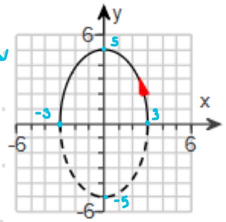
# Homework Calculas

(Week 1)

**10.1.7** Given parametric equations and parameter intervals for the motion of a particle in xy - plane below, identify the particle's path by finding a Cartesian equation for it. Graph the Cartesian equation. Indicate the portion of the graph traced by particle and direction of motion.

$$x = \underbrace{3}_{} \cos(t), y = \underbrace{5}_{} \sin(t) \quad ; 0 \le t \le \pi$$

แกน x  แกน y




$\boxed{\cos^2 t + \sin^2 t = 1}$ ------- สมการวงกลม

$\rightarrow \frac{x^2}{9} + \frac{y^2}{25} = 1$ Ans.

$x = 3\cos(t) \xrightarrow[\text{ยกกำลัง 2}]{} x^2 = 9\cos^2 t$

$y = 5\sin(t) \longrightarrow y^2 = 25\sin^2 t$

} จัดรูปใหม่

**10.1.19** Match the given parametric equations with one of the parametric curves.

$$x = 3 - \sin t, y = 5 + \cos t$$

$\sin t = 3 - x \qquad \cos t = y - 5$

$\sin^2 t = (3-x)^2 \qquad \cos^2 t = (y-5)^2$

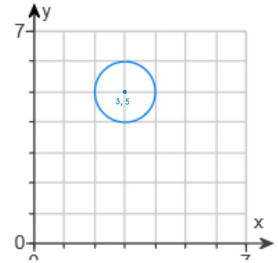
$\boxed{\cos^2 t + \sin^2 t = 1}$

หรือ $(x-h)^2 + (y-k)^2 = 1$

$\therefore (3-x)^2 + (y-5)^2 = 1$

$(x-3)^2$

พิกัด $(x,y) = (3,5)$



**10.1.35** Find a parametrization for the ray (half line) with initial point $(2,4)$ when $t = 0$ and $(-2,1)$ when $t = 1$.

$x_0, y_0$

| | พิกัด $x$ | พิกัด $y$ |
|---|---|---|
| $t = 0$ ; | 2 | 4 |
| $t = 1$ ; | -2 | 1 |

$x = x_0 + at$ —— (1)

$y = y_0 + bt$ —— (2)

(1); $-2 = 2 + a(1) \rightsquigarrow a = -4$

(2); $1 = 4 + b(1) \rightsquigarrow b = -3$

Ans $\begin{cases} (1) : x = 2 - 4t \\ (2) : y = 4 - 3t \end{cases}$

**10.2.6** Find an equation for the line tangent to the curve at the point defined by the given value of t. Also, find the value of $\frac{d^2y}{dx^2}$ at this point.

$$x = \sec^2 t - 1, y = \cos t \quad ; t = -\frac{\pi}{3} \rightsquigarrow 60^\circ$$

$y - y_0 = \underbrace{m}_{\frac{dy}{dx}}(x - x_0)$

แทนค่า $t = \frac{-\pi}{3}$ :: $x_0 = \sec^2\left(\frac{-\pi}{3}\right) - 1 = 4$

$y_0 = \cos\left(\frac{-\pi}{3}\right) = \frac{1}{2}$

$\frac{dx}{dt} = 2(\sec t)(\tan t \sec t)$

$\frac{dy}{dt} = -\sin t$

$\frac{dy}{dx} = \frac{-\cancel{\sin t}}{2\underbrace{\sec^2 t}_{\frac{1}{\cos^2 t}}\underbrace{\tan t}_{\frac{\cancel{\sin t}}{\cos t}}} = \frac{-\cos^3 t}{2} = \frac{-\cos^3\left(\frac{-\pi}{3}\right)}{2} = -\frac{1}{16}$

$\frac{d^2y}{dx^2} = \frac{-3\cos^2 t(-\sin t)}{2}$

$y - \frac{1}{2} = -\frac{1}{16}(x - 3)$

$y = \frac{-x}{16} + \frac{3}{16} + \frac{1}{2}$

$= \frac{-x}{16} + \frac{11}{16}$ Ans

want $\frac{d^2y}{dx^2}$ diff 2 ครั้ง ครั้งเดียว $\longrightarrow \frac{\frac{-3\cos^2 t(-\cancel{\sin t})}{2}}{2\underbrace{\sec^2 t}_{\frac{1}{\cos^2 t}}\underbrace{\tan t}_{\frac{\cancel{\sin t}}{\cos t}}} = \frac{3\cos^5 t}{4} = \frac{3}{128}$ Ans.

$\left(\frac{1}{2}\right)^5 = \frac{1}{32}$

**10.2.23** Find the area enclosed by the given ellipse.

$$x = a\cos t, y = b\sin t \quad ; 0 \leq t \leq 2\pi$$

$ab\pi$

**10.2.27** Find the length of the curve.

$$x = \frac{t^2}{2}, y = \frac{(2t+1)^{3/2}}{3} \quad ; 0 \leq t \leq 2$$

$f'(x) = t$ $\quad$ $g'(x) = (2t+1)^{\frac{1}{2}}$

$$L = \int_a^b \sqrt{(f'(x))^2 + (g'(x))^2}\, dt$$

$$= \int_0^2 \sqrt{(t)^2 + (2t+1)^{\frac{1}{2}\cdot 2}}\, dt$$

$$= \int_0^2 \sqrt{t^2 + 2t + 1}\, dt$$

$$= \int_0^2 \sqrt{(t+1)^2}\, dt$$

$$= \int_0^2 t + 1\, dt$$

$$= \frac{t^2}{2} + t \Big|_0^2$$

$$= \frac{(2)^2}{2} + 2 + 0$$

$= 4$ Ans.

**10.2.31** Find the area of the surface generated by revolving the curve $x = \frac{1}{4}\cos(4t), y = 6 + \frac{1}{4}\sin(4t)$ on $0 \le t \le \frac{\pi}{4}$ about the x – axis.

$$S = \int_a^b 2\pi y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$x = \frac{1}{4}\cos(4t) \rightsquigarrow \frac{dx}{dt} = -\sin(4t)$

$y = 6 + \frac{1}{4}\sin(4t) \rightsquigarrow \frac{dy}{dt} = \cos(4t)$

$$= \int_0^{\pi/4} 2\pi\left(6 + \frac{1}{4}\sin(4t)\right)\sqrt{\underbrace{(-\sin 4t)^2 + (\cos 4t)^2}_{\sin^2 4t + \cos^2 4t = 1}}\, dt$$

$\sin^2 4t + \cos^2 4t = 1 \; ; \; \sin^2 u + \cos^2 u = 1$

$$= \int_0^{\pi/4} 12\pi + \frac{1}{2}\pi\sin(4t)\, dt$$

$$= \int_0^{\pi/4} 12\pi\, dt + \int_a^b \frac{\pi}{2}\sin(\underbrace{4t}_{u})\, dt$$

$$= 12\pi t\Big|_0^{\pi/4} + -\frac{\pi}{8}\cos(4t)\Big|_0^{\pi/4}$$

$$= \Big[\underbrace{\cancel{12}^{3}\pi\left(\frac{\pi}{\cancel{4}}\right) - (0)}_{3\pi^2}\Big] + \underbrace{\Big[-\frac{\pi}{8}\underbrace{\cos\left(\cancel{4}\cdot\frac{\pi}{\cancel{4}}\right)}_{(-1)} + \frac{\pi}{8}\underbrace{\cos 0}_{(1)}\Big]}_{\frac{2\pi}{8}}$$

∴ The area of the surface $= 3\pi^2 + \frac{\pi}{4}$ Ans.

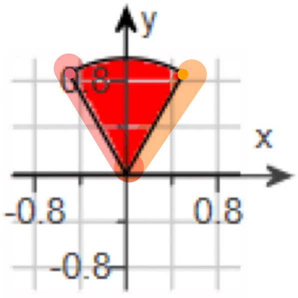
**10.3.10** Find the polar coordinates, $-\pi \le \theta \le 2\pi$ and $r \le 0$, of the following points given in Cartesian coordinates.

a. $(-1,0)$ b. $(2,0)$ c. $(0,-5)$ d. $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$r^2 = x^2 + y^2 \; ; \; \tan\theta = \frac{y}{x}$

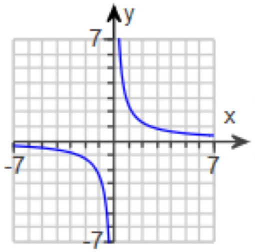
**10.3.23** Graph the sets of points whose polar coordinates satisfy the inequalities $\frac{\pi}{6} \le \theta \le \frac{5\pi}{6}$; $0 \le r \le 3$.

30 150




* เป็นแบบ ray line

**10.3.38** Convert the polar equation to a Cartesian equation. Then use a Cartesian coordinate system to graph the Cartesian equation.




$r^2 \sin 2\theta = 5$ —— โจทย์

$\sin 2\theta = 2\sin\theta\cos\theta$

* $r^2 = x^2 + y^2$ ; $r\cos\theta = x$ , $r\sin\theta = y$

$r^2 \sin 2\theta = 5$

$r^2\, 2\sin\theta\cos\theta = 5$

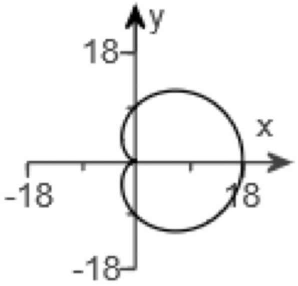
$2xy = 5$

$y = \frac{5}{2x}$ Ans.

# Homework Calculas

(Week 2)

**10.4.1** Identify the symmetries of the curve $r = 9 + 9\cos\theta$. Then sketch the curve.




curve symmetric about the x-axis

the point $(r, -\theta)$ or $(-r, \pi - \theta)$

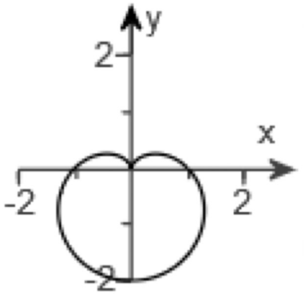
SAME

**10.4.3** Identify the symmetries of the curve $r = 1 - \sin\theta$. Then sketch the curve.

curve symmetric about the y-axis

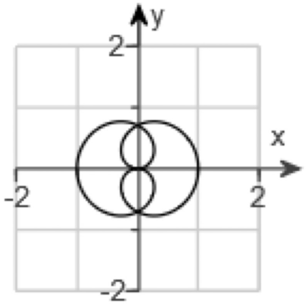
the point $(r, \theta)$ or $(r, \pi - \theta)$

SAME




**10.4.7** Identify the symmetries of the curve below. Then sketch the curve.

$$r = \sin\left(\frac{\theta}{2}\right)$$



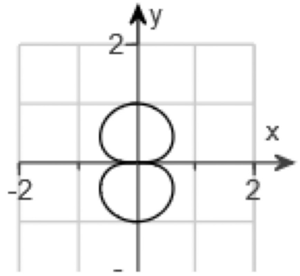

curve symmetric about the x-axis

the y-axis

the origin

**10.4.11** Identify the symmetries of the curve below. Then sketch the curve.

$$r = -\sin(\theta)$$



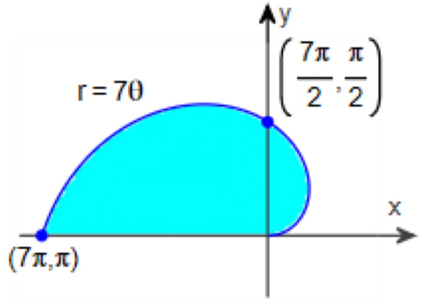

curve symmetric about the x-axis

the y-axis

the origin

**10.5.1** Find the area of the region bounded by the spiral $r = 7\theta$ for $0 \leq \theta \leq \pi$.

$$\frac{49\pi^3}{6}$$




**10.5.7** Find the area inside one loop of the lemniscate $r^2 = 9\sin 2\theta$

$A = \int_{\alpha}^{\beta} \frac{1}{2} r^2 \, d\theta$

$= \int_{0}^{\frac{\pi}{2}} \frac{1}{2} (9 \sin 2\theta) \, d\theta$

$= \frac{9}{2} \int_{0}^{\frac{\pi}{2}} \sin 2\theta \, d\theta$

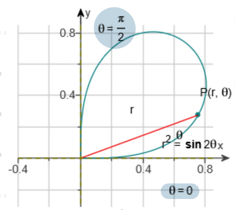
$= \frac{9}{2} \cdot \boxed{\left. \frac{-\cos(2x)}{2} \right|_{0}^{\frac{\pi}{2}}} \, d\theta$

$\frac{-\cos(2 \cdot \frac{\pi}{2})}{2} - \frac{-\cos(0)}{2}$

$= \frac{-(-1)}{2} - \frac{(-1)}{2}$

$= \frac{1}{2} + \frac{1}{2}$

$= 1$




Example.

**10.5.27** Find the length of the curve $r = \cos^3(\theta/3)$, $0 \le \theta \le \pi/4$.

$$\frac{dr}{d\theta} = 3\cos^2\left(\frac{\theta}{3}\right) \cdot -\sin\left(\frac{\theta}{3}\right) \cdot \left(\frac{1}{3}\right)$$

$$= -\cos^2\left(\frac{\theta}{3}\right) \sin\left(\frac{\theta}{3}\right)$$

$$L = \int_\alpha^\beta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

$$= \int_0^{\pi/4} \sqrt{\left(\cos^3\left(\frac{\theta}{3}\right)\right)^2 + \left(-\cos^2\left(\frac{\theta}{3}\right)\sin\left(\frac{\theta}{3}\right)\right)^2}\, d\theta$$

$$= \cos^6 x + \cos^4 x \sin^2 x \quad ; \; x = \frac{\theta}{3}$$

$$= \cos^4 x \left(\underbrace{\cos^2 x + \sin^2 x}_{1}\right)$$

$$= \int_0^{\pi/4} \sqrt{\cos^4 x}$$

$$= \int_0^{\pi/4} \cos^2\left(\frac{\theta}{3}\right)$$

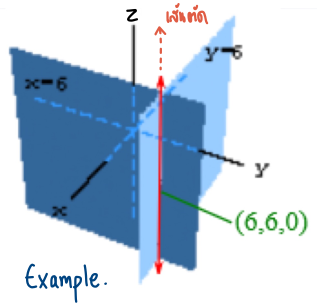
$$= \frac{1\theta}{2} + \frac{3\sin\left(\frac{2\theta}{3}\right)}{4} \Bigg|_0^{\pi/4}$$

$$= \left[\frac{1}{2}\left(\frac{\pi}{4}\right) + \frac{3\sin\left(\frac{2}{3}\cdot\frac{\pi}{4}\right)}{4}\right] - \left[\cancel{\frac{1}{2}\cdot 0} + \cancel{\frac{3\sin\left(\frac{2}{3}\cdot 0\right)}{4}}\right]$$

$$= \frac{\pi}{8} + \frac{3\sin(\pi/6)}{4}$$

**11.1.1** Give a geometric description of the set of set of points in space whose coordinates satisfy the given pair of equations.

$x = 9, y = 8$




Example.

The line through the point (9,8,0) parallel to the z-axis.

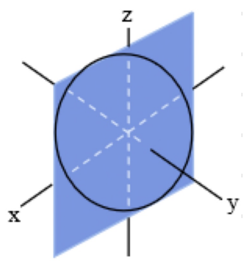
**11.1.3** Give a geometric description of the set of set of points in space whose coordinates satisfy the given pair of equations.

$y = 0, z = 0$

The x-axis Ans.

**11.1.7** Give a geometric description of the set of set of points in space whose coordinates satisfy the given pair of equations.

$$x^2 + z^2 = 16 \; (r^2), \; y = 3$$




The circle with center (0,3,0) and radius 4, parallel to the xz-plane.

# Homework Calculas

(Week 3)

**11.2.25** Express the vector $6i + 7j - 6k$ as a product of its length and direction.

$$|V| \frac{V}{|V|} \longrightarrow 11\left[\left(\frac{6}{11}\right)i + \left(\frac{7}{11}\right)j + \left(-\frac{6}{11}\right)k\right]$$

$$\sqrt{6^2 + 7^2 + (-6)^2}$$

$$\sqrt{36 + 49 + 36}$$

$$\sqrt{121} = 11$$

**11.2.39** If $\overline{AB} = i + 8j - 3k$ and B is the point (6,4,3). find A.

$$\overline{AB} = (B_x - A_x)i + (B_y - A_y)j + (B_z - A_z)k$$

$$i + 8j - 3k = (6 - A_x)i + (4 - A_y)j + (3 - A_z)k$$

$1 = 6 - A_x$

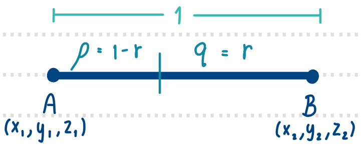
$A_x = 5$

$4 - A_y = 8$ $\quad$ $3 - A_z = -3$

$A_y = -4$ $\quad$ $A_z = 6$

$\therefore$ The point A is $\langle 5, -4, 6 \rangle$

**11.2.52** Use similar triangles to find the coordinates of the point Q that divides the segment from $P_1(x_1, y_1, z_1)$ to $P_2(x_2, y_2, z_2)$ into two lengths whose ratio is $\frac{p}{q} = r$




$$\left(\frac{x_1 + r x_2}{1+r}\right), \left(\frac{y_1 + r y_2}{1+r}\right), \left(\frac{z_1 + r z_2}{1+r}\right)$$

**11.3.1** Find the following for the vectors $u = 7i - 7j + \sqrt{2}k$ and $v = \cancel{7i+}\ {-7i} + 7j - \sqrt{2}k$

a. $v \cdot u$ , $|v|$ and $|u|$

b. the cosine of the angle between v and u

c. the scalar component of u in the direction of v

d. the vector $\text{proj}_v u$

$$v \cdot u = v_x u_x + v_y u_y + v_z u_z$$
$$= (7 \cdot (-7)) + ((-7) \cdot 7) + (\sqrt{2} \cdot (-\sqrt{2}))$$
$$= -49 - 49 - 2$$
$$= -100$$

$$|u| = \sqrt{7^2 + (-7)^2 + (\sqrt{2})^2}$$
$$= \sqrt{49 + 49 + 2} = 10$$

$$|v| = \sqrt{(-7)^2 + 7^2 + (-\sqrt{2})^2}$$
$$= \sqrt{49 + 49 + 2} = 10$$

$$\cos\theta = \frac{v \cdot u}{|v| \cdot |u|}$$
$$= \frac{-100}{10 \cdot 10}$$
$$= -1$$

$$\frac{u \cdot v}{|v|} = \frac{-100}{10} = -10$$

$$\text{proj}_v u = \left(\frac{u \cdot v}{|v|^2}\right) v$$
$$= \left(\frac{-100}{10^2}\right)(-7i + 7j - \sqrt{2}k)$$

**11.3.11** Find the angle between the vectors $u = \sqrt{3}i - 4j$ and $v = \sqrt{3}i + j - 4k$.

$$\theta = \cos^{-1}\left(\frac{u \cdot v}{|u||v|}\right)$$

$$= \cos^{-1}\left(\frac{-1}{\sqrt{19}\sqrt{20}}\right)$$

$$= 1.62$$

$$u \cdot v = (\sqrt{3}\sqrt{3}) + ((-4)(1)) + ((0)(-4))$$

$$= 3 + (-4) + 0$$

$$= -1$$

$$|u| = \sqrt{(\sqrt{3})^2 + (-4)^2}$$

$$= \sqrt{3 + 16}$$

$$= \sqrt{19}$$

$$|v| = \sqrt{(\sqrt{3})^2 + (1)^2 + (-4)^2}$$

$$= \sqrt{3 + 1 + 16}$$

$$= \sqrt{20}$$

**11.3.20** Suppose that AB is the diameter of a circle with center O and that C is a point on one of two arcs joining A and B. Show that $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are orthogonal.

**11.4.1** Find the length and direction (when defined) of $u \times v$ and $v \times u$.

**11.4.18** Find the area of the triangle determined by the points P, Q and R. Find a unit vector perpendicular to plane PQR.

$$P(-1,1,2)\ ,\ Q(2,0,1)\ ,\ R(0,2,-1)$$

**11.4.23** Let $\mathbf{u} = -14\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$, $\mathbf{v} = 7\mathbf{i} - \mathbf{j} + \mathbf{k}$, $\mathbf{w} = \mathbf{j} - 7\mathbf{k}$. Which vector, if any are (a) perpendicular? (b) Parallel?

**11.4.25** Find the magnitude of the torque exerted by F on the bolt at P if $|\overrightarrow{PQ}| = 4$ cm. and $|\mathbf{F}| = 42$ N.

# Homework Calculas

(Week 4)

**11.5.3** A line passes through the points P(-2,-9,3) and Q(-9,7,-3). Find the standard parametric equation for the line, written using the base point P(-2,-9,3) and the components of the vector $\overrightarrow{PQ}$.

**11.5.13** Find a parametrization for the line segment joining the points P(0,0,0) and $Q\left(10,8,\frac{9}{10}\right)$. Draw coordinate axes and sketch the segment, indicating the direction of increasing t for the parametrization.

**11.5.21** Find the equation for the plane through the point $P_0 = (5,4,7)$ and normal to the vector $n = 2i + 10j + 6k$

**11.5.25** Find the equation for the plane through $P_0 = (-7,-3,5)$ perpendicular to the following line.

$$x = -7 + t,\ \ y = 3 - 2t,\ \ z = -3t \qquad ;\ -\infty < t < \infty$$

**11.5.29** Find the plane determined by the intersecting lines.

$$L1 : x = -1 + t \qquad y = 2 + 3t \qquad z = 1 - 2t$$

$$L2 : x = 1 + 4s \qquad y = 1 + 2s \qquad z = 2 - 2s$$

**11.5.45** Find the distance from the plan $7x + 9y + 9z = 56$ to the plane $7x + 9y + 9z = 14$.

**11.5.47** Find the angle between the planes $4x + 5y = 3$ and $2x + 3y + 2z = -4$

**11.5.65** Given two lines in space, either they are parallel, they intersect, or they are skew (line in parallel planes). Determine whether the lines below, taken two at a time, are parallel, intersect, or are skew. If they intersect, find the point of intersection. Otherwise, find the distance between the two lines.

**11.5.71** Is the line $x = -9 - 4t$, $y = -7 + 8t$, $z = -7t$ parallel to the plane $2x+y-z = 3$? Given reasons for your answer.

**11.5.73** Find two different planes whose intersection is the line $x = -3 - 3t$, $y = -2 + t$, $z = -2 + 3t$. Write equations for each plane in the form $Ax + By + Cz = D$

# Homework Calculas

(Week 5)

**12.1.** Find the limit or indicate that it does not exist.

$$\lim_{t\to\pi}\left[\left(\sin\frac{5}{4}t\right)i+\left(\cos\frac{7}{6}t\right)j+\left(\tan\frac{t}{4}\right)k\right]$$

**12.1.9** The path $r(t) = (4\sin t)i + (4\cos t)j$ describes motion on the circle $x^2 + y^2 = 16$. Find the particle's velocity and acceleration vector at $t = \frac{\pi}{4}$ *and* $\frac{\pi}{2}$ and sketch them as vectors on the curve.

**12.1.15** The equation $r(t) = (4\cos t)i + (6\sin t)j + (6)k$ is the position of a particle in space at time t. Find the particle's velocity and acceleration vector. Then write the particle's velocity at $t = 2\pi$ as a product of its speed and direction.

**12.1.25** Find parametric equations for the line that is tangent to the given curve at the given parameter value.

$$r(t) = (\ln t)i + \left(\frac{t-1}{t+2}\right)j + (t\ln t)k \qquad ; t = t_0 = 1$$

**12.1.31** The vector function r(t) is the position of a particle in space at time t. Determine the position function.

$$r(t) = (2t\cos t)i + (2t\sin t)j + tk$$

**12.2.1** Evaluate the integral $\int_0^1 [(t^2)i + (6)j + (3t+4)k]\,dt$.

**12.2.9** Evaluate the integral.

$$\int_{0}^{\pi/2} [2\cos 5t\ i - 3\sin 2t\ j + \ sin^2 7t\ k]\ dt$$

**12.2.17** Solve the initial value problem for r as a vector function of t.

Differential equation : $\frac{d^2r}{dt^2} = \ -18k$

Initial conditions : $r(0) = 90k$ and $\left.\frac{dr}{dt}\right|_{t=0} = 15i + 15j$

**12.2.25** A projectile is fired with an initial speed of 550 m/s at an angle of elevation of 30°. Answer parts (a) through (d) below.

**12.2.39** A model train engine was moving at a constant speed on a straight horizontal track. As the engine moved along, a marble was fired into the air by a spring in the engine's smokestack. The marble, which continued to move with the same forward speed as the engine, rejoined the engine 1s after it was fired. The measure of the angle the marble's path made with the horizontal was 58°. Use the information to find how high the marble went and how fast the engine was moving

a.

$$t = \frac{2V_0 \sin\alpha}{g}$$

$$1 = \frac{2(V_0 \sin\alpha)}{32} \quad ; \; g = 32 \text{ N/ft}$$

$$V_0 \sin\alpha = 16 \longrightarrow V_0 = \frac{16}{\sin\alpha}$$

$$y_{max} = \frac{(V_0 \sin\alpha)^2}{2g} = \frac{(16)^2}{2 \times 32} = 4 \text{ ft.}$$

b.

$$R = \frac{V_0^2}{g}\sin 2\alpha$$

$$= \frac{V_0^2}{g}\, 2\sin\alpha\cos\alpha \quad (58^\circ,\ g = 32)$$

$$= \frac{\left(\frac{16}{\sin 58^\circ}\right)^2 2\sin 58^\circ \cos 58^\circ}{32}$$

$$= \frac{\left(\cancel{\frac{16}{\sin 58^\circ}}\right)\left(\frac{16}{\sin 58^\circ}\right)\cancel{2\sin 58^\circ}\cos 58^\circ}{\cancel{32}}$$

$$= 16 \underbrace{\cot 58^\circ}_{}\ (0.12)$$

$$= 1.92$$

# Homework Calculas

(Week 6)

**12.3.5** Find the curve's unit tangent vector. Also, find the length of the indicated portion of the curve.

$$r(t) = (3cos^3 t)j + (3sin^3 t)k \qquad ; 0 \le t \le \frac{\pi}{6}$$

$v(t) = (-9\cos^2 t \sin t)j + (9\sin^2 t \cos t)k$

$|v| = \sqrt{(-9\cos^2 t \sin t)^2 + (9\sin^2 t \cos t)^2}$

$= \sqrt{81\cos^4 t \sin^2 t + 81\sin^4 t \cos^2 t}$

$= \sqrt{81\cos^2 t \sin^2 t \underbrace{(\cos^2 t + \sin^2 t)}_{1}}$

$= 9\cos t \sin t$

$T = \frac{v}{|v|}$

$= \frac{(-9\cos^2 t \sin t)j + (9\sin^2 t \cos t)k}{9\cos t \sin t}$

$= (-\cos t)j + (\sin t)k$

$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\, dt$

$= \int_0^{\pi/6} 9\cos t \sin t \, dt$

$= \frac{9}{2}\sin^2 t \Big|_0^{\pi/6}$

$= \frac{9}{2}\sin^2\left(\frac{\pi}{6}\right) - \frac{9}{2}\sin^2(0)$

$= \frac{9}{2}\left(\frac{1}{2}\right)^2 - \frac{9}{2}(0)^2$

$= \frac{9}{8}$

**12.3.7** Find the curve's unit tangent vector. Also, find the length of the indicated portion of the curve.

$2t(-\sin t) + \cos t(2)$ $\qquad$ $\frac{\cancel{4}^{2}\sqrt{2}}{\cancel{3}} \cdot \frac{\cancel{3}}{\cancel{2}} t^{1/2} = 2\sqrt{2t}$

$$r(t) = (2t\cos t)i + (-2t\sin t)j + \left(\frac{4\sqrt{2}}{3}\right)t^{3/2}k \qquad ; 0 \le t \le \pi$$

$-2t(\cos t) + \sin t(-2)$

$v(t) = (-2t\sin t + 2\cos t)i + (-2t\cos t - 2\sin t)j + 2\sqrt{2t}\,k$

$|v| = \sqrt{(-2t\sin t + 2\cos t)^2 + (-2t\cos t - 2\sin t)^2 + (2\sqrt{2t})^2}$

$= \sqrt{4t^2\sin^2 t - 8t\sin t\cos t + 4\cos^2 t) + (4t^2\cos^2 t + 8t\sin t\cos t + 4\sin^2 t) + 8t^2}$

$= \sqrt{4t^2\underbrace{(\sin^2 t + \cos^2 t)}_{1} + 4\underbrace{(\cos^2 t + \sin^2 t)}_{1} + 8t}$

$= \sqrt{4t^2 + 8t + 4}$

$= \sqrt{(2t+2)^2}$

$= 2t + 2 \longrightarrow 2(t+1)$

$T = \frac{v}{|v|}$

$= \frac{(-2t\sin t + 2\cos t)i + (-2t\cos t - 2\sin t)j + 2\sqrt{2t}\,k}{2(t+1)}$

$= \frac{(-t\sin t + \cos t)i}{t+1} + \frac{(-t\cos t - \sin t)j}{t+1} + \frac{\sqrt{2t}\,k}{t+1}$

**12.3.9** Find the point on the curve $r(t) = (5\sin t)i + (-5\cos t)j + 12tk$ at a distance $52\pi$ units along the curve from the point (0,-5,0) in the direction of increasing arc length.

S:ระยะทาง

$$r(t) = (5\sin t)i + (-5\cos t)j - 12tk$$
$$v(t) = (5\cos t)i + (5\sin t)j - 12k$$
$$|v| = 13$$

$$s(t) = \int_{t_0}^{t} |v(\tau)|\, d\tau$$
$$= \int_0^t 13\, d\tau$$
$$= 13\tau \Big|_0^t$$

$$s(t) = 13t$$

$$\therefore t = \frac{s}{13}$$ ใน time

แทนค่า

$$* \; r(t(s)) = (5\sin t)i + (-5\cos t)j - 12tk$$
$$= 5\sin\left(\frac{s}{13}\right)i - 5\cos\left(\frac{s}{13}\right)j - 12\left(\frac{s}{13}\right)k$$
$$r(52\pi) = 5\sin\left(\frac{\cancel{52}^{4}\pi}{\cancel{13}}\right)i - 5\cos\left(\frac{\cancel{52}^{4}\pi}{\cancel{13}}\right)j - 12\left(\frac{\cancel{52}^{4}\pi}{\cancel{13}}\right)k$$
$$= 0i - 5j - 48\pi k$$

**12.3.14** Find the arc length parameter along the curve from the point where t = 0 by evaluating the integral $s = \int_0^t |v(\tau)|\, d\tau$. Then find the length of the indicated portion of the curve.

$$r(t) = (6 + 2t)i + (4 + 3t)j + (9 - 3t)k \qquad ; -1 \le t \le 0$$

**12.3.18** To illustrate that the length of a smooth space curve does not depend on the parameterization used to compute it, calculate the length of one turn of the helix with the following parameterizations.

$$a.\ r(t) = (\cos 4t)i + (\sin 4t)j + 4tk \qquad ; 0 \leq t \leq \frac{\pi}{2}$$

$$b.\ r(t) = \left[\cos\left(\frac{t}{2}\right)\right] i + \left[\sin\left(\frac{t}{2}\right)\right] j + \frac{t}{2}k \qquad ; 0 \leq t \leq 4\pi$$

$$c.\ r(t) = (\cos t)i - (\sin t)j - tk \qquad ; -2\pi \leq t \leq 0$$

Note that the helix shown to the right is just one example of such a helix and does not exactly correspond to the parameterizations in parts a, b or c.

* หา $s(t) = \int_{t_0}^{t} |v(\tau)|\, d\tau \longrightarrow$ แทนค่าขอบเขตจะได้ arc length (L)

**12.4.13** Find T,N and $\kappa$ for the space curve $r(t) = \frac{t^4}{4}i + \frac{t^3}{3}j$ ; $t > 0$.

$T = \frac{V}{|V|}$ , $N = \frac{dT/dt}{|dT/dt|}$ , $\kappa = \frac{1}{|V|}\left|\frac{dT}{dt}\right|$

**12.4.20** The total curvature of the portion of a smooth curve that runs from $s = s_0$ $to$ $s_1 > s_0$ can be found by integrating $\kappa$ from $s_0$ $to$ $s_1$ If the curve has some other parameter, say t, then the total curvature is $K = \int_{s_0}^{s_1} \kappa\, ds = \int_{t_0}^{t_1} \kappa \frac{ds}{dt}\, dt = \int_{t_0}^{t_1} \kappa |v|\, dt,$ where $t_0$ and $t_1$ correspond to $s_0$

and $s_1$

a. Find the total curvature of the portion of the helix $r(t) = (2\cos t)i + (2\sin t)j + tk$ ; $0 \le t \le 4\pi$

b. Find the total curvature of the parabola $y = 5x^2$ ; $-\infty < x < \infty$

$r(t) = \overset{x}{t}i + \overset{y}{5x^2}j$

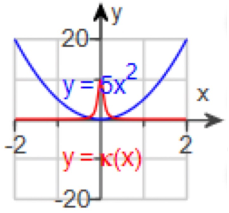
**12.4.21** Find an equation for the circle of curvature of the curve r(t) = 3t I + sin(t) j at the point $\left(\frac{3\pi}{2}, 1\right)$. (The curve parameterizes the graph of $y = \sin\left(\frac{1}{3}x\right)$ in the xy – plane.)

**12.4.23** The formula $\kappa(x) = \frac{|f''(x)|}{[1+(f'(x))^2]^{3/2}}$ expresses the curvature $\kappa(x)$ of a twice – differentiable plane curve y = f(x) as a function of x. Find the curvature function of the following curve. Then graph f(x) together with $\kappa(x)$ over the given interval.

$$f(x) = 5x^2 \quad ; -2 \le x \le 2$$

$$K(x) = \frac{|f''(x)|}{[1+(f'(x))^2]^{3/2}}$$

$$= \frac{10}{[1 + \underbrace{100x^2}_{\text{มาจาก } (10x)^2}]^{3/2}}$$



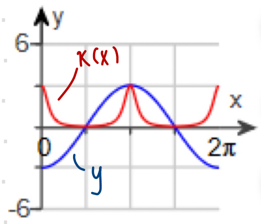

$f'(x) = 10x$
$f''(x) = 10$

**12.4.25** The formula $\kappa(x) = \frac{|f''(x)|}{[1+(f'(x))^2]^{3/2}}$ expresses the curvature of a twice – differentiable plane curve as a function of x. Find the curvature function of the curve. $y = -3\cos x$ ; $0 \le x \le 2\pi$. Then graph f(x) together with $\kappa(x)$ over the given interval.

$$K(x) = \frac{|f''(x)|}{[1+(f'(x))^2]^{3/2}}$$

$$= \frac{|3\cos(x)|}{[1+(3\sin(x))^2]^{3/2}}$$

$$= \frac{3\cos(x)}{[1+9\sin^2(x)]^{3/2}}$$




$y' = 3\sin x$
$y'' = 3\cos x$

# Homework Calculas

(Week 7)

**13.1.19** For the given function, complete parts (a) through (f) below.

$f(x,y) = 25x^2 + 9y^2$

(a) Find the function's domain. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

The domain is the entire xy-plane.

ทั้งหมด

(b) Find the function's range. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

The range is $[0, \infty)$

ค่าน้อยที่สุดที่เป็นไปได้

(c) Describe the function's level curves. Choose the correct answer below.

For $f(x,y) = 0$, the level curve is the origin. For $f(x,y) \neq 0$, the level curve are ellipses centered at the origin and major and major axes along the x, y-axes, respectively.

(d) Find the boundary of the function's domain. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

The are no boundary points.

(e) Determine if the domain is an open region, a closed region, or neither. Choose the correct answer below.

The domain is both open & closed

(f) Decide if the domain is bounded or unbounded. Choose the correct answer below.

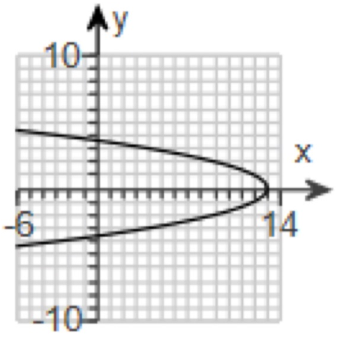
The domain is unbounded

**13.1.51** Find an equation for and sketch the graph of the level curve of the function f(x,y) that passes through the given point.

$f(x,y) = \sqrt{x + y^2 - 14}$ ; (6,3)

$$f(6,3) = \sqrt{6 + (3)^2 - 14} = \sqrt{6 + 9 - 14} = \sqrt{1} = 1$$

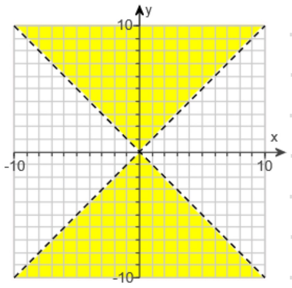
$$1 = \sqrt{x + y^2 - 14}$$
$$1^2 = x + y^2 - 14$$
$$15 = x + y^2$$ Ans.



**13.1.65** Find and sketch the domain of f. Then find an equation for the level curve or surface of the function passing through the given point.

$$f(x,y) = \sum_{n=0}^{\infty} \left(\frac{x}{y}\right)^n \quad ; (1,4)$$

y ต้องมากกว่า x ไม่งั้นจะเป็น $\infty$




$y = 4x$

$\frac{y}{x}$

**13.2.13** Find $\lim\limits_{(x,y)\to(1,10)} \dfrac{\overbrace{100x^2-20xy+y^2}^{(10x-y)^2}}{10x-y}$ ; $10x \neq y$ by rewriting the fraction first.

$= 10x - y$

แทน $x = 0$ แต่ $y \neq 0$ ; $y$

แทน $x \neq 0$ แต่ $y = 0$ ; $10x$

สรุปคือ $\underbrace{10x}_{\text{เข้าใกล้ } 1} - \underbrace{y}_{\text{เข้าใกล้ } 10} = 0$

$\therefore \lim\limits_{(x,y)\to(1,10)} = \dfrac{100x^2 - 20xy + y^2}{10x - y}$

$= 0$

**13.2.58** If $f(x_0, y_0) = 3$, what can you say about the limit below if f is continuous at $(x_0, y_0)$? If f is not continuous at $(x_0, y_0)$? Give reasons for your answers

$$\lim_{(x,y)\to(x_0,y_0)} f(x,y)$$

if it is continuous at $(x_0, y_0, z_0)$?

$$\lim_{(x,y)\to(x_0,y_0)} f(x,y) = f(x_0,y_0) = 3$$

if it is not continuous at $(x_0, y_0, z_0)$?

$$\lim_{(x,y)\to(x_0,y_0)} f(x,y) \neq f(x_0,y_0) \neq 3$$

**13.2.41** By considering different paths of approach, show that the function has no limit as (x,y)→(0,0).

$$f(x,y) = -\frac{x}{\sqrt{x^2+y^2}}$$

**13.2.40** At what points (x,y,z) in space is the function continuous?

a. $h(x,y,z) = \sqrt{\underbrace{3 - x - y - 5z}_{\geq 3}}$ — $\geq 0$

b. $h(x,y,z) = \dfrac{1}{16-\sqrt{x^2+y^2+z^2-25}}$ — $\neq 0$

ห้ามเป็น 16 และใน $\sqrt{\phantom{x}}$ ต้องมากกว่า 0

**13.3.16** Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for the following function.

$$f(x,y) = e^{4xy}\ln(3y)$$

$\frac{\partial f}{\partial x} = 4ye^{4xy}\ln(3y)$

ตัด x, y คงที่

Solⁿ $\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left(e^{4xy}\ln(3y)\right)$

$\frac{\partial f}{\partial y} =$

$= e^{4xy}\frac{\partial}{\partial x}(\ln(3y)) + \ln(3y)\frac{\partial}{\partial x}e^{4xy}$

$= e^{4xy}$

**13.3.48** Find all the second – order partial derivatives of the following function.

$$w = 5ye^{x^2-9y}$$

$$\frac{\partial^2 w}{\partial x^2} = \frac{\partial}{\partial x}(10xy)e^{x^2-9y}$$

ค่าคงที่

$$= 10xy\frac{\partial}{\partial x}e^{x^2-9y} + e^{x^2-9y}\frac{\partial}{\partial x}10xy$$

$$= 10xye^{x^2-9y}(2x) + e^{x^2-9y}10y$$

$$= 20x^2ye^{x^2-9y} + 10ye^{x^2-9y}$$

$$= (10y + 20x^2y)e^{x^2-9y}$$

$$\frac{\partial^2 w}{\partial y^2} = \frac{\partial}{\partial y}-45ye^{x^2-9y} + \frac{\partial}{\partial y}5e^{x^2-9y}$$

$$= \left(-45ye^{x^2-9y}(-9) + e^{x^2-9y}(-45)\right) + 5e^{x^2-9y}(-9)$$

$$= \left(405ye^{x^2-9y} - 45e^{x^2-9y}\right) - 45e^{x^2-9y}$$

$$= (405y - 90)e^{x^2-9y}$$

$$\frac{\partial^2 w}{\partial y \partial x}$$

เท่ากัน เนื่องจากทั้งสอง

เป็น Continuous

$$\frac{\partial^2 w}{\partial x \partial y}$$

$$\frac{\partial}{\partial y}10xye^{x^2-9y}$$

$$= 10xy\frac{\partial}{\partial y}e^{x^2-9y} + e^{x^2-9y}\frac{\partial}{\partial y}10xy$$

$$= 10xye^{x^2-9y}(-9) + 10xe^{x^2-9y}$$

$$= (10x - 9xy)e^{x^2-9y}$$

$$\frac{\partial}{\partial x}\left(5ye^{x^2-9y}\right)\, dx$$

$$= 5y\frac{\partial}{\partial x}\left(e^{x^2-9y}\right)$$

$$= 5ye^{x^2-9y}(2x)$$

$$\frac{\partial}{\partial y}\left(5ye^{x^2-9y}\right)\, dx$$

$$= 5y\frac{\partial}{\partial y}e^{x^2-9y} + e^{x^2-9y}\frac{\partial}{\partial y}5y$$

$$= 5ye^{x^2-9y}(-9) + 5e^{x^2-9y}$$

$$= -45ye^{x^2-9y} + 5e^{x^2-9y}$$

**13.3.77** Express A implicitly as a function of a, b and c and calculate $\frac{\partial A}{\partial a}$ and $\frac{\partial A}{\partial b}$.