0.      -0.03614  0.34753 -0.03149  0.44021  0.16187
  0.08592  0.08472 -0.09378 -0.00453 -0.10189 -0.07465  0.20149  0.18566
-0.08425  0.07396  0.22966 -0.08189 -0.27661 -0.6173  -0.02168  0.1117 ]
 [ 0.       0.       0.      -0.       0.18123 -0.03299  0.085   -0.10037
  0.61202 -0.62688 -0.07456 -0.15427 -0.26068  0.03846  0.10964  0.01248
 0.1368   0.03789 -0.06179 -0.00642  0.02203  0.17435  0.12303 -0.01506]
 [ 0.       0.       0.      -0.10221  0.108   -0.18157  0.24692 -0.35647
 -0.15719 -0.02234  0.12965  0.03313  0.14812  0.11113 -0.22547 -0.00567
-0.20506  0.41143  0.03723 -0.38525 -0.07332  0.16269  0.48491  0.02234]]

Matriz A Final:
[[459787.0489      -0.           0.           0.           0.          -0
.          -0.          -0.           0.          -0.          -0.
    0.          -0.           0.           0.           0.          -0.
          0.          -0.          -0.          -0.          -0.
 0.           0.     ]
 [     0.      432319.63729     -0.           0.          -0.          -0
.          -0.          -0.           0.          -0.          -0.
    0.           0.           0.           0.          -0.          -0.
       -0.           0.          -0.          -0.          -0.
 0.          -0.     ]
 [     0.           0.      442927.02636     -0.          -0.           0
.          -0.           0.          -0.          -0.          -0.
   -0.          -0.          -0.          -0.           0.          -0.
          0.           0.           0.          -0.           0.
 0.           0.     ]
 [    -0.           0.          -0.      351642.75412     -0.           0
.          -0.          -0.           0.          -0.          -0.
```

```
    0.          0.          -0.          -0.          0.          -0.
       -0.          0.          -0.          0.          0.
 0.          -0.     ]
 [     0.          0.          0.          -0.      389912.93        -0
.          -0.          0.          0.          0.          0.
    0.          -0.          0.          -0.          -0.          -0.
       -0.          0.          -0.          -0.          0.
-0.          -0.     ]
 [    -0.          -0.          0.          0.          0.      313358
.47757     -0.          -0.          0.          0.          0.
   -0.          -0.          0.          -0.          -0.          -0.
       -0.          -0.          0.          -0.          -0.
-0.          0.     ]
 [    -0.          -0.          0.          -0.          -0.          -0
.      302305.85287      0.          0.          0.          -0.
    0.          -0.          -0.          0.          -0.          -0.
       -0.          0.          -0.          0.          0.
-0.          -0.     ]
 [    -0.          -0.          -0.          0.          -0.          0
.          0.      272742.1392      -0.          -0.          0.
    0.          0.          -0.          0.          -0.          -0.
       -0.          -0.          -0.          -0.          -0.
 0.          -0.     ]
 [     0.          0.          -0.          -0.          0.          -0
.          0.          -0.      340498.67825      0.          -0.
    0.          0.          0.          -0.          -0.          0.
       -0.          -0.          0.          -0.          -0.
 0.          0.     ]
 [    -0.          0.          0.          -0.          -0.          -0
.          -0.          0.          -0.      213324.1883      -0.
    0.          -0.          -0.          -0.          -0.          0.
       -0.          -0.          0.          0.          0.
 0.          -0.     ]
 [    -0.          -0.          0.          -0.          0.          0
.          -0.          -0.          0.          0.      286156.72527
    0.          -0.          0.          0.          0.          0.
       -0.          0.          0.          -0.          -0.
 0.          0.     ]
 [    -0.          -0.          -0.          0.          -0.          -0
.          -0.          0.          0.          -0.          -0.      1
78865.9668       0.          0.          0.          0.          0.
       -0.          -0.          0.          -0.          -0.
-0.          0.     ]
 [    -0.          0.          -0.          0.          -0.          -0
.          -0.          0.          -0.          0.          -0.
   -0.      214713.95159      0.          0.          0.          -0.
         0.          -0.          0.          -0.          -0.
 0.          -0.     ]
 [     0.          0.          -0.          -0.          0.          0
.          -0.          -0.          -0.          -0.          -0.
   -0.          0.      123427.17879     -0.          0.          -0.
         0.          0.          0.          -0.          0.
-0.          -0.     ]
 [     0.          0.          0.          0.          0.          -0
.          -0.          -0.          0.          -0.          0.
   -0.          -0.          -0.       53112.35234     -0.          -0.
       -0.          0.          -0.          -0.          -0.
 0.          0.     ]
```

```
 [    -0.          -0.           0.           0.          -0.           0
.           0.           0.          -0.           0.          -0.
   -0.          -0.           0.          -0.       20394.60959      0.
          0.          -0.          -0.           0.          -0.
-0.           0.     ]
 [     0.           0.           0.          -0.           0.          -0
.           0.          -0.          -0.           0.          -0.
   -0.           0.           0.           0.          -0.      146507.07
226     -0.           0.           0.           0.           0.
 0.          -0.     ]
 [    -0.           0.          -0.          -0.          -0.          -0
.          -0.           0.          -0.           0.          -0.
   -0.          -0.           0.          -0.          -0.          -0.
       604.79341      0.           0.          -0.           0.
 0.           0.     ]
 [     0.           0.          -0.          -0.          -0.          -0
.          -0.          -0.           0.           0.          -0.
   -0.           0.           0.          -0.          -0.          -0.
        -0.        8968.72741      0.           0.           0.
 0.           0.     ]
 [     0.           0.          -0.          -0.          -0.           0
.           0.           0.          -0.           0.          -0.
   -0.           0.           0.           0.           0.          -0.
        -0.           0.      150401.6595      -0.          -0.
-0.           0.     ]
 [    -0.          -0.          -0.           0.          -0.          -0
.           0.          -0.          -0.           0.           0.
   -0.          -0.          -0.          -0.          -0.          -0.
        -0.          -0.       22747.31385      0.
-0.          -0.     ]
 [    -0.          -0.           0.           0.           0.          -0
.           0.          -0.          -0.           0.           0.
   -0.           0.           0.          -0.          -0.          -0.
        -0.           0.           0.           0.       84178.0885
-0.           0.     ]
 [     0.           0.           0.           0.          -0.          -0
.          -0.           0.           0.           0.           0.
   -0.          -0.          -0.           0.          -0.          -0.
        -0.           0.          -0.          -0.           0.        84
66.28997     -0.     ]
 [    -0.          -0.          -0.          -0.          -0.           0
.          -0.          -0.           0.          -0.          -0.
   -0.          -0.          -0.          -0.           0.          -0.
        0.           0.          -0.           0.          -0.
 0.       82526.57997]]

Matriz V Final:
[[ 0.2071  -0.39871 -0.00833  0.14918 -0.04003 -0.39384 -0.09785 -0.32138
 -0.00666 -0.10756 -0.03008  0.25634  0.02434 -0.01617  0.08637  0.44643
-0.06274  0.40338 -0.08988 -0.21307 -0.01036 -0.00264 -0.00072  0.01223]
 [ 0.16047 -0.29255 -0.00161 -0.09772 -0.0626   0.02481 -0.19996  0.29613
  0.05798  0.20285 -0.42037  0.37472  0.19973  0.19718 -0.0422  -0.22275
 0.1036  -0.07063  0.09328 -0.04418  0.22185 -0.28857 -0.25227 -0.18052]
 [-0.2071   0.39871 -0.00833 -0.14918  0.04003  0.39384 -0.09785  0.32138
 -0.00666  0.10756 -0.03008  0.25634  0.02434  0.01617  0.08637  0.44643
 0.06274  0.40338 -0.08988 -0.21307  0.01036  0.00264  0.00072  0.01223]
```

```
 [ 0.16047 -0.29255  0.00161 -0.09772 -0.0626   0.02481  0.19996  0.29613
 -0.05798  0.20285  0.42037 -0.37472 -0.19973  0.19718  0.0422   0.22275
 0.1036   0.07063 -0.09328  0.04418  0.22185 -0.28857 -0.25227  0.18052]
 [-0.20379 -0.00107 -0.15513 -0.18851 -0.10933 -0.29896 -0.44558  0.02715
 -0.09515  0.14495  0.01953 -0.24801 -0.23946  0.04495  0.01408 -0.17172
 0.38975  0.26183  0.07391 -0.01657  0.17368  0.33583  0.05457 -0.21244]
 [-0.3461  -0.24224 -0.46017 -0.01599  0.19484  0.03299  0.04076 -0.00218
  0.06456  0.0187  -0.00681 -0.01313 -0.00428 -0.04626 -0.07814  0.30936
 0.08586 -0.20921  0.32748  0.05491 -0.39091  0.02983 -0.34447 -0.16229]
 [ 0.20226 -0.14125  0.05392  0.3881   0.15729  0.29987  0.37444  0.01449
  0.00775  0.08232  0.04926  0.00199  0.1056  -0.10314 -0.05507 -0.11266
 0.29386  0.36691  0.14272  0.18798  0.07088  0.25303  0.02873 -0.3704 ]
 [-0.15696  0.28186 -0.0303   0.25731  0.10756 -0.08615  0.00786 -0.25797
 -0.12525 -0.03237  0.45707  0.33057 -0.07212  0.07665  0.00462 -0.18652
 0.09451 -0.09718  0.11645 -0.30165  0.29332 -0.17401 -0.33466 -0.10553]
 [-0.20226  0.14125  0.05392 -0.3881  -0.15729 -0.29987  0.37444 -0.01449
  0.00775 -0.08232  0.04926  0.00199  0.1056   0.10314 -0.05507 -0.11266
-0.29386  0.36691  0.14272  0.18798 -0.07088 -0.25303 -0.02873 -0.3704 ]
 [-0.15696  0.28186  0.0303   0.25731  0.10756 -0.08615 -0.00786 -0.25797
  0.12525 -0.03237 -0.45707 -0.33057  0.07212  0.07665 -0.00462  0.18652
 0.09451  0.09718 -0.11645  0.30165  0.29332 -0.17401 -0.33466  0.10553]
 [ 0.20379  0.00107 -0.15513  0.18851  0.10933  0.29896 -0.44558 -0.02715
 -0.09515 -0.14495  0.01953 -0.24801 -0.23946 -0.04495  0.01408 -0.17172
-0.38975  0.26183  0.07391 -0.01657 -0.17368 -0.33583 -0.05457 -0.21244]
 [-0.3461  -0.24224  0.46017 -0.01599  0.19484  0.03299 -0.04076 -0.00218
 -0.06456  0.0187   0.00681  0.01313  0.00428 -0.04626  0.07814 -0.30936
 0.08586  0.20921 -0.32748 -0.05491 -0.39091  0.02983 -0.34447  0.16229]
 [-0.12865 -0.08599 -0.06774  0.09617 -0.38581  0.2492   0.11844 -0.19951
 -0.54709 -0.00287 -0.17226 -0.08038  0.10802  0.4423  -0.05903 -0.04915
-0.04974  0.08817  0.2194  -0.1338  -0.06088  0.11933 -0.00037  0.23358]
 [ 0.33397  0.27025  0.48418  0.03295 -0.28641 -0.11369 -0.1743   0.02615
 -0.15067  0.08516  0.0449  -0.03323 -0.02016 -0.06826 -0.07543  0.25724
 0.15332 -0.1605   0.25973  0.10166 -0.32832  0.03399 -0.27265 -0.18317]
 [ 0.20715  0.11976  0.10004 -0.07251  0.4078  -0.11236 -0.04304  0.0258
  0.36423 -0.00098  0.04057 -0.04935  0.03566  0.46569 -0.09502 -0.09185
-0.04328  0.17858  0.42982 -0.1123  -0.11262  0.16272 -0.00072  0.32157]
 [ 0.15583 -0.05001  0.10504 -0.34018  0.0031   0.19442  0.23347 -0.12935
  0.09392 -0.26828 -0.25156 -0.15457 -0.29054 -0.112    0.20663  0.00363
-0.08879 -0.09094  0.05575 -0.41216  0.20251  0.2679  -0.31269 -0.17343]
 [-0.20715 -0.11976  0.10004  0.07251 -0.4078   0.11236 -0.04304 -0.0258
  0.36423  0.00098  0.04057 -0.04935  0.03566 -0.46569 -0.09502 -0.09185
 0.04328  0.17858  0.42982 -0.1123   0.11262 -0.16272  0.00072  0.32157]
 [ 0.15583 -0.05001 -0.10504 -0.34018  0.0031   0.19442 -0.23347 -0.12935
 -0.09392 -0.26828  0.25156  0.15457  0.29054 -0.112   -0.20663 -0.00363
-0.08879  0.09094 -0.05575  0.41216  0.20251  0.2679  -0.31269  0.17343]
 [ 0.12865  0.08599 -0.06774 -0.09617  0.38581 -0.2492   0.11844  0.19951
 -0.54709  0.00287 -0.17226 -0.08038  0.10802 -0.4423  -0.05903 -0.04915
 0.04974  0.08817  0.2194  -0.1338   0.06088 -0.11933  0.00037  0.23358]
 [ 0.33397  0.27025 -0.48418  0.03295 -0.28641 -0.11369  0.1743   0.02615
  0.15067  0.08516 -0.0449   0.03323  0.02016 -0.06826  0.07543 -0.25724
 0.15332  0.1605  -0.25973 -0.10166 -0.32832  0.03399 -0.27265  0.18317]
 [ 0.05382  0.00779 -0.01326 -0.16564  0.08262  0.09681 -0.00547 -0.33702
 -0.04893  0.55912  0.00442  0.14139 -0.09312 -0.09289  0.59275 -0.03633
-0.13589  0.00657  0.1818   0.26919  0.00786  0.00369  0.00662  0.1024 ]
 [-0.07954  0.00602 -0.00696  0.23837 -0.06507 -0.1471  -0.09578  0.26467
  0.04167  0.16202  0.11823 -0.26273  0.52856 -0.04315  0.26296  0.00703
-0.42019 -0.04654 -0.00886 -0.1996   0.10669  0.30837 -0.17247 -0.12402]
```

```
 [-0.05382 -0.00779 -0.01326  0.16564 -0.08262 -0.09681 -0.00547  0.33702
 -0.04893 -0.55912  0.00442  0.14139 -0.09312  0.09289  0.59275 -0.03633
 0.13589  0.00657  0.1818   0.26919 -0.00786 -0.00369 -0.00662  0.1024 ]
 [-0.07954  0.00602  0.00696  0.23837 -0.06507 -0.1471   0.09578  0.26467
 -0.04167  0.16202 -0.11823  0.26273 -0.52856 -0.04315 -0.26296 -0.00703
-0.42019  0.04654  0.00886  0.1996   0.10669  0.30837 -0.17247  0.12402]]


iterações k com deslocamento: 173


5 menores frequências sem contar os zeros:
ω1:  24.59254776974222
ω2:  92.01244464604494
ω3:  94.70336537381728
ω4:  142.80969710649188
ω5:  150.8221265108189

5 autovetores correspondentes:

v1:  [ 0.40338 -0.07063  0.40338  0.07063  0.26183 -0.20921  0.36691 -0.0
9718  0.36691  0.09718  0.26183  0.20921  0.08817 -0.1605   0.17858 -0.09
094  0.17858  0.09094  0.08817  0.1605   0.00657 -0.04654  0.00657  0.046
54]

v2:  [-0.00072 -0.25227  0.00072 -0.25227  0.05457 -0.34447  0.02873 -0.3
3466 -0.02873 -0.33466 -0.05457 -0.34447 -0.00037 -0.27265 -0.00072 -0.31
269  0.00072 -0.31269  0.00037 -0.27265  0.00662 -0.17247 -0.00662 -0.172
47]

v3:  [-0.08988  0.09328 -0.08988 -0.09328  0.07391  0.32748  0.14272  0.1
1645  0.14272 -0.11645  0.07391 -0.32748  0.2194   0.25973  0.42982  0.05
575  0.42982 -0.05575  0.2194  -0.25973  0.1818  -0.00886  0.1818   0.008
86]

v4:  [ 0.44643 -0.22275  0.44643  0.22275 -0.17172  0.30936 -0.11266 -0.1
8652 -0.11266  0.18652 -0.17172 -0.30936 -0.04915  0.25724 -0.09185  0.00
363 -0.09185 -0.00363 -0.04915 -0.25724 -0.03633  0.00703 -0.03633 -0.007
03]

v5:  [-0.01036  0.22185  0.01036  0.22185  0.17368 -0.39091  0.07088  0.2
9332 -0.07088  0.29332 -0.17368 -0.39091 -0.06088 -0.32832 -0.11262  0.20
251  0.11262  0.20251  0.06088 -0.32832  0.00786  0.10669 -0.00786  0.106
69]
```

6. Discussão

Os problemas dos autovalores e autovetores de uma matriz simétrica possuem inúmeras aplicações como vimos no EP1, com sistemas massa mola e agora, no EP2, com treliças planas. Nos foi proposto um método de classificação composto por um algoritmo simples e extremamente poderoso, capaz de produzir excelentes resultados. Pode-se perceber que o trabalho extenso com matrizes de grandes dimensões pode levar a programas que requerem um maior poder de processamento e tempo de execução. Dessa forma, se mostra latente a necessidade de implementações eficientes dos algoritmos de manipulação de matrizes. A linguagem Python caracteriza uma ferramenta muito útil na implementação do algoritmo proposto, no entanto apresenta um maior tempo de execução devido ao seu caráter interpretado, em comparação com linguagens compiladas como C, C++ ou Julia. O uso da biblioteca Numpy representa um salto em desempenho do algoritmo, uma vez que recorre à chamada de funções compiladas de C.