Artificial Intelligence

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Lecture 3
Uninformed Search

- Uninformed Search Strategies
- Breadth-First Search
- Uniform-Cost Search
- Depth-First Search
- Depth-Limited Search
- Iterative Deepening Search
- Bidirectional Search

Search Strategies

There are two groups of search strategies:

1. **Uninformed Search (Blind Search)**

   The strategies have no additional information about states. All they can do is generate successors and distinguish a goal state from a non-goal state. Search strategies are distinguished by the order in which nodes are expanded.

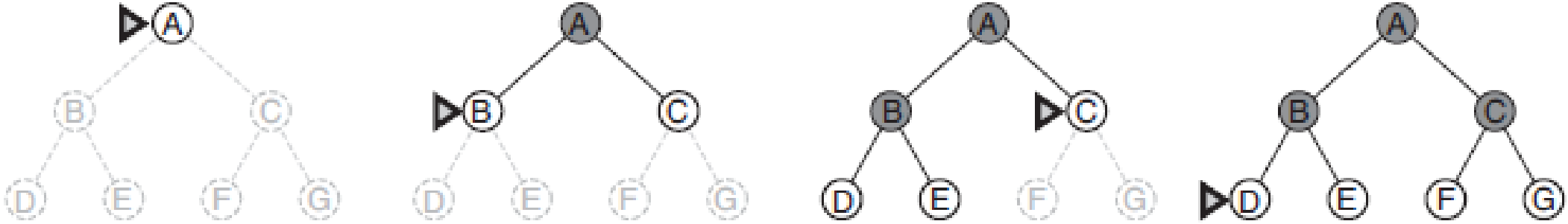
2. **Informed Search (Heuristic Search)**

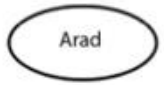
   The strategies have useful information for searching. They know whether one non-goal state is “more promising” than another.

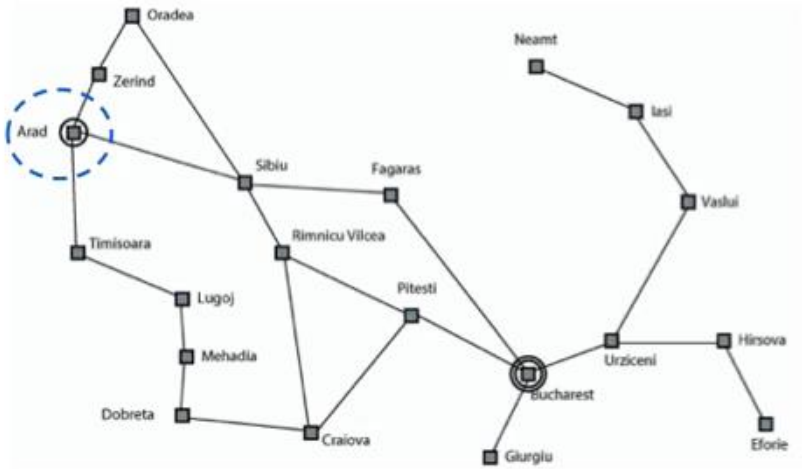
Uninformed Search Strategies

1. **Breadth-first search**

Breadth-first search is a simple strategy in which the root node is expanded first, then all the successors of the root node are expanded next, then their successors, and so on. In general, all the nodes are expanded at a given depth in the search tree before any nodes at the next level are expanded.

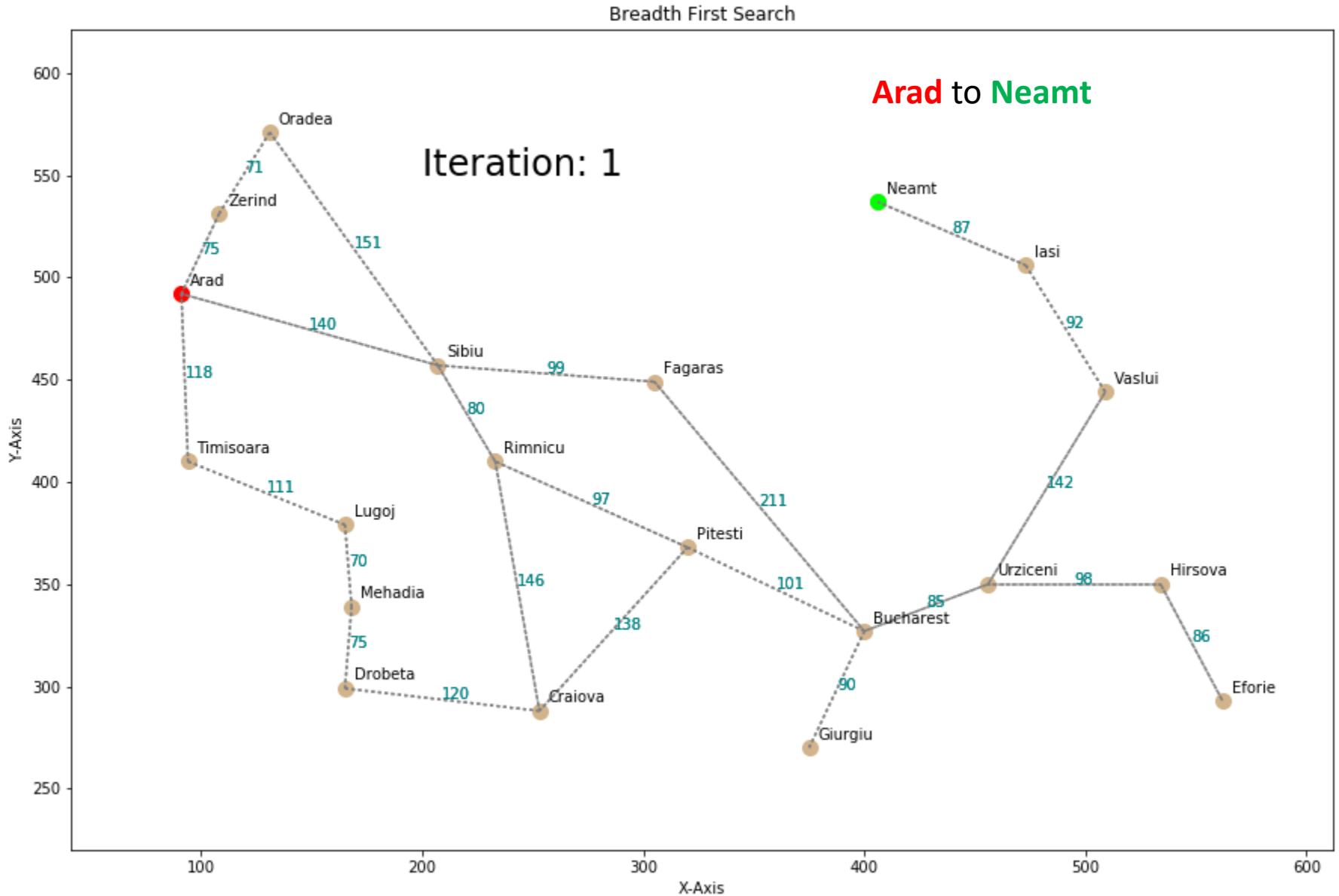






Breadth-First Search

BFS: https://youtu.be/aJnDZscuoj8



http://teleported.in/posts/ai-search-algorithms/

Uninformed Search Strategies

```
function BREADTH-FIRST-SEARCH(problem) returns a solution, or failure

    node ← a node with STATE = problem.INITIAL-STATE, PATH-COST = 0
    if problem.GOAL-TEST(node.STATE) then return SOLUTION(node)
    frontier ← a FIFO queue with node as the only element
    explored ← an empty set
    loop do
        if EMPTY?(frontier) then return failure
        node ← POP(frontier)   /* chooses the shallowest node in frontier */
        add node.STATE to explored
        for each action in problem.ACTIONS(node.STATE) do
            child ← CHILD-NODE(problem, node, action)
            if child.STATE is not in explored or frontier then
                if problem.GOAL-TEST(child.STATE) then return SOLUTION(child)
                frontier ← INSERT(child, frontier)
```

Uninformed Search Strategies

Analysis of Breadth-First Search

Completeness : Yes
Optimality : Yes (in terms of steps)
Time complexity : In the worst case, it is the time spends to generate the last node. Then the total number of nodes generated is $b + b^2 + b^3 + \cdots + b^d = O(b^d)$.
Space complexity : There will be $O(b^{d-1})$ nodes in the explored set and $O(b^d)$ nodes in the frontier. So the space complexity is $O(b^d)$, i.e., it is dominated by the size of the frontier.

b is branching factor, d is depth of the solution

Uninformed Search Strategies

Time and memory requirements for breadth-first search (scary, isn't it?)

Depth	Nodes	Time	Memory
2	110	.11 milliseconds	107 kilobytes
4	11,110	11 milliseconds	10.6 megabytes
6	10^6	1.1 seconds	1 gigabyte
8	10^8	2 minutes	103 gigabytes
10	10^{10}	3 hours	10 terabytes
12	10^{12}	13 days	1 petabyte
14	10^{14}	3.5 years	99 petabytes
16	10^{16}	350 years	10 exabytes

How does time and space functions grow?

Uninformed Search Strategies

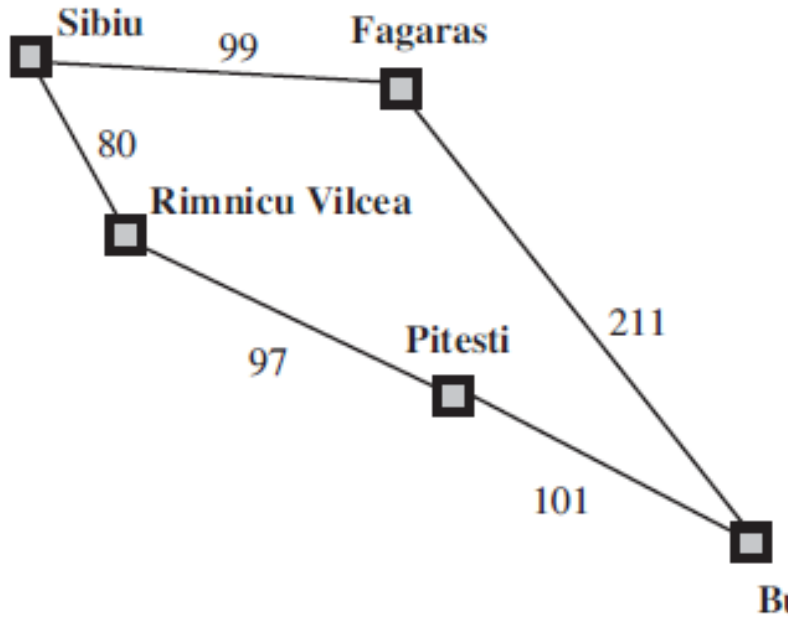
2. **Uniform-cost search**

When all step costs are equal, breadth-first search is optimal because it always expands the shallowest unexpanded node. Instead of expanding the shallowest node, uniform-cost search expands the node n with the lowest path cost g(n).

Example:

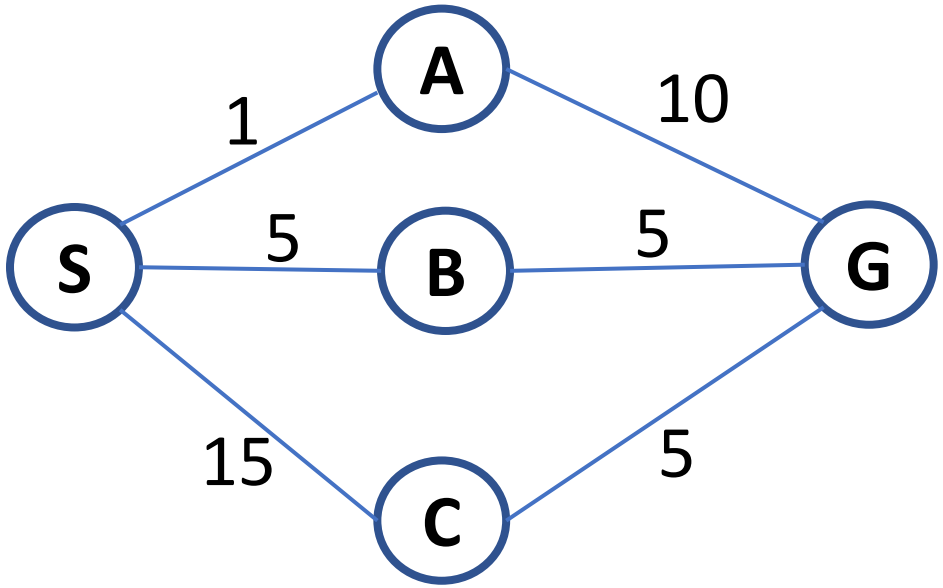
Path_Cost(S->F->B) = 310

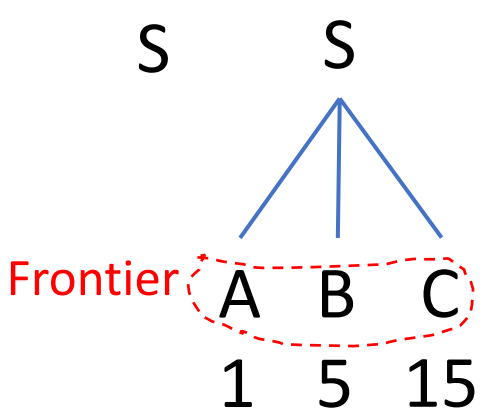
Path_Cost(S->R->P->B) = 278

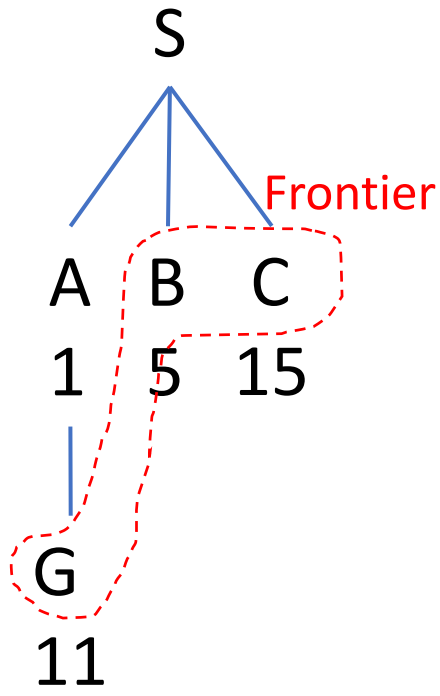


Uninformed Search Strategies

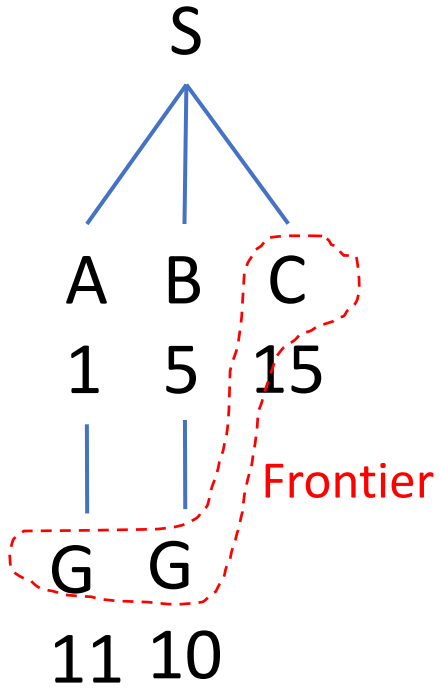
Example: Start from S to goal G.



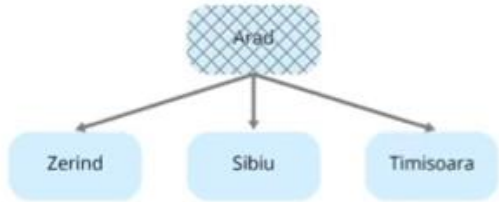


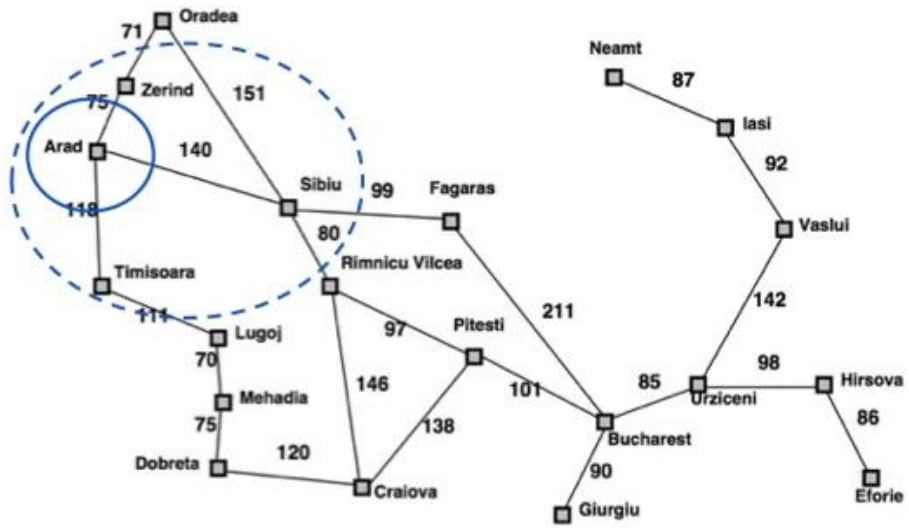


11 > 5



10 < 15





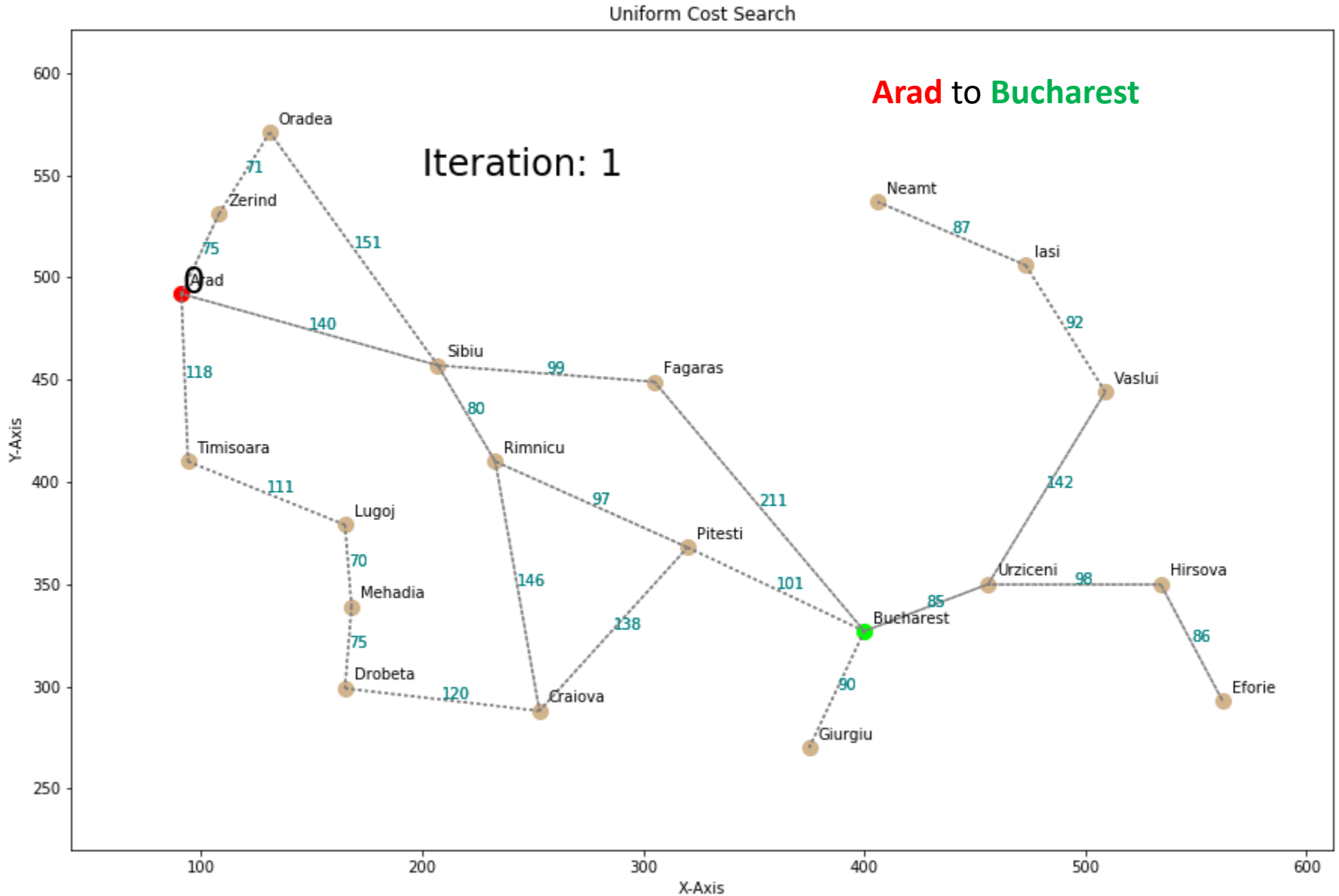
Uniform-Cost Search

Uniform-cost search: https://youtu.be/-FY7t2kqWX4

Uninformed Search Strategies

```
function UNIFORM-COST-SEARCH(problem) returns a solution, or failure

    node ← a node with STATE = problem.INITIAL-STATE, PATH-COST = 0
    frontier ← a priority queue ordered by PATH-COST, with node as the only element
    explored ← an empty set
    loop do
        if EMPTY?(frontier) then return failure
        node ← POP(frontier)   /* chooses the lowest-cost node in frontier */
        if problem.GOAL-TEST(node.STATE) then return SOLUTION(node)
        add node.STATE to explored
        for each action in problem.ACTIONS(node.STATE) do
            child ← CHILD-NODE(problem, node, action)
            if child.STATE is not in explored or frontier then
                frontier ← INSERT(child, frontier)
            else if child.STATE is in frontier with higher PATH-COST then
                replace that frontier node with child
```



http://teleported.in/posts/ai-search-algorithms/

Uninformed Search Strategies

Analysis of Uniform-cost Search

Completeness : Yes
Optimality : Yes
Time complexity : $O(b^{1+\lfloor C^*/\epsilon \rfloor})$
Space complexity : $O(b^{1+\lfloor C^*/\epsilon \rfloor})$

Where:

b is branching factor,

C* is the optimal path cost,

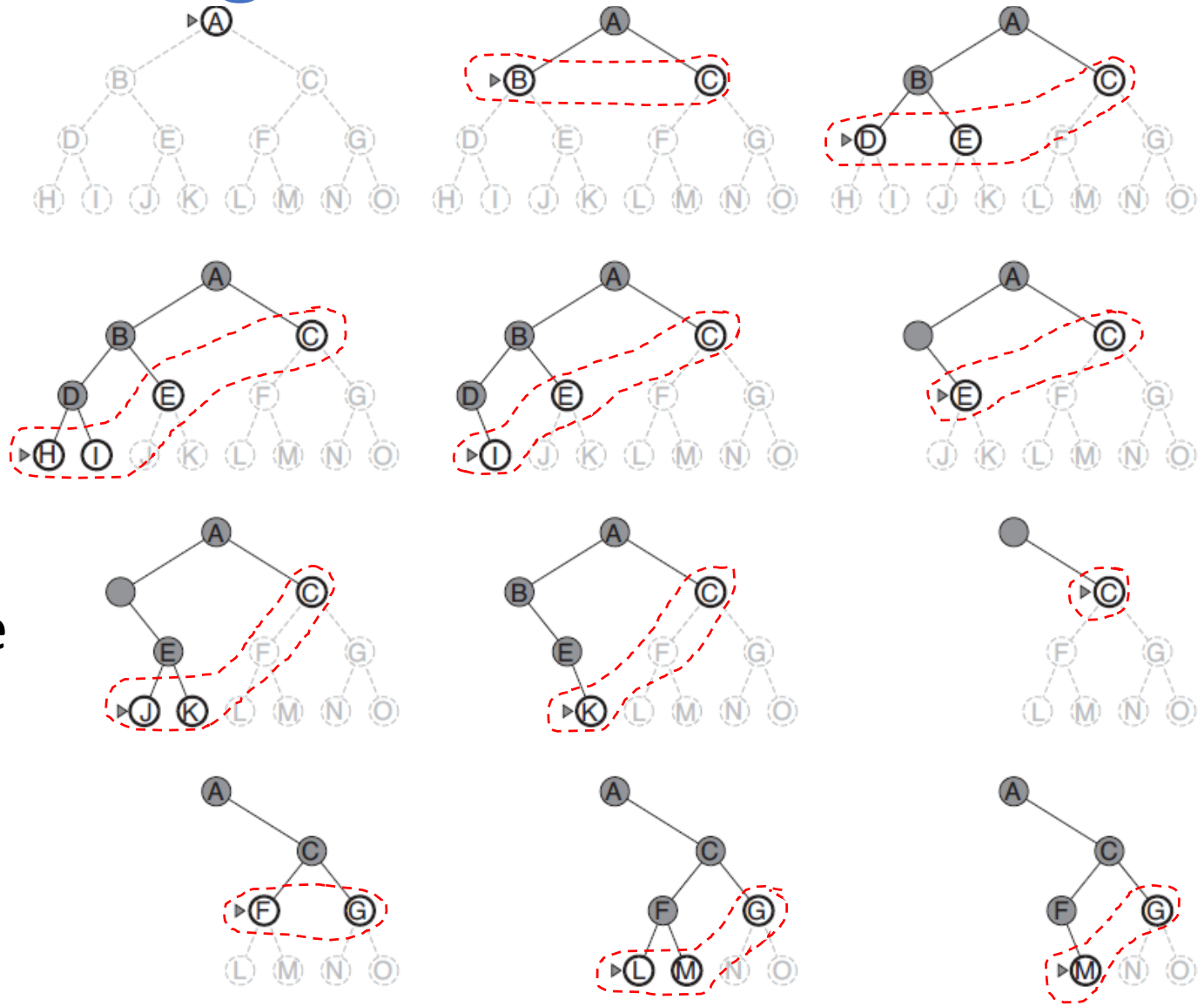
Every action costs at least ∈. If ∈ is small, the complexity will be high.

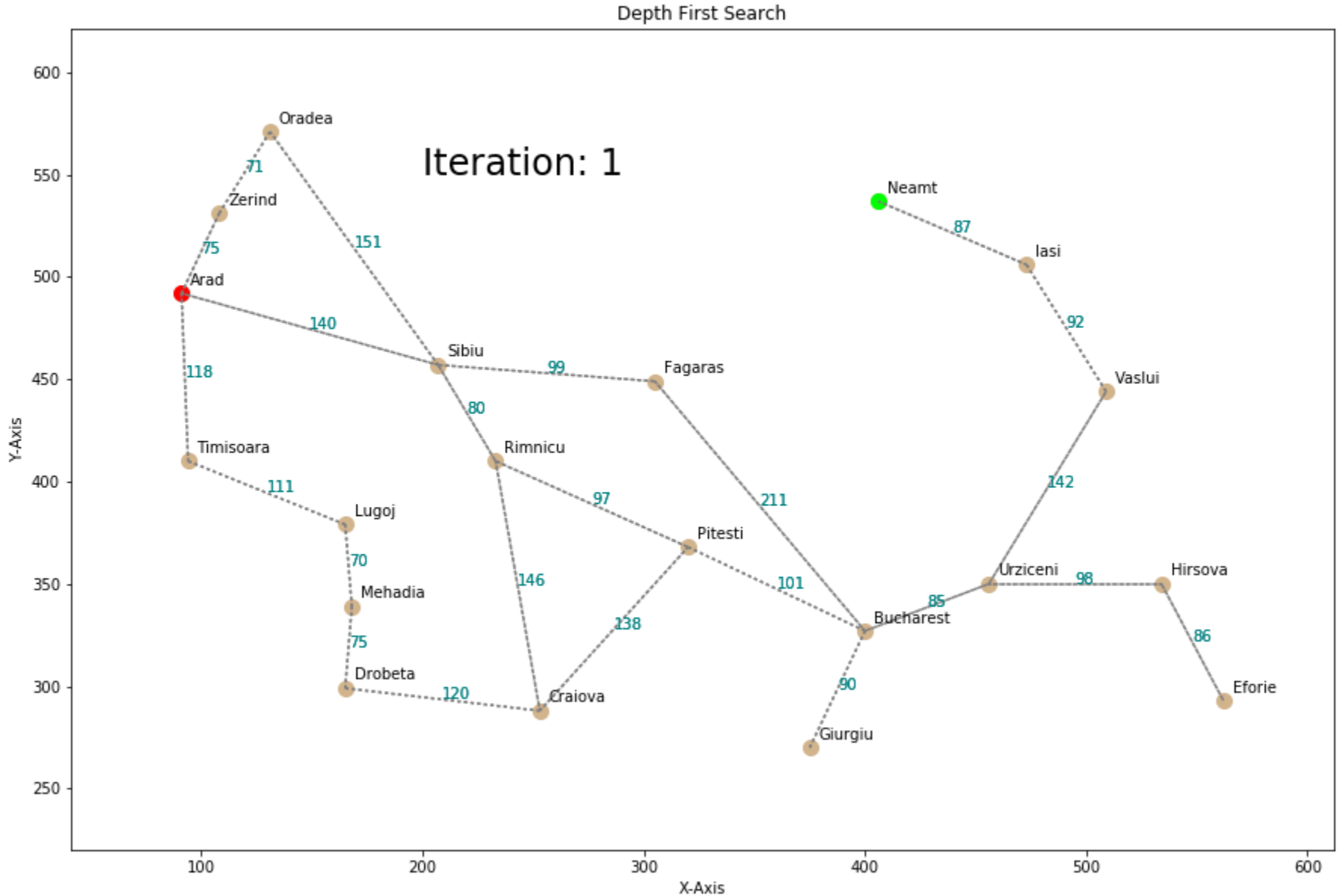
Uninformed Search Strategies

3. **Depth-First Search**

DFS always expands the deepest node in the current frontier of the search tree.

While Breadth-first-search uses a FIFO queue, depth-first search use a LIFO queue (**stack**).





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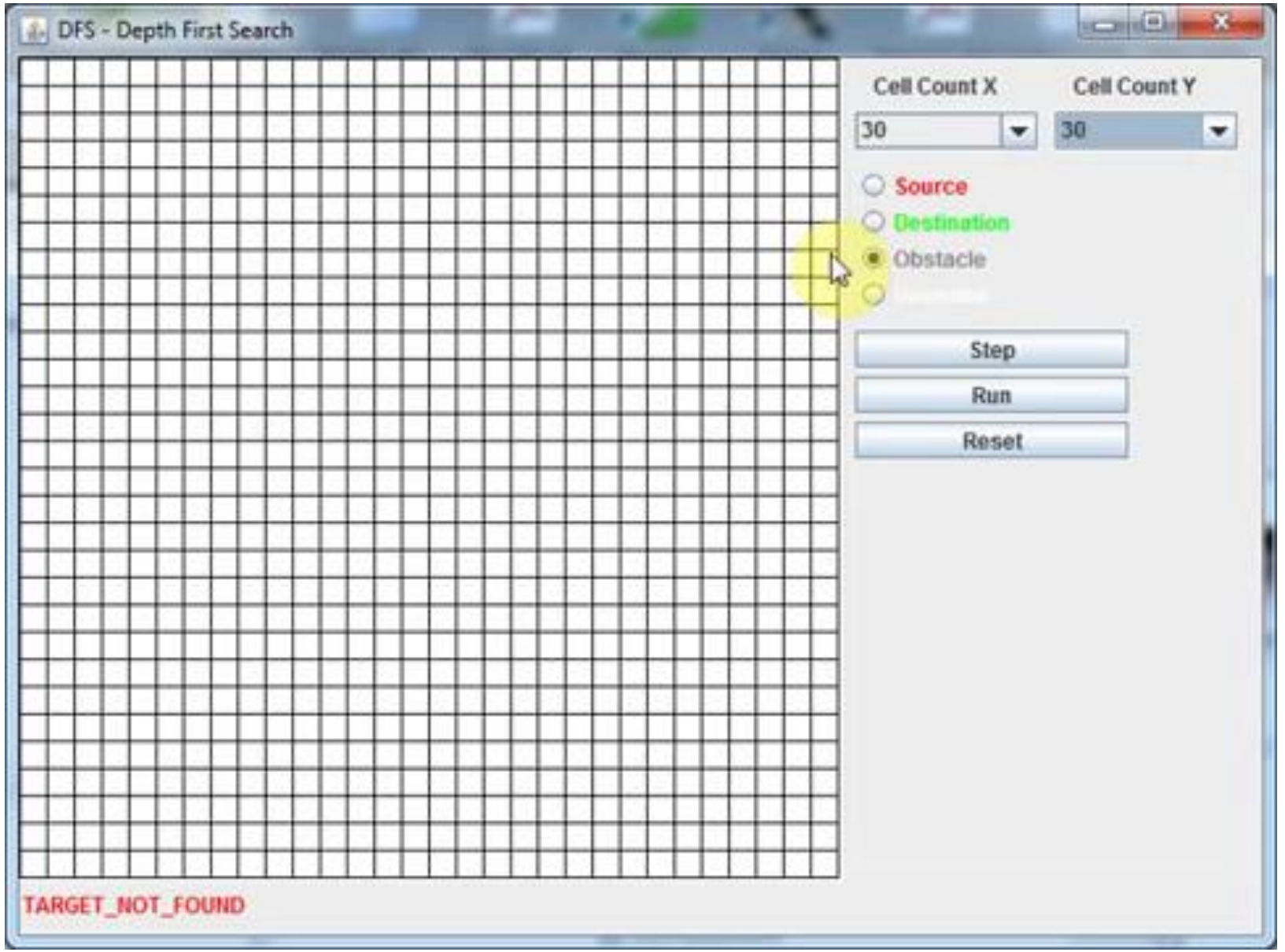
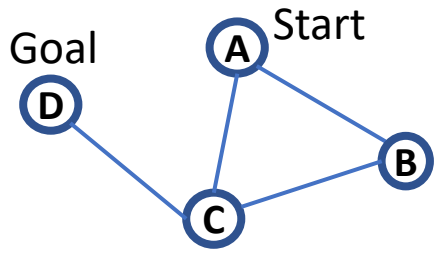


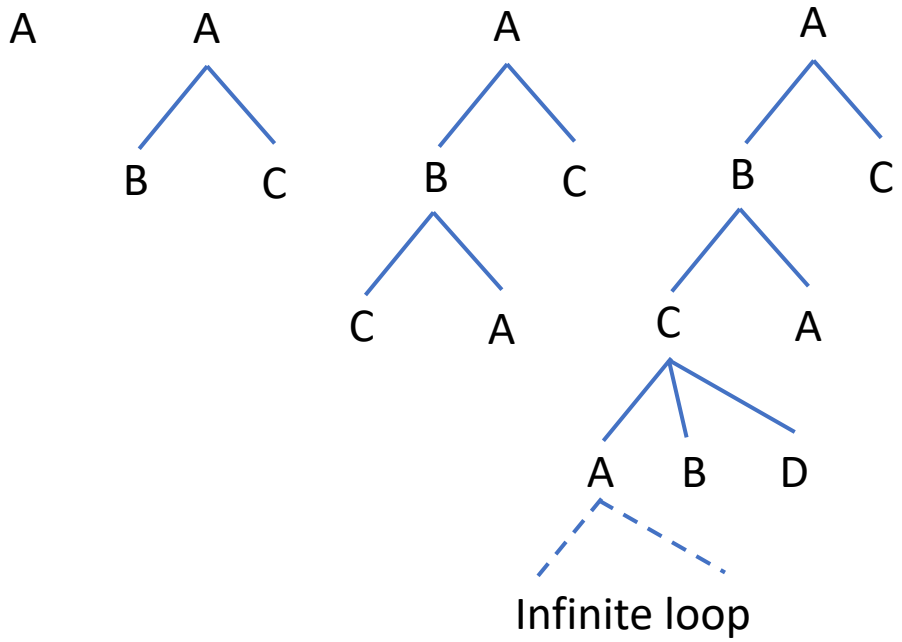
https://youtu.be/sciGbuuBJ-M

Uninformed Search Strategies

Analysis of Depth-First Search



Completeness : No (if redundant states are allowed)
Optimality : No
Time complexity : $O(b^d)$
Space complexity : O(bd) => **Good**



Where:

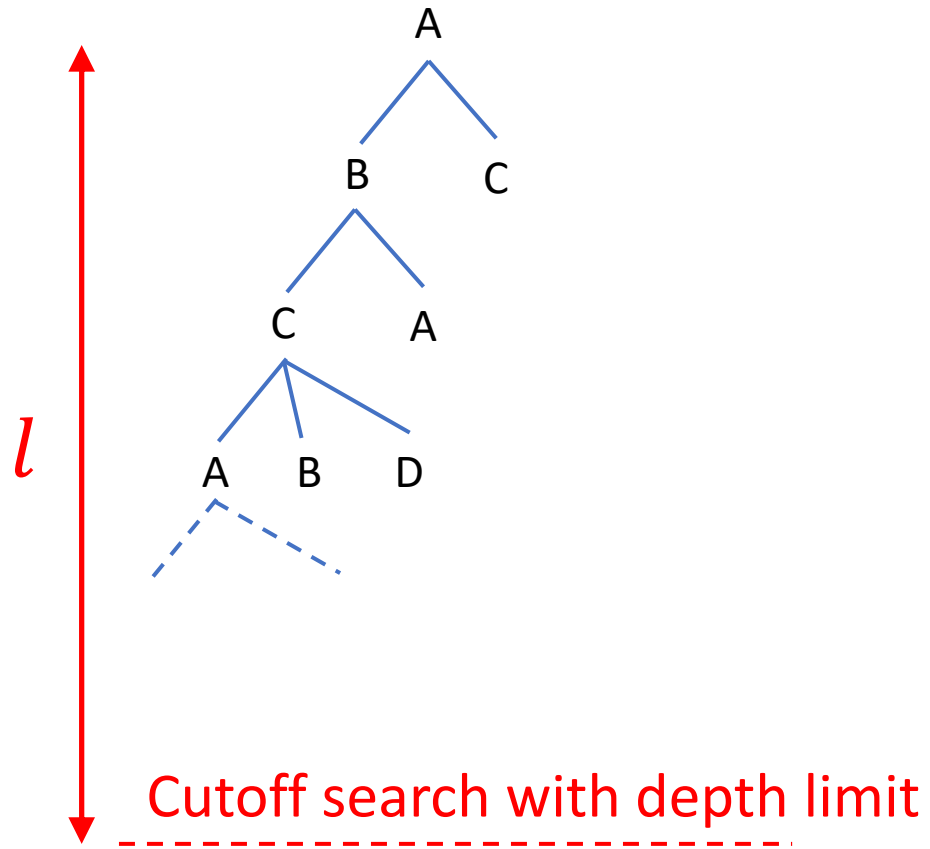
b is branching factor,

d is the max. depth of solution.

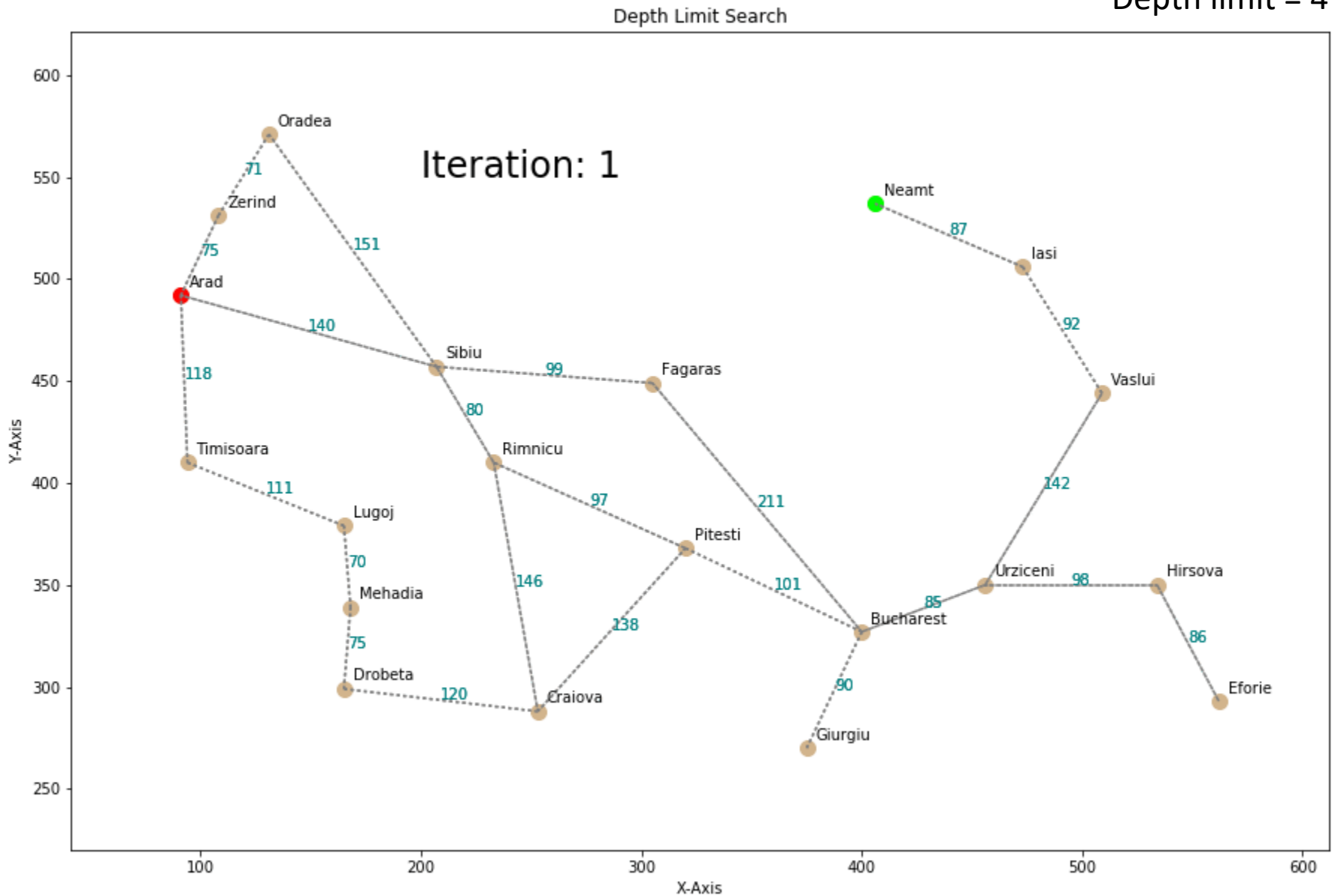
Uninformed Search Strategies

4. **Depth-Limited Search**

Incompleteness of DFS can be alleviated by a predetermined depth limit. The depth limit solves the infinite-path problem. DLS usually uses when we know the maximum solution depth of the problem.



Depth limit = 4



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Uninformed Search Strategies

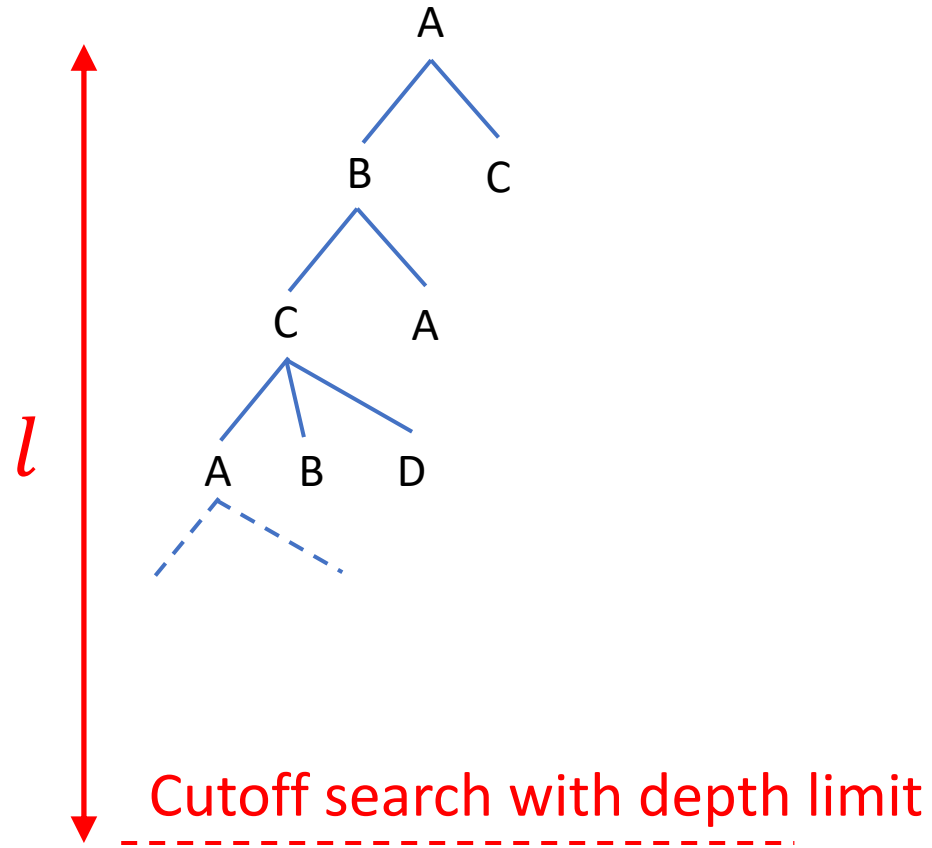
Analysis of Depth-limited Search

Completeness : Yes (if $l \geq maximum\ depth\ of\ the\ solution$)
Optimality : No
Time complexity : $O(b^l)$
Space complexity : $O(bl)$

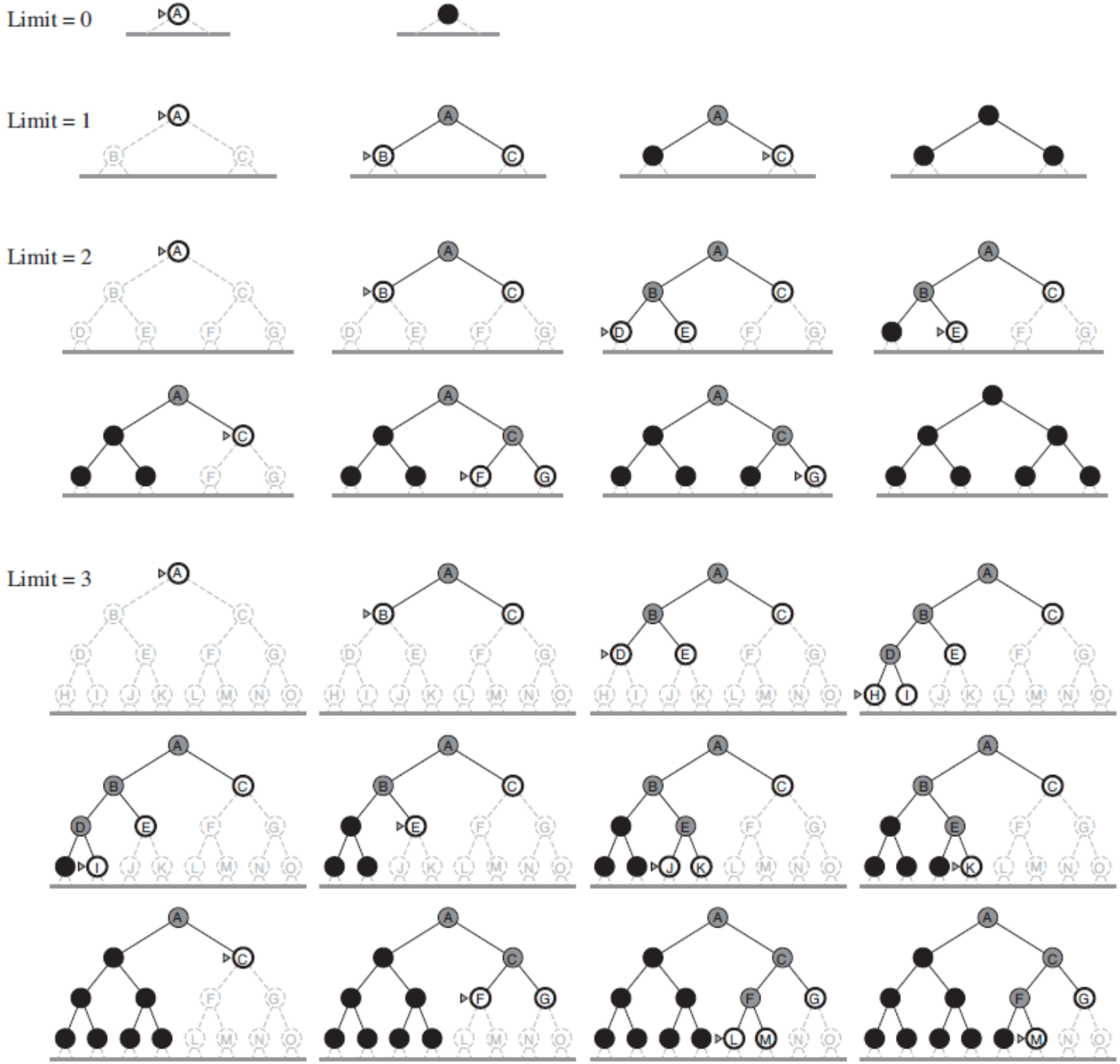
Uninformed Search Strategies

5. **Iterative Deepening Search**
(Iterative deepening depth-first search)

IDS iterates through the depth-limited search by gradually increasing the limit—first 0, then 1, then 2, and so on—until a goal is found.



IDS combines the benefits of DFS and BFS. Like DFS, its memory requirements are modest: O(bd). Like BFS, it is complete when the branching factor is finite and optimal when the path cost is a nondecreasing function of the depth of the node.


Limit = 0
Limit = 1
Limit = 2
Limit = 3

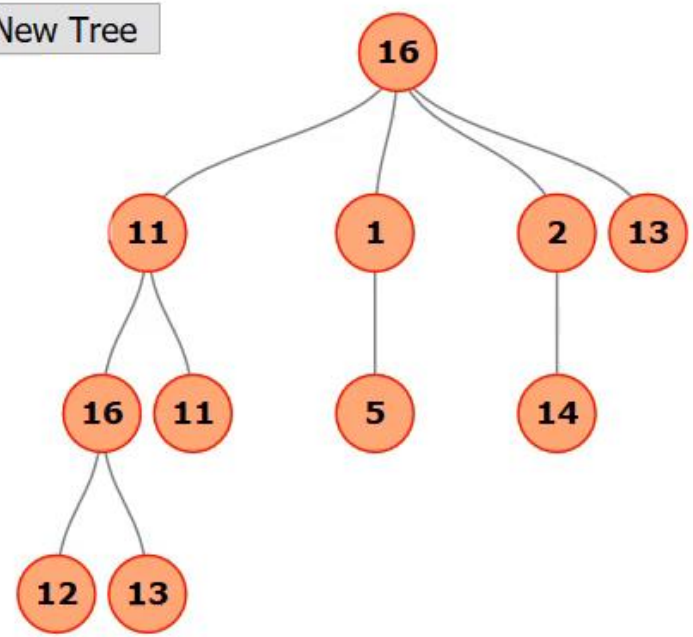
Algorithm Wiki

Recent Changes Media Ma

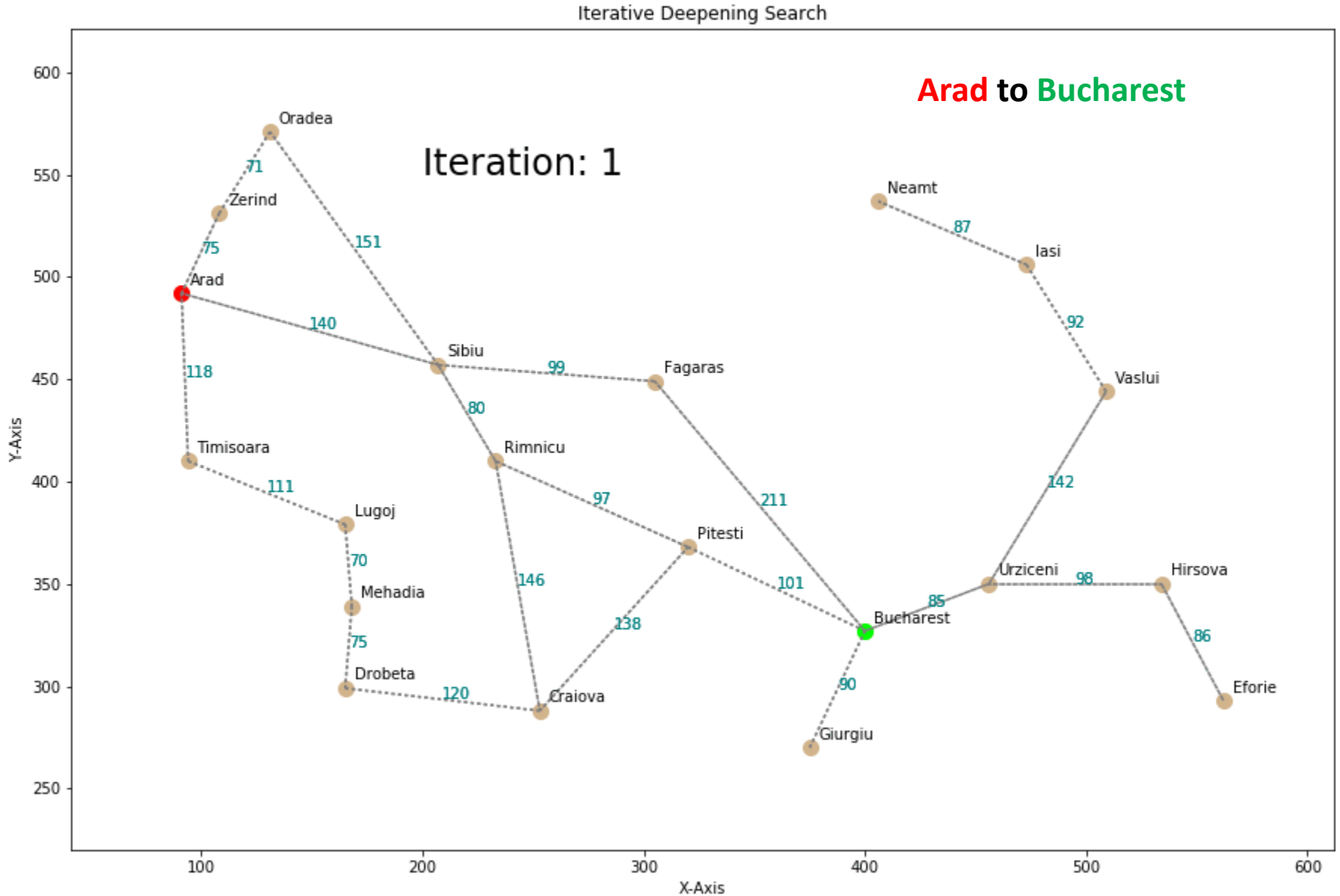
Trace: · Iterative deepening depth-first search · λ **Iterative deepening depth-first search**

```
1  function iterativeDeepeningDepthFirstSearch(node) {
2      // Repeatedly depth-first search up-to a maximum depth of 6.
3      for (var maxDepth = 1; maxDepth < 6; maxDepth++) {
4          depthFirstSearch(node, maxDepth);
5      }
6  }
7
8  function depthFirstSearch(node, maxDepth) {
9      // If reached the maximum depth, stop recursing.
10     if (maxDepth <= 0) {
11         return;
12     }
13
14     // Recurse for all children of node.
15     for (var i=0, c=node.children.length; i<c; i++) {
16         depthFirstSearch(node.children[i], maxDepth-1);
17     }
18 }
```

New Tree



http://will.thimbleby.net/algorithms/doku.php?id=algorithm:iterative_deepening_depth-first_search



http://teleported.in/posts/ai-search-algorithms/

Uninformed Search Strategies

- Iterative deepening search may seem wasteful because states are generated multiple times.
- It turns out this is not too costly.
- The reason is that in a search tree with the same (or nearly the same) branching factor at each level, most of the nodes are in the bottom level, so it does not matter much that the upper levels are generated multiple times.

Uninformed Search Strategies

In an iterative deepening search, the nodes on the bottom level (depth d) are generated once, those on the next-to-bottom level are generated twice, and so on, up to the children of the root, which are generated d times. So **the total number of nodes generated** in the worst case is

$$N(\text{IDS}) = (d)b + (d-1)b^2 + \cdots + (1)b^d \ ,$$

which give a time complexity $O(b^d)$.

However, there is some extra cost for generating the upper levels multiple times, but it is not large. For example, if b = 10 and d = 5, the numbers are

N(IDS) = 50 + 400 + 3, 000 + 20, 000 + 100, 000 = 123, 450

N(BFS) = 10 + 100 + 1, 000 + 10, 000 + 100, 000 = 111, 110 .

IDS is preferred when the search space is large and the solution depth is unknown.

Uninformed Search Strategies

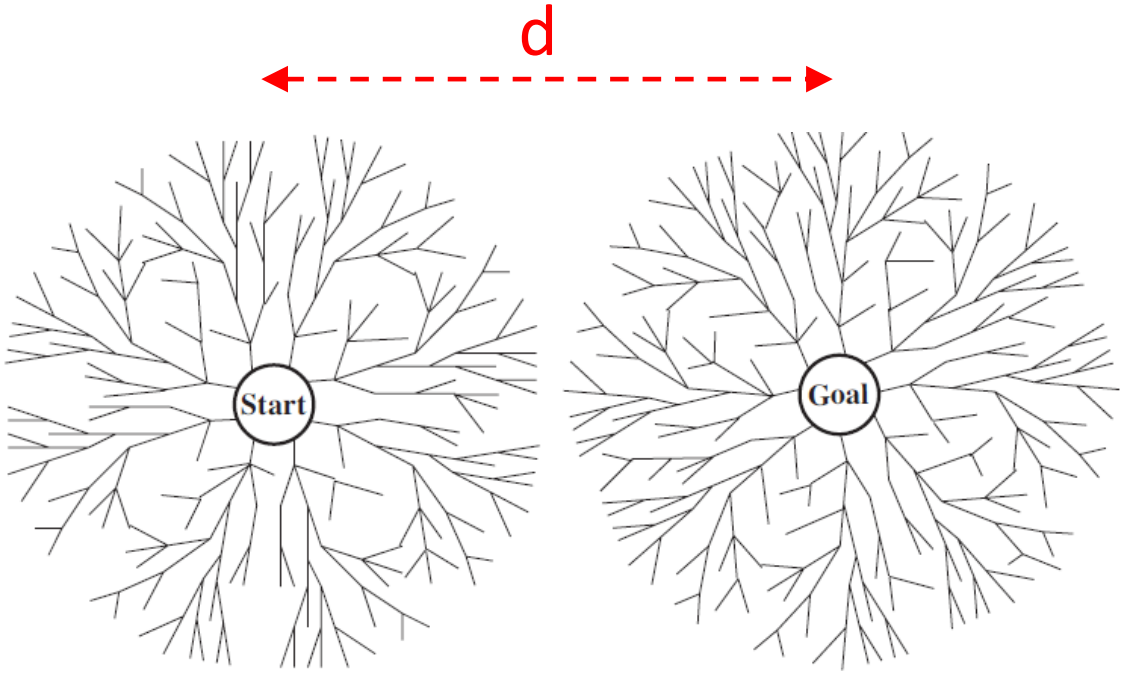
Analysis of Iterative Deepening Search

Completeness : Yes
Optimality : Yes
Time complexity : $O(b^d)$
Space complexity : $O(bd)$

Uninformed Search Strategies

5. **Bidirectional Search**

Idea : Run two simultaneous searches—one forward from the initial state and the other backward from the goal—hoping that the two searches meet in the middle.

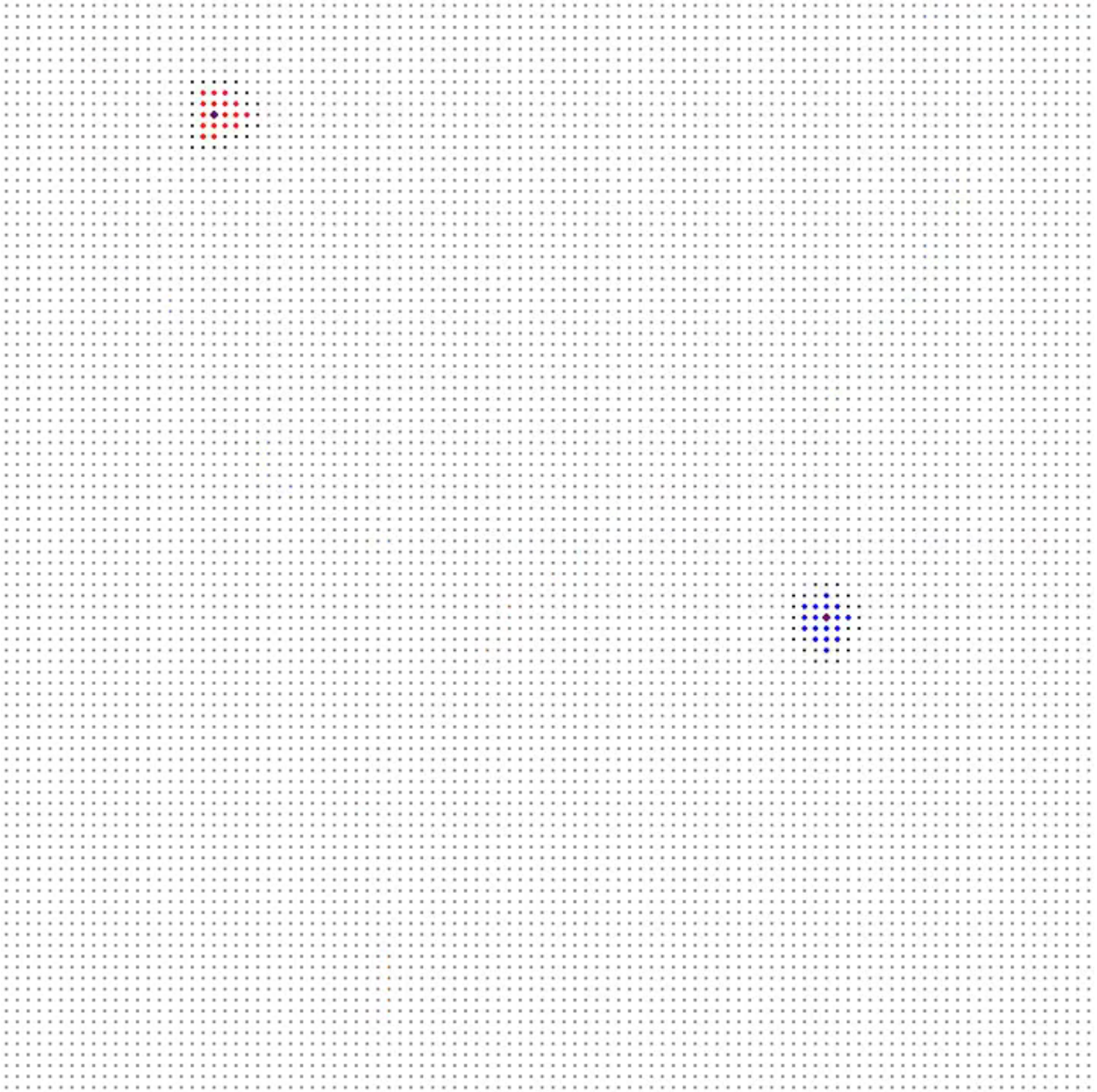


The motivation is that $b^{d/2} + b^{d/2}$ is much less than b^d.

For each search on both sides, we can use either breadth-first search or iterative deepening search to reduce the space complexity.

Bidirectional Search Simulation

https://www.youtube.com/watch?v=YtPT99c5OXc



Uninformed Search Strategies

Analysis of Iterative Deepening Search

Completeness : Yes
Optimality : Yes
(if the cost for each action is equal & monotonically increasing)
Time complexity : $O(b^{d/2})$
Space complexity : $O(b^{d/2})$ BFS, $O(bd/2)$ IDS

Uninformed Search Strategies

Comparison of uninformed search strategies

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening	Bidirectional (if applicable)
Complete?	Yes^{a}	Yes^{a,b}	No	No	Yes^{a}	Yes^{a,d}
Time	$O(b^d)$	$O(b^{1+\lfloor C^*/\epsilon \rfloor})$	$O(b^m)$	$O(b^\ell)$	$O(b^d)$	$O(b^{d/2})$
Space	$O(b^d)$	$O(b^{1+\lfloor C^*/\epsilon \rfloor})$	$O(bm)$	$O(b\ell)$	$O(bd)$	$O(b^{d/2})$
Optimal?	Yes^{c}	Yes	No	No	Yes^{c}	Yes^{c,d}

Figure 3.21 Evaluation of tree-search strategies. b is the branching factor; d is the depth of the shallowest solution; m is the maximum depth of the search tree; l is the depth limit. Superscript caveats are as follows: ^{a} complete if b is finite; ^{b} complete if step costs $\geq \epsilon$ for positive ϵ; ^{c} optimal if step costs are all identical; ^{d} if both directions use breadth-first search.