Question 2
Task 1

we know: $\vec{l} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ - light source

$c = \begin{pmatrix} 4 \\ 6 \\ 7 \end{pmatrix}$ - camera posn.

$\vec{n} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

plain = $y = 0$

Steps: ① find the reflected ray r

② find the position via $c - kr = p$, where c, r are fixed

③ we know $y=0$, solve the equations

①

$$2(\vec{n}) \cdot \langle \vec{n}, \vec{l} \rangle - \vec{l}$$

$$\Rightarrow 2\begin{pmatrix} 0 \\ b \end{pmatrix} \cdot \left\langle \begin{pmatrix} 0 \\ b \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \right\rangle - \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix}$$

②

$$\begin{pmatrix} 4 \\ 6 \\ 7 \end{pmatrix} - k\begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

③

$$4 + k = x$$
$$6 - 2k = 0$$
$$7 + 2k = z$$

$\Rightarrow k = 3$

$\Rightarrow x = 7$

$k = 3$

$z = 13$

$\Rightarrow \begin{pmatrix} 7 \\ 0 \\ 13 \end{pmatrix}$ = intersection p

Question 2
Task 2.

$$n = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} ,$$

$$\vec{l} = \frac{\vec{l}}{\|l\|} = \frac{(1,2,2)^T}{\sqrt{1^2+2^2+2^2}} = \frac{(1,2,2)^T}{3} = \begin{pmatrix} 1/3 \\ 2/3 \\ 2/3 \end{pmatrix}$$

we know $\rho_d = \frac{1}{2}$, $\rho_s = \frac{1}{2}$, $I = 1$.

$$I_{tot} = \rho_d \left(\langle \vec{n}, \vec{l} \rangle \cdot I \right) + \rho_s \left(\langle \vec{n}, \vec{v} \rangle^k \cdot I \right)$$

$$= \frac{1}{2} \left(1 \cdot \langle \vec{n}, \vec{l} \rangle \right) + \frac{1}{2} \left(1 \cdot \langle \vec{n}, \vec{v} \rangle^2 \right)$$

$$= \frac{1}{2} \left(\left(0 \cdot \frac{1}{3} + 1 \cdot \frac{2}{3} + 0 \cdot \frac{2}{3} \right) \cdot 1 \right) + \frac{1}{2} \left(1 \cdot 1 \right)$$

$$= \frac{1}{2} \cdot \frac{1}{3} + \frac{1}{2} = \frac{5}{6}.$$

Bonus Exercise 3

(1) normalise all vectors such that

$\Rightarrow \quad \|n\| = \|v\| = \|l\| = \|r\| = 1$

(2) let $\vec{h}$ be the half vector such that:

$$\vec{h} = \frac{l+v}{\|l+v\|}$$

(3) let $\vec{r} = 2(l \cdot n)n - l$

(4) $\measuredangle$ between $\vec{v}, \vec{r}$ such as:

$\cos(\phi) = \langle v, r \rangle$

$\Rightarrow$ use (3)

$\Rightarrow \cos(\phi) = \vec{v} \cdot [2(l \cdot n)n - l]$

(5) $\measuredangle$ between $\vec{n}, \vec{h}$ such as

$\cos(\theta) = \langle \vec{n}, \vec{h} \rangle$

(6) $\cos(\theta) = \langle n, \frac{l+v}{\|l+v\|} \rangle$ by definitions ✓

(7) Since they are all coplanar, $\vec{h}$ bisects angle between $\vec{l}, \vec{v}$

$\Rightarrow \frac{1}{2}\phi = \theta$ as angle of incidence = angle of reflection