Alexander-Sadiku
Fundamentals of Electric Circuits

Chapter 3
Methods of Analysis

Methods of Analysis - Chapter 3

3.1   Motivation

3.2   Nodal analysis.

3.3   Nodal analysis with voltage sources.
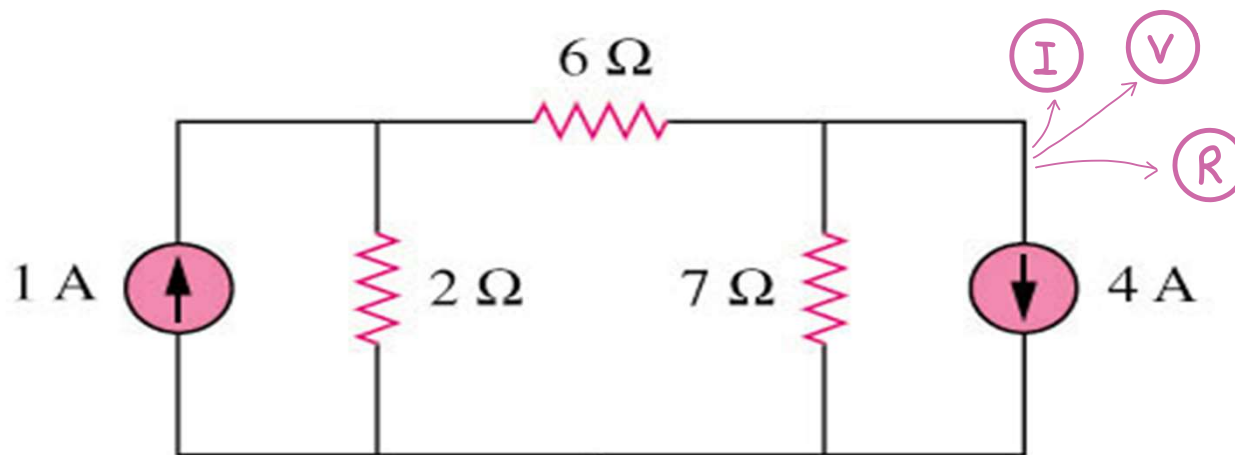
3.4   Mesh analysis.

3.5   Mesh analysis with current sources.

3.6   Nodal and mesh analysis by inspection.

3.7   Nodal versus mesh analysis.

3.1 Motivation (1)

If you are given the following circuit, how can we determine (1) the voltage across each resistor, (2) current through each resistor. (3) power generated by each current source, etc.

What are the things which we need to know in order to determine the answers?

3.1 Motivation (2)

Things we need to know in solving any resistive circuit with current and voltage sources only:

- Kirchhoff's Current Laws (KCL) ΣI
- Kirchhoff's Voltage Laws (KVL) ΣV
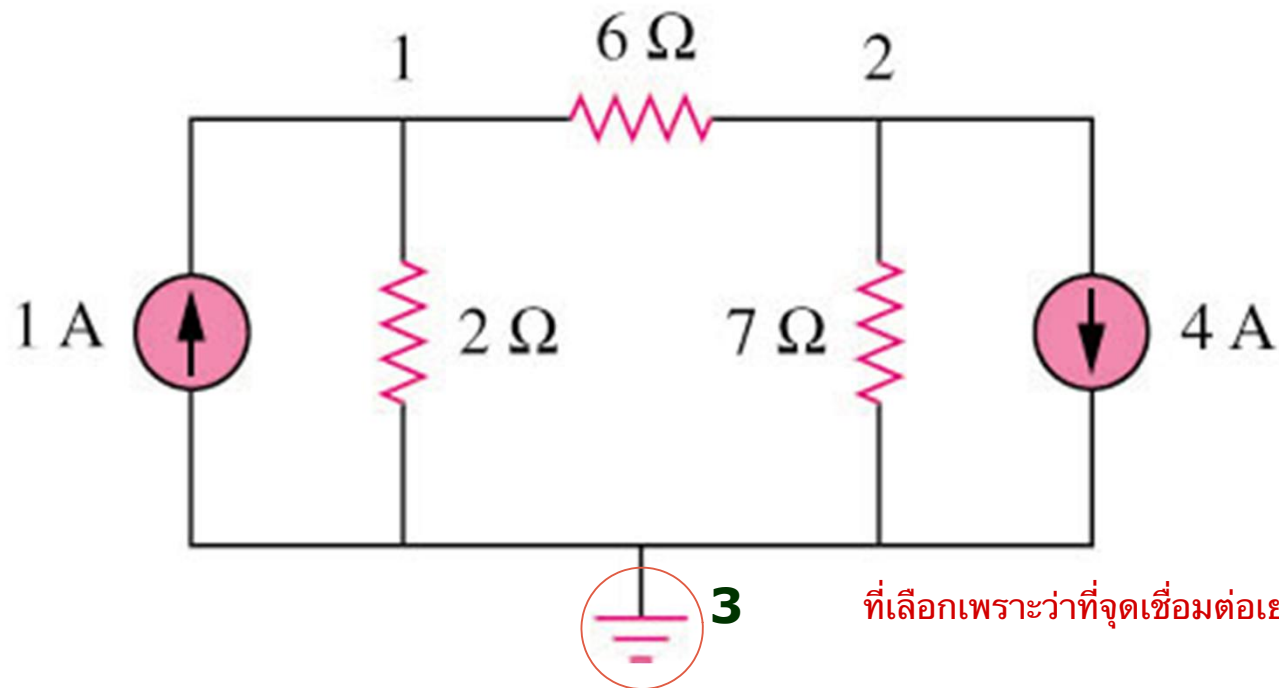
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- Ohm's Law $V = IR$

How should we apply these laws to determine the answers?

3.2 Nodal Analysis (1)

It provides a general procedure for analyzing circuits using <u>node voltages</u> as the circuit variables.

<u>Example 1</u>

เลือก NODE 3

ที่เลือกเพราะว่าที่จุดเชื่อมต่อเยอะ จะได้คิดง่ายขึ้น

3.2 Nodal Analysis (2)

Steps to determine the node voltages:

1. Select a node as the reference node.
2. Assign voltages v1,v2,…,vn-1 to the remaining n-1 nodes. The voltages are referenced with respect to the reference node.
3. Apply KCL to each of the n-1 non-reference nodes. Use Ohm's law to express the branch currents in terms of node voltages.
4. Solve the resulting simultaneous equations to obtain the unknown node voltages.
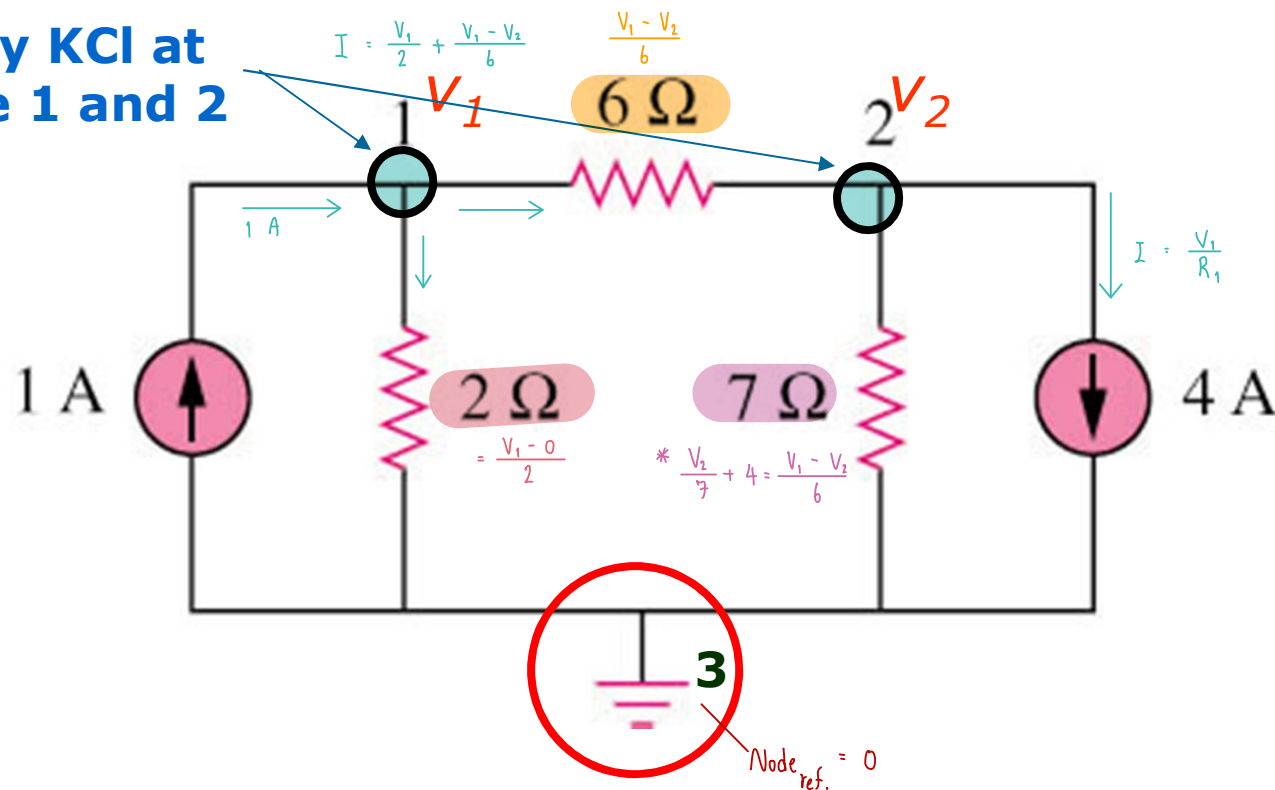
3.2 Nodal Analysis (3)
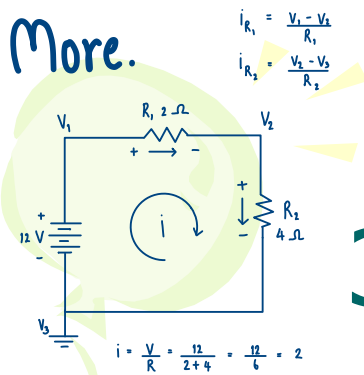
$V = IR, I = \frac{V}{R}$

Example 2 – circuit independent current source only

Apply KCl at node 1 and 2

$I = \frac{V_1}{2} + \frac{V_1 - V_2}{6}$

$\frac{V_1 - V_2}{6}$

V_1 6 Ω   2 V_2

1 A

$I = \frac{V_1}{R_1}$

2 Ω   7 Ω   4 A

$= \frac{V_1 - 0}{2}$

$* \frac{V_2}{7} + 4 = \frac{V_1 - V_2}{6}$

3

Node$_{ref.}$ = 0

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*Refer to in-class illustration, textbook, answer $v_1 = -2V$, $v_2 = -14V$

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More.

$$i_{R_1} = \frac{V_1 - V_2}{R_1}$$

$$i_{R_2} = \frac{V_2 - V_3}{R_2}$$

$$i = \frac{V}{R} = \frac{12}{2+4} = \frac{12}{6} = 2$$

Example 3 – current with dependant current source

(a)

(b)

$3 = i_x + i_1$　　$i_1 = \frac{\bar{V}_1 - \bar{V}_3}{4}$

$i_x = i_2 + i_3$　　$i_2 = \frac{\bar{V}_2 - \bar{V}_3}{8}$　　ความต่างศักย์ = $V_{สูง} - V_{ต่ำ}$

$i_1 + i_2 = 2i_x$　　$i_3 = \frac{\bar{V}_2 - 0}{4}$
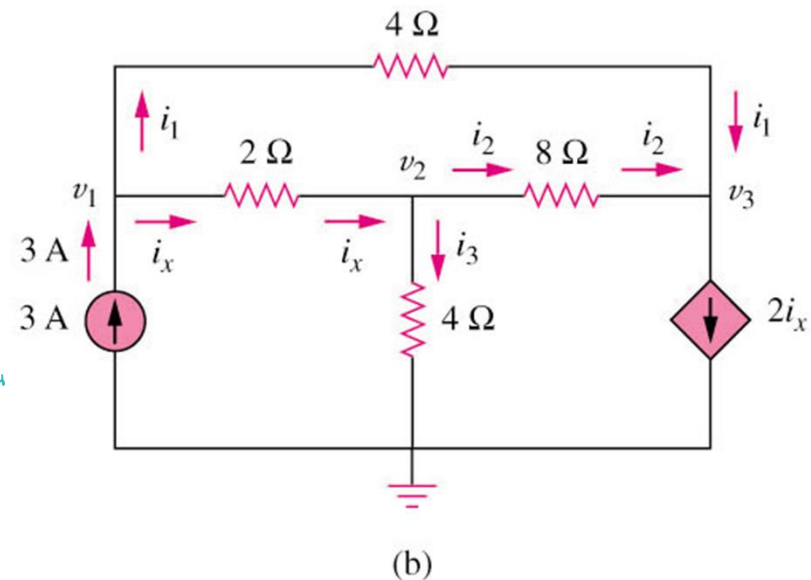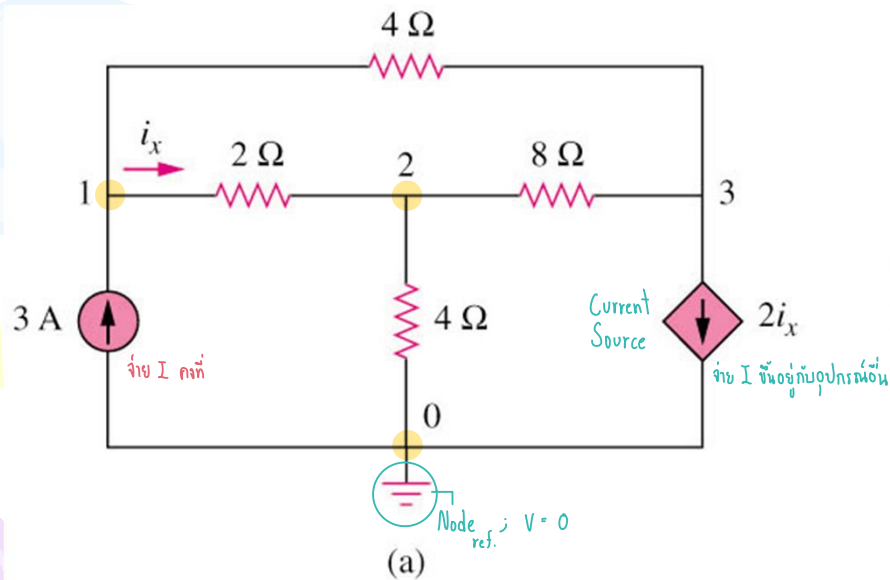
　　　　　　　$i_x = \frac{V_1 - V_2}{2}$

*Refer to in-class illustration, textbook,
answer v_1= 4.8V, v_2 = 2.4V, v_3 = -2.4V

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3.3 Nodal Analysis with Voltage Source (1)

Example 4 –circuit with independent voltage source
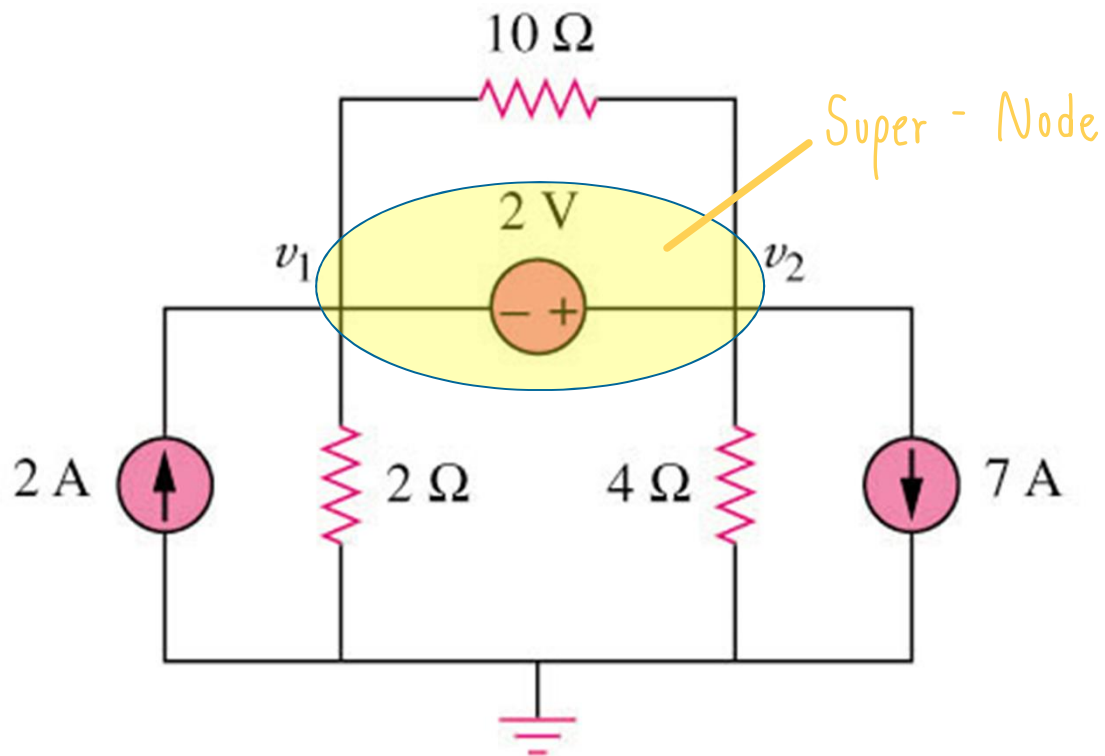
How to handle the 2V voltage source?

3.3 Nodal Analysis with Voltage Source (2)

A super-node is formed by <u>enclosing</u> a (dependent or independent) voltage source connected between two non-reference nodes and any elements connected in parallel with it.

*<u>Note</u>: We analyze a circuit with super-nodes using the same three steps mentioned above except that the super-nodes are treated differently.
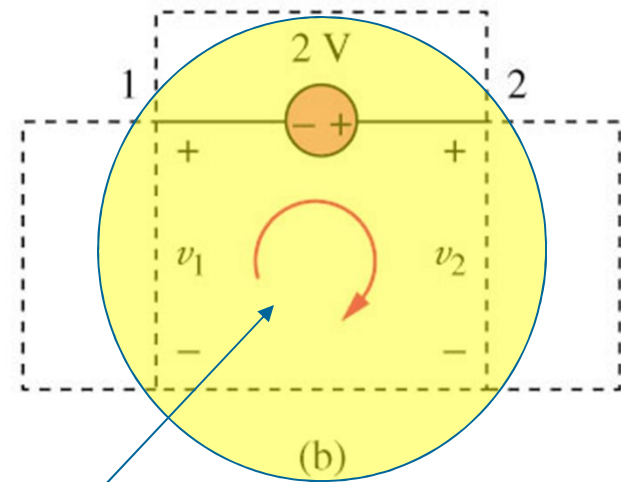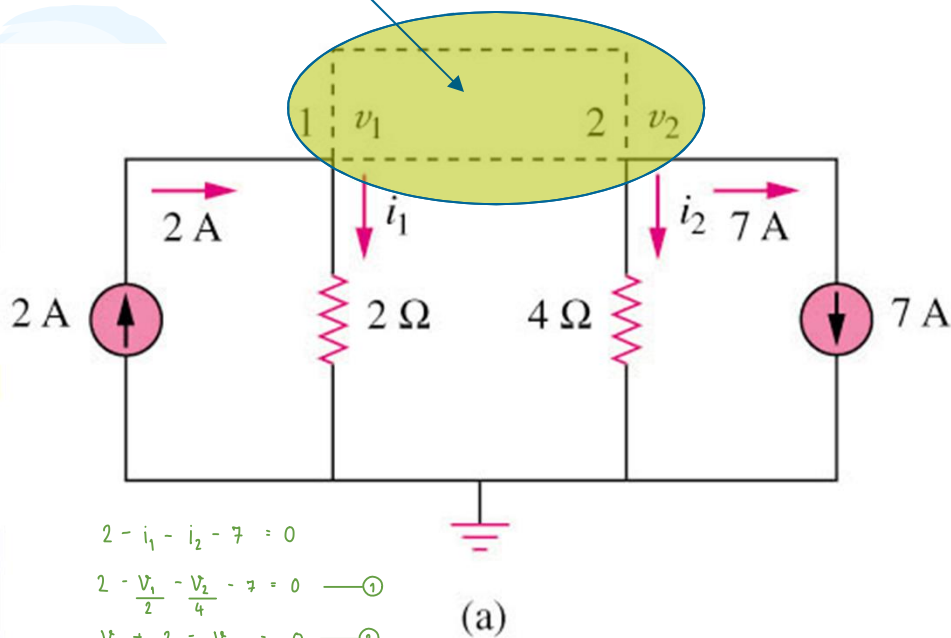
3.3 Nodal Analysis with Voltage Source (3)

Basic steps:

1. Take off all voltage sources in super-nodes and apply KCL to super-nodes.

2. Put voltage sources back to the nodes and apply KVL to relative loops.

3.3 Nodal Analysis with Voltage Source (4)

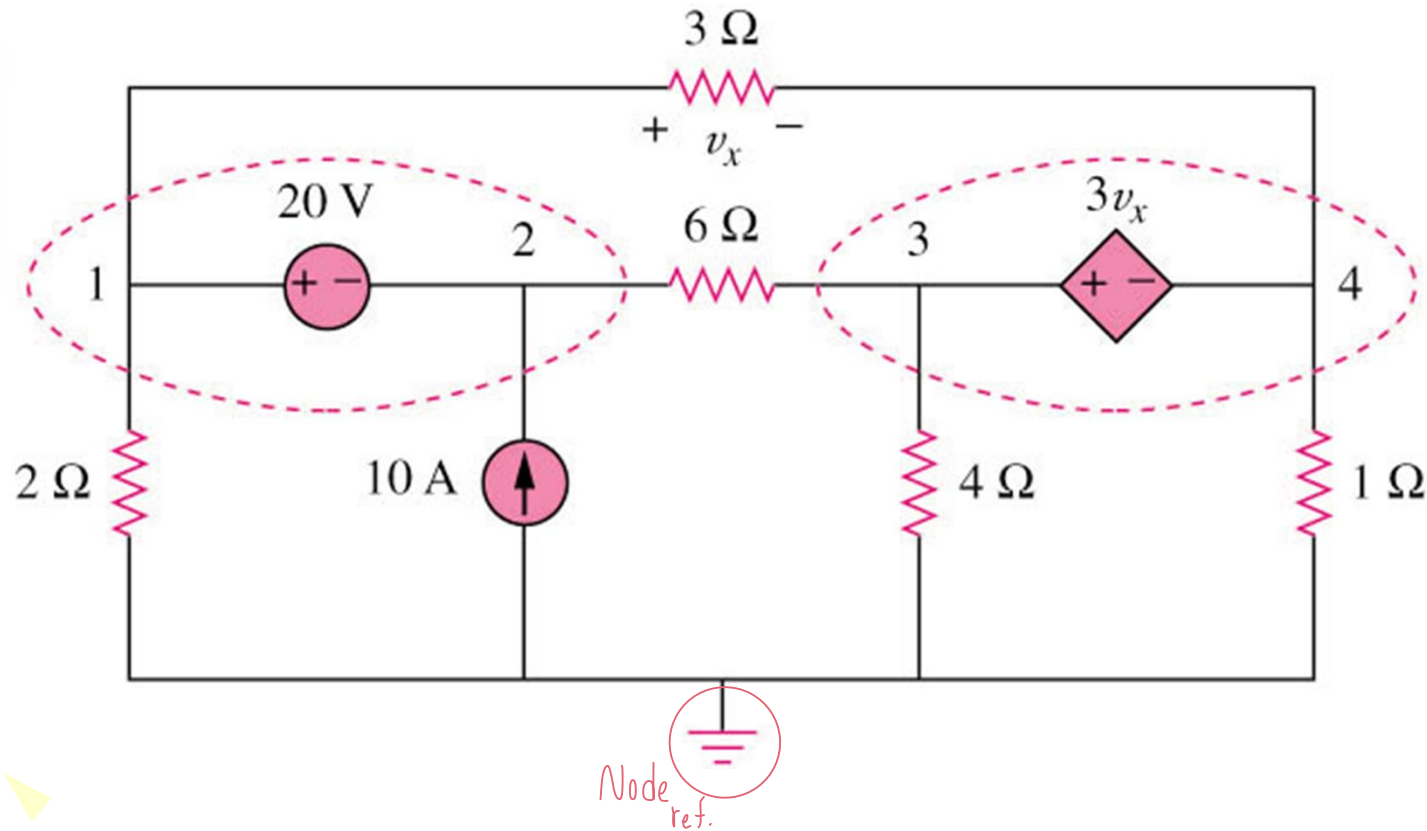**<u>Example 5</u> – circuit with independent voltage source**

Super-node => $2 - i_1 - i_2 - 7 = 0$

$2 - i_1 - i_2 - 7 = 0$

$2 - \dfrac{V_1}{2} - \dfrac{V_2}{4} - 7 = 0$ ——①

$V_1 + 2 - V_2 = 0$ ——②
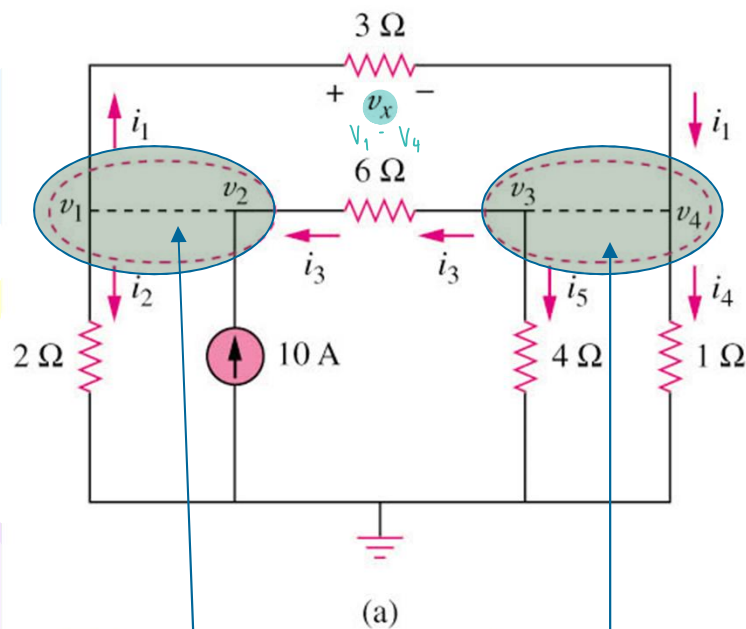
(a)

(b)

Apply KVL => $v_1 + 2 - v_2 = 0$

3.3 Nodal Analysis with Voltage Source (5)
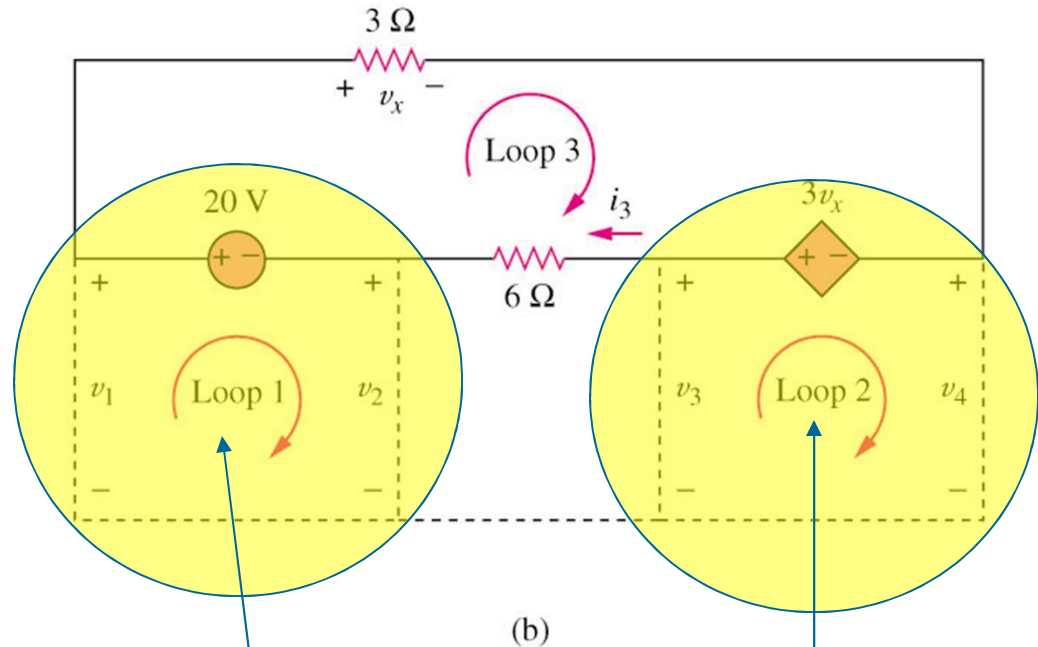
Example 6 – circuit with two independent voltage sources

3.3 Nodal Analysis with Voltage Source (6)

Example 7 – circuit with two independent voltage sources

(a)

(b)

$$\underbrace{\frac{V_3 - V_2}{6}}_{} \qquad \underbrace{\frac{V_1 - 0}{2}}_{} \qquad \underbrace{\frac{V_4 - 0}{1}}_{}$$

$$10 + i_3 = i_1 + i_2 \qquad i_1 = i_3 + i_5 + i_4 \qquad v_1 - 20 - v_2 = 0 \qquad v_3 - 3v_x - v_4 = 0$$

$$\underbrace{\frac{V_1 - V_4}{3}}_{} \qquad \underbrace{\frac{V_3 - 0}{4}}_{}$$

$$10 + i_3 = i_1 + i_2 \underbrace{-\frac{V_1 - 0}{2}}_{} \qquad\qquad① \qquad V_1 = V_2 + 20 \qquad\qquad③ \qquad 14$$

$$\underbrace{\frac{V_3 - V_1}{6}}_{} \quad \underbrace{\frac{V_1 - V_4}{3}}_{}$$

$$i_1 = i_3 + i_4 + i_5 \qquad\qquad② \qquad V_3 = V_4 - 3V_x \qquad\qquad④$$

$$\underbrace{\frac{V_4 - 0}{1}}_{} \underbrace{\frac{V_3 - 0}{4}}_{} \qquad\qquad \underset{V_1 - V_4}{\sim}$$

3.4 Mesh Analysis (1)

1. Mesh analysis provides another general procedure for analyzing circuits using <u>mesh currents</u> as the circuit variables.

สร้างสมการหากระแส
MESH : ลูปที่ไม่มีอะไรอยู่ในลูป

2. Nodal analysis applies KCL to find unknown voltages in a given circuit, while <u>mesh analysis applies KVL</u> to find unknown currents.

3. A <u>mesh</u> is a loop which does not contain any other loops within it.

3.4 Mesh Analysis (2)

Steps to determine the mesh currents:

1. Assign mesh currents i_1, i_2, ..., in to the n meshes.

2. Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.

3. Solve the resulting n simultaneous equations to get the mesh currents.

3.4 Mesh Analysis (3)

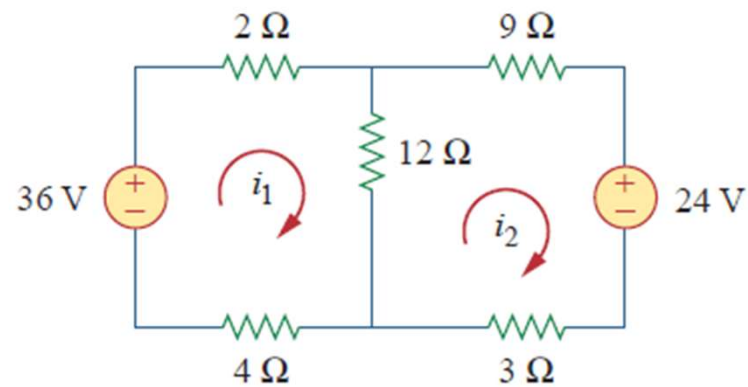**Example 8** – circuit with independent voltage sources
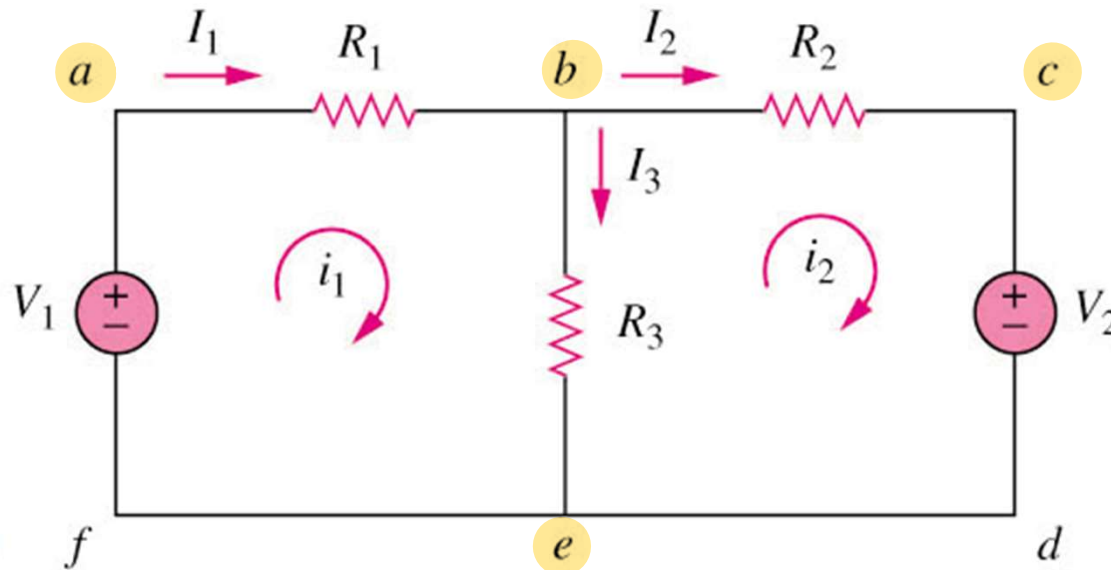
Note:

i_1 and i_2 are mesh current (imaginative, not measurable directly)

I_1, I_2 and I_3 are branch current (real, measurable directly)

$I_1 = i_1$; $I_2 = i_2$; $I_3 = i_1 - i_2$

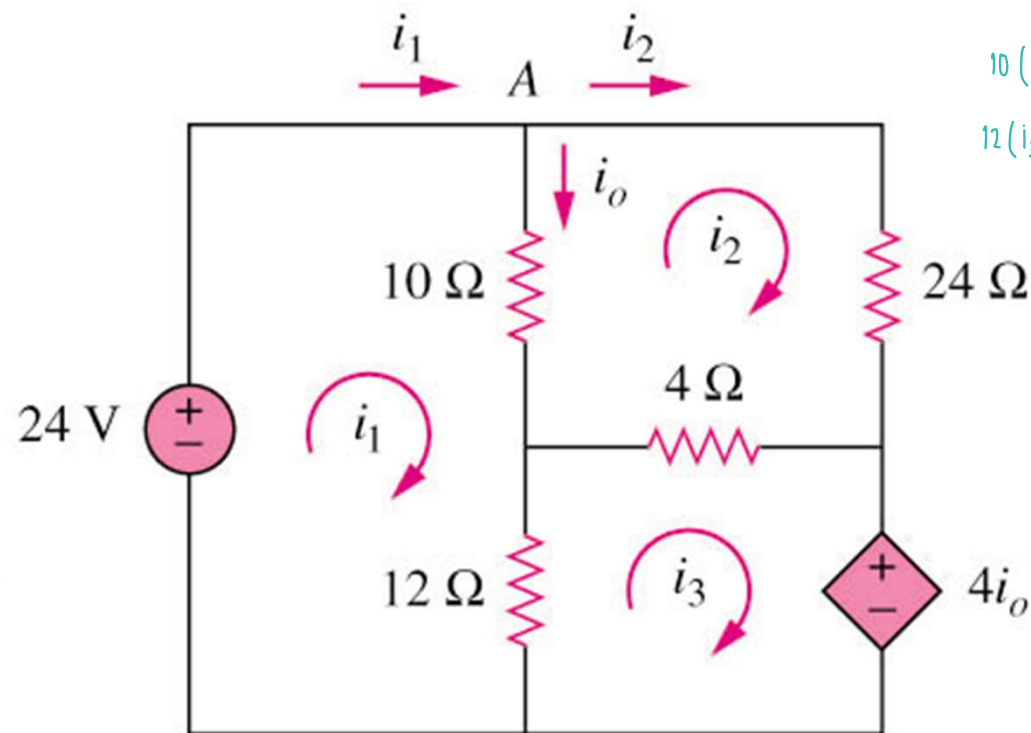*Refer to in-class illustration, textbook

3.4 Mesh Analysis

$$2\,\Omega \qquad\qquad 9\,\Omega$$

$$12\,\Omega$$

36 V i_1 i_2 24 V

$$4\,\Omega \qquad\qquad 3\,\Omega$$

$36 = 2i_1 + 12i_{1-2} + 4i_1$

$0 = 12i_{1-2} + 9i_2 + 24 + 3i_2$

3.4 Mesh Analysis (4)

circuit with dependent voltage source

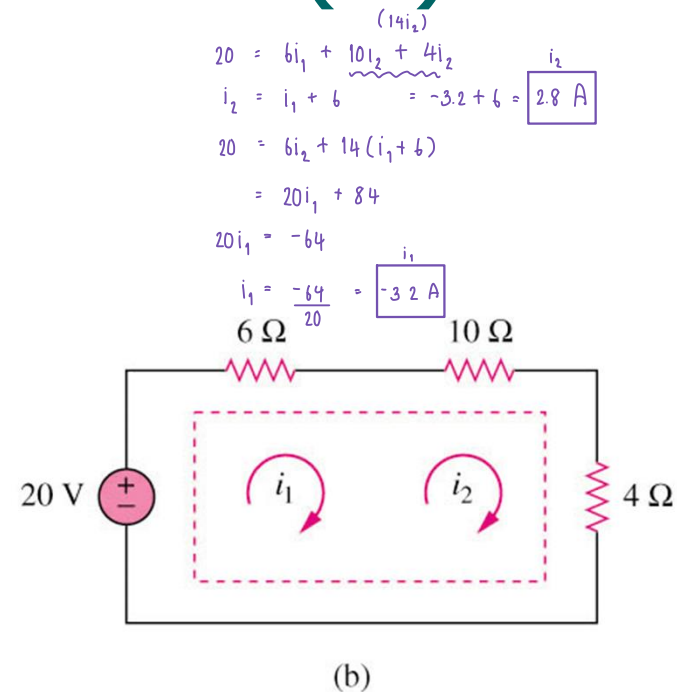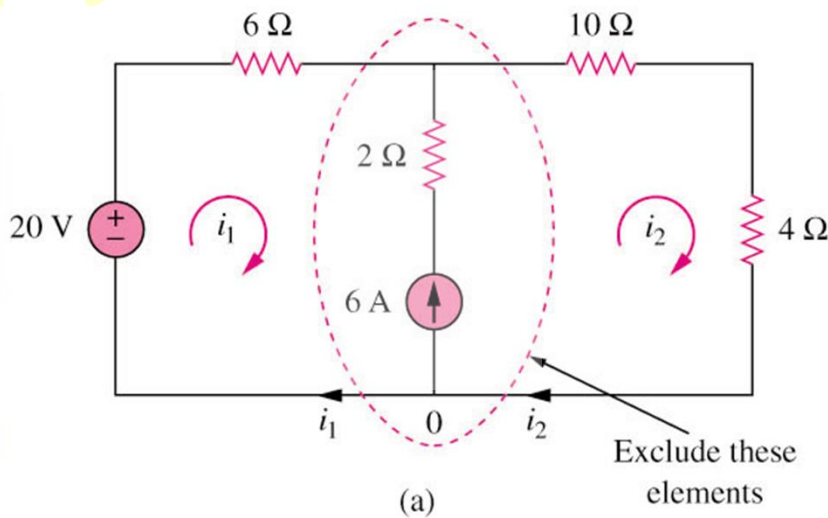$$24 = 10(i_2 - i_1) + 12(i_1 - i_3)$$

$$10(i_2 - i_1) + 24i_2 + 4(i_2 - i_3) \quad 0$$

$$12(i_3 - i_1) + 4(i_3 - i_2) + 4i_o = 0$$
$$\qquad \qquad \qquad \scriptstyle i_1 - i_2$$

*Refer to in-class illustration, textbook, answer $I_o = 1.5A$

3.5 Mesh Analysis with Current Source (1)

Circuit with current source

$$(14i_2)$$
$$20 = 6i_1 + 10i_2 + 4i_2$$
$$i_2 = i_1 + 6 \qquad = -3.2 + 6 = \boxed{2.8\ A} \quad i_2$$
$$20 = 6i_2 + 14(i_1 + 6)$$
$$= 20i_1 + 84$$
$$20i_1 = -64$$
$$i_1 = \frac{-64}{20} = \boxed{-3.2\ A} \quad i_1$$

(a)

(b)

A **super-mesh** results when two meshes have a (dependent or independent) current source in common as shown in (a). We create a super-mesh by excluding the current source and any elements connected in series with it as shown in (b).

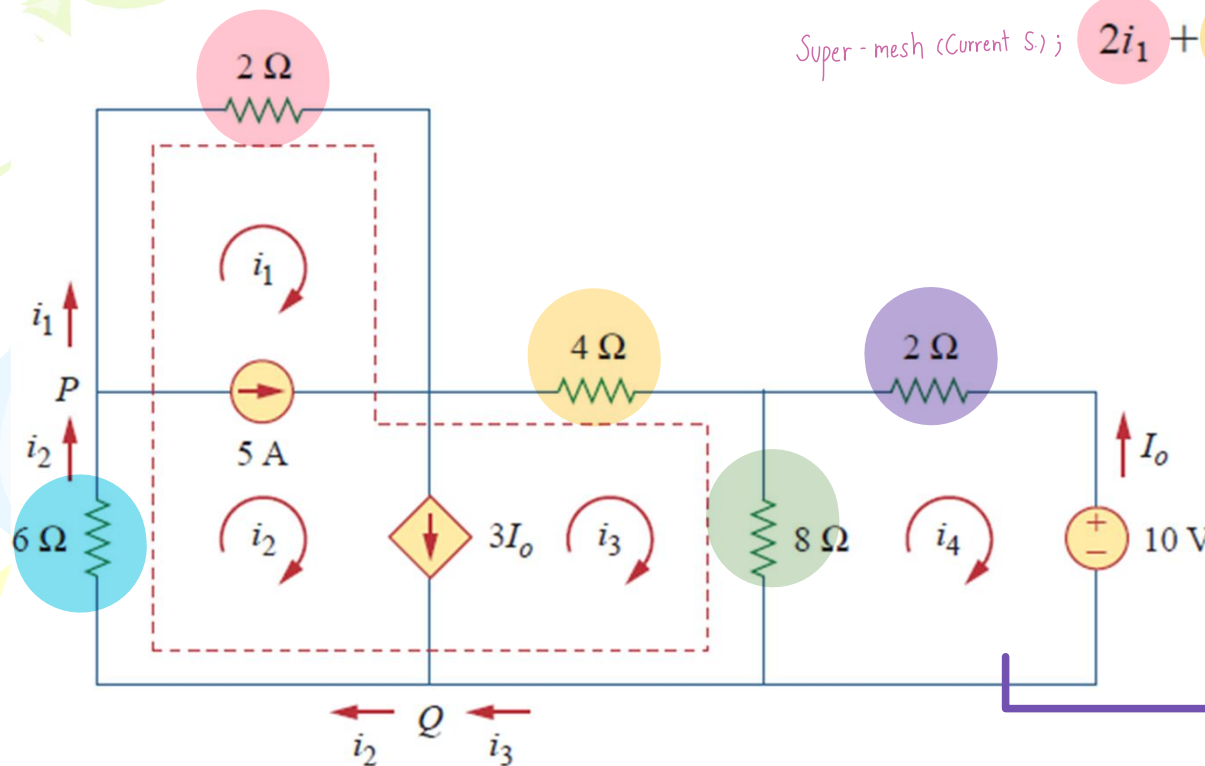*Refer to in-class illustration, textbook

3.5 Mesh Analysis with Current Source (2)

The properties of a super-mesh:

1. The current source in the super-mesh is not completely ignored; it provides the constraint equation necessary to solve for the mesh currents.

2. A super-mesh has no current of its own.

3. A super-mesh requires the application of both KVL and KCL.

3.5 Mesh Analysis with Current Source

find i_1 to i_4

Super-mesh (Current S.) ; $2i_1 + 4i_3 + 8(i_3 - i_4) + 6i_2 = 0$

$i_1 + 3i_2 + 6i_3 - 4i_4 = 0$
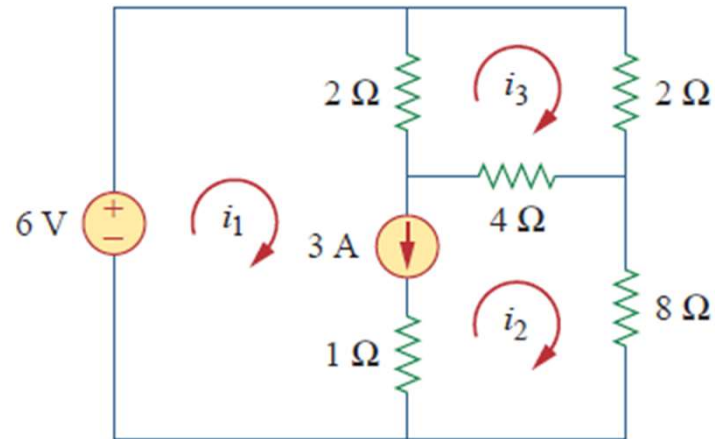
$i_2 = i_1 + 5$

$i_2 = i_3 + 3I_o$

$I_o = -i_4,$

$i_2 = i_3 - 3i_4$

$2i_4 + 8(i_4 - i_3) + 10 = 0$

$5i_4 - 4i_3 = -5$

$i_1 = -7.5$ A, $\qquad i_2 = -2.5$ A, $\qquad i_3 = 3.93$ A, $\qquad i_4 = 2.143$ A

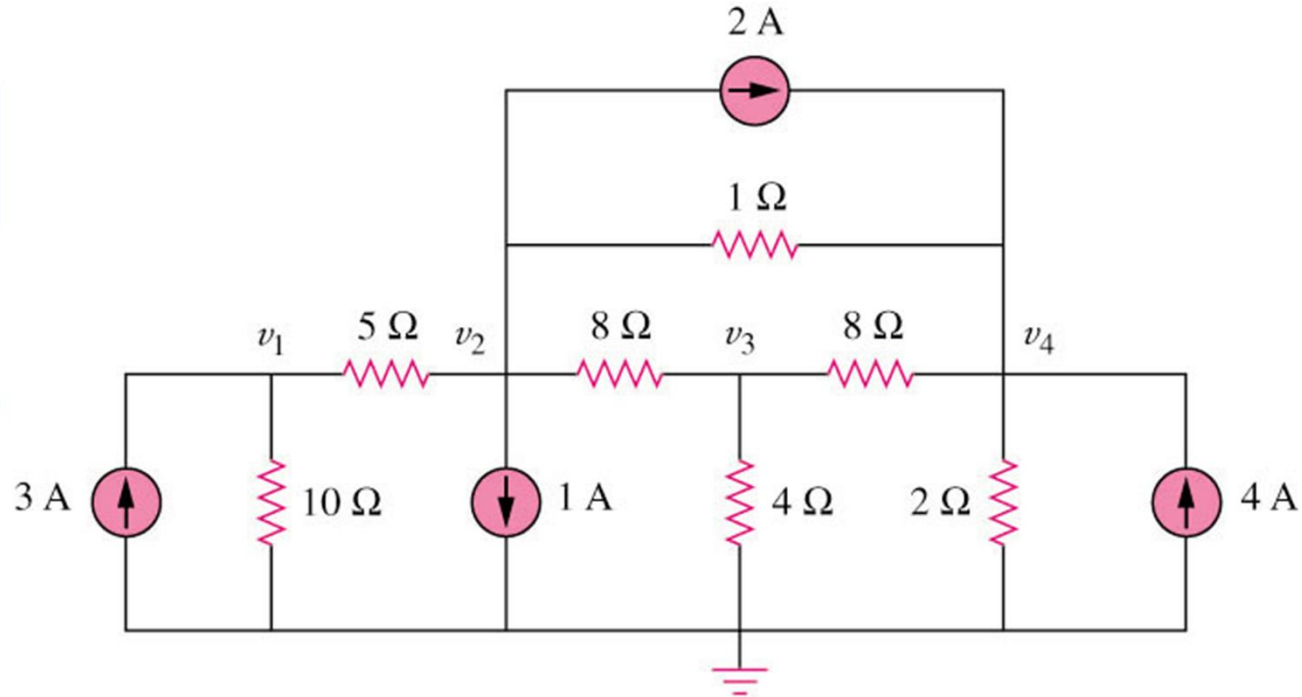# 3.5 Mesh Analysis with Current Source

3.6 Nodal and Mesh Analysis with Inspection (1)

The properties of a super-mesh:

1. The current source in the super-mesh is not completely ignored; it provides the constraint equation necessary to solve for the mesh currents.

2. A super-mesh has no current of its own.

3. A super-mesh requires the application of both KVL and KCL.
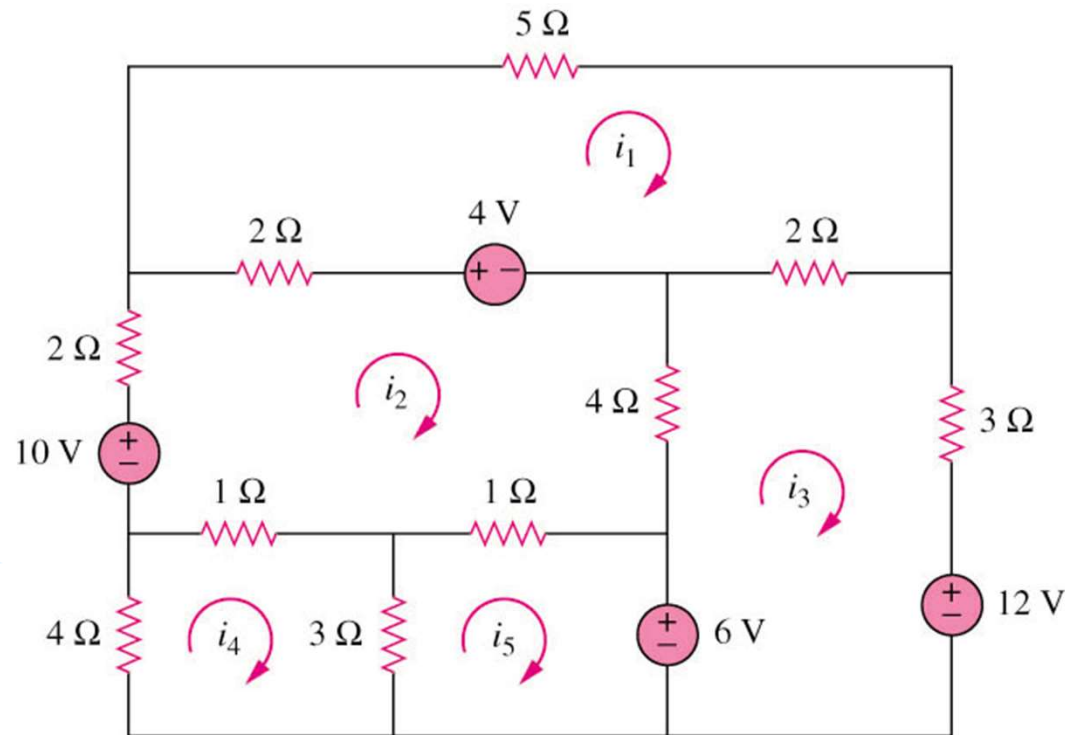
3.6 Nodal and Mesh Analysis with Inspection (2)

Example 10 – By inspection, write the nodal voltage equations for the circuit

*Refer to in-class illustration, textbook

3.6 Nodal and Mesh Analysis with Inspection (3)

Example 11 – By inspection, write the mesh-current equations for the circuit

*Refer to in-class illustration, textbook

3.7 Nodal versus Mesh Analysis (1)

To select the method that results in the smaller number of equations. For example:

1. Choose nodal analysis for circuit with fewer nodes than meshes.
    *Choose mesh analysis for circuit with fewer meshes than nodes.
    *Networks that contain many series connected elements, voltage sources, or supermeshes are more suitable for mesh analysis.
    *Networks with parallel-connected elements, current sources, or supernodes are more suitable for nodal analysis.
2. If node voltages are required, it may be expedient to apply nodal analysis. If branch or mesh currents are required, it may be better to use mesh analysis.