Theory of Computation

Exercise 1: (Mathematematic preliminary, Language, String)

1. Let $\sum = \{a, b\}$ and $L = \{aa, bb\}$. Describe $\overline{L}$ by a set notation.

2. Find five strings which are in each of the following languages.

a) $L = \{w \in \{a\}^*: |w|\ mod\ 3\ \neq |w|\ mod\ 2\ \}$

b) $L = \{w \in \{a, b\}^*: n_a(w) \geq n_b(w) + 1\}$

Where $n_a(w)$ means the number of a's in string w.

Theory of Computation

Exercise 2: (Deterministic Finite Automata - DFA)

1. Draw DFA for L1

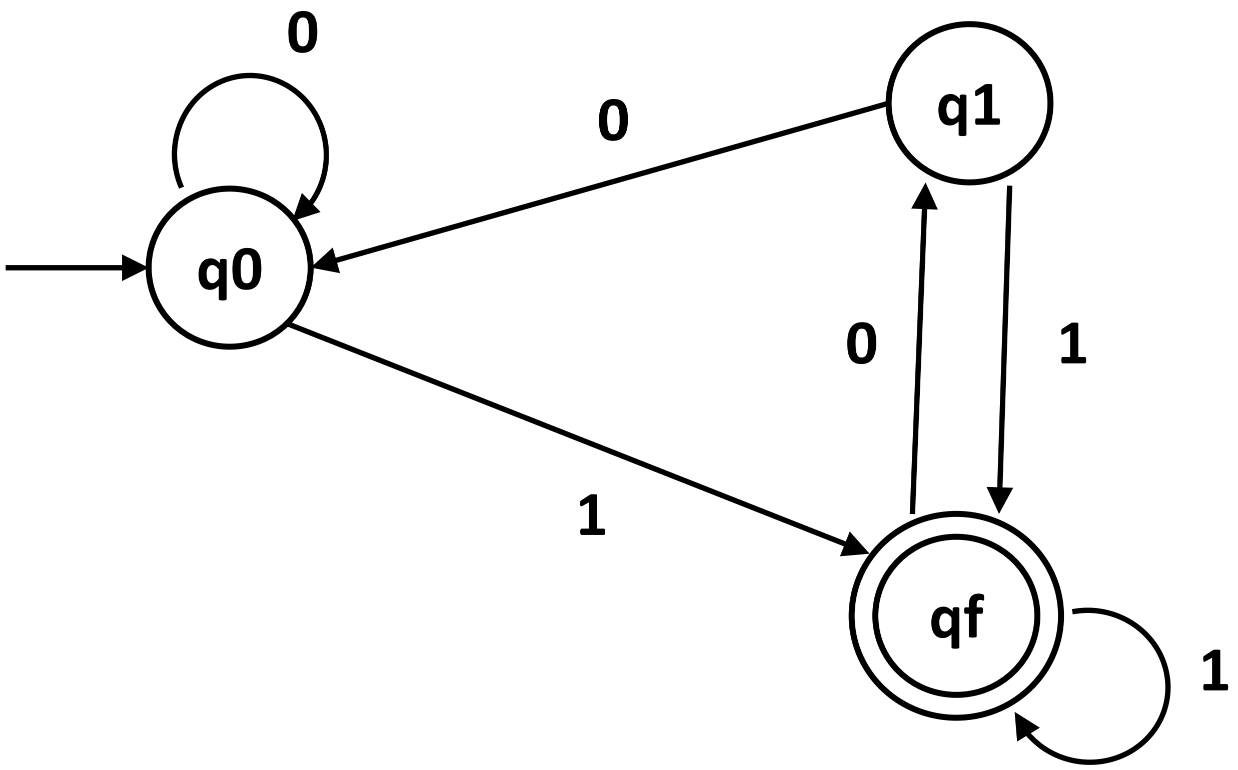
$$L1 = \{w \in \{0,1\}^*: w\ has\ no\ substring\ 11\}$$

2. Draw DFA for L2

$$L2 = \{0, 10\}^* \cup \{1\}$$

3. Find the language of DFA M.

M:



* 4. Draw DFA for L3 (Submit 1)

$$L3 = \{\, a^m\ b^n :\ m + n = 5;\ \ m\ and\ n\ \ \geq 0 \,\}$$

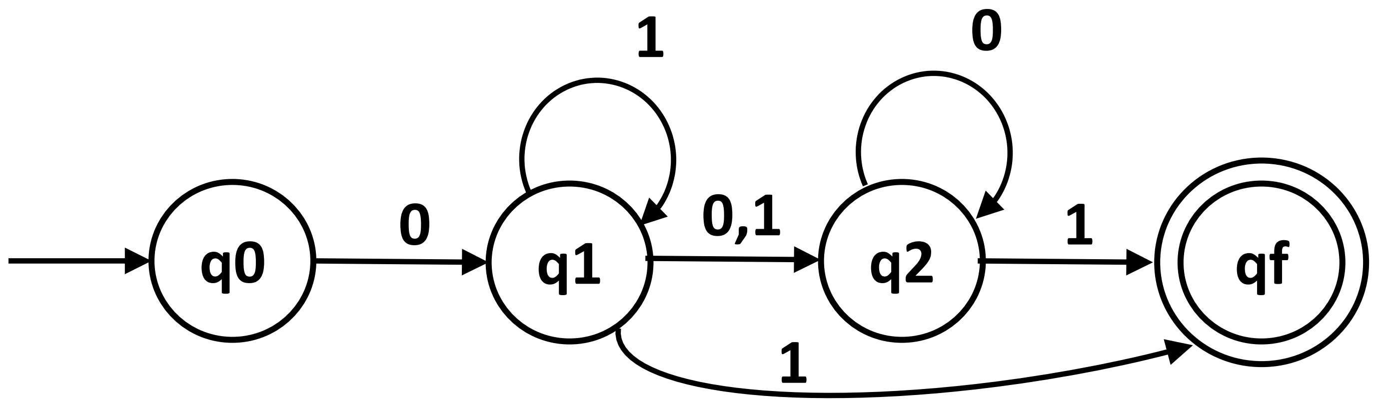
Theory of Computation

Exercise 3: (Nondeterministic Finite Automata - NFA)

1. Construct the minimal-state NFA that accepts the language $\{ab, abc\}^*$

2. Convert the following NFA to DFA

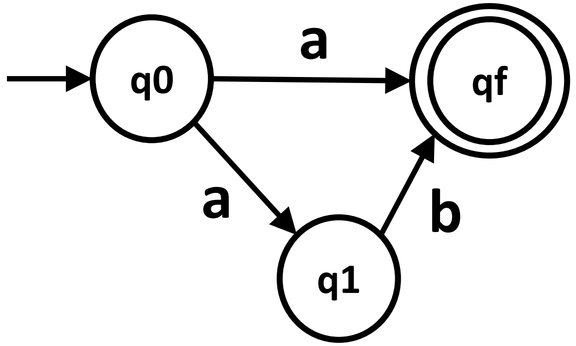
NFA



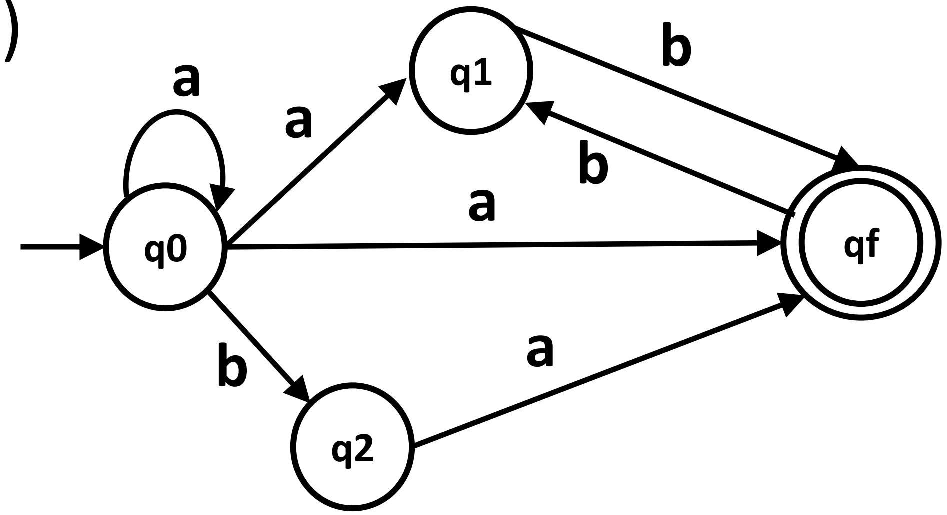
(Homework 2)

*3. What are the languages accepted by the following NFA ?

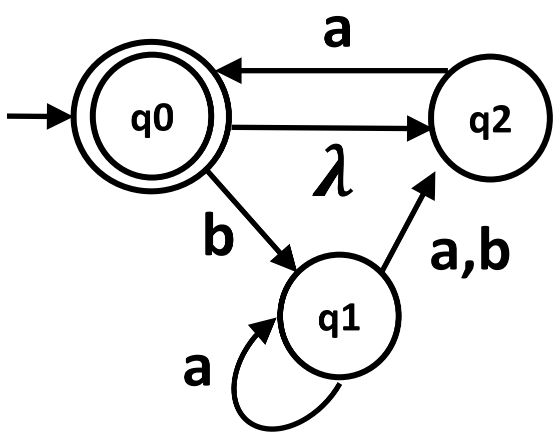
a)



b)



c)



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Exercise 4:

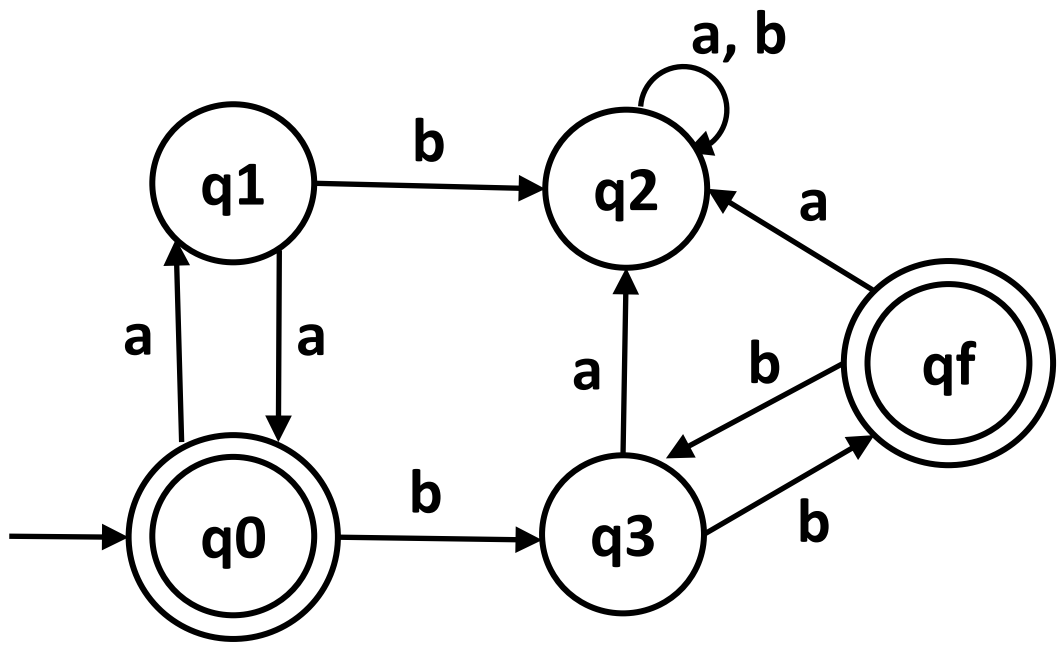
(Closure properties of Regular Language and Regular Expression)

1. Prove that the language $\{a^m b : m \geq 1 \; and \; m \neq 100\}$ is regular.

2. Find regular expression for the following language

L = $\{ w \in \{a, b\}^* : w \text{ does not end with } ab \}$

*3. Find regular expression for the following DFA. (Submit 3)



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Exercise 5:

(Regular Grammar)

1. Give the right-linear grammar for the language
   L =$\{1^n 0^m : n \geq 1 \; and \; m \geq 2\}$.

2. Give the left-linear grammar for the language L in problem 1.

(Homework 4)

*3. Draw NFA for the following grammar G.

$$
\begin{aligned}
G: A &\rightarrow xyB, \\
\mathrm{B} &\rightarrow yxC, \\
\mathrm{C} &\rightarrow xB \mid yy
\end{aligned}
$$

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Exercise 6:

(Pumping Lemma)

Prove by Pumping Lemma that the following languages are not regular.

1. L1 $=\{ 0^i 1^j \; : j \; is \; a \; multiple \; of \; i \}$

2. L2 $=\{\ 0^i 1^j\ :\ i < j\ \}$

(Homework 5)

*3. L3 = $\{\ 0^i 1^j\ :\ i \geq j\ \}$

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Exercise 7: (More exercises on Pumping Lemma)

Prove that the following languag is not Regular

$$L = \{ a^i b^j c^k : k < 2i+j \}$$

Short prove:

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Exercise 8: (Context-free grammar part 1)

1. Prove that the following grammar is ambiguous.

$$S \rightarrow S + S \mid S - S \mid S * S \mid S/S \mid c$$

2. Find CFG for the language L.

L $=\{ a^i b^j \; : i \le j \}$

*3. Find the language of the following grammar. (Homework 6)

$$G: S \rightarrow aA \mid bA \mid a \mid b$$
$$\mathrm{A} \rightarrow aS \mid bS$$

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Exercise 9: (Context-free grammar part 2)

1. Show that L(G1) ≠ L(G2).

G1 = ({S}, {a, b}, S, P1)

P1: $S \rightarrow aSb \mid SS \mid \lambda$

G2 = ({S}, {a, b}, S, P2)

P2: $S \rightarrow aSb \mid abS \mid \lambda$

2. Find CFG for the language L.

L =$\{ a^i b^j c^k : j = i + k \}$

*3. Use CYK algorithm to find whether **abab** ∈ L(G).

(Homework 7)

$$G:\ S \rightarrow AB$$
$$\mathrm{A} \rightarrow BB$$
$$\mathrm{A} \rightarrow a$$
$$\mathrm{B} \rightarrow AB$$
$$\mathrm{B} \rightarrow b$$

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Exercise 10: (Pushdown Automata)

Find Pushdown Automata for the following languages.

1. L1 ={ $a^n b^{2n}: n \geq 0$ }

2. L2 =$\{w \in \{a, b\}^*: \ n_a(w) > n_b(w)\}$

*3. L3 =$\{a^n b^m a^{n+m} : n, m \geq 1\}$ (Homework 8)

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Exercise 11_12: (Pushdown Automata & Properties of CFL)

1. Find the language of NPDA M.

M = $(\{q0, q1, qf\}, \{0,1\}, \{0,1, \$\}, \delta, q0, \$, \{qf\})$

δ: δ(q0, 0, \$) = { (q1, 0), (qf, λ) },

δ(q1, 1, 1) = { (q1, 1) },

δ(q1, 1, 0) = { (q1, 1) },

δ(q1, 0, 1) = { (qf, 1) }

2. Find PDA for the language L2.

L2 $=\{a^n b^m : n \leq m \leq 2n\}$

*3. Prove that the language L is CFL by <u>using properties of CFL</u> (DO NOT draw PDA or CFG). (Homework 9)

L3 = $\{0^i 1^j 2^k : j = i + k\}$

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Exercise 13&14: (Pumping Lemma for NonCFL)

Prove by P.L. that the following languages are not CFL.

1. L1 $=\{a^n b^{n+1} c^{n+2} : n \geq 0\}$

<u>Proof</u>: Assume that L1 is CFL. There is PDA accepts L1 with m number of states

2. L2 ={ $w \in \{a, b, c\}^*$: $n_a(w) = \min(\, n_b(w), n_c(w)\,)$ }