Homework Calculas

(Week 8)

13.4.1 For the functions $w = -3x^2 - 6y^2$, $x = \cos t$ and $y = \sin t$ express $\frac{dw}{dt}$ as a function of t, both by using the chain rule and by expressing w in terms of t and differentiating directly with respect to t. Then evaluate $\frac{dw}{dt}$ at $t = \frac{2\pi}{3}$

$\frac{\partial w}{\partial x} = -6x$, $\frac{\partial w}{\partial y} = -12y$, $\frac{dx}{dt} = -\sin t$, $\frac{dy}{dt} = \cos t$

$$\boxed{\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt}}$$

$= -6x \cdot -\sin t + -12y \cdot \cos t$

$= 6x \sin t - 12y \cos t$

แทนค่า $x = \cos t$, $y = \sin t$ $= (6\cos t)(\sin t) + (-12\sin t)(\cos t)$; $2\cos x \sin x = \sin 2x$

$= 3\sin(2t) - 6\sin(2t)$

แทนค่าในฟังก์ชันด้านบนด้วย $= -3\sin(2t) \longrightarrow -6\sin t\cos t$ Ans.

$w = -3x^2 - 6y^2$

$= -3(\cos t)^2 - 6(\sin t)^2$

$= -3\cos^2 t - 6\sin^2 t$

$\frac{dw}{dt} = -6\cos t(-\sin t) - 12\sin t\cos t$

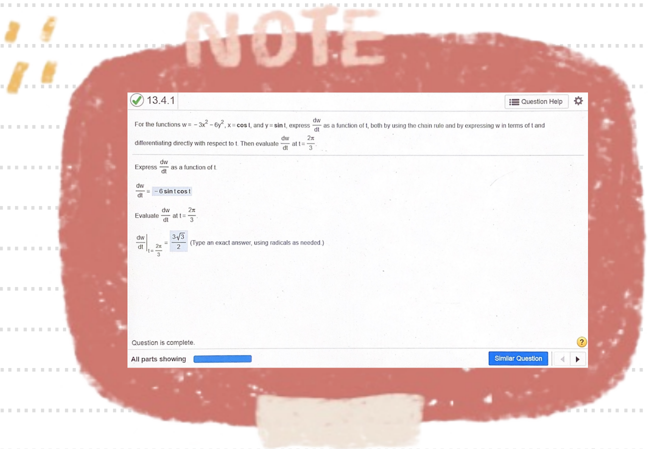
$= 6\cos t\sin t - 12\sin t\cos t$

$= -6\sin t\cos t$ SAME

$\therefore \left.\frac{dw}{dt}\right|_{t=\frac{2\pi}{3}} = -6\sin\left(\frac{2\pi}{3}\right)\cos\left(\frac{2\pi}{3}\right)$ $120°$

$= -6\left(\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{2}\right)$

$= \frac{3\sqrt{3}}{2}$ Ans.



13.4.1 Question Help

For the functions $w = -3x^2 - 6y^2$, $x = \cos t$, and $y = \sin t$, express $\frac{dw}{dt}$ as a function of t, both by using the chain rule and by expressing w in terms of t and differentiating directly with respect to t. Then evaluate $\frac{dw}{dt}$ at $t = \frac{2\pi}{3}$.

Express $\frac{dw}{dt}$ as a function of t.

$\frac{dw}{dt} = -6\sin t\cos t$

Evaluate $\frac{dw}{dt}$ at $t = \frac{2\pi}{3}$.

$\left.\frac{dw}{dt}\right|_{t=\frac{2\pi}{3}} = \frac{3\sqrt{3}}{2}$ (Type an exact answer, using radicals as needed.)

Question is complete.

All parts showing Similar Question

13.4.3 a. Express dw/dt as a function of t, both by using the Chain Rule and by expressing w in terms of t and differentiating directly with respect to t.

b. Evaluate dw/dt at the given value of t.

$$w = \frac{x}{z} + \frac{y}{z}, x = \sin^2 t, y = \cos^2 t, z = \frac{1}{t}, t = 10$$

diff $x: \frac{1}{z}$, $y: \frac{1}{z}$, $z: \frac{-(x+y)}{z^2}$; $2\sin t\cos t$; $-2\cos t\sin t$; $-\frac{1}{t^2}$

(a) $w = \frac{x+y}{z}$ แทนค่า $x = \sin^2 t$, $y = \cos^2 t$

$w = \frac{\sin^2 t + \cos^2 t}{\frac{1}{t}} = t$ (1)

$\frac{dw}{dt} = 1$

$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt}$

$= \cancel{\frac{1}{z} \cdot 2\sin t\cos t} + \cancel{\frac{1}{z} \cdot -2\cos t\sin t} + \frac{-(x+y)}{z^2} \cdot \frac{-1}{t^2}$

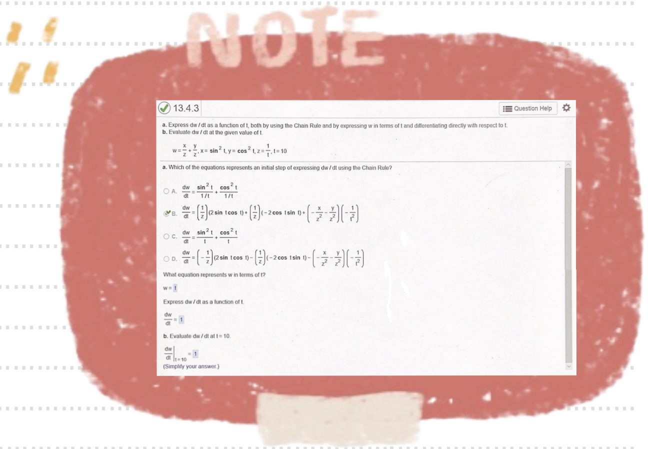
แทนค่าใน $\frac{-(x+y)}{z^2} \cdot \frac{-1}{t^2}$: $x = \sin^2 t$, $y = \cos^2 t$, $z = \frac{1}{t}$

$= \frac{-(\sin^2 t + \cos^2 t)}{(1/t)^2} \cdot$

$= -t^2 \cdot \frac{-1}{t^2}$

$= 1$

(b) $\left.\frac{dw}{dt}\right|_{t=10} = 1$



13.4.3 Question Help

a. Express dw / dt as a function of t, both by using the Chain Rule and by expressing w in terms of t and differentiating directly with respect to t.
b. Evaluate dw / dt at the given value of t.

$w = \frac{x}{z} + \frac{y}{z}$, $x = \sin^2 t$, $y = \cos^2 t$, $z = \frac{1}{t}$, $t = 10$

a. Which of the equations represents an initial step of expressing dw / dt using the Chain Rule?

A. $\frac{dw}{dt} = \frac{\sin^2 t}{1/t} + \frac{\cos^2 t}{1/t}$

B. $\frac{dw}{dt} = \left(\frac{1}{z}\right)(2\sin t\cos t) + \left(\frac{1}{z}\right)(-2\cos t\sin t) + \left(-\frac{x}{z^2} - \frac{y}{z^2}\right)\left(-\frac{1}{t^2}\right)$

C. $\frac{dw}{dt} = \frac{\sin^2 t}{t} + \frac{\cos^2 t}{t}$

D. $\frac{dw}{dt} = \left(-\frac{1}{z}\right)(2\sin t\cos t) - \left(\frac{1}{z}\right)(-2\cos t\sin t) - \left(-\frac{x}{z^2} - \frac{y}{z^2}\right)\left(-\frac{1}{t^2}\right)$

What equation represents w in terms of t?

w = t

Express dw / dt as a function of t.

$\frac{dw}{dt} = 1$

b. Evaluate dw / dt at t = 10.

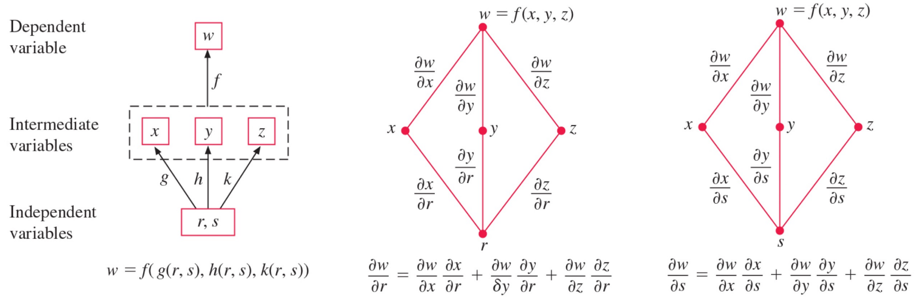
$\left.\frac{dw}{dt}\right|_{t=10} = 1$

(Simplify your answer.)

diff (x) $= y+z$ — (y) $= x+z$ — (z) $= y+x$

diff u 1, diff v 1 — diff u 1, diff v −1 — diff u v, diff v u

13.4.9 For the functions $w = xy + yz + xz$, $x = u + v$, $y = u - v$ and $z = uv$ express $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$ using the chain rule and by expressing w directly in terms of u and v before differentiating. Then evaluate $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$ at the point $(u, v) = \left(-\frac{1}{2}, -2\right)$



$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial r} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial r} \qquad \frac{\partial w}{\partial s} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial s}$$

(a) (b) (c)

$$\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial u} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial u}$$

$= (y+z)(1) + (x+z)(1) + (y+x)(v)$

$= y + 2z + x + yv + xv$

แทนค่า $x = u+v$, $y = u-v$, $z = uv$

$\cancel{u} - \cancel{v} + 2uv + \cancel{u} + \cancel{v} + uv - \cancel{v^2} + uv + \cancel{v^2}$

$= 2u + 4uv$

$$\left.\frac{\partial w}{\partial u}\right|_{(u,v) = \left(-\frac{1}{2}, -2\right)} = 2\left(-\frac{1}{2}\right) + 4\left(-\frac{1}{\cancel{2}}\right)(-\cancel{2})$$

$= 3$

$$\frac{\partial w}{\partial v} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial v} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial v}$$

$= (y+z)(1) + (x+z)(-1) + (y+x)(u)$

$= y - x + yu + xu$

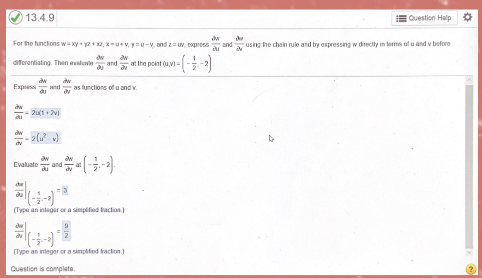
$\cancel{u} - v - \cancel{u} - v + u^2 - \cancel{uv} + u^2 + \cancel{uv}$

$= -2v + 2u^2$

$$\left.\frac{\partial w}{\partial v}\right|_{(u,v) = \left(-\frac{1}{2}, -2\right)} = (-2)(-2) + 2\left(-\frac{1}{2}\right)^2$$

$= \frac{9}{2}$

NOTE



13.4.9 Question Help

For the functions w = xy + yz + xz, x = u + v, y = u − v, and z = uv, express $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$ using the chain rule and by expressing w directly in terms of u and v before differentiating. Then evaluate $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$ at the point (u,v) = $\left(-\frac{1}{2}, -2\right)$.

Express $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$ as functions of u and v.

$\frac{\partial w}{\partial u} = 2u(1+2v)$

$\frac{\partial w}{\partial v} = 2\left(u^2 - v\right)$

Evaluate $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$ at $\left(-\frac{1}{2}, -2\right)$.

$\left.\frac{\partial w}{\partial u}\right|_{\left(-\frac{1}{2}, -2\right)} = 3$

(Type an integer or a simplified fraction.)

$\left.\frac{\partial w}{\partial v}\right|_{\left(-\frac{1}{2}, -2\right)} = \frac{9}{2}$

(Type an integer or a simplified fraction.)

Question is complete.

diff x : $2e^x \ln y$, diff y : $\frac{2e^x}{y}$ — diff u : $\frac{1}{u}$, diff v : $-\tan v$ — diff u : $\sin v$, diff v : $u\cos v$

13.4.7 Consider the functions $z = 2e^x \ln y$, $x = \ln(u\cos v)$ and $y = u\sin v$.

(a) Express $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ as functions of u and v both by using the Chain Rule and by expressing z directly in terms of u and v before differentiating.

(b) Evaluate $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ at $(u, v) = \left(8, \frac{\pi}{3}\right)$

a. , b.

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u}$$

$= 2e^x \ln y \cdot \frac{1}{u} + \frac{2e^x}{y} \cdot \sin v$

แทนค่า $x = \ln(u\cos v)$, $y = u\sin v$

$\frac{2e^{\ln(u\cos v)} \ln(u\sin v)}{u} + \frac{2e^{\ln(u\cos v)} \sin v}{u\sin v}$

$= \frac{2\cancel{u}\cos v \ln(u\sin v)}{\cancel{u}} + \frac{2\cancel{u}\cos v \cancel{\sin v}}{\cancel{u}\cancel{\sin v}}$

$= 2\cos v\left[1 + \ln(u\sin v)\right]$

$$\left.\frac{\partial z}{\partial u}\right|_{(u,v) = \left(8, \frac{\pi}{3}\right)} = 2\cos\frac{\pi}{3}\left[1 + \ln\left(8\sin\frac{\pi}{3}\right)\right]$$

$\cos 60^\circ = \frac{1}{2}$, $\sin 60^\circ = \frac{\sqrt{3}}{2}$

$= 1 + \ln(4\sqrt{3})$

NOTE

$$\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v}$$

$= 2e^x \ln y \cdot (-\tan v) + \frac{2e^x}{y} \cdot u\cos v$

แทนค่า $x = \ln(u\cos v)$, $y = u\sin v$

$\frac{-2e^{\ln(u\cos v)} \ln(u\sin v)\sin v}{\cos v} + \frac{2e^{\ln(u\cos v)} u\cos v}{u\sin v}$

$= \frac{-2u\cancel{\cos v}\ln(u\sin v)\sin v}{\cancel{\cos v}} + \frac{2\cancel{u}\cos v \cdot u\cos v}{\cancel{u}\sin v}$

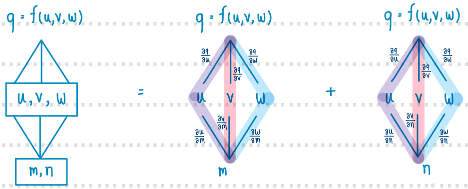
$= -2u\ln(u\sin v)\sin v + \frac{2u\cos^2 v}{\sin v}$

$$\left.\frac{\partial z}{\partial v}\right|_{(u,v) = \left(8, \frac{\pi}{3}\right)} = -2(8)\ln\left(8\sin\frac{\pi}{3}\right)\sin\frac{\pi}{3} + \frac{2(8)\cos^2\left(\frac{\pi}{3}\right)}{\sin\left(\frac{\pi}{3}\right)}$$

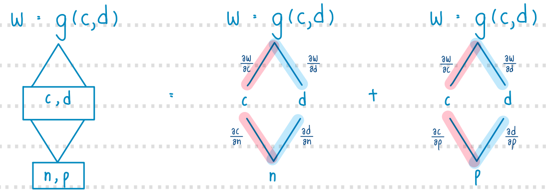
$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$, $\cos^2\left(\frac{\pi}{3}\right) = \left(\frac{1}{2}\right)^2$

$= \frac{8\sqrt{3}}{3}\left(1 - 3\ln(4\sqrt{3})\right)$

13.4.15 Draw a dependency diagram, and write a chain rule formula for $\frac{\partial q}{\partial m}$ and $\frac{\partial q}{\partial n}$ where $q = f(u, v, w),\ u = k(m, n),\ v = h(m, n)\ and\ w = g(m, n)$



13.4.19 Draw a dependency diagram and write a chain rule formula for $\frac{\partial w}{\partial n}$ and $\frac{\partial w}{\partial p}$ given the functions below. $w = g(c, d),\ c = f(n, p),\ d = k(n, p)$



13.5.7 Find ∇f at the given point.

$$f(x,y,z) = x^3 + y^3 - 3z^2 + z\ln x, \quad (1,2,3)$$

diff $x: 3x^2 + \frac{z}{x}$, $y: 3y^2$, $z: -6z + \ln x$

$$\nabla f = \frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} j + \frac{\partial f}{\partial z} k$$

$$= \frac{3x^3 + z}{x} i + 3y^2 j + (\ln x - 6z)k$$

$$\left.\frac{\partial f}{\partial x}\right|_{(1,2,3)} = \frac{3(1)^3 + 3}{1} = 6$$

$$\left.\frac{\partial f}{\partial y}\right|_{(1,2,3)} = 3(2)^2 = 12$$

$$\left.\frac{\partial f}{\partial z}\right|_{(1,2,3)} = \cancelto{0}{\ln 1} - 6(3) = -18$$

$$\left.\nabla f\right|_{(1,2,3)} = 6i + 12j - 18k$$

13.5.9 Find the gradient of $f(x,y,z) = (x^2+y^2+z^2)^{-1/2} + \ln(xyz)$ at the point (1,-2,-2).

$$\frac{\partial f}{\partial x} = (2x)\left(-\frac{1}{2}\right)(x^2+y^2+z^2)^{-3/2} + \frac{yz}{xyz}$$

$$\frac{\partial f}{\partial y} = -\frac{y}{(x^2+y^2+z^2)^{3/2}} + \frac{1}{y}$$

$$\frac{\partial f}{\partial z} = -\frac{z}{(x^2+y^2+z^2)^{3/2}} + \frac{1}{z}$$

$$\boxed{\nabla f = \frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} j + \frac{\partial f}{\partial z} k}$$

$$\left.\nabla f\right|_{(1,-2,-2)} = \left(-\frac{x}{(x^2+y^2+z^2)^{3/2}} + \frac{1}{x}\right)i + \left(-\frac{y}{(x^2+y^2+z^2)^{3/2}} + \frac{1}{y}\right)j + \left(-\frac{z}{(x^2+y^2+z^2)^{3/2}} + \frac{1}{z}\right)k$$

$$= \left(-\frac{1}{[(1)^2+(-2)^2+(-2)^2]^{3/2}} + \frac{1}{1}\right)i + \left(-\frac{-2}{\underbrace{[(1)^2+(-2)^2+(-2)^2]^{3/2}}_{(9)^{3/2} = 27}} + \frac{1}{-2}\right)j + \left(-\frac{-2}{[(1)^2+(-2)^2+(-2)^2]^{3/2}} + \frac{1}{-2}\right)k$$

$$= \frac{26}{27} i - \frac{23}{54} j - \frac{23}{54} k$$

13.5.13 Find the derivative of the function at Upper P_0 in the direction of u.

$$g(x,y) = \frac{x-y}{3xy+5},\quad P_0(-1,1),\qquad u = 12i + 5j$$

$\dfrac{\text{ล่าง diff บน} - \text{บน diff ล่าง}}{\text{ล่าง}^2}$

$$\frac{\partial g}{\partial x} = \frac{\overbrace{(3xy+5)(1) - (x-y)(3y)}^{5+3y^2}}{(3xy+5)^2}$$

$$\frac{\partial g}{\partial y} = \frac{\overbrace{(3xy+5)(-1) - (x-y)(3x)}^{-5-3x^2}}{(3xy+5)^2}$$

$$\nabla g\Big|_{(-1,1)} = \frac{(5+3y^2)i}{(3xy+5)^2} + \frac{(-5-3x^2)j}{\underbrace{(3xy+5)^2}_{4}}$$

$$= 2i - 2j$$

$$|u| = \sqrt{12^2 + 5^2}$$

$$= 13$$

$$\frac{u}{|u|} = \frac{1}{13}(12i + 5j)$$

$\therefore$ **WANT : $D_u g_{(-1,1)}$**

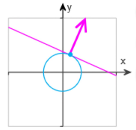
$$= \nabla g\Big|_{(-1,1)} \cdot \frac{u}{|u|}$$

$$= (2i - 2j)\cdot\left(\frac{12}{13}i + \frac{5}{13}j\right)$$

$$= \frac{24}{13} - \frac{10}{13}$$

$$= \frac{14}{13}$$

13.5.25 Sketch the curve $f(x,y) = c$, together with ∇f and the tangent line at the given point. Then write an equation for the tangent line.



$$x^2 + y^2 = 7,\quad (\sqrt{5}, \sqrt{2})$$

รัศมี

$\dfrac{\partial f}{\partial x} = 2x$, $\dfrac{\partial f}{\partial y} = 2y$

$$\nabla f = (2x)i + (2y)j$$

แทนค่า Point $(\sqrt{5}, \sqrt{2})$

$$\nabla f\Big|_{\sqrt{5},\sqrt{2}} = 2\sqrt{5}\,i + 2\sqrt{2}\,j$$

สมการ Tangent line : $f_x(x_0,y_0)(x - x_0) + f_y(x_0,y_0)(y - y_0) = 0$

$$= (2\sqrt{5})(x - \sqrt{5}) + (2\sqrt{2})(y - \sqrt{2}) = 0$$

$$2\sqrt{5}x - 10 + 2\sqrt{2}y - 4 = 0$$

$$2\sqrt{5}x + 2\sqrt{2}y = 14$$

13.5.21 Find the directions in which the function increases and decreases most rapidly at P_0. Then find the derivatives of the function in these directions.

$$f(x,y,z) = \left(\frac{x}{y}\right) - yz, \quad P_0(4,-1,2)$$

diff : x $\frac{1}{y}$, y $\frac{-x}{y^2} - z$, z $-y$

$$\nabla f\Big|_{(4,-1,2)} = \frac{1}{y}i + \left(\frac{-x}{y^2} - z\right)j - yk$$

$$= \frac{1}{-1}i + \left(\underbrace{\frac{-4}{(-1)^2}}_{-4} - 2\right)j + 1k$$

$$= -1i - 6j + k$$

$$\therefore |\nabla f| = \sqrt{f_1^2 + f_2^2 + f_3^2}$$

$$= \sqrt{(-1)^2 + (-6)^2 + (-1)^2}$$

$$= \sqrt{38}$$

$$\boxed{u = \frac{\nabla f|_{(4,-1,2)}}{|\nabla f|_{(4,-1,2)}|}}$$

$$u = -\frac{1}{\sqrt{38}}i - \frac{6}{\sqrt{38}}j + \frac{1}{\sqrt{38}}k$$

$$-u = \frac{1}{\sqrt{38}}i + \frac{6}{\sqrt{38}}j - \frac{1}{\sqrt{38}}k$$

$$\therefore D_u f = |\nabla f|\underbrace{\cancel{\cos(0)}}_{1} = \sqrt{38}$$

$$\therefore D_{-u} f = |\nabla f|\underbrace{\cancel{\cos(\pi)}}_{-1} = -\sqrt{38}$$

13.5.23 Find the directions in which the function increases and decreases most rapidly at P_0. Then find the derivatives of the function in these directions.

$$f(x,y,z) = 2\ln(xy) + \ln(yz) + 3\ln(xz), \quad P_0(1,1,1)$$

diff : x $\frac{5}{x}$, y $\frac{3}{y}$, z $\frac{4}{z}$

$$\nabla f\Big|_{(1,1,1)} = \frac{5}{x}i + \frac{3}{y}j + \frac{4}{z}k$$

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$$= 5i + 3j + 4k$$

$$\therefore |\nabla f| = \sqrt{\underbrace{5^2}_{25} + \underbrace{3^2}_{9} + \underbrace{4^2}_{16}}$$

$$= 5\sqrt{2}$$

$$u = \frac{5}{5\sqrt{2}}i + \frac{3}{5\sqrt{2}}j + \frac{4}{5\sqrt{2}}k$$

$$D_u f = 5\sqrt{2}$$

$$-u = \frac{-5}{5\sqrt{2}}i - \frac{3}{5\sqrt{2}}j - \frac{4}{5\sqrt{2}}k$$

$$D_{-u} f = -5\sqrt{2}$$

Homework Calculas

(Week 9)

13.6.3 Find the equation for (a) the tangent plane and (b) the normal line at the point P_0 (9,0,9) on the surface $6z - x^2 = 0$.

diff x : $-2x$ แทนค่าได้ -18
diff y : 0
diff z : 6

(a) $f_x(P_0)(x - x_0) + f_y(P_0)(y - y_0) + f_z(P_0)(z - z_0)$

$(-18)(x-9) + (0)(y-0) + (6)(z-9) = 0$

$-18x + 162 + 6z - 54 = 0$

$-18x + 6z + 108 = 0$

$-3x + z + 18$

(b)
$x = x_0 + f_x(P_0)t \longrightarrow x = 9 - 18t$

$y = y_0 + f_y(P_0)t \longrightarrow y = 0 + 0t$

$z = z_0 + f_z(P_0)t \longrightarrow z = 9 + 6t$

13.6.20 Find parametric equations for the line tangent to the curve of intersection of the surfaces at the given point.

Surfaces: $x^2 + y^2 = 20,$ $\quad x^2 + y^2 - z = 0$

$f(x,y,z)$ diff x : $2x$, y : $2y$, z : 0

$g(x,y,z)$ diff x : $2x$, y : $2y$, z : -1

Point: $(\sqrt{10}, \sqrt{10}, 20)$

สูตร $\boxed{V = \nabla f \times \nabla g}$ และ $\nabla f\Big|_{x,y,z} = \frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} j + \frac{\partial f}{\partial z} k$

แทนค่า

Sol² $\begin{vmatrix} i & j & k \\ 2\sqrt{10} & 2\sqrt{10} & 0 \\ 2\sqrt{10} & 2\sqrt{10} & -1 \end{vmatrix}$

$= [(-2\sqrt{10}) - 0]i - [(-2\sqrt{10}) - 0]j + [40 - 40]k$

$= -2\sqrt{10}\,i + 2\sqrt{10}\,j + 0k$

Sol¹ $\nabla f\Big|_{\sqrt{10},\sqrt{10},20} = 2x\,i + 2y\,j + 0k$

$= 2\sqrt{10}\,i + 2\sqrt{10}\,j$

$\nabla g\Big|_{\sqrt{10},\sqrt{10},20} = 2x\,i + 2y\,j - 1k$

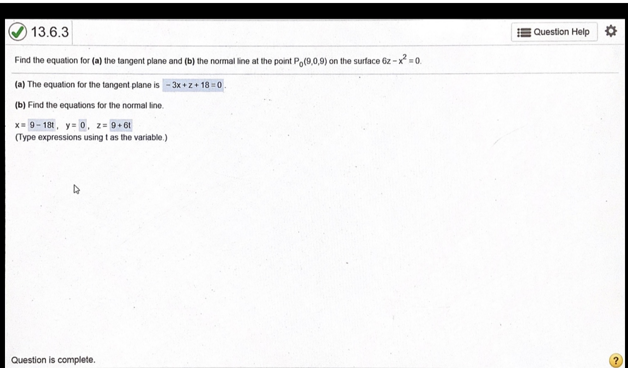
$= 2\sqrt{10}\,i + 2\sqrt{10}\,j - k$

∴ Parametric equations

$x = x_0 + f_x(P_0)t \longrightarrow x = \sqrt{10} - 2\sqrt{10}\,t$

$y = y_0 + f_y(P_0)t \longrightarrow y = \sqrt{10} + 2\sqrt{10}\,t$

$z = z_0 + f_z(P_0)t \longrightarrow z = 20 + 0t$



13.6.3 Question Help

Find the equation for **(a)** the tangent plane and **(b)** the normal line at the point $P_0(9,0,9)$ on the surface $6z - x^2 = 0$.

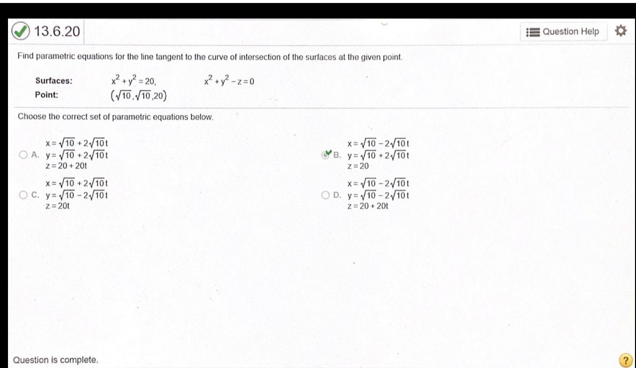
(a) The equation for the tangent plane is $-3x + z + 18 = 0$.

(b) Find the equations for the normal line.

$x = 9 - 18t$, $y = 0$, $z = 9 + 6t$

(Type expressions using t as the variable.)

Question is complete.



13.6.20 Question Help

Find parametric equations for the line tangent to the curve of intersection of the surfaces at the given point.

Surfaces: $x^2 + y^2 = 20,$ $\quad x^2 + y^2 - z = 0$

Point: $(\sqrt{10}, \sqrt{10}, 20)$

Choose the correct set of parametric equations below.

A. $x = \sqrt{10} + 2\sqrt{10}t$, $y = \sqrt{10} + 2\sqrt{10}t$, $z = 20 + 20t$

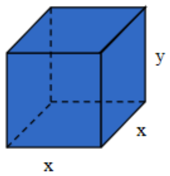
B. $x = \sqrt{10} - 2\sqrt{10}t$, $y = \sqrt{10} + 2\sqrt{10}t$, $z = 20$

C. $x = \sqrt{10} + 2\sqrt{10}t$, $y = \sqrt{10} - 2\sqrt{10}t$, $z = 20t$

D. $x = \sqrt{10} - 2\sqrt{10}t$, $y = \sqrt{10} - 2\sqrt{10}t$, $z = 20 + 20t$

Question is complete.

13.6.61 Consider a closed rectangular box with a square base, as shown in the figure. Assume x is measured with an error of at most 0.2% and y is measured with an error of at most 0.50%, so we have $\frac{|dx|}{x} \le \underbrace{0.002}_{0.2\%}$ *and* $\frac{|dy|}{y} \le \underbrace{0.005}_{0.5\%}$



a. Use a differential to estimate the relative error $\frac{|dV|}{V}$ in computing the box's volume V. — x^2y < diff x : $2xy$; diff y : x^2

b. Use a differential to estimate the relative error $\frac{|dS|}{S}$ in computing the box's surface area S. — $2x^2 + 4xy$ < diff x : $4x + 4y$; diff y : $4x$

(a.) use the total differential of $V = f(x,y)$

$$df = f_x(x,y)\,dx + f_y(x,y)\,dy$$

$$dV = 2xy\,dx + x^2\,dy$$

$$\frac{dV}{V} = \frac{1}{x^2y}\left[2xy\,dx + x^2\,dy\right]$$

$$= \frac{2}{x}dx + \frac{1}{y}dy \qquad \boxed{\le 0.9\%}$$

* error most x + y + x%

ไว้ก่อน

(b.) Surface area S : $dV = (4x+4y)dx + (4x)dy$

$$\frac{dS}{S} = \frac{4x+4y}{2x^2+4xy}dx + \frac{4x}{2x^2+4xy}dy$$

Conjugate $\frac{x}{x}$ Conjugate $\frac{y}{y}$

$$= \frac{4x^2+4xy}{2x^2+4xy}\frac{dx}{x} + \frac{4xy}{2x^2+4xy}\frac{dy}{y}$$

13.6.40 Find the linearization L(x,y) of the function f(x,y) at P_0. Then find an upper bound for the magnitude $|E|$ of the error in the approximation f(x,y) ≈ L(x,y) over the rectangle R.

diff x : $\frac{2}{x} \xrightarrow{\text{แทน } P_0} 2$

diff y : $\frac{5}{y} \to 5$

แทนค่า $P_0(1,1)$ ใน $f(x,y)$ = 0

$$f(x,y) = 2\ln x + 5\ln y \; at\; P_0(1,1)$$

$$R: |x-1| \le 0.2, \qquad |y-1| \le 0.5$$

$$L(x,y) = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$$

$$= 0 + (2)(x-1) + (5)(y-1)$$

$$= 2x - 2 + 5y - 5 \quad (-7)$$

$$|E(x,y)| \le \frac{1}{2}M\left(|x-x_0| + |y-y_0|\right)^2 \longrightarrow |E| \le \frac{1}{\cancel{2}}\,\cancel{(20)}^{10}\left(\overbrace{0.2+0.5}^{0.7}\right)^2$$

$$\le 10(0.7)^2$$

$$\le 4.9$$

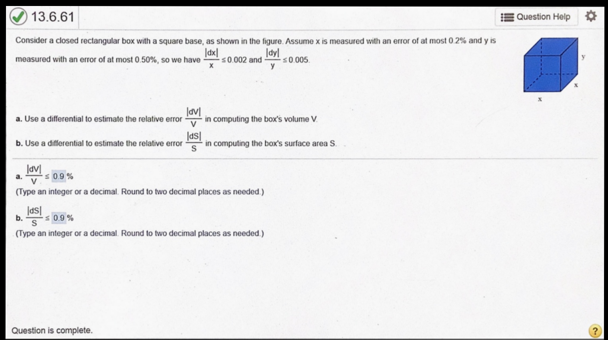
- $f_{xx} = \frac{\partial}{\partial x}\left(\frac{2}{x}\right)$ (f_x) $= -\frac{1}{x^2}$
- $f_{xy} = \frac{\partial}{\partial y}\left(\frac{2}{x}\right)$ (f_x) $= 0$
- $f_{yy} = \frac{\partial}{\partial y}\left(\frac{5}{y}\right)$ (f_y) $= -\frac{5}{y^2}$

R : $|x-1| \le 0.2$, $|y-1| \le 0.5$

x = 0.8 y = 0.5

$$|f_{xx}| = \left|-\frac{2}{x^2}\right| = \left|-\frac{2}{0.8^2}\right| = \frac{500}{64} \to \frac{125}{16} = 7.81$$

$$|f_{yy}| = \left|-\frac{5}{y^2}\right| = \left|-\frac{5}{0.5^2}\right| = \frac{500}{25} \to 20 : \text{The upper bound } M$$



13.6.61 Question Help

Consider a closed rectangular box with a square base, as shown in the figure. Assume x is measured with an error of at most 0.2% and y is measured with an error of at most 0.50%, so we have $\frac{|dx|}{x} \le 0.002$ and $\frac{|dy|}{y} \le 0.005$.

a. Use a differential to estimate the relative error $\frac{|dV|}{V}$ in computing the box's volume V.

b. Use a differential to estimate the relative error $\frac{|dS|}{S}$ in computing the box's surface area S.

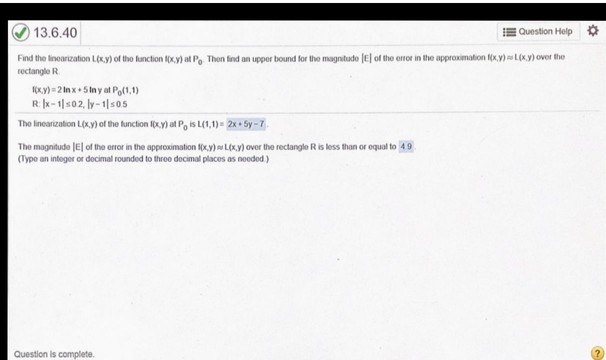
a. $\frac{|dV|}{V} \le$ 0.9 %

(Type an integer or a decimal. Round to two decimal places as needed.)

b. $\frac{|dS|}{S} \le$ 0.9 %

(Type an integer or a decimal. Round to two decimal places as needed.)

Question is complete.



13.6.40 Question Help

Find the linearization L(x,y) of the function f(x,y) at P_0. Then find an upper bound for the magnitude |E| of the error in the approximation f(x,y) ≈ L(x,y) over the rectangle R.

$f(x,y) = 2\ln x + 5\ln y$ at $P_0(1,1)$

R: $|x-1| \le 0.2$, $|y-1| \le 0.5$

The linearization L(x,y) of the function f(x,y) at P_0 is L(1,1) = 2x + 5y − 7.

The magnitude |E| of the error in the approximation f(x,y) ≈ L(x,y) over the rectangle R is less than or equal to 4.9.

(Type an integer or decimal rounded to three decimal places as needed.)

Question is complete.

13.6.58 Find the derivative of $f(x,y) = x^2 + y^2$ in the direction of the unit tangent vector of the curve $r(t) = (\cos t + t \sin t)i + (\sin t - t \cos t)j,\ t > 0.$

คำตอบคือ

2

13.7.6 Find all the local maxima, local minima, and saddle points of the function.

$$f(x,y) = x^2 - 4xy + y^2 + 6y + 1$$

$f_x : 2x - 4y = 0$

$* \ x = 2y$ แทนค่า

$f_y : -4x + 2y + 6 = 0$

ได้ $(-4)(2y) + 2y + 6 = 0$

$-6y + 6 = 0$

$y = 1,\ x = 2$

∴ There is only one critical point for $f(x,y)$ at $(2,1)$

ได้จากการแทนค่า x, y ใน $f'(x,y)$

$f_{xx} = \frac{\partial}{\partial x}(2x - 4y) = 2$

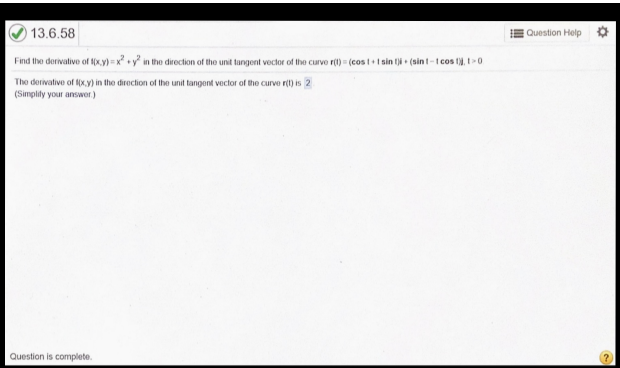
$f_{yy} = \frac{\partial}{\partial y}(-4x + 2y + 6) = 2$

$f_{xy} = \frac{\partial}{\partial x}(-4x + 2y + 6) = -4$

find $f_{xx} f_{yy} - f_{xy}^2 = (2)(2) - (-4)^2$

$= -12$

- the local maxima ▸ NONE
- the local minima ▸ NONE
- saddle points ▸ at $(2,1)$



13.6.58

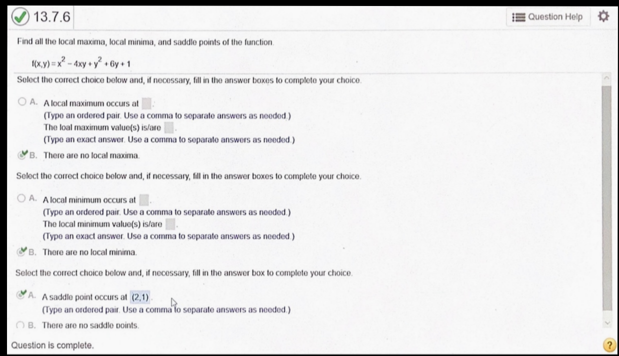
Question Help

Find the derivative of $f(x,y) = x^2 + y^2$ in the direction of the unit tangent vector of the curve $\mathbf{r}(t) = (\cos t + t \sin t)\mathbf{i} + (\sin t - t \cos t)\mathbf{j}$, $t > 0$

The derivative of f(x,y) in the direction of the unit tangent vector of the curve **r**(t) is 2.

(Simplify your answer.)

Question is complete.



13.7.6

Question Help

Find all the local maxima, local minima, and saddle points of the function.

$f(x,y) = x^2 - 4xy + y^2 + 6y + 1$

Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

A. A local maximum occurs at .
(Type an ordered pair. Use a comma to separate answers as needed.)
The loal maximum value(s) is/are .
(Type an exact answer. Use a comma to separate answers as needed.)

B. There are no local maxima.

Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

A. A local minimum occurs at .
(Type an ordered pair. Use a comma to separate answers as needed.)
The local minimum value(s) is/are .
(Type an exact answer. Use a comma to separate answers as needed.)

B. There are no local minima.

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. A saddle point occurs at (2,1).
(Type an ordered pair. Use a comma to separate answers as needed.)

B. There are no saddle points.

Question is complete.

13.7.19 Find all the local maxima, local minima, and saddle points of the function.

$$f(x,y) = 4xy - x^4 - y^4$$

$f_x : 4y - 4x^3 = 0$
$y = x^3$

$f_y : 4x - 4y^3 = 0$
$x = y^3$

แทนค่า

$4y - 4(y^3)^3 = 0$
$4y - 4y^9 = 0$
$4y(1 - y^8) = 0$
$y = 0, -1, 1$

จะได้ $x = 0$, $x = -1$, $x = 1$ $\xleftarrow{\text{แทนค่า}}$

∴ There is critical point for f(x,y) at (0,0), (-1,-1), (1,1)
ได้จากการแทนค่า x, y ใน f'(x,y)

$f_{xx} = \frac{\partial}{\partial x}(4y - 4x^3) = -12x^2$

$f_{yy} = \frac{\partial}{\partial y}(4x - 4y^3) = -12y^2$

$f_{xy} = \frac{\partial}{\partial x}(4x - 4y^3) = 4$

find $f_{xx}f_{yy} - f_{xy}^2 = (-12x^2)(-12y^2) - 4^2$
$= 144x^2y^2 - 4$

- (0,0) : -4 ⇝ the discriminant is negative
- (-1,-1) : 140 ⇝ the discriminant is positive $\xrightarrow{\text{หา MAX, MIN}}$ $f_{xx} = -12x^2 = -12$, $f(x,y) = 4xy - x^4 - y^4 = 2$
- (1,1) : 140 ⇝ the discriminant is positive
  - หา MAX, MIN: $f_{xx} = -12x^2 = -12$
  - $f(x,y) = 4xy - x^4 - y^4 = 2$

- the local maxima ▸ 2 at point (-1,-1) and (1,1)
- the local minima ▸ none
- saddle points ▸ at (0,0)

13.7.27 Find all the local maxima, local minima, and saddle points of the function.

$$f(x,y) = 5e^{-y}(x^2 + y^2) + 6$$

$f_x : \frac{10x}{e^y}$ จะได้ $x = 0$

$\downarrow$ แทนค่า

$f_y : \frac{10y - 5(x^2+y^2)}{e^y}$ จะได้ $y = 0, 2$

∴ There is only one critical point for f(x,y) at (2,1)
ได้จากการแทนค่า x, y ใน f'(x,y)

$f_{xx} = \frac{\partial}{\partial x}\left(\frac{10x}{e^y}\right) = \frac{10}{e^y}$

$f_{yy} = \frac{\partial}{\partial y}\frac{(10y - 5(x^2+y^2))}{e^y} = \frac{10 - 20y + 5x^2 + 5y^2}{e^y}$

$f_{xy} = \frac{\partial}{\partial x}(-4x + 2y + 6) = \frac{-10x}{e^y}$

find $f_{xx}f_{yy} - f_{xy}^2 = \left(\frac{10}{e^y}\right)\left(\frac{10 - 20y + 5x^2 + 5y^2}{e^y}\right) - \left(\frac{-10x}{e^y}\right)^2$

แทน point (0,0) : 100

แทน point (0,2) : $\frac{-100}{e^4}$

- the local maxima ▸ none
- the local minima ▸ $f(0,0) = 5e^{-y}(x^2+y^2) + 6$
  $= 6$
- saddle points ▸ at (0,2) the discriminant is $\frac{-100}{e^4}$

13.7.19 Question Help

Find all the local maxima, local minima, and saddle points of the function.

$f(x,y) = 4xy - x^4 - y^4$

Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

○ A. A local minimum occurs at ☐.
(Type an ordered pair. Use a comma to separate answers as needed.)
The local minimum value(s) is/are ☐.
(Type an exact answer. Use a comma to separate answers as needed.)

✔ B. There are no local minima.

Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

✔ A. A local maximum occurs at (1,1),(−1,−1).
(Type an ordered pair. Use a comma to separate answers as needed.)
The local maximum value(s) is/are 2.
(Type an exact answer. Use a comma to separate answers as needed.)

○ B. There are no local maxima.

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

✔ A. A saddle point occurs at (0,0).
(Type an ordered pair. Use a comma to separate answers as needed.)

Question is complete.

13.7.27 Question Help

Find all the local maxima, local minima, and saddle points of the function.

$f(x,y) = 5e^{-y}(x^2 + y^2) + 6$

Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

○ A. A local maximum occurs at ☐.
(Type an ordered pair. Use a comma to separate answers as needed.)
The local maximum value(s) is/are ☐.
(Type an exact answer. Use a comma to separate answers as needed.)

✔ B. There are no local maxima.

Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

✔ A. A local minimum occurs at (0,0).
(Type an ordered pair. Use a comma to separate answers as needed.)
The local minimum value(s) is/are 6.
(Type anexact answer. Use a comma to separate answers as needed.)

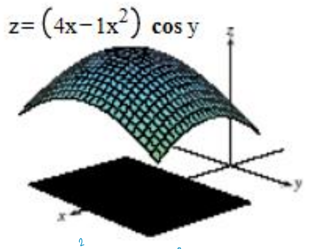
○ B. There are no local minima.

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

✔ A. A saddle point occurs at (0,2).
(Type an ordered pair. Use a comma to separate answers as needed.)

Question is complete.

13.7.37 Find the absolute maximum and minimum of the function $f(x,y) = (4x - 1x^2)\cos y$ on the rectangular plate $1 \le x \le 3, -\frac{\pi}{4} \le y \le \frac{\pi}{4}$



The absolute maximum : 4

The absolute minimum : $\frac{3\sqrt{2}}{2}$

$z = (4x - x^2)\cos y$ แทนค่าได้ 4

$f_x = (4 - 2x)\cos y$ — $4 - 2x = 0$ ($x = 2$)

$\cos y = 0$: no y-value

$f_y = -(4x - x^2)\sin y$ — $-(4x - x^2) = 0$ ($x = 0, 4$)

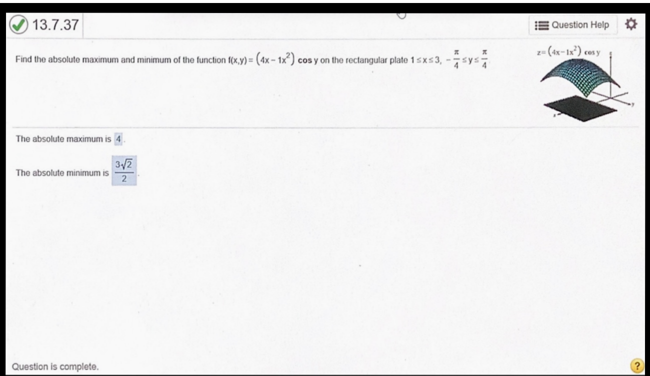
$\sin y = 0$ ($y = 0$)

$f\left(1, -\frac{\pi}{4}\right) = (4(1) - (1)^2)\left(\cos\left(-\frac{\pi}{4}\right)\right) = \frac{3\sqrt{2}}{2}$

$f\left(1, \frac{\pi}{4}\right) = (4(1) - (1)^2)\left(\cos\left(\frac{\pi}{4}\right)\right) = \frac{3\sqrt{2}}{2}$

$f\left(3, -\frac{\pi}{4}\right) = (4(3) - (3)^2)\left(\cos\left(-\frac{\pi}{4}\right)\right) = \frac{3\sqrt{2}}{2}$

$f\left(3, \frac{\pi}{4}\right) = (4(3) - (3)^2)\left(\cos\left(\frac{\pi}{4}\right)\right) = \frac{3\sqrt{2}}{2}$



13.7.37

Question Help

Find the absolute maximum and minimum of the function $f(x,y) = (4x - 1x^2)$ **cos** y on the rectangular plate $1 \le x \le 3, -\frac{\pi}{4} \le y \le \frac{\pi}{4}$

z= (4x−1x²) cos y

The absolute maximum is 4.

The absolute minimum is $\frac{3\sqrt{2}}{2}$.

Question is complete.

13.7.46 For what values of the constant k does the Second Derivative Test guarantee that $f(x,y) = x^2 + kxy + y^2$ will have a saddle point at (0,0)? A local minimum at (0,0)? For what values of k is the Second Derivative Test inconclusive? Give reasons for your answers.

If $f_{xx}f_{yy} - f_{xy}^2 < 0$ at (a,b)

$f_{xx} = 2$, $f_{xy} = k$, $f_{yy} = 2$

$\therefore k > 2$ or $k < -2$

If $f_{xx} > 0$ and $f_{xx}f_{yy} - f_{xy}^2 > 0$ at (a,b)

$\therefore -2 < k < 2$

If $f_{xx}f_{yy} - f_{xy}^2 = 0$ at (a,b)

$\therefore k = 2$ or $k = -2$

Ans. ข้างบน !

Homework Calculas

(Week 10)

14.1.2 Evaluate

$$\int_0^5 \int_{-2}^{-1} (4x+y)dx\,dy$$

$$= \int_0^5 \left[\int_{-2}^{-1} (4x+y)\,dx \right] dy$$

$$= \int_0^5 \left[2x^2 + yx \right]_{-2}^{-1} dy$$

$$\left(2(-1)^2 + y(-1)\right) - \left(2(-2)^2 + y(-2)\right)$$

$$= (2-y) - (8-2y)$$

$$= -6+y$$

$$= \int_0^5 (-6+y)\,dy$$

$$= -6y + \frac{y^2}{2} \Big|_0^5$$

$$-6(5) + \frac{(5)^2}{2} - 0$$

$$= -30 + \frac{25}{2}$$

$$= \frac{-35}{2}$$

14.1.19 Evaluate the double integral over the given region R.

$$\iint_R xy\cos y\,dA \qquad R: -2 \le x \le 2, 0 \le y \le \pi$$

$$\int_0^\pi \int_{-2}^{2} xy\cos y\,dx\,dy$$

$$= \int_0^\pi \left[\frac{x^2 y\cos y}{2} \right]_{-2}^{2} dy$$

$$2^2 y\cos y - (-2)^2 y\cos y$$

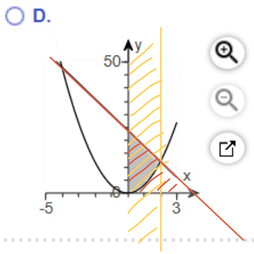
$$= 0$$

$$= \int_0^\pi 0\,dy$$

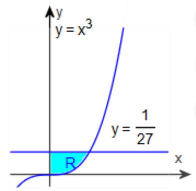
$$= 0$$

14.2.2 Sketch the described region of integration.

$$0 \le x \le 2, \qquad 3x^2 \le y \le -6x + 24$$



14.2.9 Write an iterated integral for $\int_R \int dA$ over the region R described to the right using



a. vertical cross-sections,

b. horizontal cross-sections.

(a.) $\iint_R dA = \int_{a}^{b} \int_{g_1(x)}^{g_2(x)} f(x,y)\, dy\, dx = 1 \quad ; a \le x \le b, \ g_1(x) \le y \le g_2(x)$

$a = 0$, $b = \frac{1}{3}$, $g_1(x) = x^3$, $g_2(x) = \frac{1}{27}$

$x^3 \le y \le \frac{1}{27}$

$0 \le x \le \frac{1}{3}$

(b.) $\iint_R dA = \int_{a}^{b} \int_{h_1(x)}^{h_2(x)} f(x,y)\, dx\, dy = 1 \quad ; a \le x \le b, \ h_1(x) \le y \le h_2(x)$

$a = 0$, $b = \frac{1}{27}$, $h_1(x) = 0$, $h_2(x) = y^{1/3}$

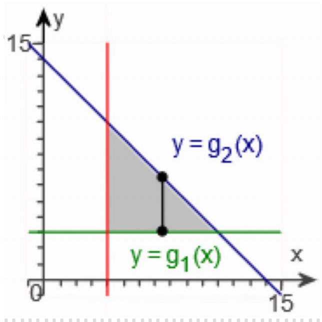
$0 \le x \le y^{1/3}$

$0 \le y \le \frac{1}{27}$

14.2.17 (a) Write an iterated integral for $\int_R \int dA$ over the region bounded by $y = 8 - x$, $y = 3$ and $x = 1$ using vertical cross-sections.

(b) Write an iterated integral for $\int_R \int dA$ over the region bounded by $y = 8 - x$, $y = 3$ and $x = 1$ using horizontal cross-sections.

a.



$g_1(x) \leq y \leq g_2(x)$

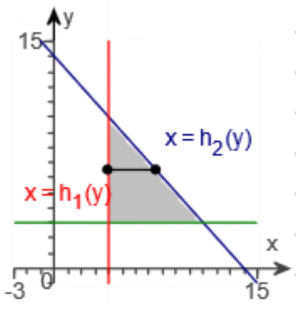
$3 \leq y \leq 8 - x$

$a \leq x \leq b$

$1 \leq x \leq 5$

$\therefore \int_a^b \int_{g_1(x)}^{g_2(x)} dy\,dx = \int_1^5 \int_3^{8-x}$

b.



$h_1(x) \leq x \leq h_2(x)$

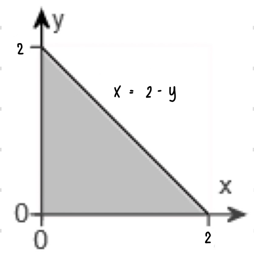
$1 \leq x \leq 8 - y$

$c \leq y \leq d$

$3 \leq y \leq 7$

$\therefore \int_a^b \int_{h_1(x)}^{h_2(x)} dy\,dx = \int_3^7 \int_1^{8-y} dx\,dy$

14.2.26 Integrate $f(x, y) = x^2 + y^2$ over the triangular region with vertices (0,0), (2,0)and (0,2).



$$\boxed{\int_0^2 \int_0^{2-y} (x^2 + y^2)\,dx\,dy}$$

$$= \int_0^2 \left[\frac{1}{3}(x^3 + 3xy^2)\right]_0^{2-y} dy$$

$$\frac{1}{3}\left((2-y)^3 + 3(2-y)y^2\right)$$

$$= \frac{1}{3}(-4y^3 + 12y^2 - 12y + 8)$$

$$= \int_0^2 \frac{1}{3}(-4y^3 + 12y^2 - 12y + 8)\,dy$$

$$= \frac{1}{3}(-y^4 + 4y^3 - 6y^2 + 8y)\Big|_0^2$$

$$= \frac{8}{3}$$

point at $(4\sqrt{3}, 4)$

$x^2 + y^2 \leq 64$

14.2.75 Integrate $f(x,y) = \sqrt{64 - x^2}$ over the smaller sector cut from the disk $x^2 + y^2 \leq 64$ by the rays $\theta \overset{\cos}{=} \frac{\pi}{6}$ and $\theta \overset{\cos}{=} \frac{\pi}{2}$ → $x = r\cos\theta,\ y = r\sin\theta$

$30°$, $y = \frac{x}{\sqrt{3}}$; $90°$, $y = 0$

The surface $z = f(x,y)$ is defined as $\iint_R f(x,y)\,dA$

$$A(x) = \int_{y=g_1(x)}^{y=g_2(x)} f(x,y)\,dy$$

$$V = \int_a^b A(x)\,dx = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\,dy\,dx$$

$$* \int_0^{4\sqrt{3}} \int_{x/\sqrt{3}}^{\sqrt{64-x^2}} \sqrt{64 - x^2}\,dy\,dx$$

$$= \int_0^{4\sqrt{3}} \left[y\sqrt{64-x^2} \right]_{x/\sqrt{3}}^{\sqrt{64-x^2}} dx$$

$$\left(\sqrt{64-x^2}\sqrt{64-x^2}\right) - \left(\frac{x}{\sqrt{3}}\sqrt{64-x^2}\right)$$

$$= 64 - x^2 - \frac{x\sqrt{64-x^2}}{\sqrt{3}}$$

$$= \int_0^{4\sqrt{3}} 64 - x^2 - \frac{x\sqrt{64-x^2}}{\sqrt{3}}\,dx$$

$$= 64x - \frac{x^3}{3} + \frac{\sqrt{64-x^2}^3}{3\sqrt{3}} \Bigg|_0^{4\sqrt{3}}$$

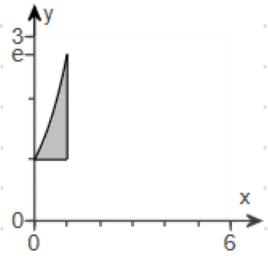
$$= \left(\overset{256\sqrt{3}}{64(4\sqrt{3})} - \frac{\overset{64\sqrt{3}}{(4\sqrt{3})^3}}{3} + \frac{\overset{4^3}{\sqrt{64-(4\sqrt{3})^2}^3}}{3\sqrt{3}} \right) - \left(\cancel{64(0)} - \cancel{\frac{(0)^3}{3}} + \frac{\overset{8^3}{\sqrt{64-(0)^2}^3}}{3\sqrt{3}} \right)$$

$$* \frac{2304 - 192 + 64}{3\sqrt{3}} = \frac{1792\sqrt{3}}{9} \qquad * \frac{512\sqrt{3}}{9}$$

$$= \frac{1280\sqrt{3}}{9} \rightsquigarrow \frac{1280}{3\sqrt{3}}$$

14.2.37 Sketch the region of integration and write an equivalent double integral with the order of integration reversed.

$$\int_0^1 \int_1^{e^x} 6\,dy\,dx$$



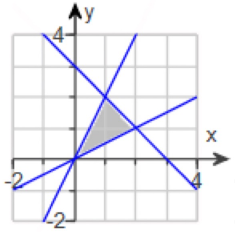
$\ln y \leq x \leq 1$

$1 \leq y \leq e$

$$\therefore \int_1^e \int_{\ln y}^1 6\,dx\,dy$$

14.3.11 Sketch the region bounded by the given lines and curves. Then express the region's area as an iterated double integral and evaluate the integral.

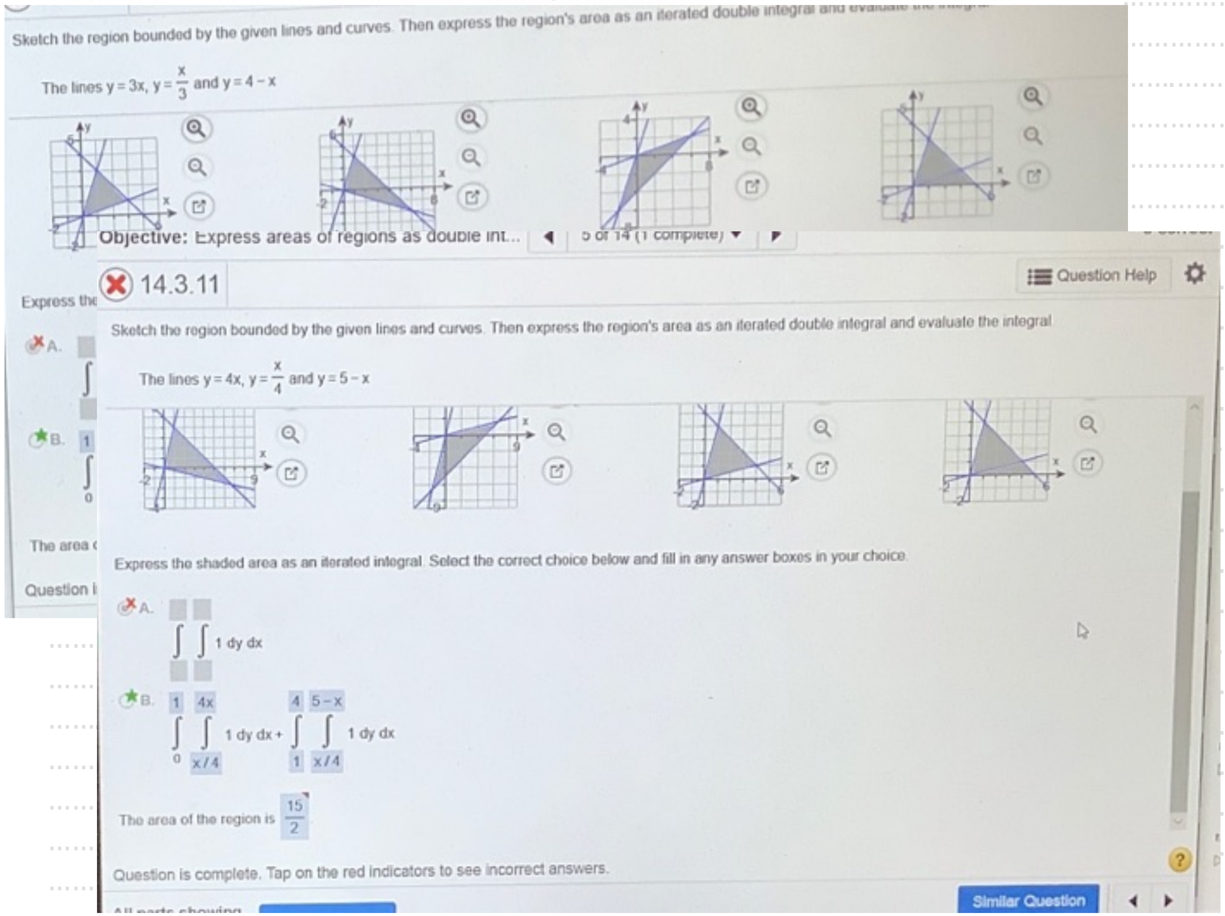
$$\textit{The lines } y = 2x, y = \frac{x}{2} \textit{ and } y = 3 - x$$



The shaded area as an iterated integral

$$= \int_0^1 \int_{x/2}^{2x} 1 \, dy \, dx + \int_1^2 \int_{x/2}^{3-x} 1 \, dy \, dx$$

The area of the region is $\frac{3}{2}$



14.3.21 Find the average height of the paraboloid $z = x^2 + y^2$ over the square $0 \le x \le 3, 0 \le y \le 3$.

$$\frac{1}{A} \iint_R (x^2 + y^2) \, dx \, dy \longrightarrow \int_0^3 \left(\frac{x^3}{3} + y^2 x\right) dy$$

$A = 3 \cdot 3 = 9$

$$= \left[\frac{x^3}{3} + y^2 x\right]_0^3$$

$$= 9 + 3y^2$$

$$\longrightarrow \frac{1}{9} \int_0^3 (9 + 3y^2) \, dx$$

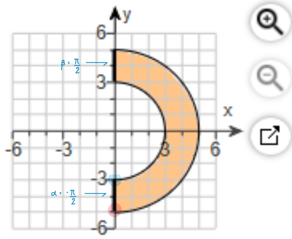
$$= \frac{1}{9} (9y + y^3) \Big|_0^3$$

$$= \frac{54}{9} \rightsquigarrow 6$$

Homework Calculas

(Week 11)

14.4.2 Describe the given region in polar coordinates.



in the from $R: g_1(\theta) \le r \le g_2(\theta)$, $\alpha \le \theta \le \beta$

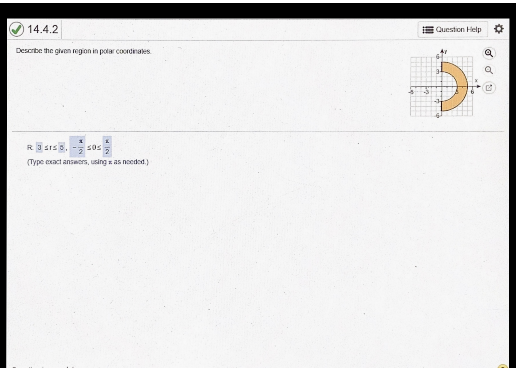
where $\beta - \alpha \le 2\pi$; $r \geq 0$

$\frac{\pi}{2} - \left(-\frac{\pi}{2}\right) = \pi$

$$\boxed{\therefore R: 3 \le r \le 5 \ , \ -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}}$$

$g_1(\theta) = 3$

$g_2(\theta) = 5$



14.4.2

Question Help

Describe the given region in polar coordinates.

R: $3 \le r \le 5$, $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

(Type exact answers, using π as needed.)

14.4.9 Change the Cartesian integral $\int_{-2}^{2} \int_{0}^{\sqrt{4-x^2}} dy\, dx$ into an equivalent polar integral. Then evaluate the polar integral.

เมื่อแทนค่าได้ 1

$$\iint_R f(x,y)\, dx\, dy = \iint_G f(r\cos\theta, r\sin\theta)\, r\, dr\, d\theta$$

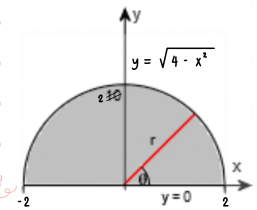
$$\therefore \int_0^{\pi}\int_0^{2} r\, dr\, d\theta = \int_0^{\pi} \left[\frac{r^2}{2}\right]_0^2 d\theta$$

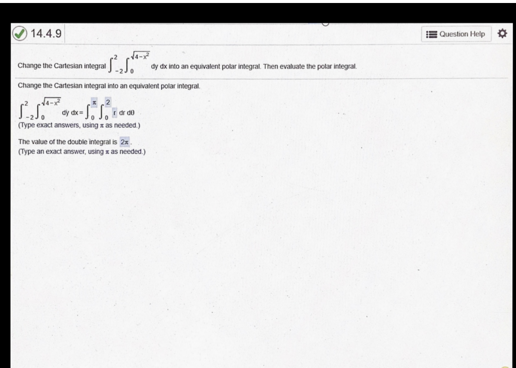
$$= \int_0^{\pi} 2\, d\theta$$

$$2\int_0^{\pi} d\theta = 2\left[\theta\right]_0^{\pi}$$

$$= 2\pi$$

The upper limit for θ is π.





14.4.9

Question Help

Change the Cartesian integral $\int_{-2}^{2} \int_{0}^{\sqrt{4-x^2}} dy\, dx$ into an equivalent polar integral. Then evaluate the polar integral.

Change the Cartesian integral into an equivalent polar integral.

$\int_{-2}^{2} \int_{0}^{\sqrt{4-x^2}} dy\, dx = \int_{0}^{\pi} \int_{0}^{2} r\, dr\, d\theta$

(Type exact answers, using π as needed.)

The value of the double integral is 2π.

(Type an exact answer, using π as needed.)

14.4.17 Change the Cartesian integral into an equivalent polar integral. Then evaluate the polar integral.

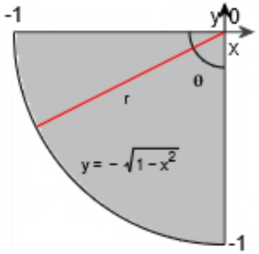
$$\int_{-1}^{0}\int_{-\sqrt{1-x^2}}^{0} \frac{10}{1+\sqrt{x^2+y^2}}\, dy\, dx$$

จะได้ r ด้วยทฤษฎี $\iint_R dx\, dy = \iint_G r\, dr\, d\theta$

$$\int_{\pi}^{\frac{3\pi}{2}}\int_{0}^{1} \frac{10r}{1+r}\, dr\, d\theta = \int_{\pi}^{\frac{3\pi}{2}} \left(10(r-\ln(r+1))\right)\Big|_0^1\, d\theta$$

$$= \int_{\pi}^{\frac{3\pi}{2}} 10(1-\ln 2)\, d\theta$$

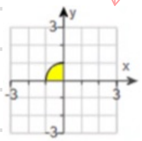
$$= (1-\ln 2)\frac{\cancel{10}^{\,5}\pi}{\cancel{2}}$$



14.4.23 Sketch the region of integration and convert the polar integral to a Cartesian integral or sum of integrals. Do not evaluate the integral.

$$\int_{\frac{\pi}{2}}^{\pi}\int_{0}^{1} r^3 \sin\theta\cos\theta\, dr\, d\theta$$

$r(r\sin\theta)(r\cos\theta)\, dr\, d\theta = yx\, dy\, dx$



$0 \leq r \leq 1$ and $\frac{\pi}{2} \leq \theta \leq \pi$

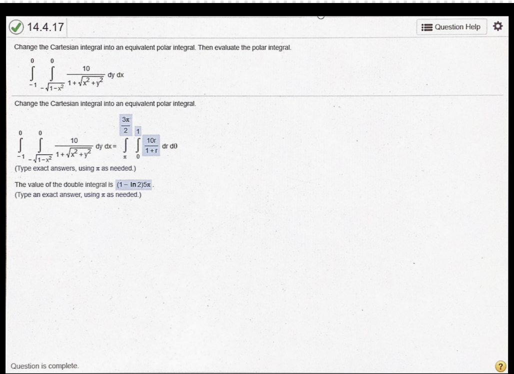
$y = -\sqrt{1-x^2}$

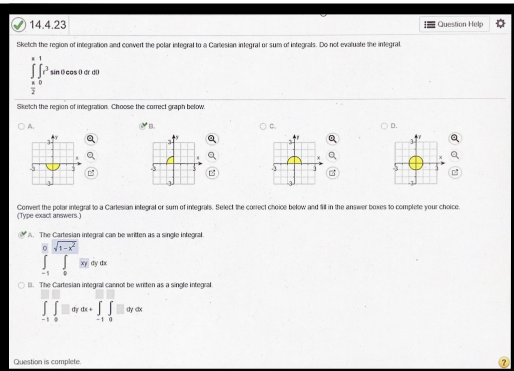
ดังนั้น $-\sqrt{1-x^2} \leq y \leq 0$ for $0 \leq x \leq 1$

∴ rewrite the expression using x and y

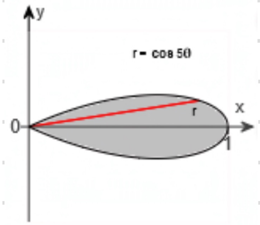
Therefor, the integral rewritten in Cartesian coordinate is $\int_{0}^{1}\int_{-\sqrt{1-x^2}}^{0} xy\, dy\, dx.$

$$= \int_{-1}^{0}\int_{0}^{\sqrt{1-x^2}} xy\, dy\, dx$$





14.4.29 Find the area enclosed by one leaf of the rose $r = 9\cos(5\theta)$



Set $r = 0$; $0 = 9\cos(5\theta)$

$\theta = \frac{\pi}{10}$

$$\int_{-\frac{\pi}{10}}^{\frac{\pi}{10}}\int_0^{9\cos(5\theta)} r\,dr\,d\theta = \int_{-\frac{\pi}{10}}^{\frac{\pi}{10}}\left[\frac{r^2}{2}\right]_0^{9\cos(5\theta)} d\theta$$

$$= \int_{-\frac{\pi}{10}}^{\frac{\pi}{10}} \frac{81}{2}\cos^2(5\theta)\,d\theta$$

$$= \frac{81}{2}\left[\frac{\theta}{2}+\frac{\sin(10\theta)}{20}\right]_{-\frac{\pi}{10}}^{\frac{\pi}{10}}$$

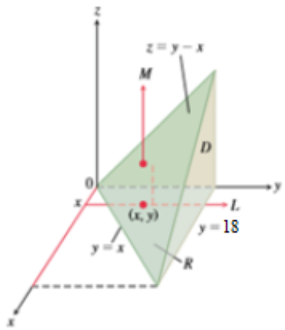
$\frac{\pi}{10}$

$$= \frac{81\pi}{20}$$

14.5.1 Find the volume of the tetrahedron shown using the order dz dx dy. The tetrahedron is bounded by the planes $z = 0, y = 18, x = 0$ *and* $z = y - x$.

$0 \le y \le 18$

$0 \le x \le y$



$$\int_0^{18}\int_0^{y}\left(\int_0^{y-x} dz\right)dx\,dy = \int_0^{18}\int_0^{y}\left([z]_0^{y-x}\right)dx\,dy$$

$$= \int_0^{18}\left(\int_0^{y}(y-x)\,dx\right)dy$$

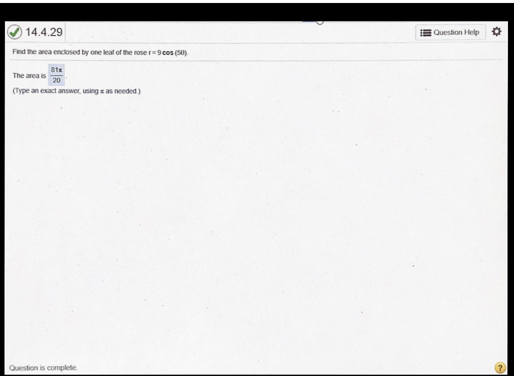
$$= \int_0^{18}\left[yx-\frac{x^2}{2}\right]_0^{y} dy$$

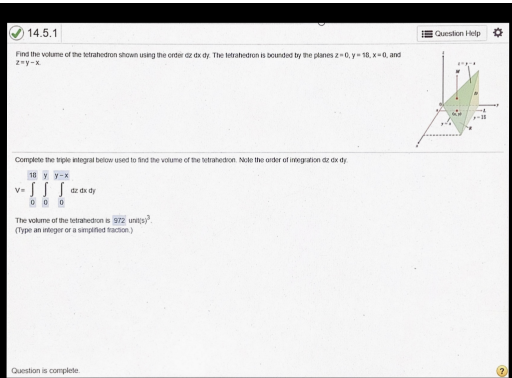
$$= \int_0^{18}\frac{y^2}{2}\,dy$$

$$= \left.\frac{y^3}{6}\right|_0^{18}$$

$$= \frac{5832}{6}$$

$= 972$ unit(s)^3 ※





14.5.11 Evaluate the integral.

$$\int_0^3 \int_0^{3\pi} \int_0^{\pi} y \cos z \, dx \, dy \, dz$$

$$\int_0^3 \int_0^{3\pi} \int_0^{\pi} y \cos z \, dx \, dy \, dz = \int_0^3 \int_0^{3\pi} \left[xy \cos z \right]_{x=0}^{x=\pi} dy \, dz$$

$$= \int_0^3 \int_0^{3\pi} \pi y \cos z \, dy \, dz$$

$$= \pi \int_0^3 \left[\frac{y^2}{2} \cos z \right]_{y=0}^{y=3\pi}$$

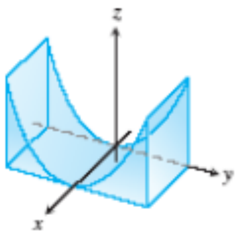
$$= \pi \int_0^3 \frac{9\pi^2}{2} \cos z \, dz$$

$$= \frac{9\pi^3}{2} \int_0^3 \cos z \, dz$$

$$= \frac{9\pi^3}{2} \left[\sin z \right] \Big|_0^3$$

$$= \frac{9\pi^3}{2} \sin 3$$

14.5.23 Find the volume of the region between the cylinder $z = 3y^2$ and the xy-plane that is bounded by the planes $x = 0, x = 2, y = -2$ and $y = 2$



$$\int_0^2 \int_{-2}^{2} \int_0^{3y^2} dz \, dy \, dx = \int_0^2 \int_{-2}^{2} \left[z \right]_0^{3y^2} dy \, dx$$

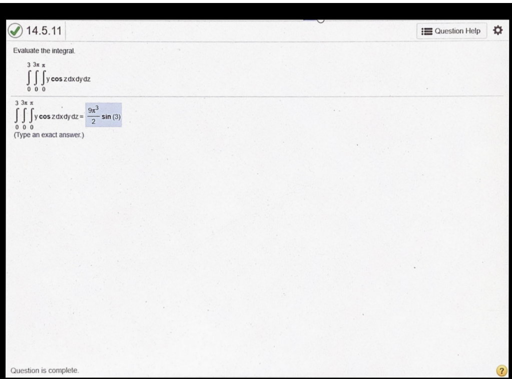
$$= \int_0^2 \int_{-2}^{2} 3y^2 \, dy \, dx$$

$$= \int_0^2 \left[y^3 \right]_{-2}^{2} dx$$

$(2)^3 - (-2)^3 = 16$

$$= \left[16x \right] \Big|_0^2$$

$$= 32$$



14.5.11 Question Help

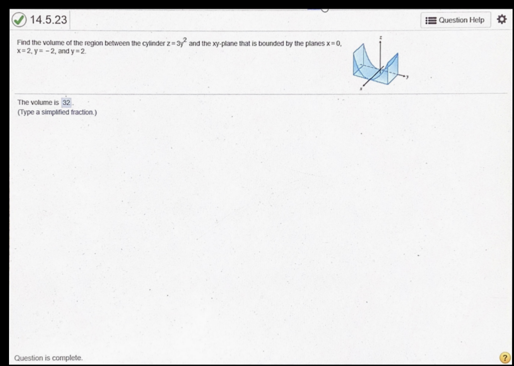
Evaluate the integral.

$$\int_0^3 \int_0^{3\pi} \int_0^{\pi} y \cos z \, dx \, dy \, dz$$

$$\int_0^3 \int_0^{3\pi} \int_0^{\pi} y \cos z \, dx \, dy \, dz = \frac{9\pi^3}{2} \sin(3)$$

(Type an exact answer.)

Question is complete.



14.5.23 Question Help

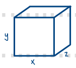
Find the volume of the region between the cylinder $z = 3y^2$ and the xy-plane that is bounded by the planes $x = 0$, $x = 2$, $y = -2$, and $y = 2$.

The volume is 32.

(Type a simplified fraction.)

Question is complete.

14.5.37 Find the average value of $F(x,y,z) = x^2 + 5$ over the cube in the first octant bounded by the coordinate planes and the planes $x = 1, y = 1\ and\ z = 1$



ปริมาตรลูกบาศก์ = 1 Unit^3

$$\int_0^1\int_0^1\int_0^1 (x^2+5)\,dx\,dy\,dz = \int_0^1\int_0^1 \left[\frac{x^3}{3}+5x\right]_0^1 dy\,dz$$

$$= \int_0^1\int_0^1 \frac{16}{3}\,dy\,dz$$

$$= \int_0^1 \left[\frac{16y}{3}\right]_0^1 dz$$

$$= \int_0^1 \frac{16}{3}\,dz$$

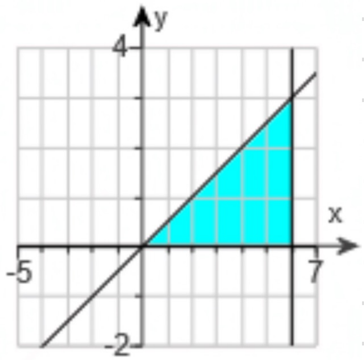
$$= \left[\frac{16z}{3}\right]_0^1$$

$$= \frac{16}{3}$$

$\therefore$ With the volume of cube ; $\dfrac{1}{\text{volume of D}}\iiint_D F\,dV$

$$= \frac{\frac{16}{3}}{1} \rightsquigarrow \frac{16}{3}$$

14.5.41 Evaluate the integral $\int_0^9\int_0^3\int_{2y}^6 \frac{5\cos(x^2)}{4\sqrt{z}}\,dx\,dy\,dz$ by changing the order of integration in an appropriate way.



$$\int_0^9\int_0^3\int_{2y}^6 \frac{5\cos(x^2)}{4\sqrt{z}}\,dx\,dy\,dz = \int_0^9\int_0^6\int_0^{\frac{x}{2}} \frac{5\cos(x^2)}{4\sqrt{z}}\,dy\,dx\,dz$$

$$= \int_0^9\int_0^6 \left[\frac{5\cos(x^2)}{4\sqrt{z}}\,y\right]_0^{\frac{x}{2}} dx\,dz$$

$$= \int_0^9\int_0^6 \frac{5\cos(x^2)}{8\sqrt{z}}\,x\,dx\,dz$$

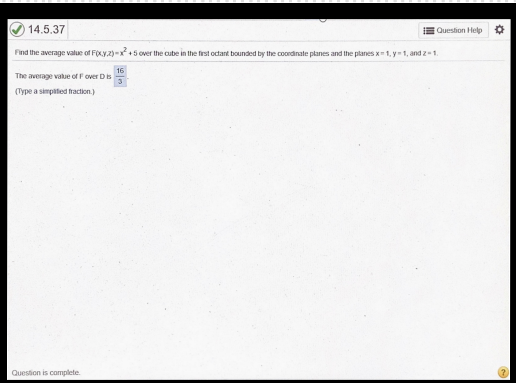
$$= \int_0^9\int_0^{36} \frac{5\cos(u)}{8\sqrt{z}}\left(\frac{du}{2}\right) dz$$

$$= \int_0^9 \left[\frac{5}{16\sqrt{z}}\sin u\right]_0^{36} dz$$

$$= \int_0^9 \frac{5}{16\sqrt{z}}\sin 36\,dz$$

$$= \left[\frac{5}{8}\sin 36\sqrt{z}\right]_0^9$$

$$= \frac{15}{8}\sin 36$$



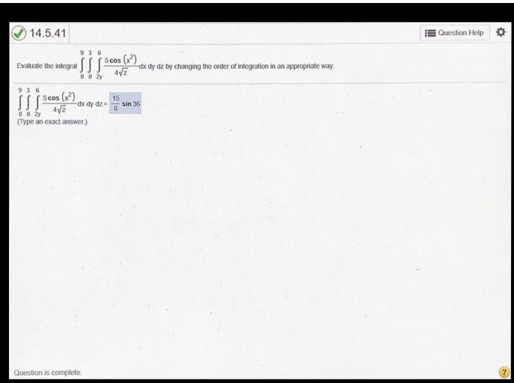
14.5.37 Question Help

Find the average value of $F(x,y,z) = x^2 + 5$ over the cube in the first octant bounded by the coordinate planes and the planes $x = 1$, $y = 1$, and $z = 1$.

The average value of F over D is $\frac{16}{3}$.

(Type a simplified fraction.)

Question is complete.



14.5.41 Question Help

Evaluate the integral $\int_0^9\int_0^3\int_{2y}^6 \frac{5\cos(x^2)}{4\sqrt{z}}\,dx\,dy\,dz$ by changing the order of integration in an appropriate way.

$\int_0^9\int_0^3\int_{2y}^6 \frac{5\cos(x^2)}{4\sqrt{z}}\,dx\,dy\,dz = \frac{15}{8}\sin 36$

(Type an exact answer.)

Question is complete.

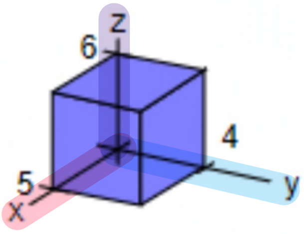
Homework Calculas

(Week 12)

14.7.21 Sketch the graph described by the following spherical coordinates in three-dimensional space.

$0 \leq \rho \cos\theta \sin\phi \leq 5,$ $0 \leq \rho \sin\theta\, sin\phi \leq 4,$ $0 \leq \rho \cos\phi \leq 6$

* ตามความสูงที่กำหนด



14.7.23 Evaluate the cylindrical coordinate integral $\int_0^{\pi}\int_0^{1}\int_r^{\sqrt{3-r^2}} dz\, r\, dr\, d\theta$.

$$\int_0^{\pi}\int_0^{1}\int_r^{\sqrt{3-r^2}} dz\, r\, dr\, d\theta = \int_0^{\pi}\int_0^{1}\left([z]_r^{\sqrt{3-r^2}}\right) r\, dr\, d\theta$$

$$= \int_0^{\pi}\int_0^{1}\left(r\sqrt{3-r^2} - r^2\right) dr\, d\theta$$

$$= \int_0^{\pi}\int_0^{1} r\sqrt{3-r^2}\, dr\, d\theta - \int_0^{\pi}\int_0^{1} r^2\, dr\, d\theta$$

แทนเป็น u

$\frac{du}{dr} = -2r$

$dr = -\frac{1}{2r}$

$$= \int_0^{\pi}\int_3^{2}\sqrt{u}\, du\, d\theta - \int_0^{\pi}\left[\frac{r^3}{3}\right]_0^1 d\theta$$

$$= -\frac{1}{2}\int_0^{\pi}\left[\frac{2u^{3/2}}{3}\right]_3^2 d\theta - \int_0^{\pi}\frac{1}{3}\, d\theta$$

$$= -\frac{1}{3}\int_0^{\pi}\left(2\sqrt{2} - 3\sqrt{3}\right) d\theta - \frac{1}{3}[\theta]_0^{\pi}$$

จะได้ π

$$= \frac{3\sqrt{3} - 2\sqrt{2}}{3}[\theta]\Big|_0^{\pi} - \frac{1}{3}\pi$$

$$= \frac{\pi(3\sqrt{3} - 2\sqrt{2} - 1)}{3}$$

Ans.

14.7.27 Evaluate the cylindrical coordinate integral.

$$\int_0^{\pi}\int_0^{1}\int_r^{\frac{1}{\sqrt{2-r^2}}} 3\,dz\,r\,dr\,d\theta.$$

$$\int_r^{\frac{1}{\sqrt{2-r^2}}} 3\,dz = 3\cdot z\Big|_r^{\frac{1}{\sqrt{2-r^2}}}$$

$$= 3\cdot\left[\frac{1}{\sqrt{2-r^2}} - r\right]$$

$$\int_0^{\pi}\int_0^{1}\int_r^{\frac{1}{\sqrt{2-r^2}}} 3\,dz\,r\,dr\,d\theta = \int_0^{\pi}\int_0^{1} 3\cdot\left[\frac{1}{\sqrt{2-r^2}} - r\right] r\,dr\,d\theta$$

$$3\int_0^1 \frac{r}{\sqrt{2-r^2}} - r^2\,dr$$

$$-\sqrt{2-r^2}\Big|_0^1 - \frac{r^3}{3}\Big|_0^1$$

$$= \sqrt{2} - \frac{4}{3}$$

ดึงค่าคงที่ออกมา

ได้ 2π

$$= 3\left(\sqrt{2} - \frac{4}{3}\right)\int_0^{\pi} d\theta$$

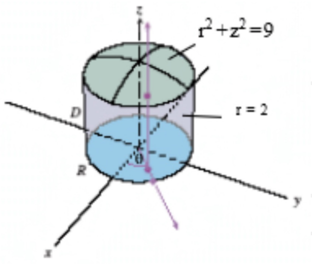
$$= \pi(3\sqrt{2} - 4)$$

$r^2 + z^2 = 16$

$r^2 = 4$

14.7.33 Let D be the region bounded below by the plane $z = 0$, above by the sphere $x^2 + y^2 + z^2 = 16$, and on the sides by the cylinder $x^2 + y^2 = 4$. Set up the triple integrals in cylindrical coordinates that give the volume of D using the following orders of integration.

a. $dz\;dr\;d\theta$ ***b.*** $dr\;dz\;d\theta$ ***c.*** $d\theta\;dz\;dr$



a. $dz\;dr\;d\theta$

The path enters D at $z = 0$

The path leaves D at $z = \sqrt{16-r^2}$

The path enters R at $r = 0$

The path leaves R at $r = 2$

$$\therefore \int_0^{2\pi}\int_0^{2}\int_0^{\sqrt{16-r^2}} r\,dz\,dr\,d\theta \quad \text{Ans.}$$

b. $dr\;dz\;d\theta$

The path enters the cylinder at $r = 0$

The path leaves the cylinder at $r = 2$

หาค่า z ; $(2)^2 + z^2 = 16 \longrightarrow z = 2\sqrt{3}$ ※

The lower limit is $\theta = 0$

The upper limit is $\theta = 2\pi$

The path enters the region at $r = 0$

The path leaves the region at $r = \sqrt{16-r^2}$

The lowest z-value is $z = 2\sqrt{3}$

The highest value of $z = 4$

The lower limit is $\theta = 0$

The upper limit is $\theta = 2\pi$

$$\therefore \int_0^{2\pi}\int_0^{\sqrt{12}}\int_0^{2} r\,dr\,dz\,d\theta + \int_0^{2\pi}\int_{\sqrt{12}}^{4}\int_0^{\sqrt{16-z^2}} r\,dr\,dz\,d\theta \quad \text{Ans.}$$

c. $d\theta\;dz\;dr$

The path enters D at $\theta = 0$

The path reaches its starting point at $\theta = 2\pi$

The path enters R at $z = 0$

The path leaves R at $z = \sqrt{16-r^2}$

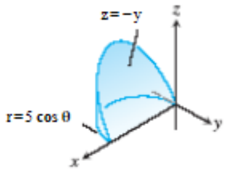
The lower limit for r is 0

The upper limit for r is 2

$$\therefore \int_0^{2}\int_0^{\sqrt{16-r^2}}\int_0^{2\pi} r\,d\theta\,dz\,dr \quad \text{Ans.}$$

$z = -r\sin\theta$

14.7.67 Find the volume of the solid bounded by $z = -y$, the xy-plane and $r = 5\cos\theta$.



$$\int_{-\pi/2}^{0}\int_{0}^{5\cos\theta}\int_{0}^{-r\sin\theta} dz\, r\, dr\, d\theta = \int_{-\pi/2}^{0}\int_{0}^{5\cos\theta} [z]_{0}^{-r\sin\theta}\, r\, dr\, d\theta$$

$$= \int_{-\pi/2}^{0}\int_{0}^{5\cos\theta} -r^2 \sin\theta\, dr\, d\theta$$

$$= -\int_{-\pi/2}^{0}\left[\frac{r^3}{3}\right]_{0}^{5\cos\theta} \sin\theta\, d\theta$$

แทน u^3 $\quad \sin = \frac{-d}{d\theta}\cos\theta$

$$= -\int_{-\pi/2}^{0} \frac{125}{3}\cos^3\theta\, \sin\theta\, d\theta \quad -\frac{1}{\sin\theta}\, d\theta$$

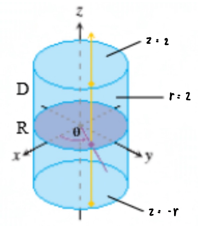
$\frac{du}{d\theta} = -\sin$

$$= -\frac{125}{3}\int_{-\pi/2}^{0} u^3\, du$$

$$= \frac{125}{3}\left[\frac{u^4}{4}\right]_{0}^{1}$$

$$= \frac{125}{12}$$

14.7.85 Find the average value of the function $f(r,\theta,z) = r$ over the region bounded by the cylinder r=2 and between the planes z=−2 and z=2.



Averge value $= \frac{1}{\text{Volume of D}} \iiint_D F\, dV$ $\longrightarrow V = \frac{4}{3}$

$dV = dz\, r\, dr\, d\theta$

ปริมาตร $V = \pi r^2 h$

$= \pi(2)^2(4)$

$= 16\pi$

$$= \int_{0}^{2\pi}\int_{0}^{2}\int_{-2}^{2} r^2\, dz\, dr\, d\theta$$

$$= \int_{0}^{2\pi}\int_{0}^{2} r^2 [z]_{-2}^{2}\, dr\, d\theta$$

$$= \int_{0}^{2\pi}\int_{0}^{2} 4r^2\, dr\, d\theta$$

$$= \int_{0}^{2\pi} \frac{4r^3}{3}\Big|_{0}^{2}\, d\theta$$

$$= \frac{32}{3}\theta\Big|_{0}^{2\pi}$$

$$= \frac{64\pi}{3}$$

14.8.1 Solve the system $u = x - y$, $v = 2x + 2y$ for x and y in terms of u and v. Then find the value of the Jacobian $\frac{\partial(x,y)}{\partial(u,v)}$. Find the image under the transformation of the triangular region with vertices (0,0) , (2,2) , and (2, −2) in the xy-plane. Sketch the transformed region in the uv-plane.

$y = x - u$

$v = 2x + 2(x - u)$

$\boxed{x = \frac{2u+v}{4}, \quad y = \frac{-2u+v}{4}}$

$\frac{\partial x}{\partial u} = \frac{1}{2}, \frac{\partial x}{\partial v} = \frac{1}{4} \qquad \frac{\partial y}{\partial u} = -\frac{1}{2}, \frac{\partial y}{\partial v} = \frac{1}{4}$

$J(u,v) = \frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial y}{\partial u}\frac{\partial x}{\partial v}$

$= \left(\frac{1}{2}\cdot\frac{1}{4}\right) - \left(\frac{-1}{2}\cdot\frac{1}{4}\right)$

$= \frac{1}{4}$

หาสมการ P_1 ถึง P_2

$(0,0)$ to $(2,-2)$: $y = \frac{-3}{2}x$

$(2,2)$ to $(2,-2)$: $x = 1$

$x \quad y$

$\frac{2u+v}{4} = \frac{-2u+v}{4}$

$u = 0$

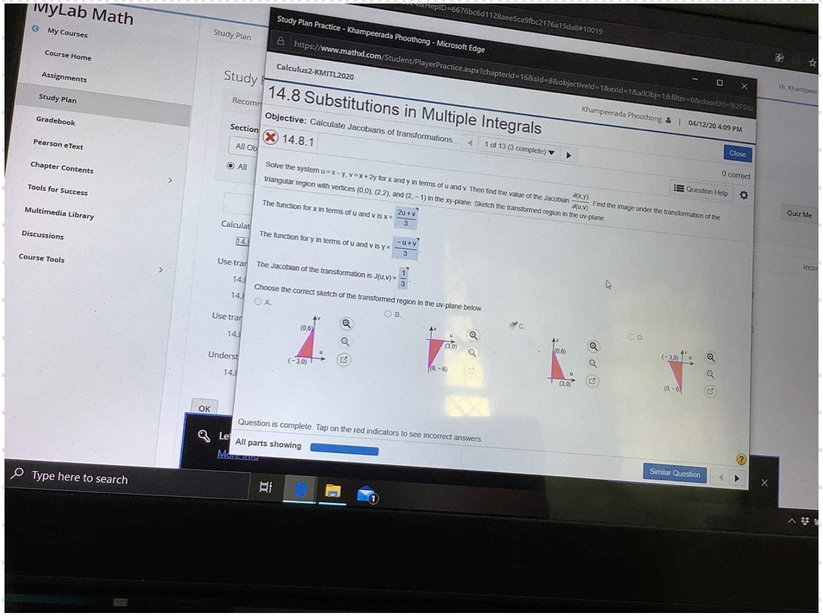
$y \quad -\frac{3}{2}x$

ถ้าเป็น $y = \frac{-3}{2}x$: $\frac{-2u+v}{4} = \frac{-3}{2}\left(\frac{2u+v}{4}\right)$

$v = 0$

ถ้าเป็น $x = 1$: $\frac{2u+v}{4} = 1$

$v = 4 - 2u$



14.8.5 Evaluate the integral directly by integration with respect to x and y.

$$\int_0^2 \int_{x=\frac{y}{6}}^{x=\left(\frac{y}{6}\right)+1} \frac{6x-y}{6}\, dxdy$$

แทน u, $v = \frac{y}{6}$

$\int_{y/6}^{(y/6)+1} \frac{6x-y}{6}\, dx = \left[\frac{x^2}{2} - \frac{xy}{6}\right]_{y/6}^{(y/6)+1}$

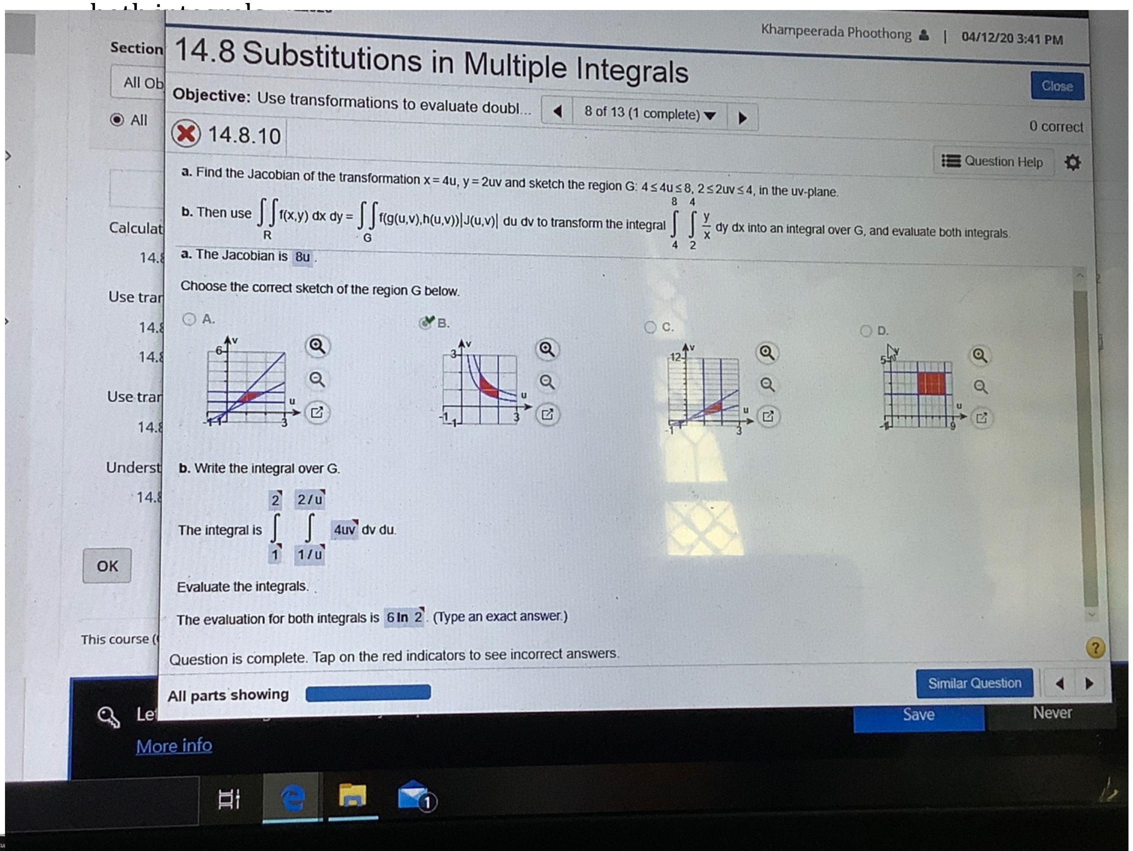
$= \frac{1}{2}$

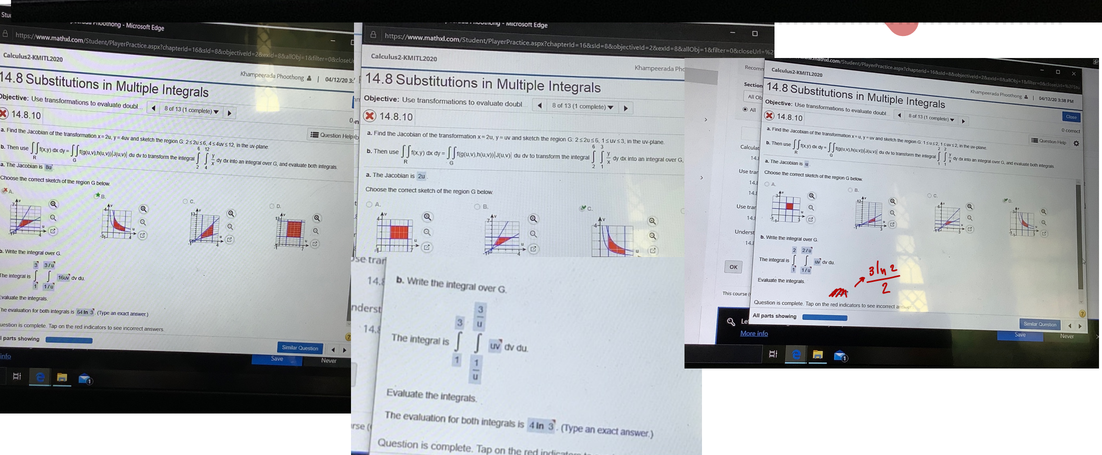
$\int_0^2 \frac{1}{2}\, dy = 1$

$\therefore \int_0^{1/3}\int_0^1 6u\, du\, dv = 1$

14.8.10 (a) Find the Jacobian of the transformation $x = 4u, y = 2uv$ and sketch the region $G: 4 \le 4u \le 8, 2 \le 2uv \le 4$.

(b) Then use $\iint_R f(x,y)dxdy = \iint_G f(g(u,v),h(u,v))|J(u,v)|dudv$ to transform the integral $\int_4^8 \int_2^4 \frac{y}{x} dy\, dx$ into an integral over G, and evaluate both integrals.





14.8.19 Evaluate the given integral over the solid ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \le 9$. (Hint: Let x = au, y = bv and z = cw. Then integrate over an appropriate region in uvw-space.)

$\frac{(au)^2}{a^2} + \frac{(bv)^2}{b^2} + \frac{(cw)^2}{c^2}$

$$\iiint 4|xyz|\, dxdydz$$

Note

$$J(u,v,w) = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{vmatrix} = \begin{vmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{vmatrix} = abc$$

$$\iiint_R F(x,y,z)dxdydz = \iiint_G H(u,v,w)|J(u,v,w)|\, dudvdw$$

$$= \iiint_G |au\ bv\ cw|\ |abc|\ dudvdw$$

$\rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$

$$= a^2b^2c^2 \iiint_G |uvw|\ dudvdw$$

สรุป $6 \cdot a^2b^2c^2 \int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}}\int_0^{3} (\rho\sin\phi\cos\theta)(\rho\sin\phi\sin\theta)(\rho\cos\phi)\rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$

ค่าคงที่

$= 6 \cdot a^2b^2c^2 \int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}}\int_0^{3} \rho^5 \sin^3\phi\cos\phi\sin\theta\cos\theta\, d\rho\, d\phi\, d\theta$

$\sin^3\phi\cos\phi\sin\theta\cos\theta \left[\frac{\rho^6}{6}\right]_0^3$ $\frac{243}{2}$

$= \frac{243}{2} \cdot 6 \cdot a^2b^2c^2 \int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} \sin^3\phi\cos\phi\sin\theta\cos\theta\, d\phi\, d\theta$

$\sin\theta\cos\theta\left[\frac{u^4}{4}\right]_0^1 = \frac{1}{4}\sin\theta\cos\theta$

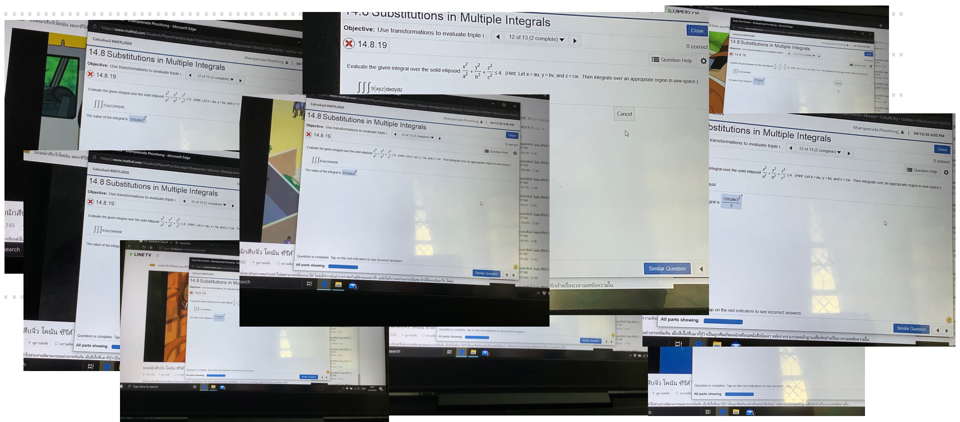
$= \frac{1}{4} \cdot \frac{243}{2} \cdot 6 \cdot a^2b^2c^2 \int_0^{\frac{\pi}{2}} \sin\theta\cos\theta\, d\theta$

$\left[\frac{u^2}{2}\right]_0^1 = \frac{1}{2}$

$= \frac{729a^2b^2c^2}{8}$

ลืม x 4

คำตอบคือ $486(abc)^2$



Homework Calculas

(Week 13)

15.1.11 Evaluate $\int_c (xy + x + z)\, ds$ *along the curve* $r(t) = 2ti + tj + (6 - 2t)k, 0 \le t \le 1.$

$$\int_c f(x,y,z)\, ds = \int_a^b f(g(t), h(t), k(t))\, |v(t)|\, dt$$

$v(t) = 2i + j - 2k$

$|v(t)| = \sqrt{2^2 + 1^2 + (-2)^2}$

$= 3$

$$= \int_0^1 f(2t, t, 6-2t)(3)\, dt$$

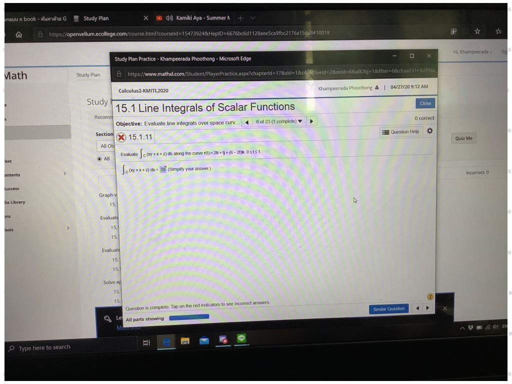
$$= \int_0^1 ((2t)(t) + t + (6-2t))(3)\, dt$$

$$= 3\int_0^1 (2t^2 - t + 6)\, dt$$

$$= 3\left[\frac{2t^3}{3} - \frac{t^2}{2} + 6t\right]_0^1$$

$$= 3\left[\frac{2(1)^3}{3} - \frac{(1)^2}{2} + 6(1)\right] - 0$$

$$= \frac{37}{2}$$



15.1.15 Integrate $f(x, y, z) = x + \sqrt{y} - z^3$ over the path from (0,0,0) to (1,1,1) given by

$$C_1 : r(t) = ti + t^2 j, 0 \le t \le 1$$

$$C_2 : r(t) = i + j + tk, 0 \le t \le 1$$

$$\int_{C_1} (x + \sqrt{y} - z^3)\, ds = \int_0^1 \overset{2t}{f(t, t^2, 0)}\, |v_1(t)|\, dt$$

Find $|v_1(t)| = \sqrt{\left(\frac{dx_1}{dt}\right)^2 + \left(\frac{dy_1}{dt}\right)^2 + \left(\frac{dz_1}{dt}\right)^2}$ 0 → 0, t → 1, $t^2 \to 2t$

$= \sqrt{(1)^2 + (2t)^2 + (0)^2}$

$= \sqrt{4t^2 + 1}$

$$= \int_0^1 2t\sqrt{4t^2+1}\, dt$$

$$= \left[\frac{(4t^2+1)^{3/2}}{6}\right]_0^1$$

$$= \frac{5\sqrt{5} - 1}{6}$$

$$\int_{C_2} (x + \sqrt{y} - z^3)\, ds = \int_0^1 \overset{2 - t^3}{f(1, 1, t)}\, |v_2(t)|\, dt$$

Find $|v_2(t)| = \sqrt{\left(\frac{dx_2}{dt}\right)^2 + \left(\frac{dy_2}{dt}\right)^2 + \left(\frac{dz_2}{dt}\right)^2}$ t → 1, 1 → 0, 1 → 0

$= \sqrt{(0)^2 + (0)^2 + (1)^2}$

$= 1$

$$= \int_0^1 (2 - t^3)\, dt$$

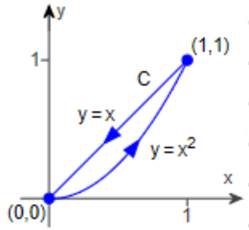
$$= \left[2t - \frac{t^4}{4}\right]_0^1$$

$$= \frac{7}{4}$$

$$\therefore \int_C (x + \sqrt{y} - z^3)\, ds = \frac{5\sqrt{5} - 1}{6} + \frac{7}{4}$$

$$= \frac{10\sqrt{5} + 19}{12}$$

15.1.25 Evaluate $\int_C \left(7x + 5\sqrt{y}\right)$ ds where C is given in the accompanying figure.



$* \int_C f\, ds = \sum \int_{C_n} f\, ds$

$ds = |v_1(t)|\, dt$

$= \sqrt{\left(\frac{dx_1}{dt}\right)^2 + \left(\frac{dy_1}{dt}\right)^2}\; dt$

$= \sqrt{1^2 + 2t^2}\; dt$

$= \sqrt{4t^2+1}\; dt$

$\int_{C_1} (7x + 5\sqrt{y})\, ds = \int_0^1 (7t + 5\sqrt{t^2})(\sqrt{4t^2+1})\, dt$

$= \int_0^1 12t\sqrt{4t^2+1}\; dt$

$= \int_0^5 \left(\frac{12}{7}\right)\sqrt{u}\, du$

$= \cancel{\left[\frac{18}{7}u^{3/2}\right]_1^5}$

$= \frac{90}{7}\sqrt{5} - \frac{18}{7}$

Ans.

$5\sqrt{5} + \frac{41}{6}\sqrt{2} - 1$

$ds = |v_2(t)|\, dt$

$= \sqrt{(-1)^2 + (-1)^2}\; dt$

$= \sqrt{2}\; dt$

$\int_{C_2} (7x + 5\sqrt{y})\, ds = \int_0^1 \left(7(1-t) + 5\sqrt{1-t}\right)\sqrt{2}\; dt$

$= 7\sqrt{2}\int_0^1 (1-t)\, dt + \sqrt{2}\int_0^1 5\sqrt{1-t}\; dt$

$\frac{7\sqrt{2}}{2}$ $\qquad \frac{10}{3}\sqrt{2}$

$\therefore \int_C (7x + 5\sqrt{y})\, ds = \frac{90}{7}\sqrt{5} - \frac{18}{7} + \frac{7\sqrt{2}}{2} + \frac{10}{3}\sqrt{2}$

$= \frac{90}{7}\sqrt{5} + \frac{41\sqrt{2}}{6} - \frac{18}{7}$

15.1.29 Integrate f(x,y) = x + y over the curve $C: x^2 + y^2 = 36$ in the first quadrant from (6,0) to (0,6)

$r(t) = (6\cos t)i + (6\sin t)j \;;\; 0 \le t \le \frac{\pi}{2}$

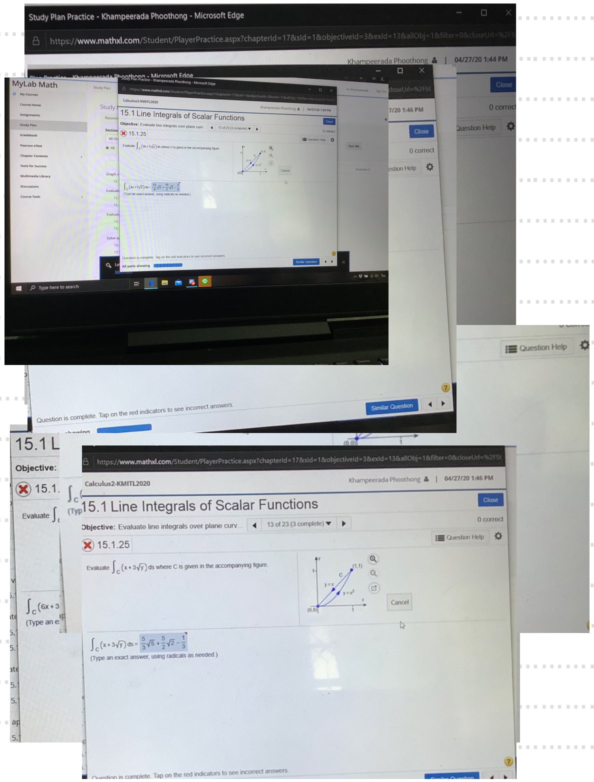
$|v(t)| = 6$

$\int_C f(x,y)\, ds = \int_0^{\pi/2} f(\cos t, \sin t)(6)\, dt$

$= 36\int_0^{\pi/2} (\cos t + \sin t)\, dt$

$[\sin t - \cos t]_0^{\pi/2} = 2$

$= 72$



15.2.2 Find the gradient filed of the function $f(x, y, z) = ln\sqrt{3x^2 + 4y^2 + z^2}$

$\boxed{u = \sqrt{3x^2 + 4y^2 + z^2}}$

$\frac{\partial}{\partial u}(\ln u) = \frac{1}{u}$

$\nabla f = \frac{\partial f}{\partial u}\frac{\partial u}{\partial x} i + \frac{\partial f}{\partial u}\frac{\partial u}{\partial y} j + \frac{\partial f}{\partial u}\frac{\partial u}{\partial z}$

$\frac{3x}{3x^2 + 4y^2 + z^2}$ $\quad$ $\frac{4y}{3x^2 + 4y^2 + z^2}$ $\quad$ $\frac{z}{3x^2 + 4y^2 + z^2}$

$= \frac{1}{3x^2 + 4y^2 + z^2}(3xi + 4yj + zk)$

15.2.21 Find the work done by F over the curve in the direction of increasing t.

$ti + (\sin t)j + (\cos t)k$

$$F = zi + xj + yk, r(t) = (\sin t)i + (\cos t)j + tk \qquad ; 0 \leq t \leq 2\pi$$

$\frac{dr}{dt} = \frac{d}{dt}(\sin t)i + \frac{d}{dt}(\cos t)j + \frac{d}{dt}tk$

$= (\cos t)i - (\sin t)j + k$

* $F \cdot \frac{dr}{dt} = t\cos t - \sin^2 t + \cos t$

* $W = \int_a^b F \cdot \frac{dr}{dt}\, dt \xrightarrow{\text{Ans.}} -\pi$

$= \int_0^{2\pi} (t\cos t - \sin^2 t + \cos t)\, dt$

$\sin t \Big|_0^{2\pi} = 0$

$\frac{t}{2} - \frac{\sin(2t)}{4}\Big|_0^{2\pi} = \pi$

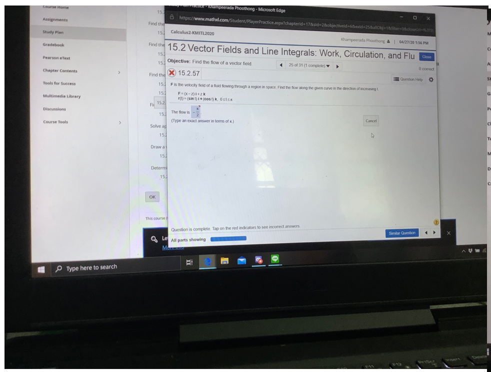
$\cos t + t\sin t \Big|_0^{2\pi} = 0$

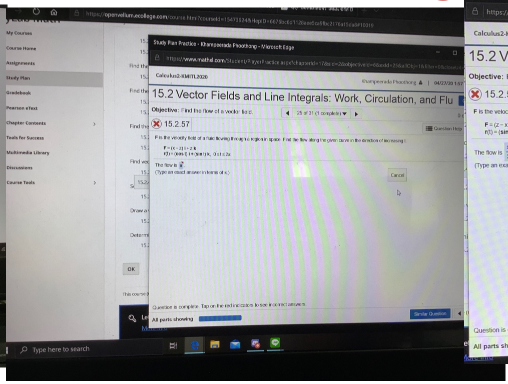
15.2.57 F is the velocity field of a fluid flowing through a region in space. Find the flow along the given curve in the direction of increasing t.

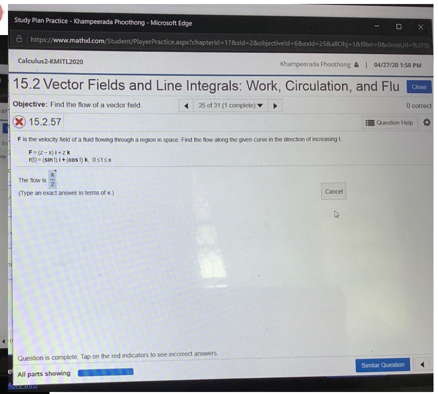
$$F = (z - x)i + xk$$

$$r(t) = (\sin t)i + (\cos t)k \ \ , 0 \le t \le \pi$$

Ans 0







Homework Calculas

(Week 14)

15.3.1 Determine if the field $F = \overset{M(x,y,z)}{11yz\,i} + \overset{N(x,y,z)}{11xz\,j} + \overset{P(x,y,z)}{11xy}\,k$ is conservative or not conservative.

F is conservative and only if $\underbrace{\frac{\partial P}{\partial y} = \frac{\partial N}{\partial z}}_{11x}, \underbrace{\frac{\partial M}{\partial z} = \frac{\partial P}{\partial x}}_{11y}, \underbrace{\frac{\partial N}{\partial x} = \frac{\partial M}{\partial y}}_{11z}$

15.3.3 Determine if the field $F = 2y\,i + 2(x+z)j - 2y\,k$ is conservative or not conservative.

* ใช้ทฤษฎีเดียวกับข้อที่ 1 แค่เช็คว่า diff แล้วเท่ากันรึป่าว

15.3.11 Find a potential function f for the field F.

$$F = \underbrace{(\ln x + \sec^2(2x+2y))}_{M} i + \underbrace{\left(\sec^2(2x+2y) + \frac{3y}{y^2+z^2}\right)}_{N} j + \underbrace{\left(\frac{3z}{y^2+z^2}\right)}_{P} k$$

F is conservative and only if $\frac{\partial P}{\partial y} = \frac{\partial N}{\partial z}$, $\frac{\partial M}{\partial z} = \frac{\partial P}{\partial x}$, $\frac{\partial N}{\partial x} = \frac{\partial M}{\partial y}$

$\frac{\partial P}{\partial y} = -\frac{6zy}{(y^2+z^2)^2}$, $\frac{\partial N}{\partial z} = -\frac{6zy}{(y^2+z^2)^2}$

$\frac{\partial M}{\partial z} = 0$, $\frac{\partial P}{\partial x} = 0$

$\frac{\partial N}{\partial x} = 4\sec^2(2x+2y)\tan(2x+2y)$, $\frac{\partial M}{\partial y} = 4\sec^2(2x+2y)\tan(2x+2y)$

$$\boxed{\begin{aligned} \frac{\partial f}{\partial x} &= \ln x + \sec^2(2x+2y) \\ \frac{\partial f}{\partial y} &= \sec^2(2x+2y) + \frac{3y}{y^2+z^2} \\ \frac{\partial f}{\partial z} &= \frac{3y}{y^2+z^2} \end{aligned}}$$

$$\int \frac{\partial f}{\partial x}\,dx = \int (\ln x + \sec^2(2x+2y))\,dx$$

$$f(x,y,z) = x\ln x - x + \frac{1}{2}\tan(2x+2y) + g(y,z)$$

$$\frac{\partial f}{\partial y} = \sec^2(2x+2y) + \underbrace{\frac{3y}{y^2+z^2}}_{\frac{\partial g}{\partial y}}$$

$$\int \frac{\partial g}{\partial y}\,dy = \int \frac{3y}{y^2+z^2}\,dy$$

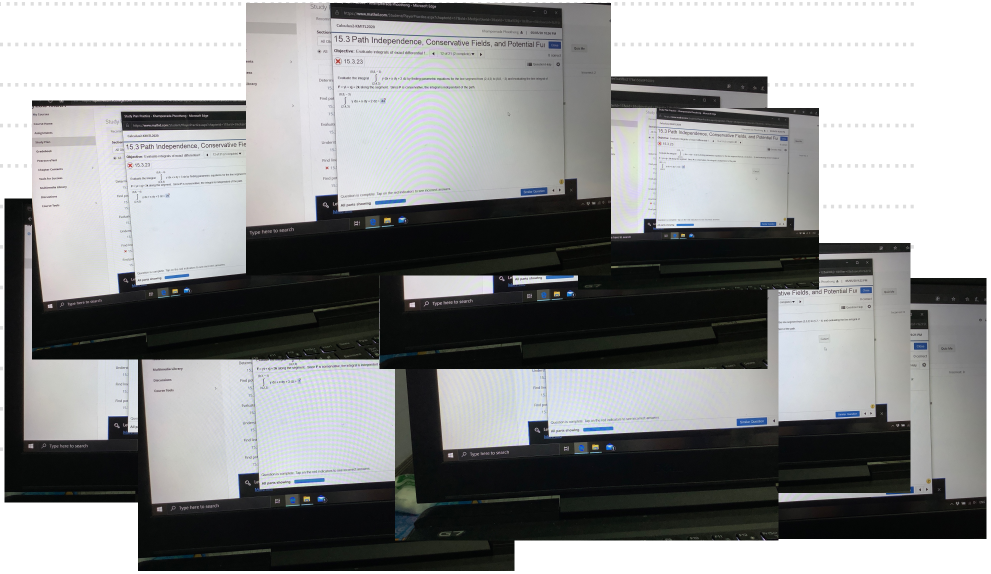
$$g(y,z) = \frac{3}{2}\ln(y^2+z^2) + h(z)$$

$$\frac{\partial f}{\partial z} = \frac{3y}{y^2+z^2} + \underbrace{\frac{\partial h}{\partial z}}_{0}$$; h(z) is a constant function

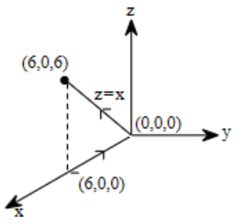
$$\therefore f(x,y,z) = x\ln x - x + \frac{1}{2}\tan(2x+2y) + \frac{3}{2}\ln(y^2+z^2) + C$$

15.3.23 Evaluate the integral $\int_{(5,3,1)}^{(8,8,-3)} y\,dx + x\,dy + 4\,dz$ by finding parametric equations for the line segment from (5,3,1) to (8,8,-3) and evaluating the line integral of $F = yi + xj + 4k$ along the segment. Since F is conservative, the integral is independent of the path.

33 Ans.



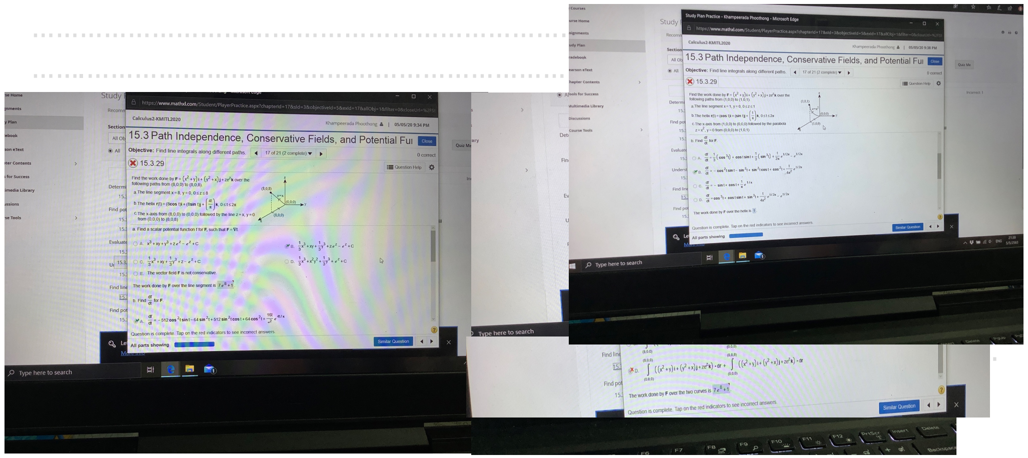
15.3.29 Find the work done by $F = (x^2 + y)\,i + (y^2 + x)\,j + ze^z\,k$ over the following paths from (6,0,0) to (6,0,6).



a. The line segment x = 6, y = 0, ≤ z ≤ 6

b. The helix $r(t) = (6\cos t)i + (6\sin t)j + \left(\frac{3t}{\pi}\right)k$,0≤t≤2π

c. The x-axis from (6,0,0) to (0,0,0) followed by the line z = x, y = 0 from (0,0,0) to (6,0,6)



15.4.1 Find the k-component of (curl F) for the following vector field on the plane.

$$F = \overset{M}{(x+2y)}i + \overset{N}{(8xy)}j$$

$\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}$

$\frac{\partial}{\partial x}(8xy) \quad \frac{\partial}{\partial y}(x+2y)$

$(\text{curl } F) \cdot k = 8y - 2$

15.4.9 Verify the conclusion of Green's Theorem by evaluating both sides of each of the two forms of Green's Theorem for the field $F = xi - 7yj$. Take the domains of integration in each case to be the disk $R: x^2 + y^2 \leq a^2$ and its bounding circle $C: r = (a\cos t)i + (a\sin t)j, 0 \leq t \leq 2\pi$.

ทำ diff $a \sin t \, dt$ $-a\cos t \, dt$

จะได้ $M = a\cos t$, $N = -7a\sin t$

$$\oint_C M\,dy - N\,dx = \boxed{\iint_R \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}\right) dx\,dy}$$

$$= \int_{t=0}^{t=2\pi} \left[(a\cos t)(a\cos t\,dt) - (-7a\sin t)(-a\sin t\,dt)\right]$$

$$= \int_0^{2\pi} (a^2\cos^2 t\,dt - 7a^2\sin^2 t\,dt)$$

$$= -6a^2\pi^{*}$$ ···· flux

พื้นที่วงกลม πr^2 หรือ $a^2\pi$ ซึ่งเราได้ $-6a^2\pi$, $-6\iint_R dx\,dy$

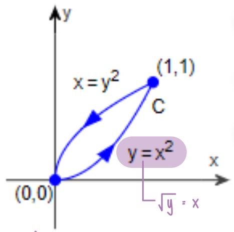
$$\oint_C M\,dy + N\,dx = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy$$

$$= \int_{t=0}^{t=2\pi} \left[(a\cos t)(-a\sin t\,dt) + (-7a\sin t)(a\cos t\,dt)\right]$$

$$= \left.\frac{a^2\cos^2 t}{2} - \frac{7a^2\cos^2 t}{2}\right|_{t=0}^{t=2\pi}$$

$= 0$ ···· circulation

15.4.15 Use Green's Theorem to find the counterclockwise circulation and outward flux for the field F and curve C.



$\sqrt{y} = x$

* ค่าในแกน y มีค่า 0,1

$$F = \underbrace{(5xy + 2y^2)}_{M}i + \underbrace{(5x - 2y)}_{N}j$$

$\frac{\partial M}{\partial x} = 5y$ $\frac{\partial N}{\partial y} = -2$

$$\oint_C F \cdot n\,ds = \oint_C M\,dy - N\,dx = \iint_R \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}\right) dx\,dy$$

$$= \iint_R (5y - 2)\,dx\,dy$$

เปลี่ยน

$$\int_{y=0}^{y=1}\int_{x=y^2}^{x=\sqrt{y}} (5y-2)\,dx\,dy$$

$$= \int_0^1 \left[5xy - 2x\right]_{y^2}^{\sqrt{y}} dy$$

$$= \int_0^1 5y^{3/2} - 2\sqrt{y} - 5y^3 + 2y^2\,dy$$

$$= \left.\frac{10y^{5/2}}{5} - \frac{4y^{3/2}}{3} - \frac{5y^4}{4} + \frac{2y^3}{3}\right|_0^1$$

the outward flux is $= \frac{1}{12}$

$$\oint_C F \cdot T\,ds = \oint_C M\,dx + N\,dy = \iint_R \left(\underbrace{\frac{\partial N}{\partial x}}_{5} - \underbrace{\frac{\partial M}{\partial y}}_{5x+4y}\right) dx\,dy$$

$$= \iint_R \left[5 - (5x + 4y)\right] dx\,dy$$

$$= \int_{y=0}^{y=1}\int_{x=y^2}^{x=\sqrt{y}} \left[5 - (5x + 4y)\right] dx\,dy$$

$$= \int_0^1 \left[-\frac{5x^2}{2} - 4xy - 5x\right]_{y^2}^{\sqrt{y}} dy$$

$$= \int_0^1 \left[-\frac{5y}{2} - 4y^{3/2} + 5\sqrt{y} + \frac{5y^4}{2} + 4y^3 - 5y^2\right] dy$$

$$= \left[-\frac{5y^2}{4} - \frac{8y^{5/2}}{5} + \frac{10y^{3/2}}{3} + \frac{5y^5}{10} + \frac{4y^4}{4} - \frac{5y^3}{3}\right]_0^1$$

the counterclockwise circulation is $\frac{19}{60}$

15.4.25 Find the work done by $F = 5xy^3 i + 9x^2 y^2 j$ in moving a particle once counterclockwise around the curve C: the boundary of the "triangular" region in the first quadrant enclosed by the x-axis, the line x = 1, and the curve $y = x^3$.

$\frac{\partial M}{\partial y} = 15xy^3$ $\quad$ $\frac{\partial N}{\partial x} = 18y^2 x$

$0 \le y \le x^3 \; , \; 0 \le x \le 1$

$$\oint_C F \cdot T \, ds = \oint_C M\,dx + N\,dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dx\,dy$$

$$= \iint_R (18y^2 x - 15xy^2)\, dxdy$$

$$= \iint_R 3xy^2 \, dxdy$$

เปลี่ยน

$$\int_0^1 \left[\int_0^{x^3} 3xy^2 \, dx \right] dx = \int_0^1 \left[xy^3 \right]_0^{x^3}$$

$$= \int_0^1 x^{10} \, dx$$

$$= \left. \frac{x^{11}}{11} \right|_0^1$$

$$= \frac{1}{11}$$

15.4.34 Use the Green's Theorem area formula shown on the right to find the area of the region enclosed by the given curves.

One arch of the cycloid $x = 2t - 2\sin t$, $y = 2 - 2\cos t$ and the x-axis.

Green's Theorem Area Formula

$$\text{Area of R} = \frac{1}{2}\oint_C x\,dy - y\,dx$$

คำตอบ คือ

12π