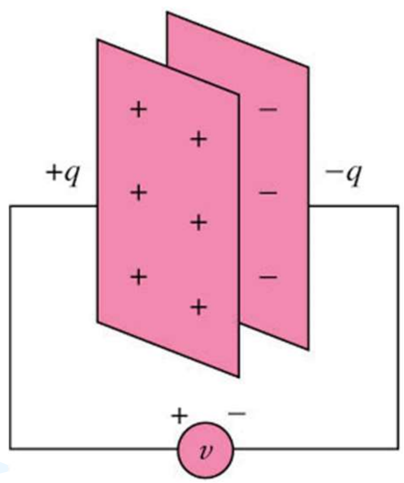
Alexander-Sadiku
Fundamentals of Electric Circuits

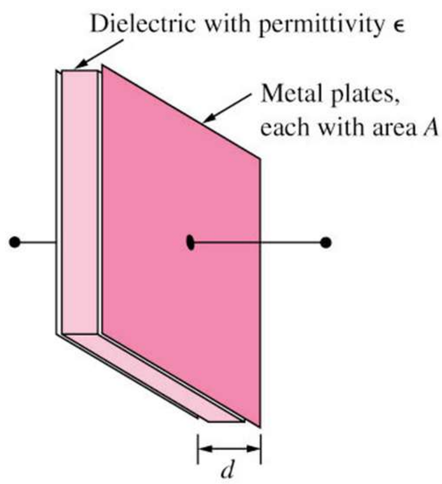
Chapter 6
Capacitors and Inductors

Capacitors and Inductors
Chapter 6

6.1 Capacitors (1)

- A capacitor is a passive element designed to **store energy** in its **electric field.**

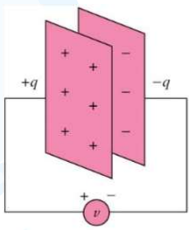




- A **capacitor** consists of two conducting plates separated by an insulator (or dielectric).

6.1 Capacitors (2)

- **Capacitance** *C* is the ratio of the charge *q* on one plate of a capacitor to the voltage difference *v* between the two plates, measured in farads (F).



$$q = C\,v$$ and $$C = \frac{\varepsilon\,A}{d}$$

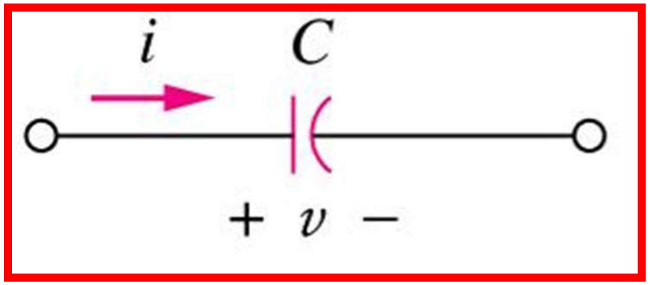
- Where ε is the permittivity of the dielectric material between the plates, A is the surface area of each plate, d is the distance between the plates.
- Unit: F, pF (10^{-12}), nF (10^{-9}), and **μF (10^{-6})**

6.1 Capacitors (3)

- If ***i*** is flowing into the +ve terminal of C

การทำงานของ Capacitor มี 2 อย่าง *

  - Charging => ***i*** is +ve
  - Discharging => ***i*** is −ve



- The current-voltage relationship of capacitor according to above convention is

$$\boxed{i = C\frac{dv}{dt}}$$

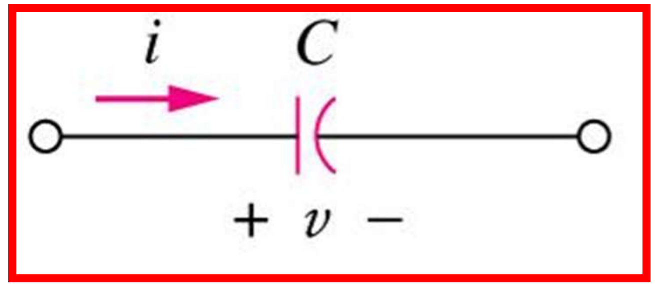
อัตราการเปลี่ยนแปลงความต่างศักย์เทียบกับเวลา

and

$$\boxed{v = \frac{1}{C}\int_{t_0}^{t} i\,dt + v(t_0)}$$

6.1 Capacitors (4)

- The energy, ***w***, stored in the capacitor is

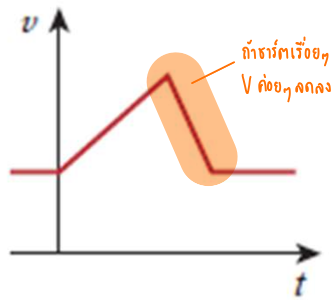


$$w = \frac{1}{2} C v^2$$

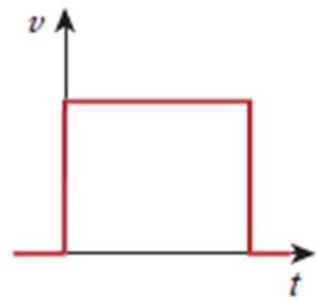
(หน่วย J)

- A capacitor is
  - an **open circuit** to dc (dv/dt = 0).
  - its voltage **cannot change** ทันทีทันใด **abruptly.**

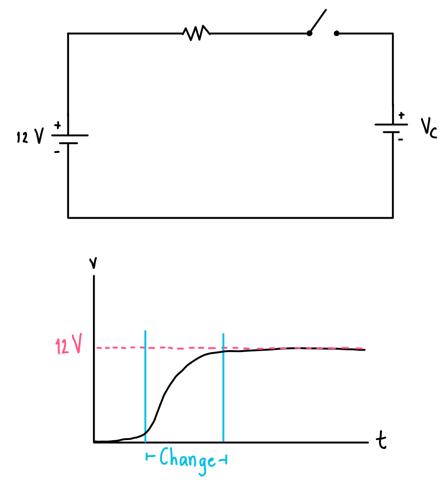
6.1 Capacitors (5)

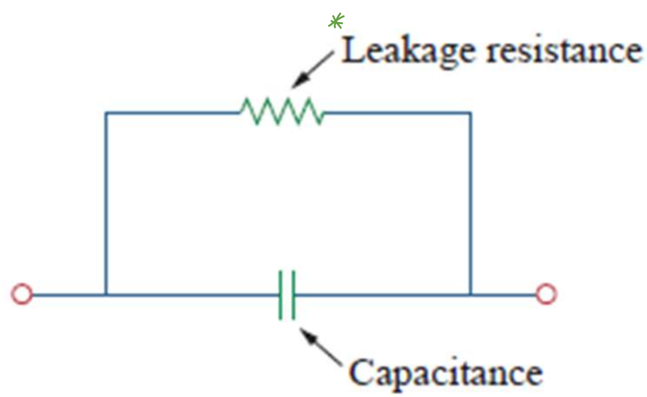


(a)



(b)





6.1 Capacitors (6)

(a) Calculate the charge stored on a 3-pF capacitor with 20 V across it.
(b) Find the energy stored in the capacitor.

Solution:

(a) Since $q = Cv$,

$$q = 3 \times 10^{-12} \times 20 = 60\ \text{pC}$$

(b) The energy stored is

$$w = \frac{1}{2}Cv^2 = \frac{1}{2} \times 3 \times 10^{-12} \times 400 = 600\ \text{pJ}$$

6.1 Capacitors (7)

The voltage across a 5-μF capacitor is

$$v(t) = 10 \cos 6000t \text{ V}$$

Calculate the current through it.

Solution:

By definition, the current is

$$i(t) = C\frac{dv}{dt} = 5 \times 10^{-6} \frac{d}{dt}(10 \cos 6000t)$$

$$= -5 \times 10^{-6} \times 6000 \times 10 \sin 6000t = -0.3 \sin 6000t \text{ A}$$

6.1 Capacitors (8)

Determine the voltage across a 2-μF capacitor if the current through it is

$$i(t) = 6e^{-3000t}\ \text{mA}$$

Assume that the initial capacitor voltage is zero.

Solution:

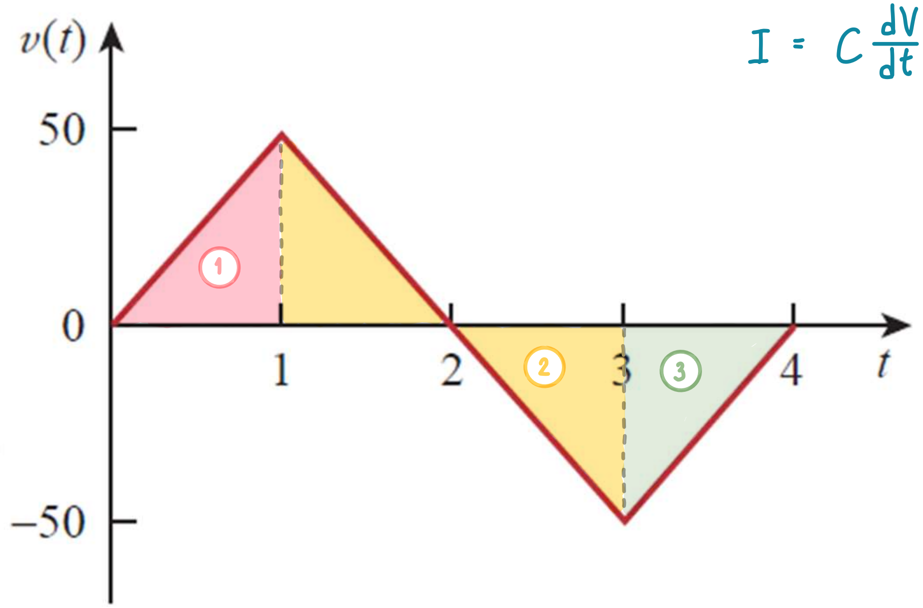
Since $v = \dfrac{1}{C}\displaystyle\int_0^t i\,dt + v(0)$ and $v(0) = 0$,

$$v = \frac{1}{2 \times 10^{-6}} \int_0^t 6e^{-3000t}\,dt \cdot 10^{-3}$$

$$= \frac{3 \times 10^3}{-3000} e^{-3000t} \Big|_0^t = (1 - e^{-3000t})\ \text{V}$$

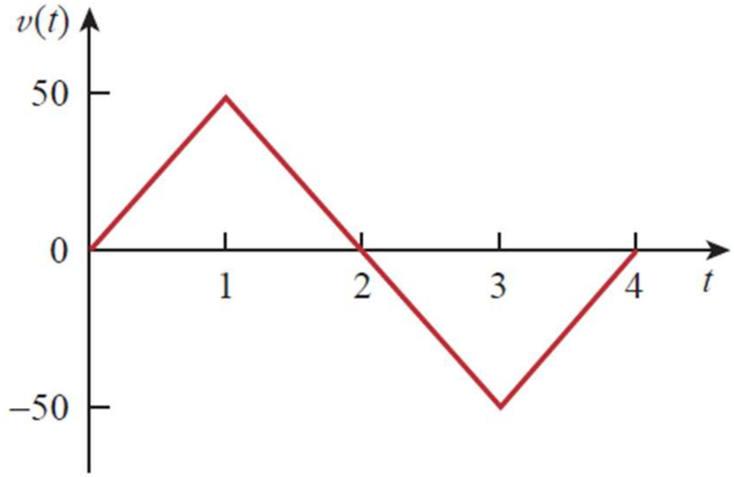
6.1 Capacitors (9)

Determine the current through a 200-μF capacitor whose voltage is shown in Fig. 6.9.



$I = C\frac{dV}{dt}$

6.1 Capacitors (10)



The voltage waveform can be described mathematically as

$$v(t) = \begin{cases} 50t \text{ V} & 0 < t < 1 \\ 100 - 50t \text{ V} & 1 < t < 3 \\ -200 + 50t \text{ V} & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

6.1 Capacitors (11)

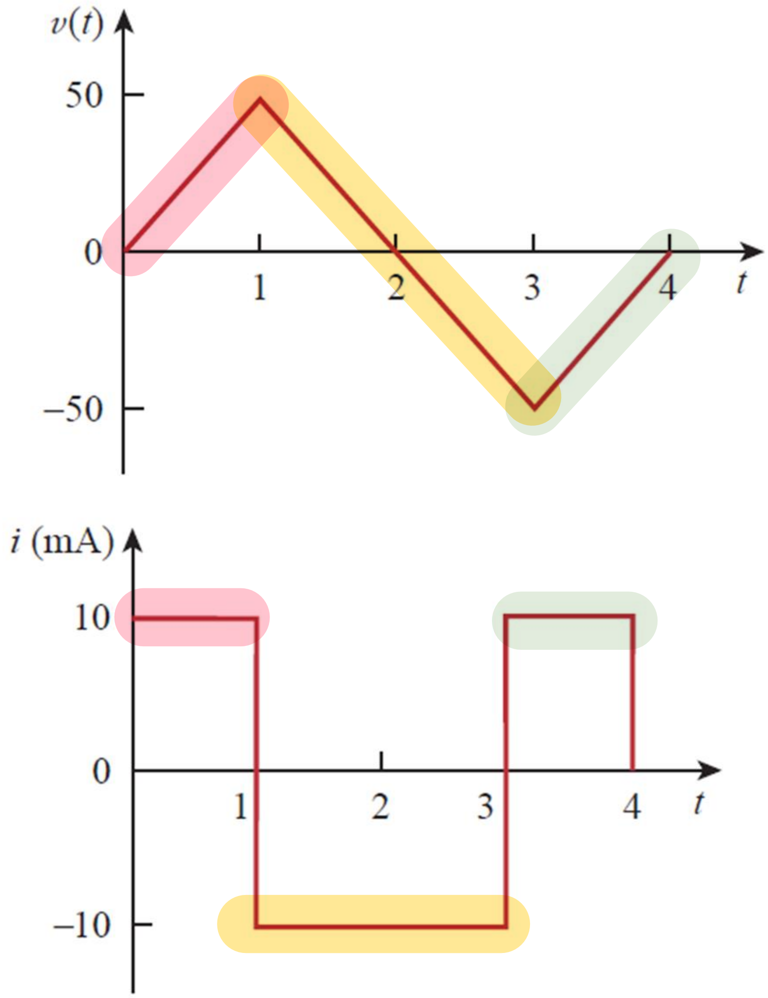
Since $i = C\,dv/dt$ and $C = 200\ \mu\text{F}$, we take the derivative of v to obtain

$$i(t) = 200 \times 10^{-6} \times \begin{cases} 50 & 0 < t < 1 \\ -50 & 1 < t < 3 \\ 50 & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

$$= \begin{cases} 10\text{ mA} & 0 < t < 1 \\ -10\text{ mA} & 1 < t < 3 \\ 10\text{ mA} & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

Thus the current waveform is as shown in Fig. 6.10.

6.1 Capacitors (12)



6.1 Capacitors (13)

Example

1×10^{-3} F

An initially uncharged 1-mF capacitor has the current shown below across it.

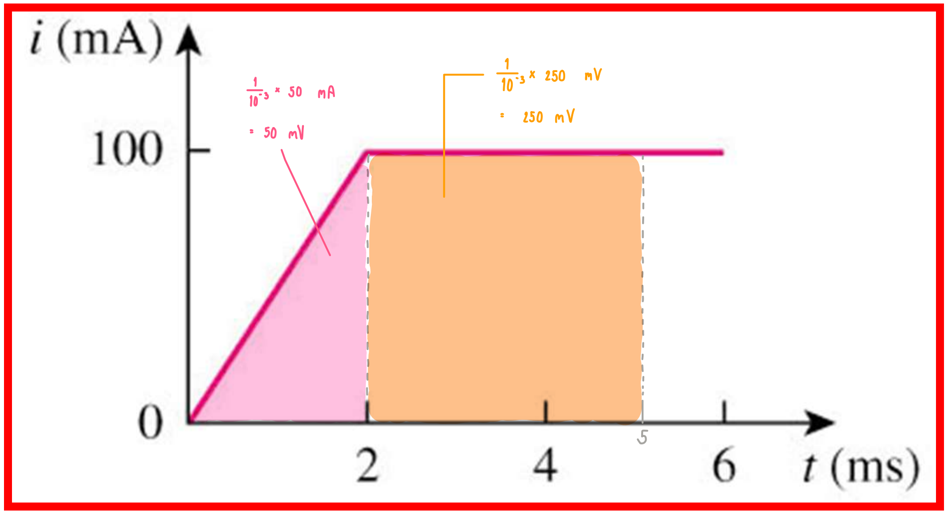
Calculate the voltage across it at t = 2 ms and t = 5 ms.

$$i = C\frac{dV}{dt}$$

$$cdV = idV$$

$$dt = \frac{1}{C}idt$$

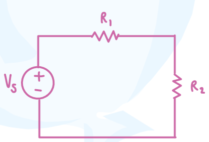
$$V = \frac{1}{C}\int idt$$ ------- พื้นที่ใต้กราฟ



6.1 Capacitors (14)

Obtain the energy stored in each capacitor in Fig. 6.12(a) under dc conditions.

$$W = \frac{1}{2}CV^2$$

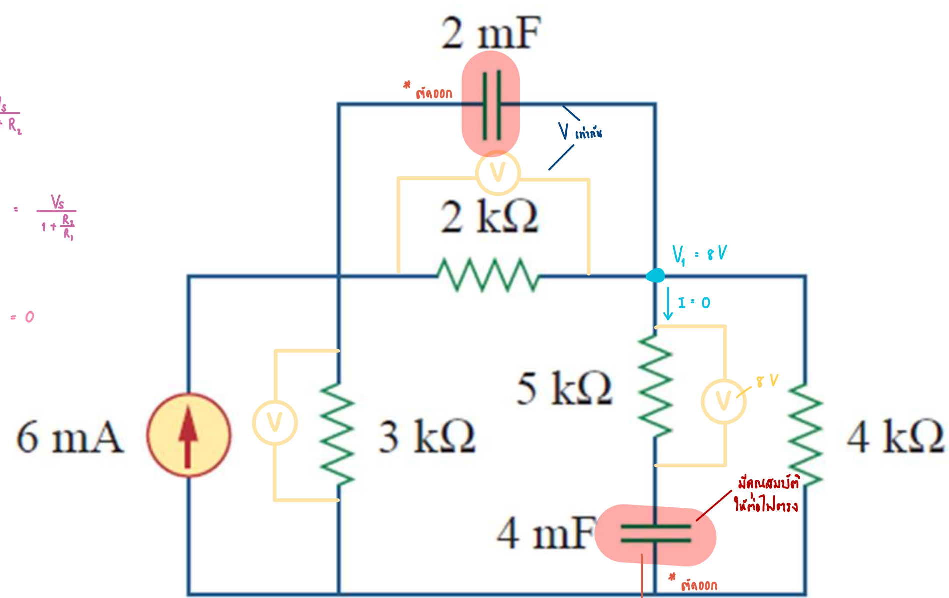


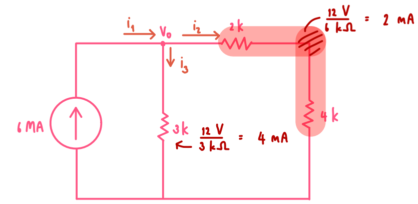
$$i = \frac{V_s}{R_1 + R_2}$$

$$V_{R_1} = i \times R_1 = \frac{V_s}{R_1 + R_2} \times R_1 = \frac{V_s}{\frac{R_1}{R_1} + \frac{R_2}{R_1}} = \frac{V_s}{1 + \frac{R_2}{R_1}}$$

$$V_{R_2} = i \times R_2 = \frac{V_s}{\frac{R_1}{R_2} + \frac{R_2}{R_2}} = \frac{V_s}{1 + \frac{R_1}{R_2}}$$

$$\frac{5k}{\infty} = 0$$





$$i_1 = i_2 + i_3$$

$$6\text{ mA} = \frac{V_0}{6k} + \frac{V_0}{3k}$$

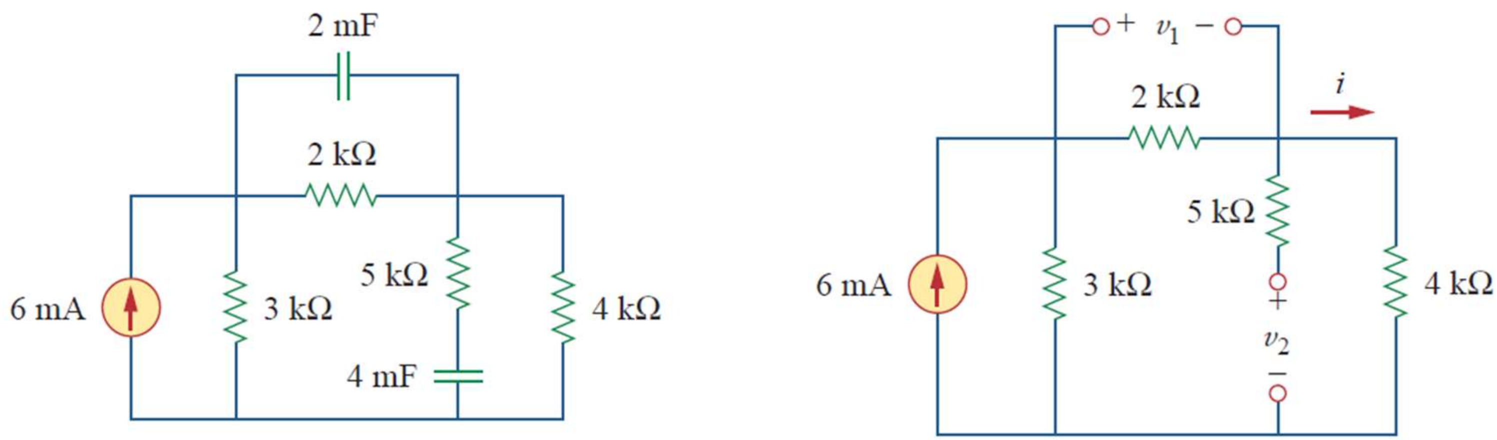
$$36 = V_0 + 2V_0$$

$$3V_0 = 36$$

$$V_0 = 12\text{ V}$$

\# Resistor มาก ∝ Capacitor มีไฟฟ้ามาก

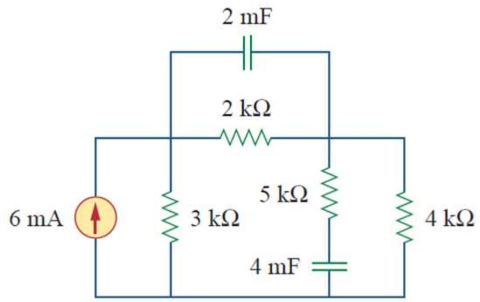
6.1 Capacitors (15)

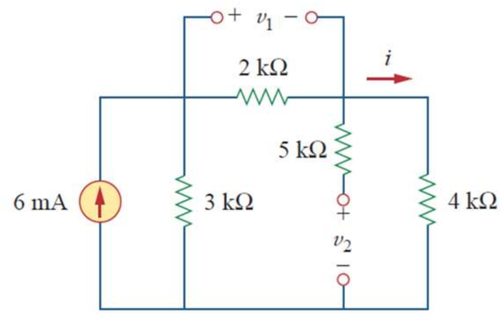


Under dc conditions, we replace each capacitor with an open circuit, as shown in Fig. 6.12(b). The current through the series combination of the 2-kΩ and 4-kΩ resistors is obtained by current division as

$$i = \frac{3}{3 + 2 + 4}(6\text{ mA}) = 2\text{ mA}$$

6.1 Capacitors (16)





Hence, the voltages v_1 and v_2 across the capacitors are

$$v_1 = 2000i = 4 \text{ V} \qquad v_2 = 4000i = 8 \text{ V}$$

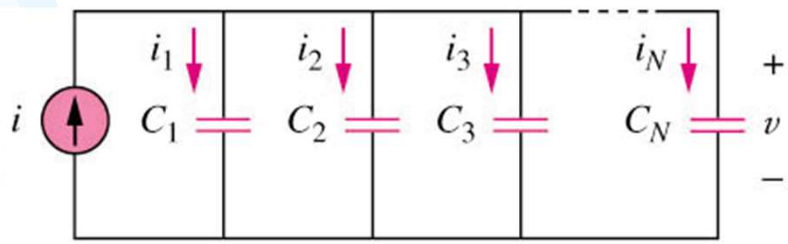
and the energies stored in them are

$$w_1 = \frac{1}{2}C_1v_1^2 = \frac{1}{2}(2 \times 10^{-3})(4)^2 = 16 \text{ mJ}$$

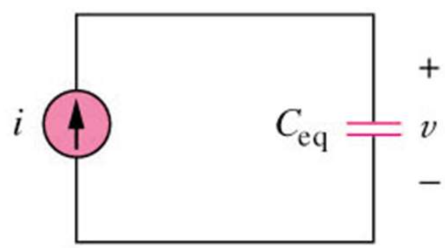
$$w_2 = \frac{1}{2}C_2v_2^2 = \frac{1}{2}(4 \times 10^{-3})(8)^2 = 128 \text{ mJ}$$

6.2 Series and Parallel Capacitors (1)

- The equivalent capacitance of *N* **parallel-connected** capacitors is the sum of the individual capacitances.



(a)

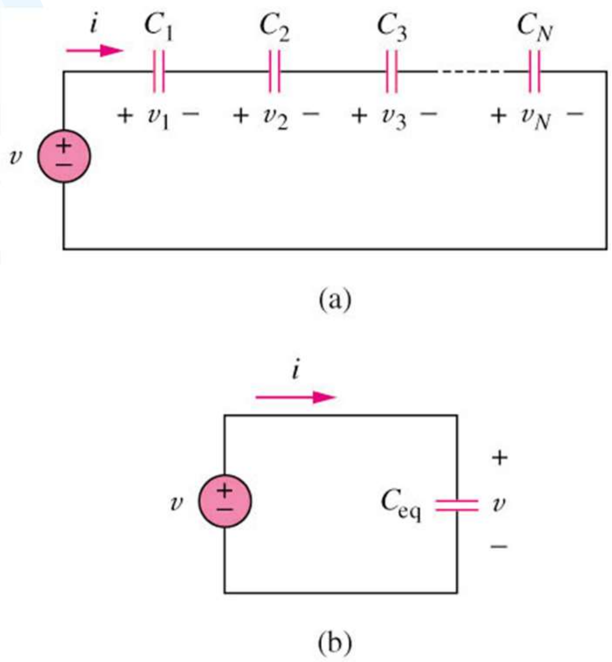


(b)

$$C_{eq} = C_1 + C_2 + \ldots + C_N$$

6.2 Series and Parallel Capacitors (2)

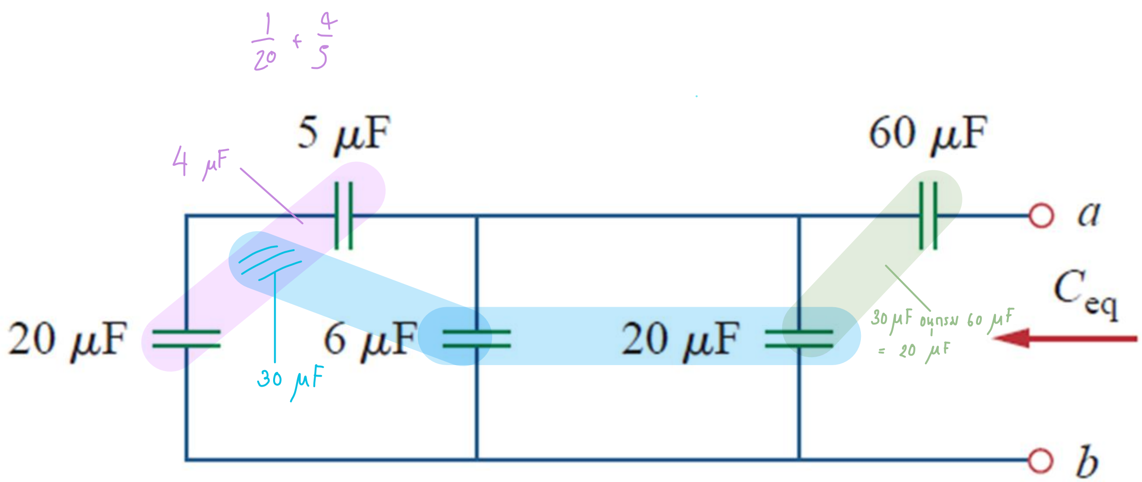
- The equivalent capacitance of *N* **series-connected** capacitors is the reciprocal of the sum of the reciprocals of the individual capacitances.



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_N}$$

6.2 Series and Parallel Capacitors (4)

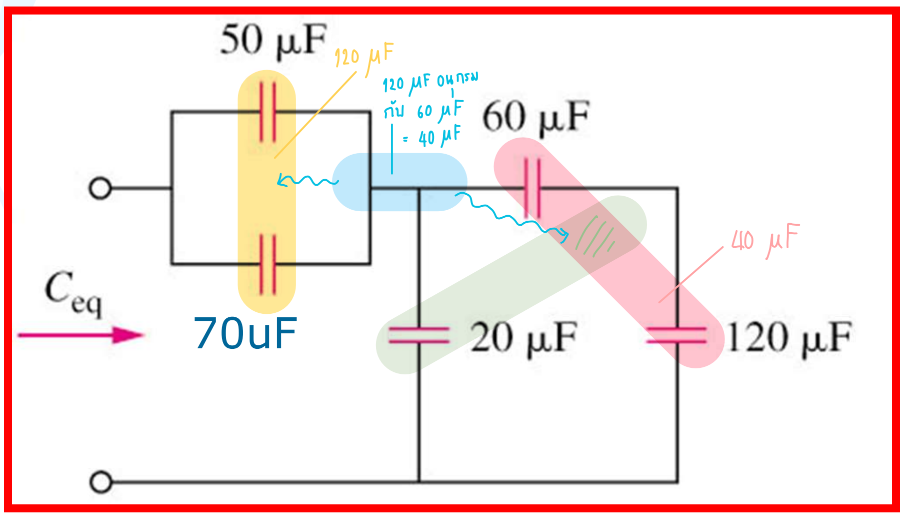
Find the equivalent capacitance seen between terminals a and b of the circuit in Fig. 6.16.



6.2 Series and Parallel Capacitors (5)

Example 3

Find the equivalent capacitance seen at the terminals of the circuit in the circuit shown below:



Answer:

C_{eq} = <u>40μF</u>

6.2 Series and Parallel Capacitors (6)

For the circuit in Fig. 6.18, find the voltage across each capacitor.

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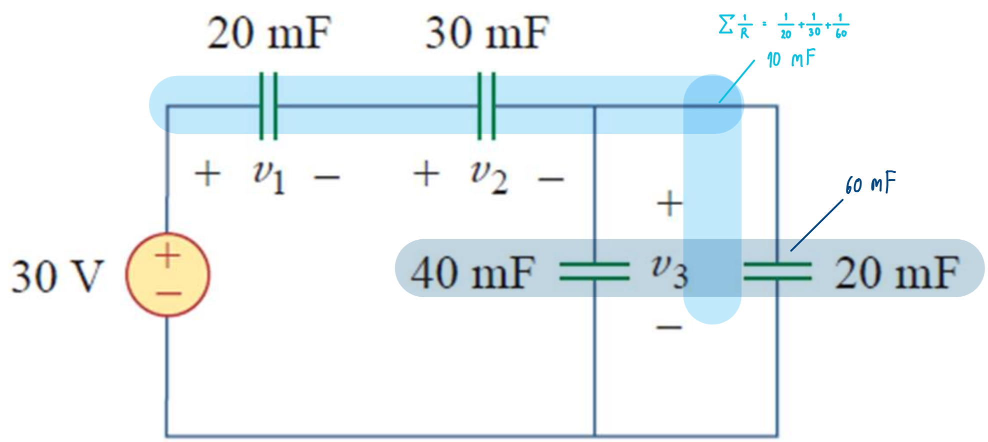
$q = CV$

$W = \frac{1}{2}CV^2$

$i = C\frac{dV}{dt}$



$W = \frac{1}{2}(10\ m)(30^2)$
$= 4.5\ J$

$q = (10\ m)(30)$
$= 300\ mC$

$\therefore C_{eq} = 10\ mF$

$30\ V = V_1 + V_2 + V_3$

$= \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$

$\frac{300\ mC}{20\ mF} = 15\ V$

$\frac{300\ mC}{30\ mF} = 10\ V$

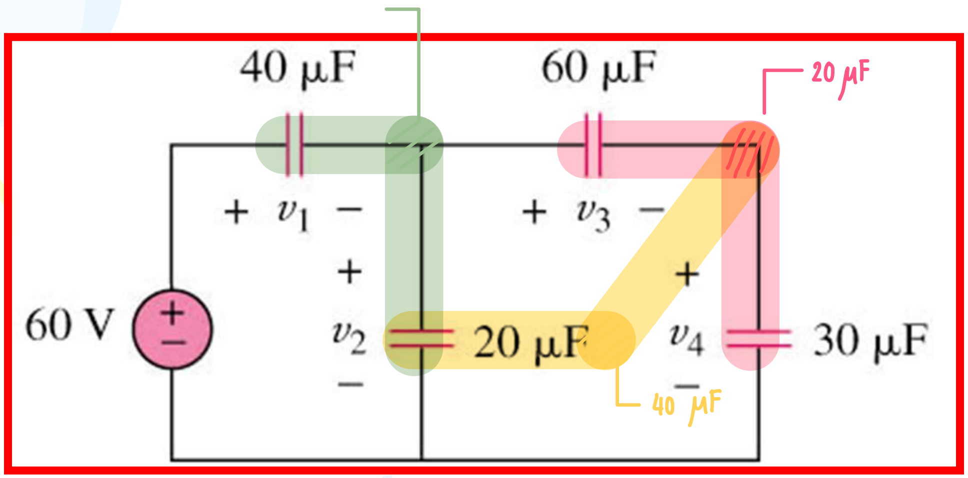
$\frac{300\ mC}{60\ mF} = \boxed{5\ V}$

$30 - V_1 - V_2 = 5\ V$

6.2 Series and Parallel Capacitors (7)

Example 4

Find the voltage across each of the capacitors in the circuit shown below:



Answer:

v_1 = 30V

v_2 = 30V

v_3 = 10V

v_4 = 20V

$Q = CV_2$

$= 20\,\mu F \times 30V$

$= 600\,\mu C$

$V_3 = \frac{600\,\mu C}{60}$

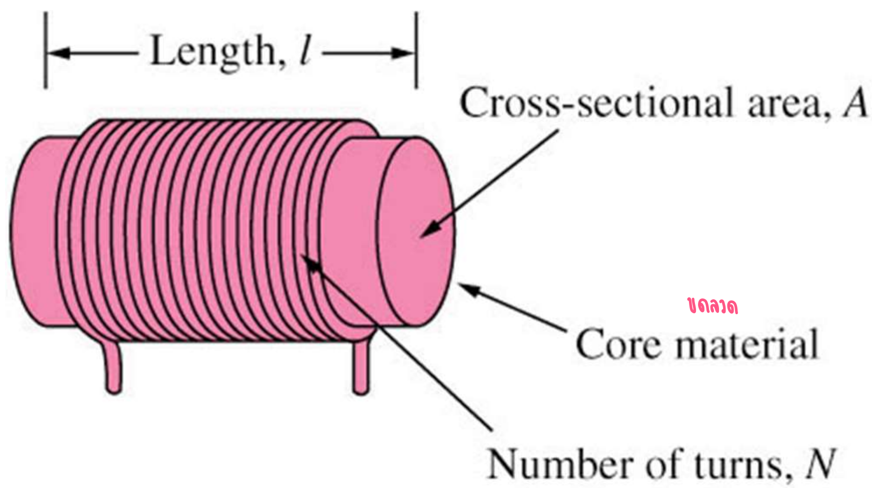
$= 10\ V$

$V_4 = \frac{600\,\mu C}{30}$

$= 20\ V$

6.3 Inductors (1)

- An inductor is a passive element designed to store energy in its magnetic field.



- An inductor consists of a coil of conducting wire.

6.3 Inductors (2)

- Inductance is the property whereby an inductor exhibits opposition to the change of current flowing through it, measured in henrys (H).

$$v = L\frac{di}{dt}$$

การเปลี่ยนแปลงของกระแสไฟฟ้า
เทียบกับเวลา

and

หน่วย

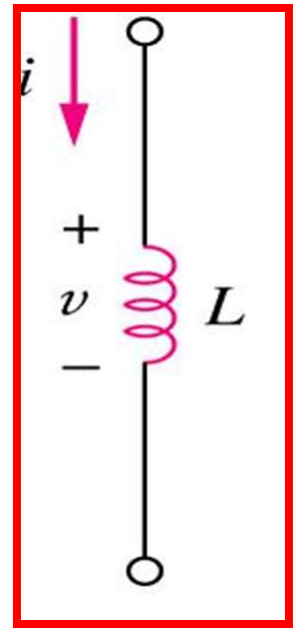
$$L = \frac{N^2 \mu A}{l}$$

- The unit of inductors is Henry (H), mH (10^{-3}) and μH (10^{-6}).

6.3 Inductors (3)

- The current-voltage relationship of an inductor:

$$i = \frac{1}{L}\int_{t_0}^{t} v(t)\,dt + i(t_0)$$



* กระแสตรง (DC) : เป็น short circuit

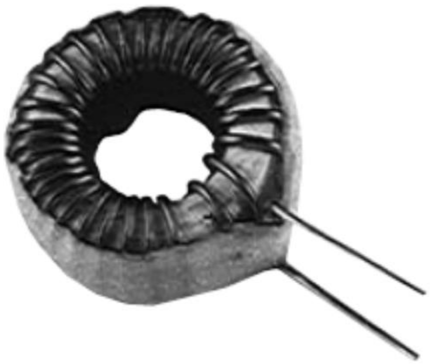
- The power stored by an inductor:

$$w = \frac{1}{2} L\, i^2$$

- An inductor acts like a short circuit to dc (di/dt = 0) and its current **cannot change abruptly.**

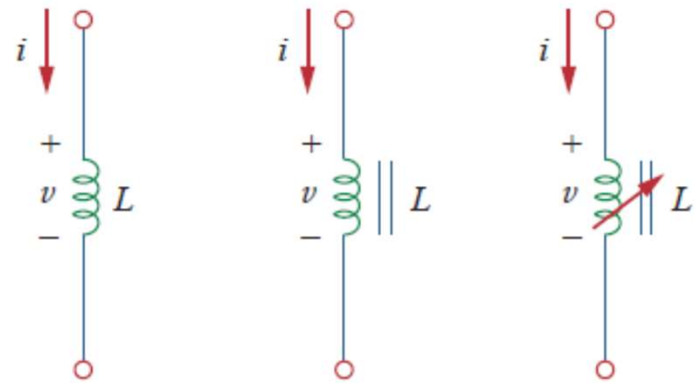
ductors (3)

(a)



(b)

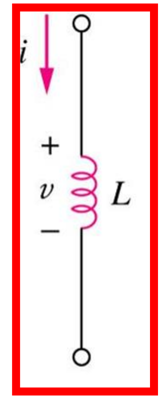
(c)



(a) (b) (c)

6.3 Inductors (4)

The current through a 0.1-H inductor is $i(t) = 10te^{-5t}$ A. Find the voltage across the inductor and the energy stored in it.



Solution:

Since $v = L\,di/dt$ and $L = 0.1$ H,

$$v = 0.1\frac{d}{dt}(10te^{-5t}) = e^{-5t} + t(-5)e^{-5t} = e^{-5t}(1 - 5t) \text{ V}$$

The energy stored is

$$w = \frac{1}{2}Li^2 = \frac{1}{2}(0.1)100t^2e^{-10t} = 5t^2e^{-10t} \text{ J}$$

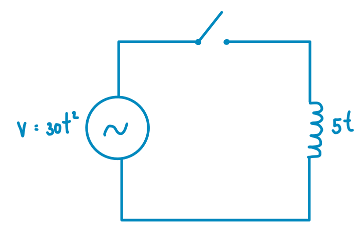
6.3 Inductors (5)

Find the current through a 5-H inductor if the voltage across it is

$$v(t) = \begin{cases} 30t^2, & t > 0 \\ 0, & t < 0 \end{cases}$$

Also, find the energy stored at $t = 5$ s. Assume $i(v) > 0$.

Solution:



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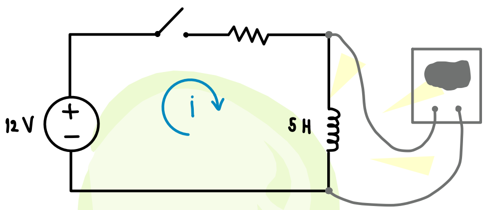
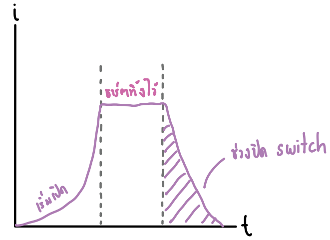
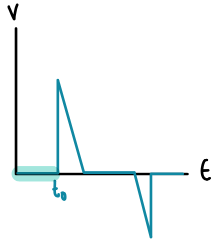
Since $i = \dfrac{1}{L}\displaystyle\int_{t_0}^{t} v(t)\, dt + i(t_0)$ and $L = 5$ H,

$$i = \frac{1}{5}\int_0^t 30t^2\, dt + 0 = 6 \times \frac{t^3}{3} = 2t^3 \text{ A}$$







6.3 Inductors (6)

The power $p = vi = 60t^5$, and the energy stored is then

$$w = \int p\,dt = \int_0^5 60t^5\,dt = 60\frac{t^6}{6}\bigg|_0^5 = 156.25 \text{ kJ}$$

Alternatively, we can obtain the energy stored using Eq. (6.24), by writing

$$w\big|_0^5 = \frac{1}{2}Li^2(5) - \frac{1}{2}Li(0) = \frac{1}{2}(5)(2\times 5^3)^2 - 0 = 156.25 \text{ kJ}$$

as obtained before.

6.3 Inductors (7)

Example 5

The terminal voltage of a 2-H inductor is

$$v = 10(1-t)\ \mathrm{V}$$

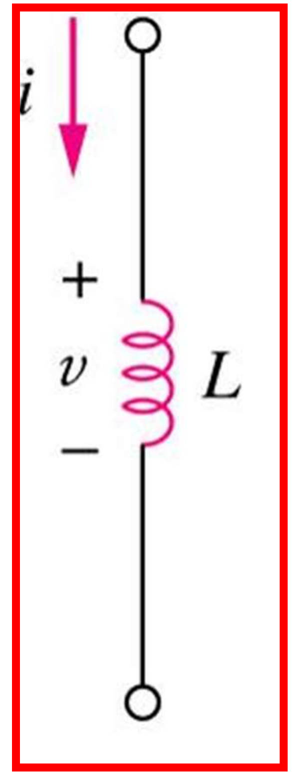
Find the current flowing through it at t = 4 s and the energy stored in it within 0 < t < 4 s.

Assume i(0) = 2 A.

Answer:

i(4s) = -18A

w(4s) = 320J



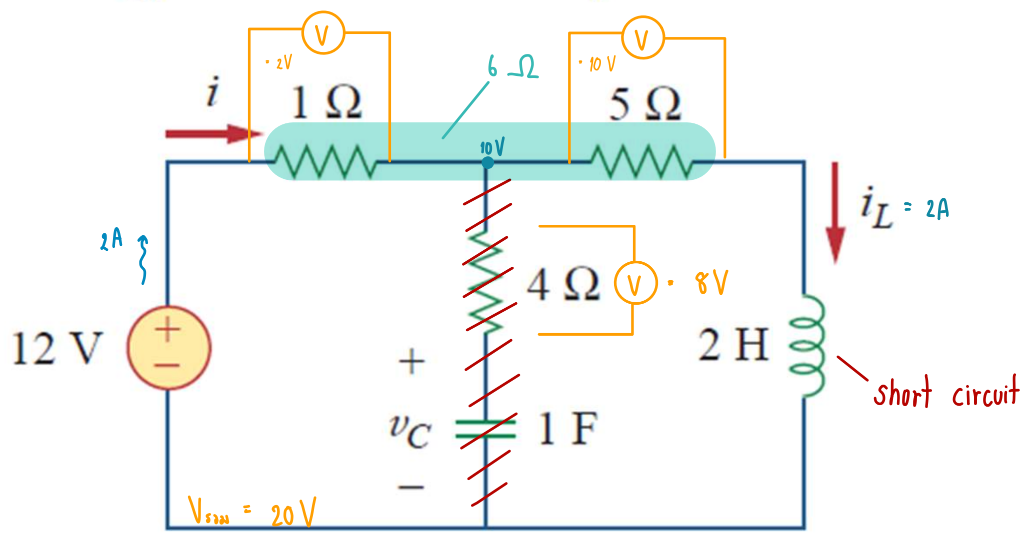
$$i = \frac{1}{L}\int_{t_0}^{t} v\,dt + i_0 = \frac{1}{2}\int_0^4 (10(1-t))\,dt + 2$$

$$= 5\left(t - \frac{t^2}{2}\right)\Big|_0^4 + 2 = 5\left(4 - \frac{4^2}{2} + 2\right) = 5(-4) + 2 = -20 + 2 = -18 \ \#$$

$$W = \frac{1}{2}Li^2\Big|_{t=4} - \frac{1}{2}Li^2\Big|_{t=0} = \frac{1}{2}\times 2\times(-18)^2 - \frac{1}{2}\times 2\times 2^2 = 324 - 4 = 320$$

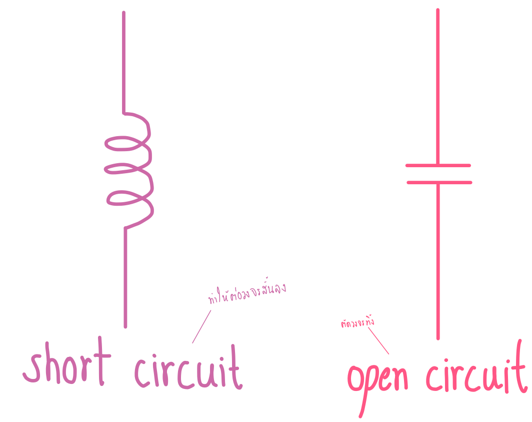
6.3 Inductors (8)

Consider the circuit in Fig. 6.27(a). Under dc conditions, find: (a) i, v_C, and i_L, (b) the energy stored in the capacitor and inductor.



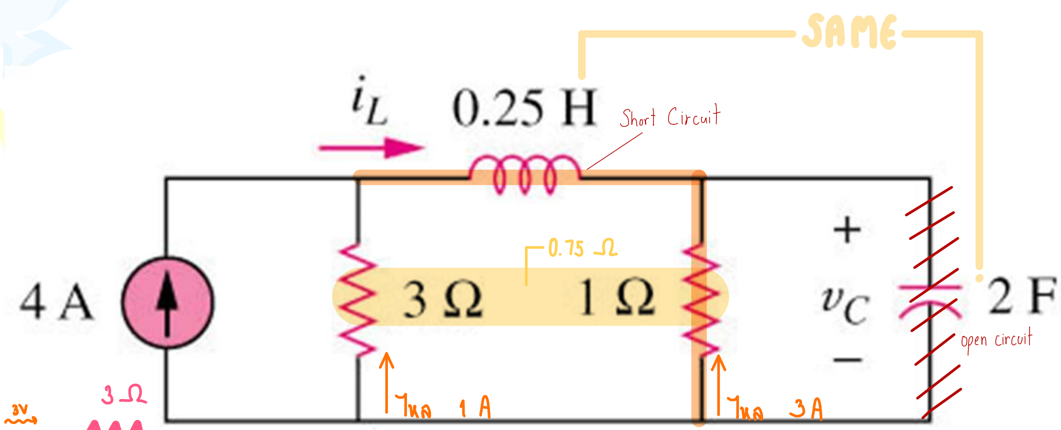
(a)

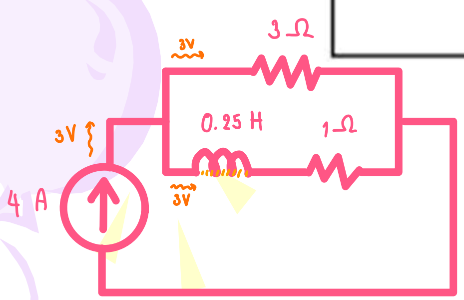
6.3 Inductors (9)



Example 6

Determine v_C, i_L, and the energy stored in the capacitor and inductor in the circuit of circuit shown below under dc conditions.





Answer:

i_L = 3A

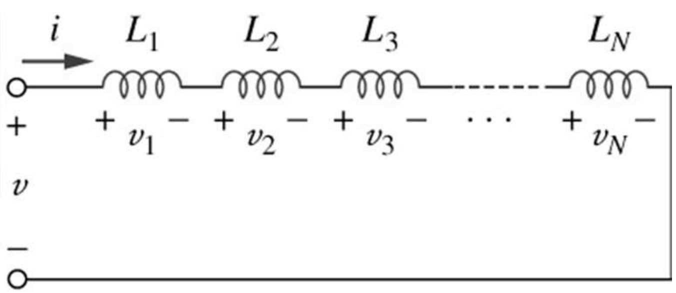
v_C = 3V

w_L = 1.125J

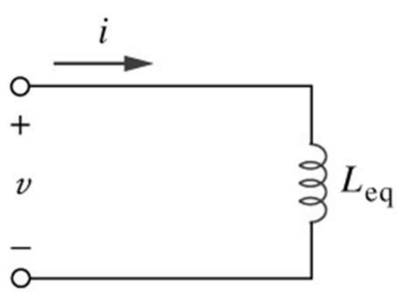
w_C = 9J

6.4 Series and Parallel Inductors (1)

- The equivalent inductance of **series-connected** inductors is the sum of the individual inductances.



(a)

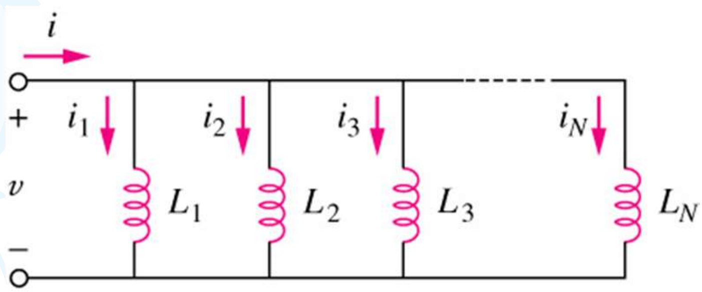


(b)

$$L_{eq} = L_1 + L_2 + \ldots + L_N$$

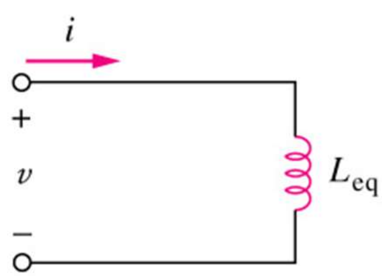
6.4 Series and Parallel Inductors (2)

- The equivalent capacitance of **parallel** inductors is the reciprocal of the sum of the reciprocals of the individual inductances.



(a)

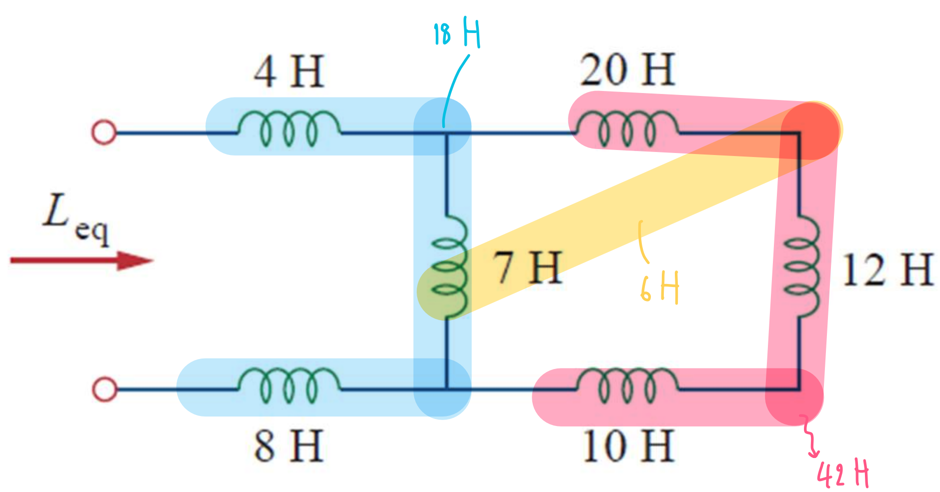
$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots + \frac{1}{L_N}$$



(b)

6.4 Series and Parallel Capacitors (3)

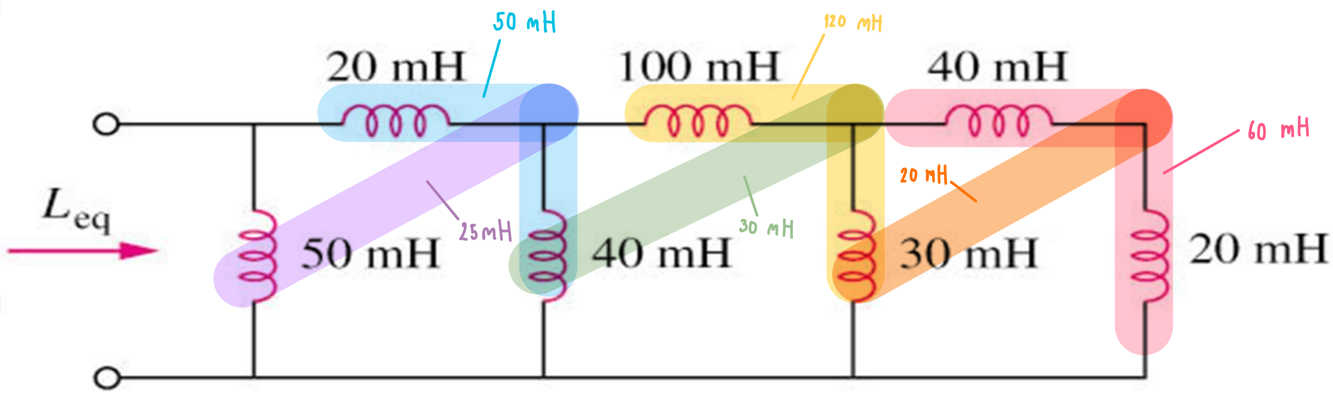
Find the equivalent inductance of the circuit



6.4 Series and Parallel Capacitors (3)

Example 7

Calculate the equivalent inductance for the inductive ladder network in the circuit shown below:

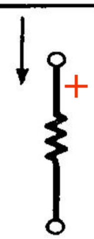


Answer:

L_{eq} = 25mH

6.4 Series and Parallel Capacitors (4)

- Current and voltage relationship for R, L, C

Circuit element	Units	Voltage	Current	Power
 **Resistance**	ohms (Ω)	$v = Ri$ **(Ohm's law)**	$i = \frac{v}{R}$	$p = vi = i^2 R$
 **Inductance**	henries (H)	$v = L\frac{di}{dt}$	$i = \frac{1}{L}\int v\,dt + k_1$	$p = vi = Li\frac{di}{dt}$
 **Capacitance**	farads (F)	$v = \frac{1}{C}\int i\,dt + k_2$	$i = C\frac{dv}{dt}$	$p = vi = Cv\frac{dv}{dt}$