8 Tests of Hypotheses Based on a Single Sample

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8.1 Hypotheses and Test Procedures

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Introduction

- Parameter can be estimated from sample data either by
  - single number (Point Estimate) or
  - entire interval of plausible values (Confidence Interval)
- However, objective of investigation is not to estimate parameter but to decide which of two contradictory claims about parameter is correct
- Methods for accomplishing this comprise the part of statistical inference called

การทดสอบสมมติฐาน (Hypothesis Testing)

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Hypotheses and Test Procedures

- **Statistical Hypothesis,** or just *hypothesis*, is a claim or assertion either about
  - value of single parameter (population characteristic or characteristic of probability distribution),
  - values of several parameters, or about the form of entire probability distribution.

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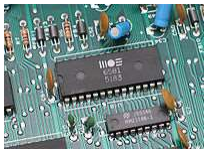
Hypotheses and Test Procedures

- One example of a hypothesis is claim $\mu = 0.75$, where μ is the true average inside diameter of a certain type of PVC pipe.

- Another example is statement $p < 0.10$, where p is proportion of defective circuit boards among all circuit boards produced by a certain manufacturer.



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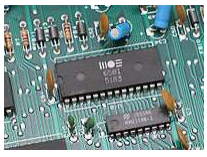
Hypotheses and Test Procedures

- In any hypothesis-testing problem, there are two contradictory hypotheses under consideration.
- One hypothesis might be
  - the claim $\mu = 0.75$ and
  - the other $\mu \neq 0.75$, or
- the two contradictory statements might be
  - $p \geq 0.10$ and
  - $p < 0.10$.



- The objective is to decide, based on sample information, which of the two hypotheses is **correct**.

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Hypotheses and Test Procedures

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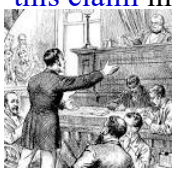
- There is familiar analogy to this in criminal trial.
- One claim is assertion that accused individual is innocent.
- In the U.S. judicial system, this is claim that is initially believed to be true.
- Only in the face of strong evidence to contrary should jury reject this claim in favor of alternative assertion that accused is guilty.



- In this sense, claim of innocence is favored or protected hypothesis, and burden of proof is placed on those who believe in the alternative claim.

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Hypotheses and Test Procedures

- Similarly, in testing statistical hypotheses, problem will be formulated so that one of claims is initially favored.

- This initially favored claim will not be rejected in favor of alternative claim unless sample evidence contradicts it and provides strong support for alternative assertion.

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Hypotheses and Test Procedures

Definition

- **Null Hypothesis,** denoted by H_0, is claim that is initially assumed to be true ("prior belief" claim).
- **Alternative Hypothesis,** denoted by H_a, is assertion that is contradictory to H_0.
- Null hypothesis will be rejected in favor of the alternative hypothesis only if sample evidence suggests that ***H_0* is false**.
- If **sample** does not strongly contradict H_0, we will continue to believe in plausibility of null hypothesis.
- **Two possible conclusions** from hypothesis-testing analysis are then ***reject H_0*** or ***fail to reject H_0***.

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Hypotheses and Test Procedures

- **Test of hypotheses** is method for using sample data to decide whether null hypothesis should be rejected.

- Thus we might test
  H_0: $\mu = 0.75$ against the alternative
  H_a: $\mu \neq 0.75$.
- Only if sample data strongly suggests that μ is something other than 0.75 should null hypothesis be rejected.
- In absence of such evidence, H_0 should not be rejected, since it is still quite plausible.

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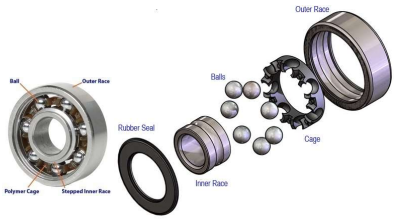
Hypotheses and Test Procedures

- Sometimes investigator does not want to accept particular assertion unless and until data can provide strong support for assertion.
- As an example, suppose company is considering putting new type of coating on bearings that it produces.



- True average wear life with current coating is known to be 1000 hours.
- With μ denoting true average life for new coating, company would not want to make a change unless evidence strongly suggested that μ exceeds 1000.

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Hypotheses and Test Procedures

- Appropriate problem formulation would involve testing H_0: $\mu = 1000$ against H_a: $\mu > 1000$.
- Conclusion that change is justified is identified with H_a, and it would take conclusive evidence to justify rejecting H_0 and switching to the new coating.
- Scientific research often involves trying to decide whether current theory should be replaced by more plausible and satisfactory explanation of phenomenon under investigation.

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Hypotheses and Test Procedures

- Conservative approach is to identify current theory with H_0 and researcher's alternative explanation with H_a.
- Rejection of current theory will then occur only when evidence is much more consistent with new theory.
- In many situations, H_a is referred to as "researcher's hypothesis," since it is claim that the researcher would really like to validate.

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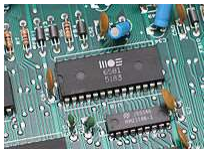
Hypotheses and Test Procedures

- The word ***null*** means "of no value, effect, or consequence," which suggests that H_0 should be identified with the hypothesis of no change (from current opinion), no difference, no improvement, and so on.



- Suppose, for example, that 10% of all circuit boards produced by a certain manufacturer during a recent period were defective.
- Engineer has suggested a change in the production process in the belief that it will result in reduced defective rate.

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Hypotheses and Test Procedures

- Let p denote true proportion of defective boards resulting from changed process.
- Then research hypothesis, on which burden of proof is placed, is assertion that $p < 0.10$.
- Thus alternative hypothesis is H_a: $p < 0.10$.

- In our treatment of hypothesis testing, H_0 will generally be stated as equality claim.
- If θ denotes **parameter of interest**, the null hypothesis will have the form H_0: $\theta = \theta_0$, where θ_0 is specified number called ***null value*** of parameter (value claimed for θ by null hypothesis).

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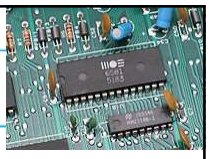
Hypotheses and Test Procedures



- As example, consider circuit board situation just discussed.
- Suggested alternative hypothesis was H_a: $p < 0.10$, claim that defective rate is reduced by process modification.

- Natural choice of H_0 in this situation is claim that $p \geq 0.10$, according to which new process is either no better *or* worse than one currently used.

- We will instead consider H_0: $p = 0.10$ versus H_a: $p < 0.10$.

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Hypotheses and Test Procedures

- Rationale for using this simplified null hypothesis is that any reasonable decision procedure for deciding between

$$H_0: p = 0.10 \text{ and } H_a: p < 0.10$$

will also be reasonable for deciding between the claim that $p \geq 0.10$ and H_a.

- The use of simplified H_0 is preferred because it has certain technical benefits, which will be apparent shortly.

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Hypotheses and Test Procedures

- Alternative to null hypothesis $H_0: \theta = \theta_0$ will look like one of following three assertions:

1. $H_a: \theta > \theta_0$ (in which case implicit null hypothesis is $\theta \leq \theta_0$),

2. $H_a: \theta < \theta_0$ (in which case implicit null hypothesis is $\theta \geq \theta_0$), or

3. $H_a: \theta \neq \theta_0$

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Hypotheses and Test Procedures

- For example, let σ denote standard deviation of distribution of inside diameters (inches) for certain type of metal sleeve.
- If decision was made to use sleeve unless sample evidence conclusively demonstrated that $\sigma > 0.001$, the appropriate hypotheses would be

$$H_0: \sigma = 0.001 \text{ versus } H_a: \sigma > 0.001.$$

- The number θ_0 that appears in both H_0 and H_a (separates the alternative from the null) is called the **null value.**

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Test Procedures

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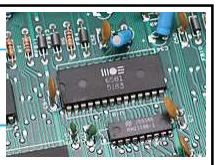
Test Procedures



- **Test procedure** is a rule, based on **sample data**, for deciding whether to reject H_0.
- Test of H_0: $p = 0.10$ versus H_a: $p < 0.10$ in circuit board problem might be based on examining random sample of $n = 200$ boards.
- Let X denote number of defective boards in the sample, a binomial random variable; x represents observed value of X.

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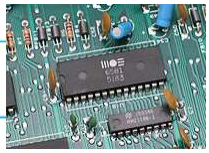
Test Procedures



- If H_0 is true, $E(X) = np = 200(0.10) = 20$, whereas we can expect fewer than 20 defective boards if H_a is true.
- Value x just a bit below 20 does not strongly contradict H_0, so it is reasonable to reject H_0 only if x is substantially less than 20.
- One such test procedure is to reject H_0 if $x \leq 15$ and not reject H_0 otherwise.

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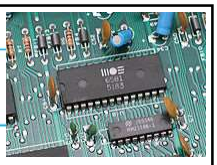
Test Procedures



This procedure has two constituents:

(1) ***Test Statistic***, or function of sample data used to make a decision, and

(2) ***Rejection Region*** consisting of those x values for which H_0 will be rejected in favor of H_a.

- For rule just suggested, the rejection region consists of $x = 0, 1, 2, \ldots,$ and 15.

- H_0 will not be rejected if $x = 16, 17, \ldots, 199,$ or 200.

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Test Procedures

A test procedure is specified by the following:

1. **Test Statistic,** a function of the sample data on which the decision (reject H_0 or do not reject H_0) is to be based

2. **Rejection Region,** the set of all test statistic values for which H_0 will be rejected

- Null hypothesis will then be rejected if and only if the observed or computed test statistic value falls in the rejection region.

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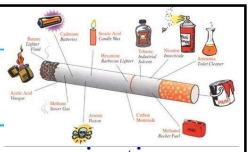
Example :



- Suppose cigarette manufacture claims that average nicotine content μ of brand B cigarettes in (at most) 1.5 mg.
- It would be unwise to reject manufacturer's claim without strong contradictory evidence, so an appropriate problem formulation is to test

  $H_0 : \mu = 1.5$ versus $H_a : \mu > 1.5$.
- Consider decision rule based on analyzing a random sample of 32 cigarettes.
- Let $\bar{X}$ denote the sample average nicotine content.
- If H_0 is true, $E(\bar{X}) = \mu = 1.5$ whereas

  If H_0 is false, we expect $\bar{X}$ to exceed 1.5
- Thus we might use $\bar{X}$ as a **test statistic** along with **rejection region** $\bar{x} \geq 1.6$

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Errors in Hypothesis Testing

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