Artificial Intelligence

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Lecture 6
First-Order Logic

- First-order logic
- FOL Syntax and Semantics
- Quantifiers
- Connections between ∀ and ∃
- First-order logic for reflex agent

Recall: Problems of the propositional agent

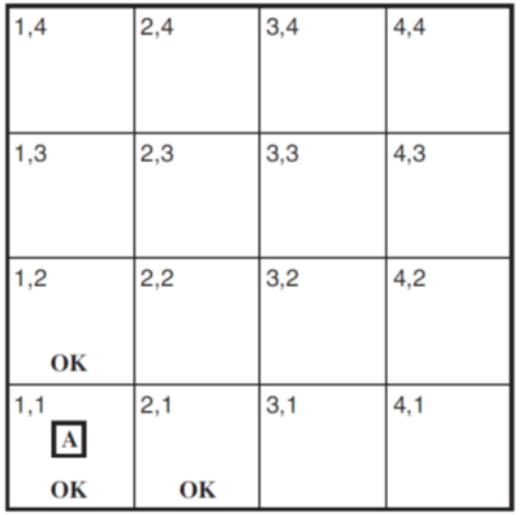
1. There are too many propositions to handle.

For example, The simple rule "Don't go forward if the Wumpus is in front of the agent" can be stated in propositional logic by 64 rules, i.e., 16 squares x 4 directions = 64.

2. Since the world can change configuration at each time step, we must specify time in the inference rules.

$$A^{0}_{1,1} \wedge East^{0}_{A} \wedge \neg W_{2,1} \Rightarrow Forward^{0}$$

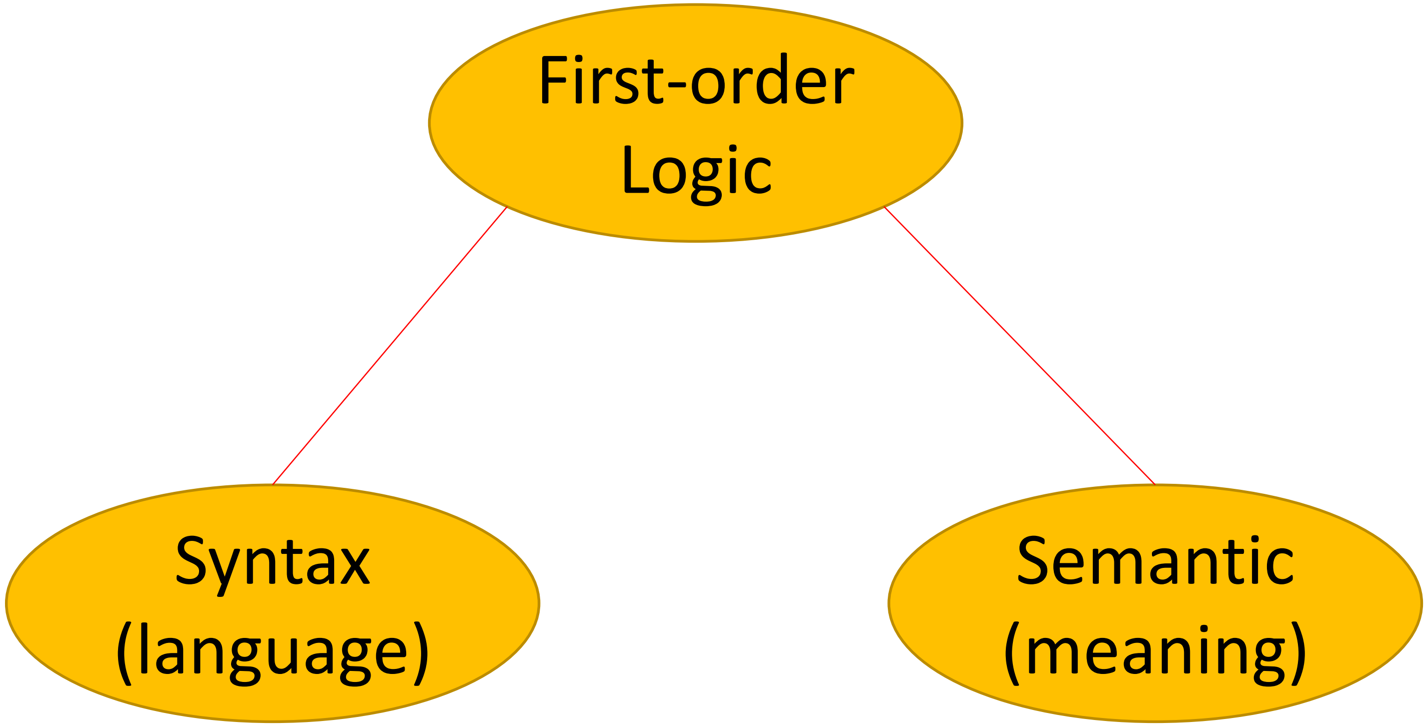
$$A^{6}_{1,1} \wedge East^{6}_{A} \wedge \neg W_{2,1} \Rightarrow TurnLeft^{6}$$



First-Order Logic

- Propositional logic is too puny a language to represent knowledge of complex environments in a concise way.
- We examine first-order logic which is sufficiently expressive to represent a knowledge.
- First-order logic can express the properties of entire collections of objects rather than having to enumerate the objects by name.

- Similar to other logics, First-order logic composes of syntax and semantic.



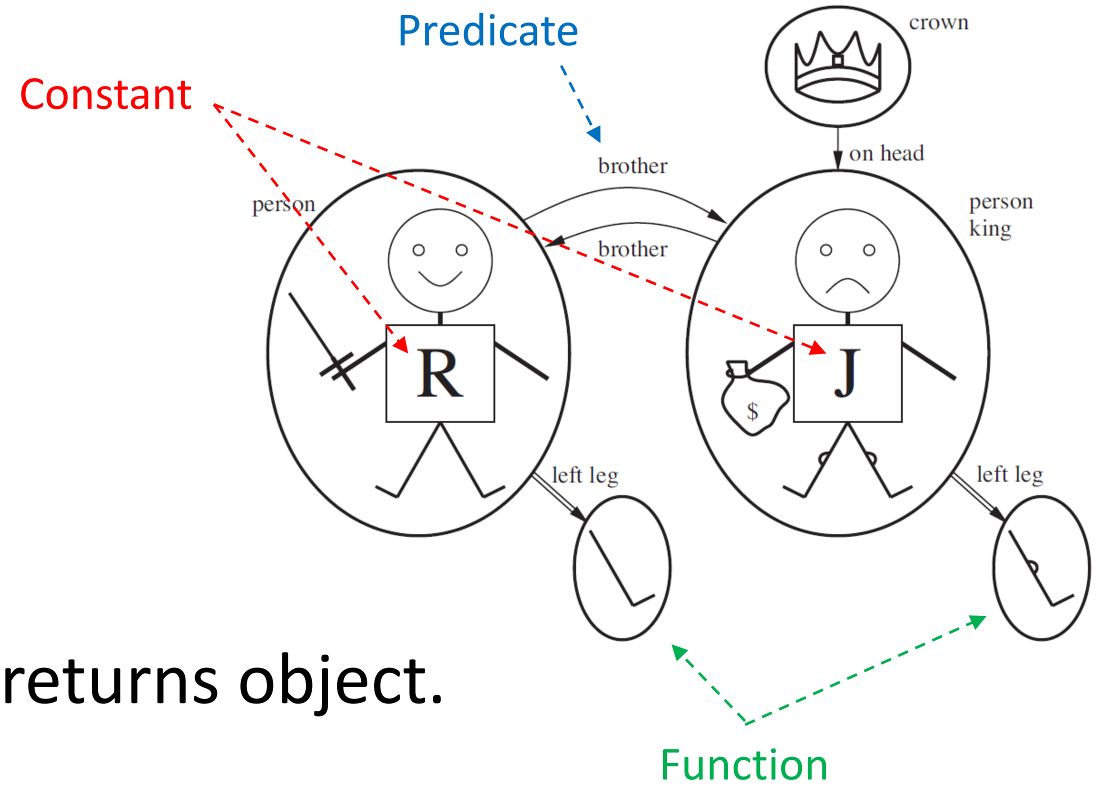
Syntax and Semantics

- **Symbol** consists of constant symbol, predicate symbol, and function symbol.
  - Constant symbol: object

    Ex: Richard , John, etc.
  - Predicate symbol: relation

    Ex: Brother, Friend, etc.
  - Function symbol: function that returns object.

    Ex: LeftLeg, etc.



Syntax and Semantics

- **Term** is a logical expression that refers to an object; e.g., KingJohn, LeftLegOf(John). Constant symbols are also terms.
- **Atomic sentence** represents a relationship between objects :- formed from a predicate symbol followed by a parenthesized list of terms.

  For example,

  Brother(Richard, John), Married(FatherOf(Richard), MotherOf(John))
- **Complex sentence** :- logical connectives of atomic sentences.

  Brother(Richard, John) $\wedge$ Brother(John, Richard),

  $\neg$King(Richard) $\Rightarrow$ King(John)

The syntax of first-order logic with equality, specified in Backus–Naur form.

$$
\begin{array}{rcl}
\mathit{Sentence} & \rightarrow & \mathit{AtomicSentence} \mid \mathit{ComplexSentence} \\
\mathit{AtomicSentence} & \rightarrow & \mathit{Predicate} \mid \mathit{Predicate}(\mathit{Term},\ldots) \mid \mathit{Term} = \mathit{Term} \\
\mathit{ComplexSentence} & \rightarrow & (\ \mathit{Sentence}\ ) \mid [\ \mathit{Sentence}\ ] \\
& \mid & \neg\ \mathit{Sentence} \\
& \mid & \mathit{Sentence} \wedge \mathit{Sentence} \\
& \mid & \mathit{Sentence} \vee \mathit{Sentence} \\
& \mid & \mathit{Sentence} \Rightarrow \mathit{Sentence} \\
& \mid & \mathit{Sentence} \Leftrightarrow \mathit{Sentence} \\
& \mid & \mathit{Quantifier}\ \mathit{Variable},\ldots\ \mathit{Sentence} \\
\\
\mathit{Term} & \rightarrow & \mathit{Function}(\mathit{Term},\ldots) \\
& \mid & \mathit{Constant} \\
& \mid & \mathit{Variable} \\
\\
\mathit{Quantifier} & \rightarrow & \forall \mid \exists \\
\mathit{Constant} & \rightarrow & A \mid X_1 \mid \mathit{John} \mid \cdots \\
\mathit{Variable} & \rightarrow & a \mid x \mid s \mid \cdots \\
\mathit{Predicate} & \rightarrow & \mathit{True} \mid \mathit{False} \mid \mathit{After} \mid \mathit{Loves} \mid \mathit{Raining} \mid \cdots \\
\mathit{Function} & \rightarrow & \mathit{Mother} \mid \mathit{LeftLeg} \mid \cdots \\
\textsc{Operator Precedence} & : & \neg, =, \wedge, \vee, \Rightarrow, \Leftrightarrow
\end{array}
$$

Predicate gives you true or false based on your input(s). While, a function gives you an output per your input(s).

- **Quantifier** : express the properties of entire collections of objects, rather than having to enumerate the objects by name.

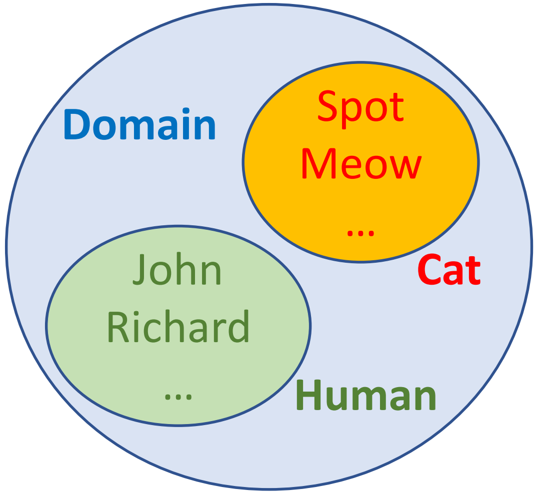
  1. **Universal Quantification ($\forall$)**

$$\forall_x\, \boldsymbol{Cat(x) \Rightarrow Mammal(x)}$$

For all object x, if x is a cat then x is mammal.

If there is no any cat in our domain, it is fine, i.e., the sentence is still true.

Problem: If we represent “All cats are mammal” by $\forall_x Cat(x) \land\ Mammal(x)$, we will not capture what we need since the representing sentence means the following:



$$[Cat(Spot) \land\ Mammal(Spot)] \land$$
$$[Cat(Meow) \land\ Mammal(Meow)] \land$$
$$[Cat(John) \land\ Mammal(John)] \land$$
$$[Cat(Richard) \land\ Mammal(Richard)] \land$$
$$\ldots\ldots \quad \ldots\ldots \quad \ldots\ldots$$

Too strong sentence (Always false)

2. Existential Quantification (∃)

$$\exists_x \boldsymbol{Sister(x, spot) \wedge Cat(x)}$$

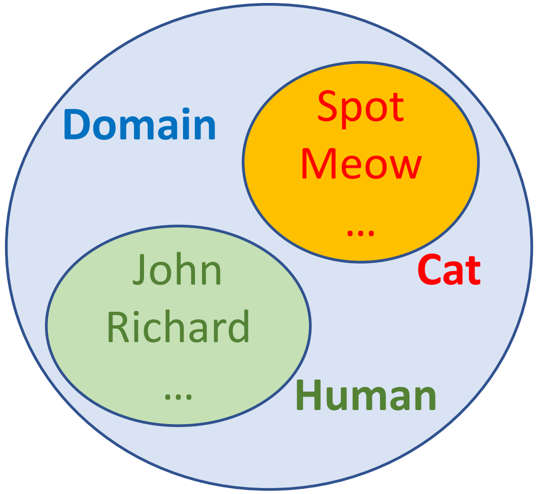
There is a sister of spot who is a cat.

If there is no sister of spot who is a cat, this sentence must be false.

Problem: If we represent "There is a sister of spot who is a cat" by

$$\exists_x Sister(x, spot) \Rightarrow Cat(x),$$

we will not capture what we need since the representing sentence means the following:



$$[Sister(Spot, Spot) \Rightarrow Cat(Spot)] \vee$$
$$[Sister(Meow, Spot) \Rightarrow Cat(Meow)] \vee$$
$$[Sister(John, Spot) \Rightarrow Cat(John)] \vee$$
$$[Sister(Richard, Spot) \Rightarrow Cat(Richard)] \vee$$
....

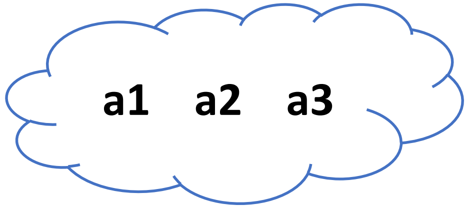
Too weak sentence
(Always true)

- Example: “Some cats are intelligent”.

$$\exists_x Cat(x) \Rightarrow Intel(x)$$

What’s wrong with the above FOL sentence ?

Domain



Name	Cat / Dog ?	Intelligent
a1	Cat	No
a2	Cat	No
a3	Dog	Yes

$$\underbrace{\underbrace{[Cat(a1) \Rightarrow Intel(a1)] \lor [Cat(a2) \Rightarrow Intel(a2)]}_{\text{False}} \lor \underbrace{[\overbrace{Cat(a3)}^{\text{False}} \Rightarrow Intel(a3)]}_{\text{True}}}_{\text{True}}$$

This FOL sentence always true even though there is no cat that is intelligent.

https://www.youtube.com/watch?v=2juspgYR7as

- Example: "Every student in this class has visited Africa or America".

Domain

People
(may be student or not student)

student(x) : x is student in this class

vaf(x) : x has visited Africa

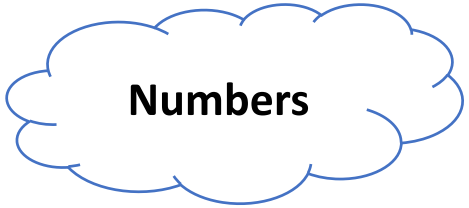
vam(x) : x has visited America

Find FOL sentence for the above sentence.

https://www.youtube.com/watch?v=2juspgYR7as

- Example: “Some prime number is even number”.

Domain



prime(x) : x is prime number

even(x) : x is even number

Find FOL sentence for the above sentence.

3. Nested Quantification

- "For all x and all y, if x is the parent of y then y is a child of x"

$$\forall_{x,y}\ \boldsymbol{Parent(x,y) \Longrightarrow Child(y,x)}$$

- "Everybody loves somebody"

$$\forall_x \exists_y\ \boldsymbol{Loves(x,y)}$$

- "There is someone who is loved by everybody"

$$\exists_y \forall_x\ \boldsymbol{Loves(x,y)}$$

- "Everybody is loved by somebody"
- "Somebody loves everybody"
- "Nobody loves everyone"

The order of quantification can change the meaning of the sentence.

Exercise 1

- Give FOL sentence for

"Gold and silver ornaments are precious"

Let G(x) : x is a gold ornament,

S(X) : x is a silver ornament,

P(X) : x is precious.

A) $\forall_{\boldsymbol{x}}\ (\boldsymbol{P}\ (\boldsymbol{x}) \Longrightarrow \boldsymbol{G}(\boldsymbol{x}) \wedge \boldsymbol{S}(\boldsymbol{x}))$

B) $\forall_{\boldsymbol{x}}\ (\boldsymbol{G}(\boldsymbol{x}) \wedge \boldsymbol{S}(\boldsymbol{x}) \Longrightarrow \boldsymbol{P}\ (\boldsymbol{x}))$

C) $\exists_{\boldsymbol{x}}\ (\boldsymbol{G}(\boldsymbol{x}) \wedge \boldsymbol{S}(\boldsymbol{x}) \Longrightarrow \boldsymbol{P}\ (\boldsymbol{x}))$

D) $\forall_{\boldsymbol{x}}\ (\boldsymbol{G}(\boldsymbol{x}) \vee \boldsymbol{S}(\boldsymbol{x}) \Longrightarrow \boldsymbol{P}\ (\boldsymbol{x}))$

Exercise 2

- Find FOL for the following sentence

“Every teacher is liked by some students”

A) $\forall_x \boldsymbol{teacher}\ (\boldsymbol{x}) \Rightarrow \exists_y(\boldsymbol{student}(\boldsymbol{y}) \Rightarrow \boldsymbol{Likes}(\boldsymbol{y},\boldsymbol{x}))$

B) $\forall_x \boldsymbol{teacher}\ (\boldsymbol{x}) \Rightarrow \exists_y(\boldsymbol{student}(\boldsymbol{y}) \wedge \boldsymbol{Likes}(\boldsymbol{y},\boldsymbol{x}))$

C) $\exists_y \forall_x \boldsymbol{teacher}\ (\boldsymbol{x}) \Rightarrow\ (\boldsymbol{student}(\boldsymbol{y}) \wedge \boldsymbol{Likes}(\boldsymbol{y},\boldsymbol{x}))$

D) $\forall_x\ (\ \boldsymbol{teacher}\ (\boldsymbol{x}) \wedge \exists_y[\boldsymbol{student}(\boldsymbol{y}) \Rightarrow \boldsymbol{Likes}(\boldsymbol{y},\boldsymbol{x})]\ )$

Exercise 3

- Find FOL for the following sentence

  "Some boys are taller than all girls."

A) $\exists_x\, \boldsymbol{boy}\,(\boldsymbol{x}) \Rightarrow \forall_{\boldsymbol{y}}(\boldsymbol{girl}(\boldsymbol{y}) \wedge \boldsymbol{taller}(\boldsymbol{x}, \boldsymbol{y}))$

B) $\exists_x\, \boldsymbol{boy}\,(\boldsymbol{x}) \wedge \forall_{\boldsymbol{y}}(\boldsymbol{girl}(\boldsymbol{y}) \wedge \boldsymbol{taller}(\boldsymbol{x}, \boldsymbol{y}))$

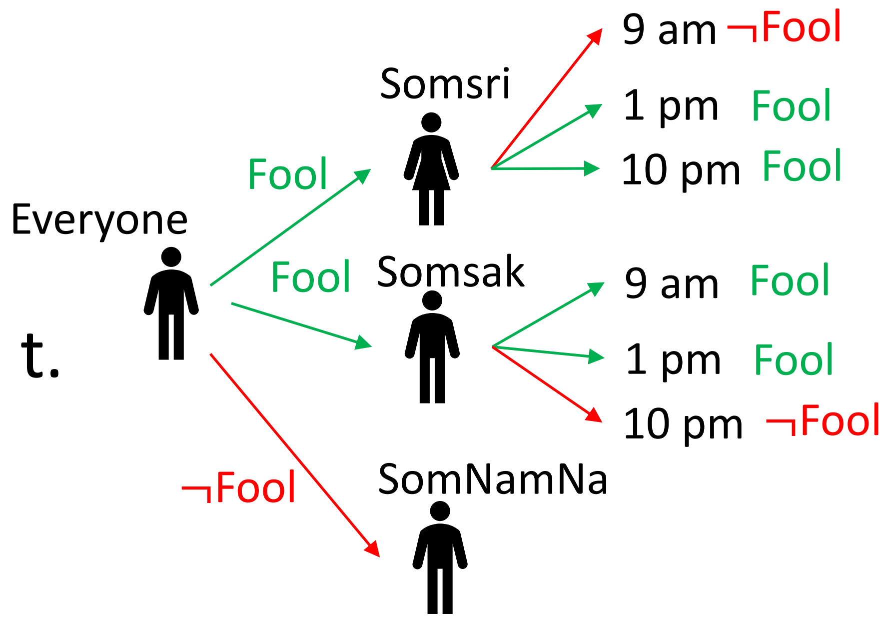
C) $\exists_x\, \boldsymbol{boy}\,(\boldsymbol{x}) \Rightarrow \forall_{\boldsymbol{y}}(\boldsymbol{girl}(\boldsymbol{y}) \Rightarrow \boldsymbol{taller}(\boldsymbol{x}, \boldsymbol{y}))$

D) $\exists_x\, \boldsymbol{boy}\,(\boldsymbol{x}) \wedge \forall_{\boldsymbol{y}}[\boldsymbol{girl}(\boldsymbol{y}) \Rightarrow \boldsymbol{taller}(\boldsymbol{x}, \boldsymbol{y})]$

Exercise 4

- Let F(x,y,t) : person x can fool person y at time t.

$$\forall_x \exists_y \exists_t \left(\neg \boldsymbol{F}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{t})\right)$$



A) Everyone can fool some person at some time.

B) No one can fool everyone all the time.

C) Everyone can not fool some person all the time.

D) No person can fool some person at some time.

Exercise 5

- Find FOL sentence for

  "Everyone who loves all animal is loved by someone."

A) $\forall_{x,y}\ [\boldsymbol{Animal}(\boldsymbol{y}) \Longrightarrow \boldsymbol{Loves}(\boldsymbol{x},\boldsymbol{y})] \Longrightarrow \exists_{\boldsymbol{z}}\boldsymbol{Loves}(\boldsymbol{z},\boldsymbol{x})$

B) $\forall_{x,y}\ [\boldsymbol{Animal}(\boldsymbol{y}) \wedge \boldsymbol{Loves}(\boldsymbol{x},\boldsymbol{y})] \Longrightarrow \exists_{\boldsymbol{z}}\boldsymbol{Loves}(\boldsymbol{z},\boldsymbol{x})$

C) $\forall_{x,y}[\boldsymbol{Animal}(\boldsymbol{y}) \Longrightarrow \boldsymbol{Loves}(\boldsymbol{x},\boldsymbol{y})] \wedge \exists_{\boldsymbol{z}}\boldsymbol{Loves}(\boldsymbol{z},\boldsymbol{x})$

Connections between ∀ and ∃

Two quantifiers are actually intimately connected with each other through negation.

$$\forall x \;\; \neg Likes(x, Parsnips) \quad \text{is equivalent to} \quad \neg\exists x \;\; Likes(x, Parsnips)\,.$$

$$\forall x \;\; Likes(x, IceCream) \quad \text{is equivalent to} \quad \neg\exists x \;\; \neg Likes(x, IceCream)\,.$$

The De Morgan rules for quantified and unquantified sentences are as follows:

$$\begin{array}{llllll}
\forall x \;\; \neg P & \equiv & \neg\exists x \;\; P & \qquad \neg(P \lor Q) & \equiv & \neg P \land \neg Q \\
\neg\forall x \;\; P & \equiv & \exists x \;\; \neg P & \qquad \neg(P \land Q) & \equiv & \neg P \lor \neg Q \\
\forall x \;\; P & \equiv & \neg\exists x \;\; \neg P & \qquad P \land Q & \equiv & \neg(\neg P \lor \neg Q) \\
\exists x \;\; P & \equiv & \neg\forall x \;\; \neg P & \qquad P \lor Q & \equiv & \neg(\neg P \land \neg Q)\,.
\end{array}$$

Equity

We can use the equality symbol to signify that two terms refer to the same object.

- **Father (John)=Henry**, says that the object referred to by Father (John) and the object referred to by Henry are the same.

We can also use equality for counting in first-order logic

- "Spot has at least two sisters"

$$\exists_{x,y}\ \boldsymbol{Sister(Spot, x) \land Sister(Spot, y) \land \neg(x = y)}$$

Kinship domain

Kinship domain represents the relationship between objects.

- Domain mother

$$\forall_{m,c}\ \boldsymbol{Mother(c) = m \iff Female(m) \land Parent(m, c)}$$

Assertions and queries in first-order logic

- Sentences are added to a knowledge base using TELL, exactly as in propositional logic.
- Such sentences are called assertions.

- For example, we can assert that John is a king, Richard is a person, and all kings are persons:

$$\textsc{Tell}(KB,\ King(John))\ .$$
$$\textsc{Tell}(KB,\ Person(Richard))\ .$$
$$\textsc{Tell}(KB,\ \forall x\ \ King(x)\ \Rightarrow\ Person(x))\ .$$

- We can ask questions of the knowledge base using ASK. For example,

$\textsc{Ask}(KB,\ King(John))$ returns true.

$\textsc{Ask}(KB,\ Person(John))$ also returns true.

First-order logic for reflex agent

We will take a look how first-order logic represents the rules in a Wumpus world.

Rules : $\boldsymbol{Percept \Rightarrow Action}$

Example:

"If the agent senses a glitter, it should do a grab in order to pick up the gold" can be represented by the sentence in first-order logic as follow:

$$\forall_{s,b,u,c,t}\ \boldsymbol{Percept([s, b, Glitter, u, c], t) \Rightarrow Action(Grab, t)}$$

We can reduce some facts into a shorter representation and then use it in the following rules.

$$\forall_{s,b,u,c,t}\ \boldsymbol{Percept([s, b, Glitter, u, c], t) \Rightarrow At_{Gold}\ (t)}$$

$$\forall_{t}\ \boldsymbol{At_{Gold}\ (t) \Rightarrow Action(Grab, t)}$$

Deducing Hidden Properties of the World

1. Causal Rules

   "Squares adjacent to Wumpus are smelly"

   "Squares adjacent to pit are breezy"

   These two rules can be represented by the following causal rules:

$$\forall_{l1,l2,t}\ \boldsymbol{At(Wumpus, l, t) \wedge Adjacent(l1, l2) \Rightarrow Smelly(l2)}$$

$$\forall_{l1,l2,t}\ \boldsymbol{At(Pit, l, t) \wedge Adjacent(l1, l2) \Rightarrow Breezy(l2)}$$

2. Diagnostic Rules

$$\forall_{l,t}\ \boldsymbol{At(Agent, l, t) \wedge Breezy(l) \Rightarrow Action(TurnLeft, t)}$$