Artificial Intelligence

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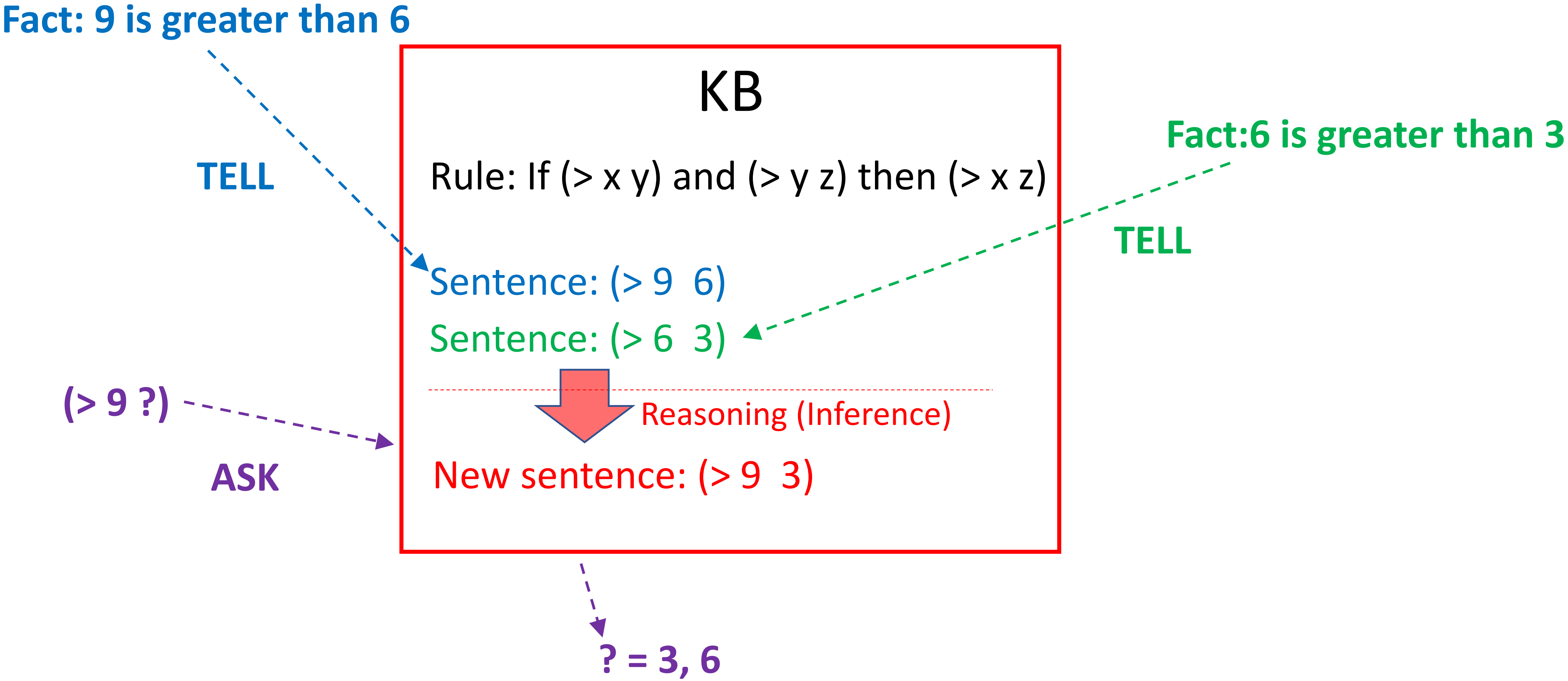
Lecture 5
Logical Agents

- Knowledge-based agents
- Propositional logic
- Semantic of propositional logic
- Proof by inference
- Proof by resolution

Knowledge-based agents

- Knowledge-based agent is the agent that has the reasoning process that operates on internal knowledge.
- The central component of a knowledge-based agent is its knowledge base, or KB, which is a set of sentences.
- Each **sentence** is expressed in a special form and represents some fact about the world.
- Knowledge-based agent can operate by being told or learning new knowledge about the environment and they can adapt to changes in the environment by updating the relevant knowledge.
- **TELL** is a way to add new sentences to the knowledge base.
- **ASK** is a way to query what is known.


Fact: 9 is greater than 6
TELL
KB
Rule: If (> x y) and (> y z) then (> x z)
Fact:6 is greater than 3
TELL
Sentence: (> 9 6)
Sentence: (> 6 3)
(> 9 ?)
Reasoning (Inference)
ASK
New sentence: (> 9 3)
? = 3, 6

function KB-AGENT(*percept*) **returns** an *action*
  persistent: *KB*, a knowledge base
    t, a counter, initially 0, indicating time

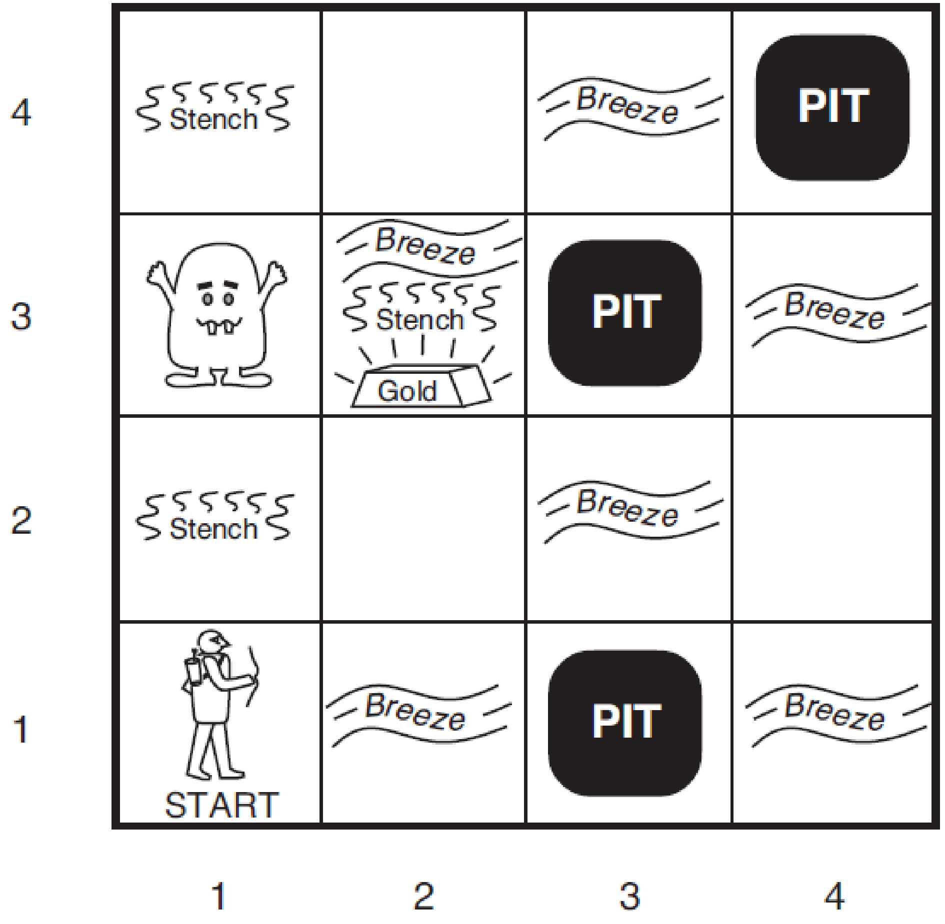
  TELL(*KB*, MAKE-PERCEPT-SENTENCE(*percept*, *t*))
  action ← ASK(*KB*, MAKE-ACTION-QUERY(*t*))
  TELL(*KB*, MAKE-ACTION-SENTENCE(*action*, *t*))
  t ← *t* + 1
  return *action*

Constructs a sentence asserting that the agent perceived the given percept at the given time.

Constructs a sentence that asks what action should be done at the current time.

Constructs a sentence asserting that the chosen action was executed.

Case study : The Wumpus World



- The wumpus world is a cave consisting of rooms connected by passageways.
- Lurking somewhere in the cave is the terrible **wumpus**, a beast that cannot move but eats anyone who enters its room.
- The wumpus can be shot by an agent, but the agent has only one **arrow**.
- Some rooms contain bottomless **pits** that will trap anyone who wanders into these rooms (except for the wumpus, which is too big to fall in).
- The agent has to find path that leads to a heap of **gold** and safely return to the origin (1,1).
- Actions : { Forward, TurnLeft, TurnRight, Grab, Shoot, Climb }
- Percepts: { Stench, Breeze, Glitter, Bump, Scream }
- Environment : { Sequential, Partial observable, discrete, static, single-agent }

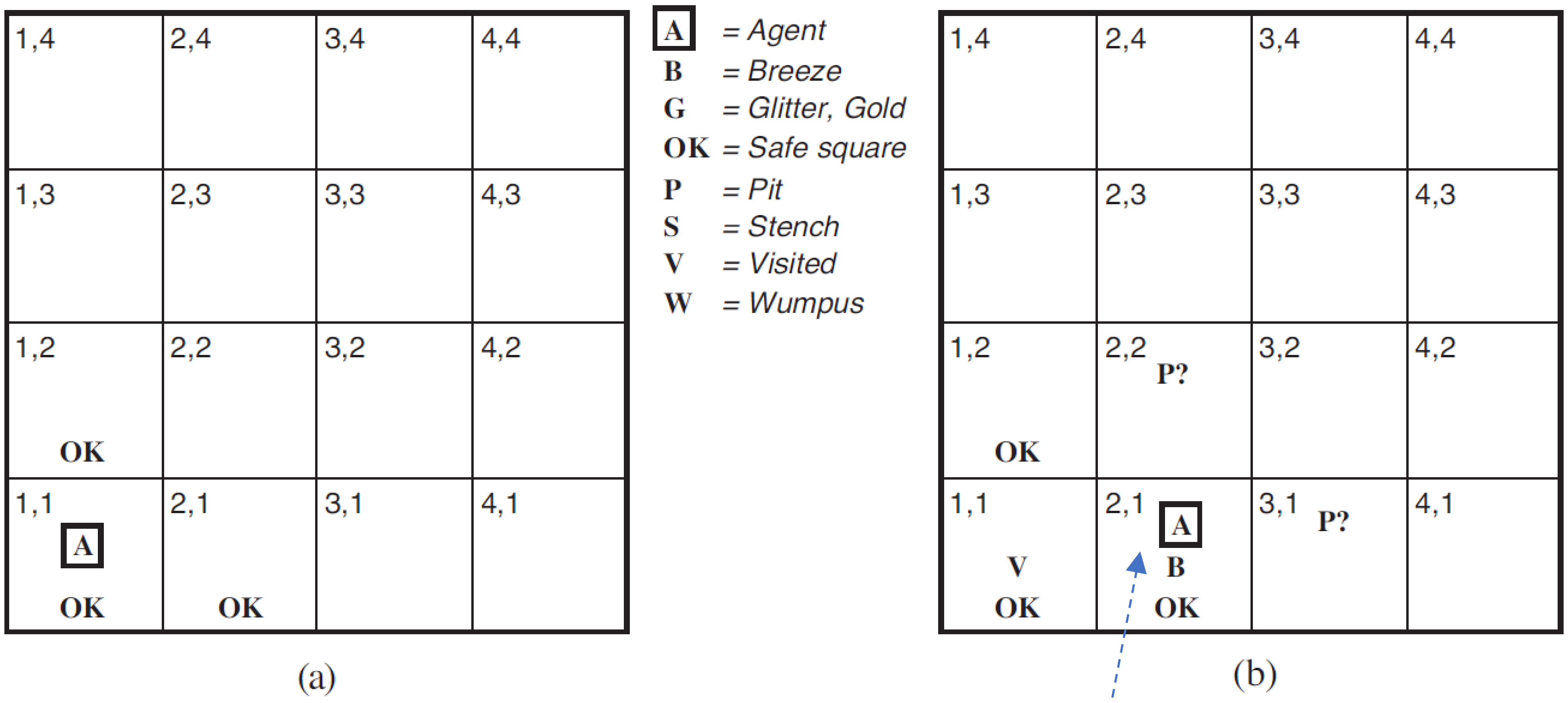


Figure 7.3 The first step taken by the agent in the wumpus world. (a) The initial situation, after percept $[None, None, None, None, None]$. (b) After one move, with percept $[None, Breeze, None, None, None]$.

The prudent agent will turn around, go back to [1,1], and then proceed to [1,2]

Assume that the agent turns and moves to [2,3].

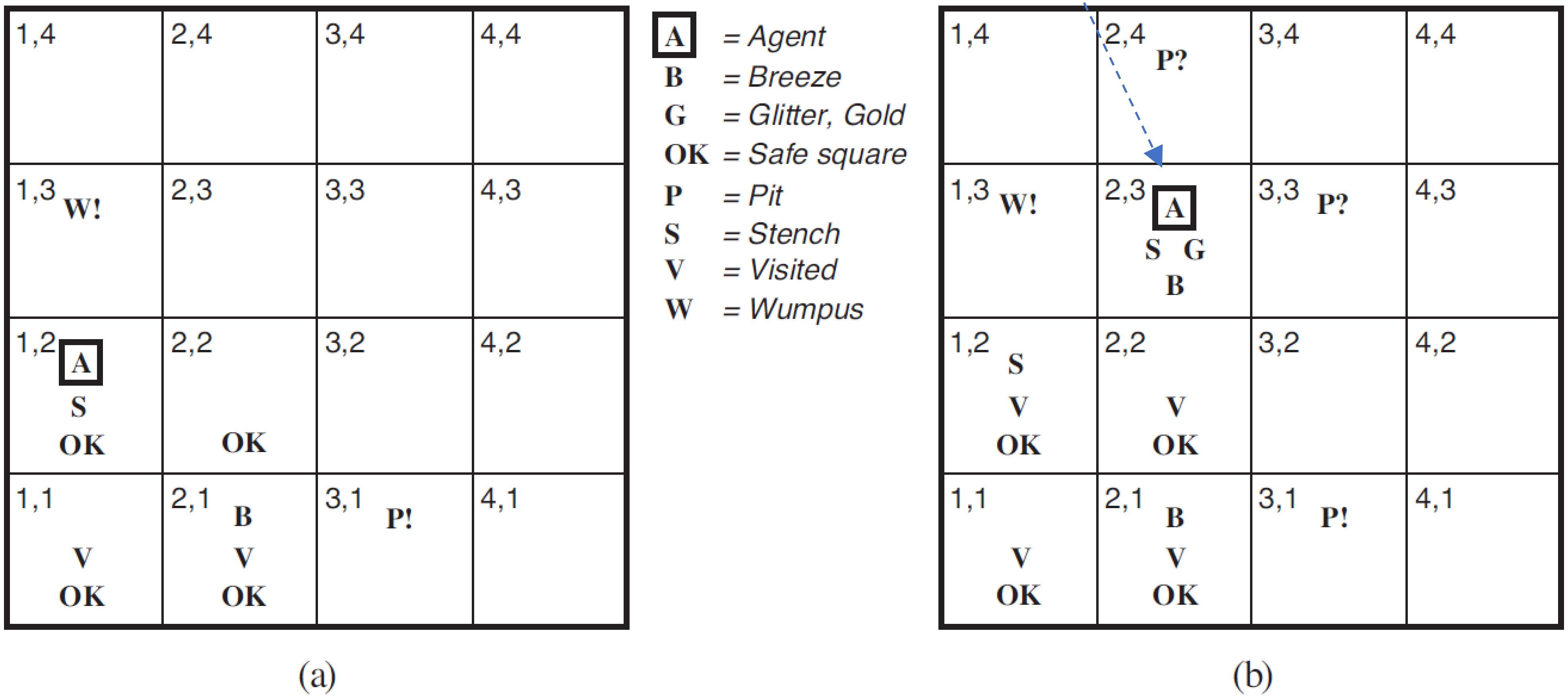
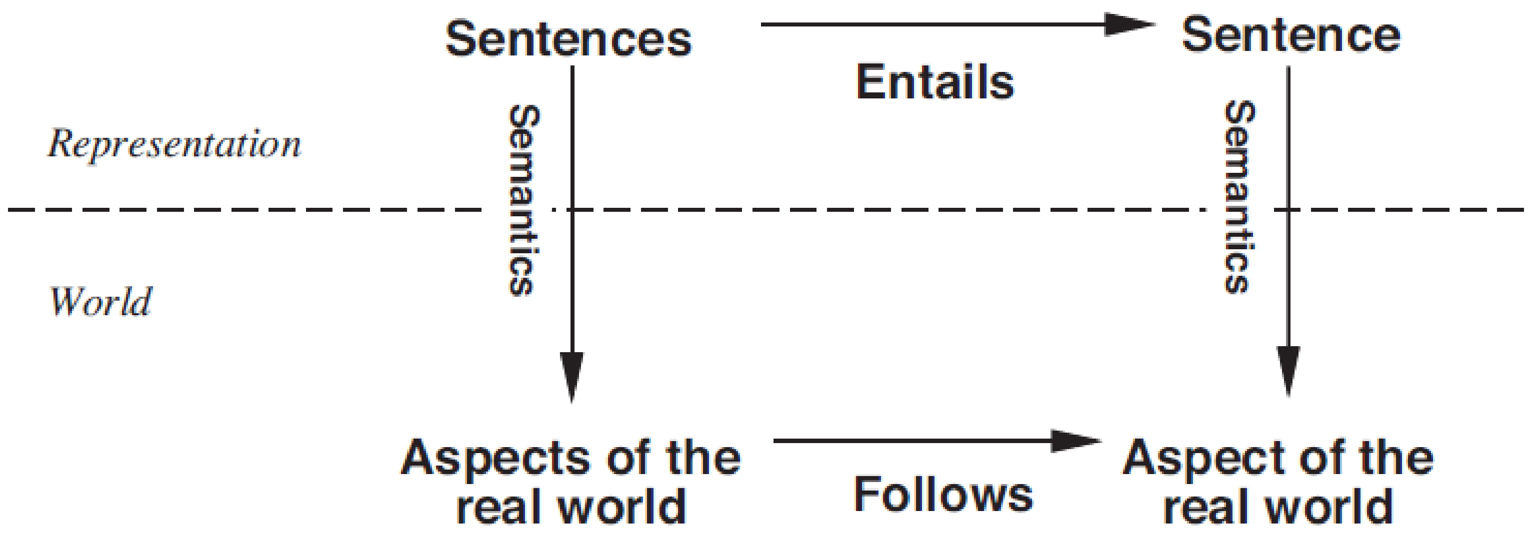


Figure 7.4 Two later stages in the progress of the agent. (a) After the third move, with percept $[Stench, None, None, None, None]$. (b) After the fifth move, with percept $[Stench, Breeze, Glitter, None, None]$.

Reasoning

- Reasoning is the process of constructing new physical configurations (sentences) from old ones.
- Proper reasoning should ensure that the new configuration represents facts that actually **follow** from the fact that the old configurations represent. In other words, new sentence **entails** the old sentences.



Propositional Logic: A Very Simple Logic

Syntax: The **syntax** of propositional logic defines the allowable sentences.

- The **atomic sentences** consist of a single proposition symbol.
  - **Symbol** represents proposition (fact) that can be true or false; e.g., $\mathbf{W_{1,3}}$ to stand for the proposition that the wumpus is in [1,3]. Symbol is atomic, i.e., W, 1, and 3 are not meaningful parts of the symbol.
  - There are **two proposition symbols** with fixed meanings: **True** is the always-true proposition and **False** is the always-false proposition

- **Complex sentences** are constructed from simpler sentences, using parentheses and logical connectives.

There are five logical connectives in common use:

- **¬ (not)** A sentence such as $\neg W_{1,3}$ is called the negation of $W_{1,3}$.
- **∧ (and)** A sentence whose main connective is ∧, such as $W_{1,3} \wedge P_{3,1}$, is called a conjunction.
- **∨ (or)** A sentence using ∨, such as $(W_{1,3} \wedge P_{3,1}) \vee W_{2,2}$, is a disjunction of the disjuncts $(W_{1,3} \wedge P_{3,1})$ and $W_{2,2}$.
- **⇒ (implies)** A sentence such as $(W_{1,3} \wedge P_{3,1}) \Rightarrow \neg W_{2,2}$ is called an implication (or conditional). Its premise or antecedent is $(W_{1,3} \wedge P_{3,1})$, and its conclusion or consequent is $\neg W_{2,2}$.
- **⇔ (if and only if)** The sentence $W_{1,3} \Leftrightarrow \neg W_{2,2}$ is a biconditional. Some other books write this as ≡.

$$
\begin{array}{rcl}
\mathit{Sentence} & \rightarrow & \mathit{AtomicSentence} \mid \mathit{ComplexSentence} \\
\mathit{AtomicSentence} & \rightarrow & \mathit{True} \mid \mathit{False} \mid P \mid Q \mid R \mid \ldots \\
\mathit{ComplexSentence} & \rightarrow & (\ \mathit{Sentence}\ ) \mid [\ \mathit{Sentence}\ ] \\
& \mid & \neg\ \mathit{Sentence} \\
& \mid & \mathit{Sentence} \land \mathit{Sentence} \\
& \mid & \mathit{Sentence} \lor \mathit{Sentence} \\
& \mid & \mathit{Sentence} \;\Rightarrow\; \mathit{Sentence} \\
& \mid & \mathit{Sentence} \;\Leftrightarrow\; \mathit{Sentence} \\
\\
\text{OPERATOR PRECEDENCE} & : & \neg, \land, \lor, \Rightarrow, \Leftrightarrow
\end{array}
$$

Figure 7.7 A BNF (Backus–Naur Form) grammar of sentences in propositional logic, along with operator precedences, from highest to lowest.

Note that we can use parenthesis in propositional logic along with other operators.

Semantics

- The semantics defines the rules for determining the truth of a sentence with respect to a particular model.
- The truth of a sentence can be evaluated recursively like a **truth table**.

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
false	*false*	*true*	*false*	*false*	*true*	*true*
false	*true*	*true*	*false*	*true*	*true*	*false*
true	*false*	*false*	*false*	*true*	*false*	*false*
true	*true*	*false*	*true*	*true*	*true*	*true*

- For example, the sentence $\neg P_{1,2} \wedge (P_{2,2} \vee P_{3,1})$

  when $P_{1,2}$ = *false*, $P_{2,2}$ = *false*, $P_{3,1}$ = *true*, gives

  true ∧ (*false* ∨ *true*)= *true* ∧ *true* =*true*.

Seven inference rules for propositional logic

- Modus Ponens

$$\frac{\propto \rightarrow \beta, \alpha}{\beta}$$

- And-Elimination

$$\frac{\propto_1 \wedge \propto_2 \wedge \ldots \wedge \propto_n}{\propto_i} \quad ; 1 \leq i \leq n$$

- And-Introduction

$$\frac{\propto_1, \propto_2, \ldots, \propto_n}{\propto_1 \wedge \propto_2 \wedge \ldots \wedge \propto_n}$$

- Or-Introduction

$$\frac{\propto_i}{\propto_1 \vee \propto_2 \vee \ldots \vee \propto_n} \quad ; 1 \leq i \leq n$$

- Double-negation Elimination

$$\frac{\neg\neg\propto}{\propto}$$

- Unit Resolution

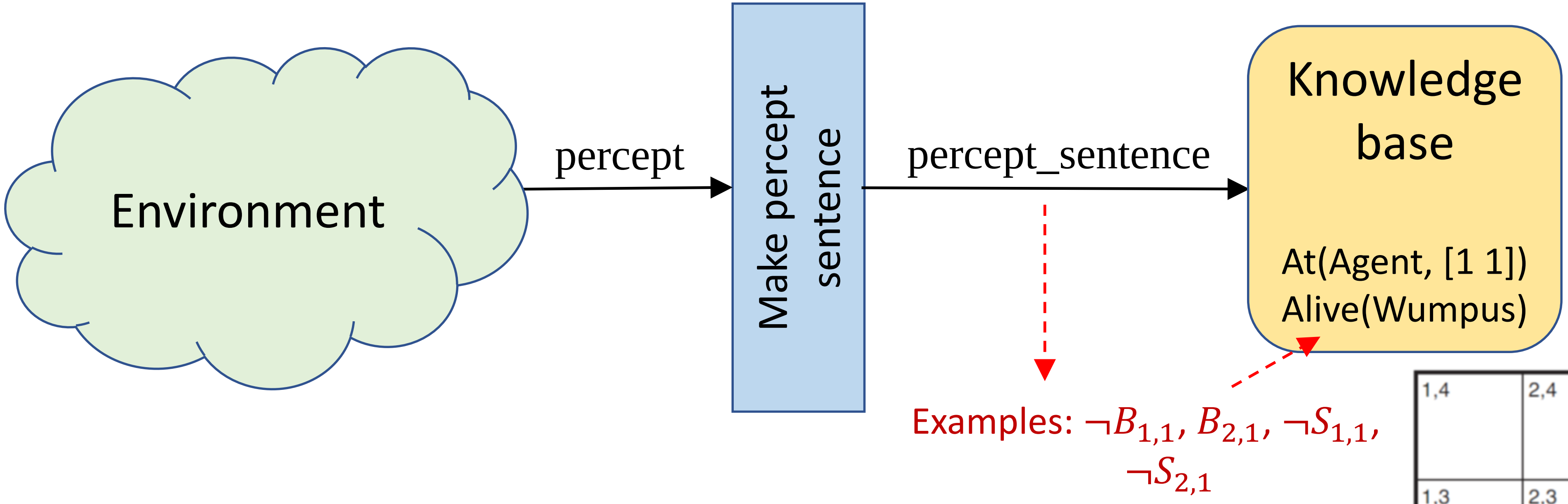
$$\frac{\propto \vee \beta \ , \neg\beta}{\propto}$$

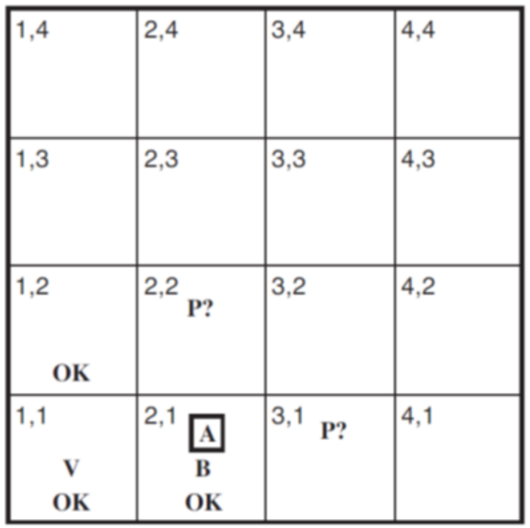
- Resolution

$$\frac{\propto \vee \beta \ , \neg\beta \vee \gamma}{\propto \vee \gamma}$$

- The agent must start out with some knowledge of environment. For example, agent's position is [1,1], wumpus is alive.

- Knowledge base is told new percept sentences or inferred sentences at each time step.





Example

- Let us see how these inference rules and equivalences can be used in the wumpus world.
- We start with the knowledge base containing R1 through R5 and show how to prove **$\neg P_{1,2}$**, that is, there is **no pit in [1,2]**.

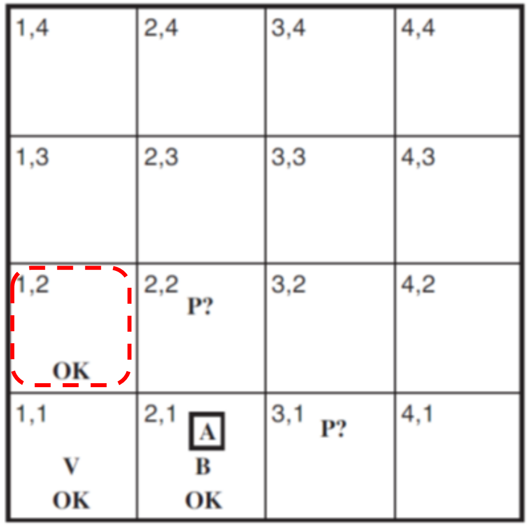
$$R_1: \quad \neg P_{1,1} .$$

$$R_2: \quad B_{1,1} \quad \Leftrightarrow \quad (P_{1,2} \lor P_{2,1}) .$$

$$R_3: \quad B_{2,1} \quad \Leftrightarrow \quad (P_{1,1} \lor P_{2,2} \lor P_{3,1}) .$$

$$R_4: \quad \neg B_{1,1} .$$

$$R_5: \quad B_{2,1} .$$



Proof by inference $\neg P_{1,2}$: There is no pit in [1,2].

- First, we apply biconditional elimination to R2 to obtain:

$$R_6 : \quad (B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})) \wedge ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1}) .$$

- Then we apply And-Elimination to R6 to obtain:

$$R_7 : \quad ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1}) .$$

- Logical equivalence for contrapositives gives:

$R_8 : \quad (\neg B_{1,1} \Rightarrow \neg(P_{1,2} \vee P_{2,1}))$. This is called "**inference**".

- Now we can apply Modus Ponens with R8 and the percept R4 (i.e., $\neg B_{1,1}$), to obtain

$$R_9 : \quad \neg(P_{1,2} \vee P_{2,1}) .$$

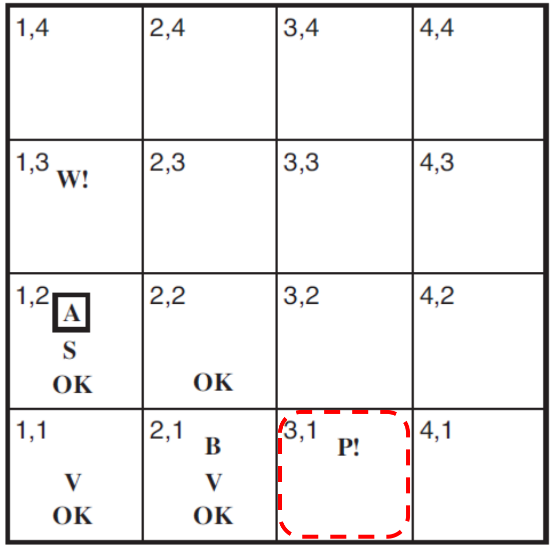
- Finally, we apply De Morgan's rule, giving the conclusion

$$R_{10} : \quad \neg P_{1,2} \wedge \neg P_{2,1} .$$

Proof by resolution : $P_{3,1}$: There is a pit at [3,1]

Let us consider the steps which the agent returns from [2,1] to [1,1] and then goes to [1,2], where it perceives a stench, but no breeze. We add the following facts to the KB:

$$R_{11}: \quad \neg B_{1,2}\,.$$
$$R_{12}: \quad B_{1,2} \;\Leftrightarrow\; (P_{1,1} \vee P_{2,2} \vee P_{1,3})\,.$$



By the same process that led to R10 earlier, we can derive

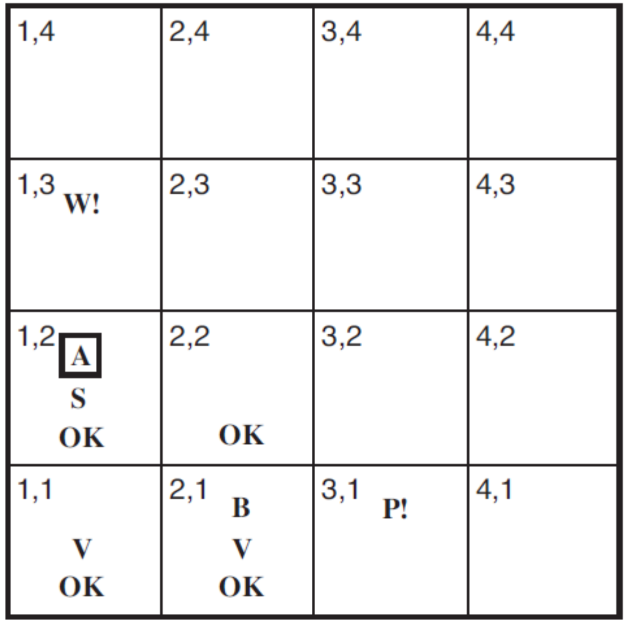
$$R_{13}: \quad \neg P_{2,2}\,.$$
$$R_{14}: \quad \neg P_{1,3}\,.$$

We can also apply biconditional elimination to R3, followed by Modus Ponens with R5, to obtain R15

$$R_{15}: \quad P_{1,1} \vee P_{2,2} \vee P_{3,1}\,.$$

Now using the resolution rule: the literal $\neg P_{2,2}$ in R13 resolves with the literal $P_{2,2}$ in R15 to give the resolvent.

$$R_{16}: \quad P_{1,1} \vee P_{3,1} .$$



Similarly, the literal $\neg P_{1,1}$ in R1 resolves with the literal $P_{1,1}$ in R16 to give

$$R_{17}: \quad P_{3,1} .$$

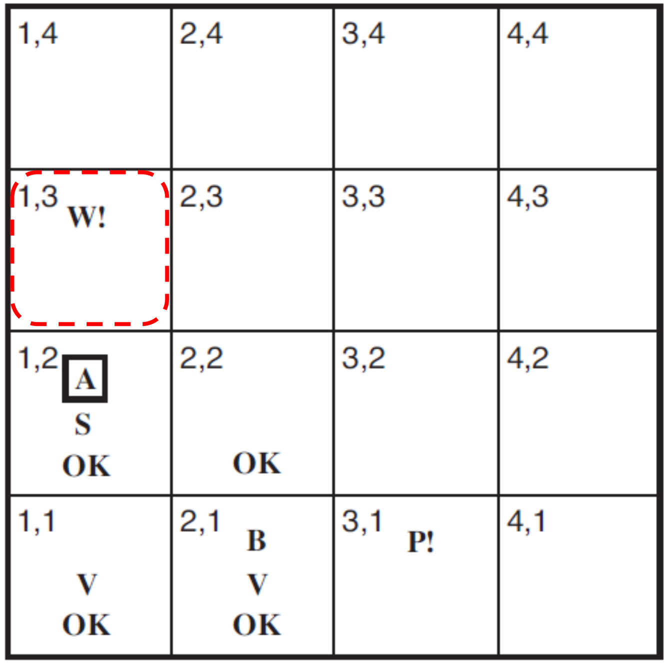
Proof by resolution : $W_{1,3}$: There is the **Wumpus at [1,3]**

$$R_1 : \neg S_{1,1} \Rightarrow \neg W_{1,1} \wedge \neg W_{1,2} \wedge \neg W_{2,1}$$

$$R_2 : \neg S_{2,1} \Rightarrow \neg W_{1,1} \wedge \neg W_{2,1} \wedge \neg W_{2,2} \wedge \neg W_{3,1}$$

$$R_3 : \neg S_{1,2} \Rightarrow \neg W_{1,1} \wedge \neg W_{1,2} \wedge \neg W_{2,2} \wedge \neg W_{1,3}$$

$$R_4 : S_{1,2} \Rightarrow W_{1,3} \vee W_{1,2} \vee W_{2,2} \vee W_{1,1}$$



Finding the Wumpus:

1. Agent starts out at [1,1] and gets the percept $\neg S_{1,1}$. By using R1 and Modus Ponens, we obtain

$$\neg W_{1,1} \wedge \neg W_{1,2} \wedge \neg W_{2,1}$$

2. Applying And-Elimination:

$$\neg W_{1,1} \quad \neg W_{1,2} \quad \neg W_{2,1}$$

3. Agent moves to [2,1] and gets percept $\neg S_{2,1}$.

   By using R2, Modus Ponens, And-Elimination, we obtain

$$\neg W_{2,2} \quad \neg W_{2,1} \quad \neg W_{3,1} \quad \neg W_{1,1}$$

4. Agent moves to [1,1] then [1,2] and get $S_{1,2}$.

   By using R4 and Modus Ponens, we obtain

$$W_{1,3} \lor W_{1,2} \lor W_{2,2} \lor W_{1,1}$$

5. Applying the unit resolution, where $\propto$ is $W_{1,3} \lor W_{1,2} \lor W_{2,2}$ and β is $W_{1,1}$.

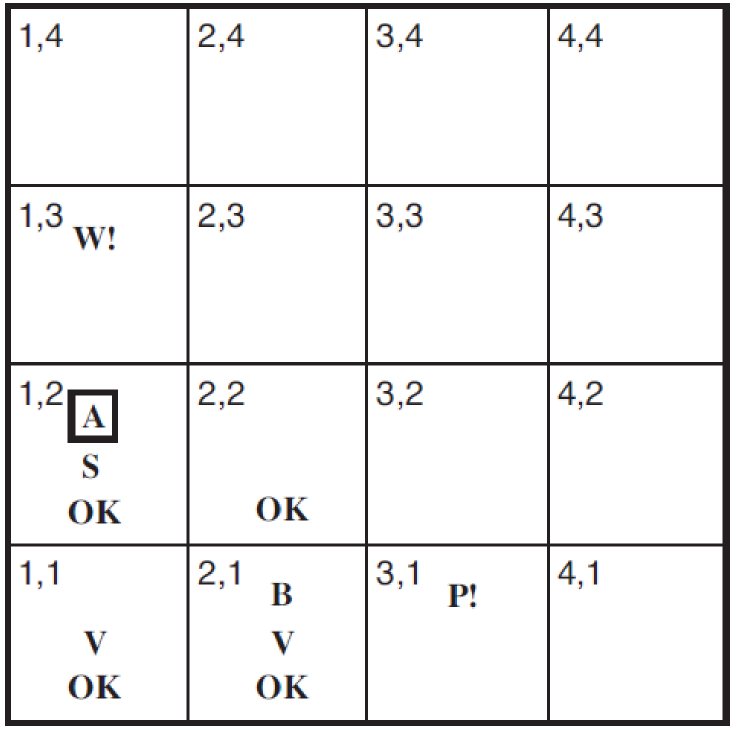
$$W_{1,3} \lor W_{1,2} \lor W_{2,2}$$

6. Applying the unit resolution again , where $\propto$ is $W_{1,3} \lor W_{1,2}$ and β is $W_{2,2}$.

$$W_{1,3} \lor W_{1,2}$$

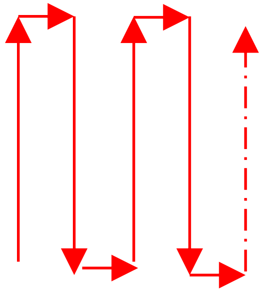
7. One more resolution where $\propto$ is $W_{1,3}$ and β is $W_{1,3}$.

$$W_{1,3}$$



Exercise : Find a set of positions where agent visits.

Walk pattern



Translating knowledge into action

We need additional rules that relate the current state of the world to the actions the agent should take.

For example,

$$A_{1,1} \wedge East_A \wedge \neg W_{2,1} \Rightarrow Forward$$

$$A_{1,2} \wedge North_A \wedge W_{1,3} \Rightarrow TurnRight$$

$$A_{1,2} \wedge North_A \wedge W_{1,3} \wedge Alive(W) \Rightarrow Shoot$$

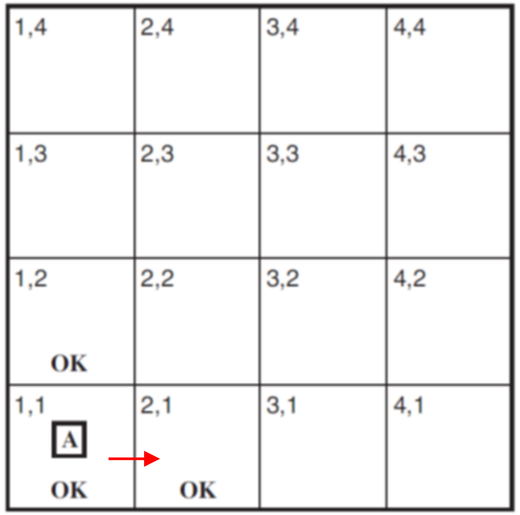
Problems of the propositional agent

1. There are too many propositions to handle.

For example, The simple rule "Don't go forward if the Wumpus is in front of the agent" can be stated in propositional logic by 64 rules.

2. Since the world can change configuration at each time step, we must specify time in the inference rules.

(Suppose that, when agent wants to turn, it always turns left.)



$$A^0_{1,1} \wedge East^0_A \wedge \neg W_{2,1} \Rightarrow Forward^0$$

$$A^6_{1,1} \wedge East^6_A \wedge \neg W_{2,1} \Rightarrow TurnLeft^6$$