

Two-Phase Friction

Experimental data indicates that the frictional pressure drop in a boiling channel is substantially higher than that for a single-phase channel with the same length and mass flow rate. Explanations for this include an apparent increased surface roughness due to bubble formation on the heated surface and increased flow velocities. The standard approach to correlating two-phase frictional losses is to assume the total system mass flow rate is due to a saturated liquid and then multiply by an empirical correction factor $\Phi_{\ell o}^2$ called the *two-phase multiplier*. The local frictional loss would then be

$$-\frac{\partial \mathcal{P}}{\partial z}\bigg|_{friction} = \frac{f_f}{D_e} \frac{G^2}{2\rho_f g_c} \Phi_{\ell o}^2 \quad (1)$$

A simple functional form for $\Phi_{\ell o}^2$ can be derived by assuming homogeneous flow. For a constant mass flux, velocity increases in a boiling channel as density decreases. If we assume the increase in the frictional loss is due solely to the increase in velocity, then

$$-\frac{\partial \mathcal{P}}{\partial z}\bigg|_{friction} = \frac{f_f}{D_e} \frac{G^2}{2\rho_{2\phi} g_c} \quad (2)$$

or

$$-\frac{\partial \mathcal{P}}{\partial z}\bigg|_{friction} = \frac{f_f}{D_e} \frac{G^2}{2\rho_f g_c} \left(\frac{\rho_f}{\rho_{2\phi}} \right) \quad (3)$$

such that the two phase multiplier is

$$\Phi_{\ell o}^2 = \frac{\rho_f}{\rho_{2\phi}} \quad (4)$$

The two-phase density is defined as

$$\rho_{2\phi} = \alpha_\ell \rho_f + \alpha_g \rho_g \quad (5)$$

where we have assumed the phases are at equilibrium. The volume fraction under the assumption of homogeneous flow can be obtained from the Fundamental Void-Quality-Slip Relation

$$\alpha_g = \frac{1}{1 + \frac{1-x}{x} \frac{\rho_g}{\rho_f}} \quad (6)$$

and

$$\alpha_\ell = 1 - \alpha_g = \frac{\frac{(1-x)}{x} \frac{\rho_g}{\rho_f}}{1 + \frac{(1-x)}{x} \frac{\rho_g}{\rho_f}} \quad (7)$$

The two-phase density is then

$$\rho_{2\phi} = \frac{\rho_g}{1 + \frac{(1-x)}{x} \frac{\rho_g}{\rho_f}} + \frac{\frac{(1-x)}{x} \rho_g}{1 + \frac{(1-x)}{x} \frac{\rho_g}{\rho_f}} = \frac{\rho_g / x}{1 + \frac{(1-x)}{x} \frac{\rho_g}{\rho_f}} \quad (8)$$

$$\rho_{2\phi} = \frac{\rho_g}{x + (1-x) \frac{\rho_g}{\rho_f}} \quad (9)$$

$$\frac{1}{\rho_{2\phi}} = \frac{x}{\rho_g} + \frac{(1-x)}{\rho_f} \quad (10)$$

Note, that Equation 10 implies

$$\nu = x\nu_g + (1-x)\nu_f \quad (11)$$

which is a familiar result. Substituting Equation 10 into Equation 4, gives

$$\Phi_{\ell o}^2 = 1 - x + x \frac{\rho_f}{\rho_g} = 1 + x \left(\frac{\rho_f}{\rho_g} - 1 \right) \quad (12)$$

giving for the *Homogeneous Multiplier*

$$\Phi_{\ell o}^2 = 1 + \frac{\nu_{fg}}{\nu_f} x \quad (13)$$

This simple model suggests that the two-phase multiplier varies with quality and therefore position along the channel. As an example, at 1000 psia $\nu_{fg}/\nu_f = 19.6$, which would imply $\Phi_{\ell o}^2$ increases rapidly with flow quality. Typical flow qualities in steam generators and BWR cores are on the order of 10 to 20 %. The corresponding two-phase frictional loss would then be 2 - 4 times that in an equivalent single-phase system.

In practice, the two-phase multiplier is given empirically as a function of pressure, flow and quality with a number of correlations available in the literature. An alternate approach by Martinelli and Nelson correlates an average value of the two-phase multiplier over the boiling height,

$$\overline{\Phi_{\ell o}^2} = \frac{1}{x} \int_0^x \Phi_{\ell o}^2 dx \quad (14)$$

in terms of pressure and exit quality. The Martinelli-Nelson two-phase multiplier is given in Figure 1 below. The total frictional drop in a boiling channel would then be the sum of the single-phase and two-phase components.

Forms or Local Losses

The two-phase pressure loss due to local flow obstructions is treated in a manner similar to the frictional losses. We define a two-phase, local loss multiplier Ψ such that

$$\Delta P_{local} = K \frac{G^2}{2\rho_f g_c} \Psi \quad (15)$$

For design purposes it has been found that the Homogeneous Multiplier given as Equation 13 does an adequate job of correlating local two-phase losses.

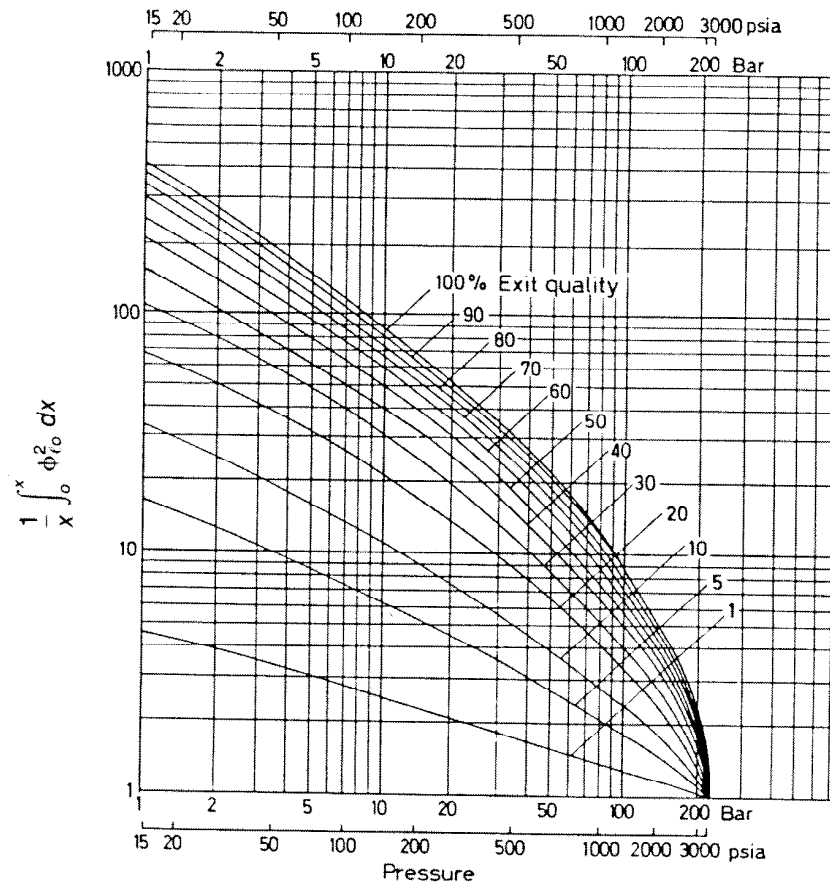


Figure 1: Martinelli-Nelson Average Two-Phase Friction Multiplier
(Taken from Figure 11-16, Todreas and Kazimi)

Pressure Drop in a Two-Phase Channel

Consider a boiling channel of constant cross sectional area. We assume the fluid enters the channel subcooled, with two phase flow beginning at some point H_o up the channel. The point H_o is called the *non boiling height* and is equivalent to the bubble departure point. The total steady-state pressure drop in the channel can be obtained by integrating the single and two-phase conservation equations up the channel.

Mixture Mass Equation

$$\frac{\partial}{\partial z}(GA_x) = 0 \Rightarrow GA_x = \dot{m} = \text{constant} \quad (1)$$

which for a uniform area channel implies G is constant.

Mixture Energy Equation

$$\dot{m} \frac{\partial h}{\partial z} = q'(z) \quad (2)$$

which may be integrated to give for the enthalpy distribution up the channel

$$h(z) = h(0) + \frac{1}{\dot{m}} \int_0^z q'(z') dz' \quad (3)$$

Mixture Momentum Equation

$$\frac{1}{g_c} \frac{\partial}{\partial z} \left\{ \frac{G^2 (1-x)^2}{\alpha_\ell \rho_\ell} + \frac{G^2 x^2}{\alpha_g \rho_g} \right\} = - \frac{\partial P}{\partial z} - \frac{\tau_{w2\phi} P_w}{A_x} - \rho \frac{g}{g_c} \quad (4)$$

Integrate over the channel height

$$- \int_0^H \frac{\partial P}{\partial z} dz = \int_0^H \frac{1}{g_c} \frac{\partial}{\partial z} \left\{ \frac{G^2 (1-x)^2}{\alpha_\ell \rho_\ell} + \frac{G^2 x^2}{\alpha_g \rho_g} \right\} dz + \int_0^H \frac{\tau_{w2\phi} P_w}{A_x} dz + \int_0^H \rho \frac{g}{g_c} dz \quad (5)$$

and examine the integrals one at a time

Channel Pressure Drop

$$- \int_0^H \frac{\partial P}{\partial z} dz = -[P(H) - P(0)] = \Delta P_{\text{channel}} \quad (6)$$

Acceleration Pressure Drop

$$\int_0^H \frac{1}{g_c} \frac{\partial}{\partial z} \left\{ \frac{G^2 (1-x)^2}{\alpha_\ell \rho_\ell} + \frac{G^2 x^2}{\alpha_g \rho_g} \right\} dz \equiv \Delta P_{\text{acceleration}} \quad (7)$$

The integral of the acceleration term is broken up into integrals over the boiling and non boiling heights, such that

$$\Delta P_{acceleration} = \int_{H_o}^H \frac{1}{g_c} \frac{\partial}{\partial z} \left\{ \frac{G^2 (1-x)^2}{\alpha_\ell \rho_\ell} + \frac{G^2 x^2}{\alpha_g \rho_g} \right\} dz + \int_0^{H_o} \frac{1}{g_c} \frac{\partial}{\partial z} \left\{ \frac{G^2}{\rho_\ell} \right\} dz \quad (8)$$

$$\begin{aligned} \Delta P_{acceleration} &= \frac{G^2}{g_c} \left\{ \left(\frac{(1-x)^2}{\alpha_\ell \rho_\ell} + \frac{x^2}{\alpha_g \rho_g} \right)_{exit} - v_{\ell d} \right\} + \frac{G^2}{g_c} \{v_{\ell d} - v_{in}\} \\ &= \frac{G^2}{g_c} \left\{ \left(\frac{(1-x)^2}{\alpha_\ell \rho_\ell} + \frac{x^2}{\alpha_g \rho_g} \right)_{exit} - v_{in} \right\} \end{aligned} \quad (9)$$

Friction and Forms Losses

The integrals of the friction and forms loss terms are also broken up into integrals over the boiling and non boiling heights giving

$$\int_0^H \frac{\tau_w P_w}{A_x} \Big|_{2\phi} dz \equiv \Delta P_{friction} = \int_{H_o}^H \frac{\tau_w P_w}{A_x} \Big|_{2\phi} dz + \int_0^{H_o} \frac{\tau_w P_w}{A_x} \Big|_{1\phi} dz \quad (10)$$

Single Phase Component

$$\int_0^{H_o} \frac{\tau_w P_w}{A_x} dz = \int_0^{H_o} \left\{ \frac{f}{D_e} \frac{\rho v^2}{2 g_c} + \sum_j K_j \delta(z - z_j) \frac{\rho v^2}{2 g_c} \right\} dz \quad (11)$$

$$\int_0^{H_o} \frac{\tau_w P_w}{A_x} dz \cong \frac{\bar{f} H_o}{D_e} \frac{G^2}{2 \bar{\rho} g_c} + \sum_{z_j \in H_o} K_j \frac{G^2}{2 \bar{\rho} g_c} \quad (12)$$

where the average fluid properties are over the non boiling height H_o .

Two-Phase Component

$$\int_{H_o}^H \frac{\tau_w P_w}{A_x} dz = \int_{H_o}^H \left\{ \frac{f}{D_e} \frac{\rho v^2}{2 g_c} \Phi_{\ell o}^2 + \sum_j K_j \delta(z - z_j) \frac{\rho v^2}{2 g_c} \Psi \right\} dz \quad (13)$$

Assuming the fluid properties are equal to the saturation properties and constant over the *boiling height* and letting $H_B = H - H_o$

$$\int_{H_o}^H \frac{\tau_w P_w}{A_x} dz \cong \frac{f_f}{D_e} \frac{G^2}{2 \rho_f g_c} \int_{H_o}^H \Phi_{\ell o}^2 dz + \sum_{z_j \in H_B} K_j \frac{G^2}{2 \rho_f g_c} \Psi_j \quad (14)$$

$$\frac{f_f}{D_e} \frac{G^2}{2\rho_f g_c} \int_{H_o}^H \Phi_{\ell o}^2 dz = \frac{f_f H_B}{D_e} \frac{G^2}{2\rho_f g_c} \frac{1}{H_B} \int_{H_o}^H \Phi_{\ell o}^2 dz = \frac{f_f H_B}{D_e} \frac{G^2}{2\rho_f g_c} \overline{\Phi_{\ell o}^2} \quad (15)$$

Note, the average two-phase multiplier in this development is the average over the boiling height and as such is not strictly equal to the Martinelli-Nelson Multiplier. However, the Martinelli-Nelson multiplier can be assumed a reasonable approximation. It can be shown, that if the quality varies linearly over the boiling height, the Martinelli-Nelson multiplier is equivalent to that obtained by averaging over the height.

$$\Delta P_{friction} = \frac{f_f H_B}{D_e} \frac{G^2}{2\rho_f g_c} \overline{\Phi_{\ell o}^2} + \sum_{z_j \in H_B} K_j \frac{G^2}{2\rho_f g_c} \Psi_j + \frac{\bar{f} H_o}{D_e} \frac{G^2}{2\bar{\rho} g_c} + \sum_{z_j \in H_o} K_j \frac{G^2}{2\bar{\rho} g_c} \quad (16)$$

If the inlet subcooling is small, the friction and forms losses may be approximated as

$$\Delta P_{friction} \cong \frac{f_f}{D_e} \frac{G^2}{2\rho_f g_c} \left(H_o + H_B \overline{\Phi_{\ell o}^2} \right) + \frac{G^2}{2\rho_f g_c} \left(\sum_{j \in H_B} K_j \Psi_j + \sum_{j \in H_o} K_j \right) \quad (17)$$

Elevation Losses

$$\Delta P_{elev} = \bar{\rho} \frac{g}{g_c} H_{core} \quad (18)$$

The elevation losses require knowledge of the void distribution and typically require numerical integration.

The total channel pressure drop is the sum of the acceleration, friction, local (forms) and the elevation terms, i.e.

$$\Delta P_{channel} = \Delta P_{acceleration} + \Delta P_{friction} + \Delta P_{local} + \Delta P_{elev} \quad (19)$$

and requires knowledge of the boiling and non boiling heights. In a simple single channel analysis under the equilibrium model assumptions, the non boiling height is obtained directly from the energy balance

$$h_f = h(0) + \frac{1}{\dot{m}} \int_0^{H_o} q'(z') dz' . \quad (20)$$

Otherwise, the non boiling height is taken to be the bubble departure point and determined as described in earlier sections.

Example:

A Boiling Water Reactor has operating characteristics given below. For the given data, determine the total core pressure drop. Assume the heat flux is uniform axially, and the void fraction varies linearly over the boiling height. You may also assume an equilibrium model for the flow quality.

Core Averaged Heat Flux	144,032 Btu/hr-ft ²
Active Fuel Height	150 inches
Bundle Height	176 inches
Rod Diameter	0.493 inches
Rod Pitch	0.640 inches
Mass Flux	1.42 x 10 ⁶ lbm/hr-ft ²
Core Inlet Temperature	532 F
System Pressure	1035 psia
Grid Loss Coefficient	1
Number of Grids	7
Core Entrance and Exit Loss Coefficient	3.5

SOLUTION

The total core pressure drop can be determined from the integrated two-phase momentum equation and is the sum of the acceleration, friction, local (forms) and the elevation terms, i.e.

$$\Delta P_{channel} = \Delta P_{acceleration} + \Delta P_{friction} + \Delta P_{local} + \Delta P_{elev}.$$

where:

$$\Delta P_{acceleration} = \frac{G^2}{g_c} \left\{ \left(\frac{(1-x)^2}{\alpha_l \rho_l} + \frac{x^2}{\alpha_g \rho_g} \right)_{exit} - v_{in} \right\} \quad (1)$$

$$\Delta P_{friction} \cong \frac{f_f}{D_e} \frac{G^2}{2\rho_f g_c} \left(H_o + H_B \overline{\Phi_{lo}^2} \right) \quad (2)$$

$$\Delta P_{local} \cong \frac{G^2}{2\rho_f g_c} \left(\sum_{j \in H_B} K_j \Psi_j + \sum_{j \in H_o} K_j \right) \quad (3)$$

$$\Delta P_{elev} = \bar{\rho} \frac{g}{g_c} H_{core} \quad (4)$$

Acceleration Pressure Drop

To determine the acceleration drop, requires the quality and void fraction at the core exit. The inlet specific volume is given directly in terms of the core inlet temperature, i.e. $v_{in} = v(532 F) = 0.02123 \text{ ft}^3/\text{lbm}$. To determine the fluid conditions at the core exit requires the core exit enthalpy. This is obtained from the energy balance

$$h_{exit} = h_{in} + \frac{\overline{q''} \pi D H}{G A_x} \quad (5)$$

The flow area for an arbitrary flow channel is given by

$$\begin{aligned}
A_x &= S^2 - \pi D^2 / 4 \\
&= 0.640^2 - (\pi)(0.493^2) / 4 \\
&= 0.219 \text{ in}^2
\end{aligned} \tag{6}$$

For the given data, the core exit enthalpy is

$$\begin{aligned}
h_{exit} &= 526.8 + \frac{(144,032)(\pi)(0.493/12)(150/12)}{(1.42 \times 10^6)(0.219 / 144)} \\
&= 634.4 \text{ Btu/lbm}
\end{aligned} \tag{7}$$

giving for the core exit quality

$$\begin{aligned}
x_{exit} &= \frac{h_{exit} - h_f}{h_{fg}} \\
&= \frac{634.4 - 547.85}{643.75} \\
&= 0.1344
\end{aligned} \tag{8}$$

The core exit void fraction can be obtained from the Zuber-Findlay correlation

$$\alpha = \frac{1}{C_o \left[1 + \frac{\rho_g}{\rho_\ell} \frac{1-x}{x} \right] + \frac{\rho_g V_{gj}}{Gx}} \tag{9}$$

For high pressure steam water flows

$$C_o = 1.13 \tag{10a}$$

$$V_{gj} = 1.41 \left(\frac{\sigma g g_c (\rho_\ell - \rho_g)}{\rho_\ell^2} \right)^{1/4} \tag{10b}$$

At 1035 psia, the drift velocity is

$$\begin{aligned}
V_{gj} &= 1.41 \left(\frac{\sigma g g_c (\rho_\ell - \rho_g)}{\rho_\ell^2} \right)^{1/4} \\
&= 1.41 \left(\frac{(0.0013)(32.17)^2 (46.05 - 2.329)}{(46.05)^2} \right)^{1/4} \\
&= 0.575 \text{ ft/sec}
\end{aligned} \tag{11}$$

giving for the core exit void fraction

$$\begin{aligned}
\alpha &= \frac{1}{C_o \left[1 + \frac{\rho_g}{\rho_\ell} \frac{1-x}{x} \right] + \frac{\rho_g V_{gj}}{Gx}} \\
&= \frac{1}{(1.13) \left[1 + \frac{(2.329)}{(46.05)} \frac{1-0.1344}{0.1344} \right] + \frac{(2.329)(0.575 \times 3600)}{(1.42 \times 10^6)(0.1344)}} \\
&= 0.656
\end{aligned} \tag{12}$$

The acceleration pressure drop is then

$$\begin{aligned}
\Delta P_{acceleration} &= \frac{G^2}{g_c} \left\{ \left(\frac{(1-x)^2}{\alpha_\ell \rho_\ell} + \frac{x^2}{\alpha_g \rho_g} \right)_{exit} - v_{in} \right\} \\
&= \frac{(1.42 \times 10^6)^2}{(4.17 \times 10^8)} \left\{ \frac{(1-0.1344)^2}{(1-0.656)(46.05)} + \frac{(0.1344)^2}{(0.656)(2.329)} - 0.02123 \right\} \\
&= 183.2 \text{ lbf/ft}^2 = 1.27 \text{ lbf/in}^2
\end{aligned} \tag{13}$$

Frictional Pressure Drop

The frictional drop is in terms of the friction factor f which is a function of the Reynolds number, the non boiling height and the two-phase friction multiplier. A reasonable approximation for the friction factor in rod bundles is the friction factor in smooth tubes from the Moody Chart. The Reynolds number is

$$Re = \frac{GD_e}{\mu} \tag{14}$$

The equivalent diameter is

$$D_e = \frac{4A_x}{P_w} = \frac{4[S^2 - \pi D^2/4]}{\pi D} \tag{15}$$

$$D_e = \frac{4(0.219)}{\pi(0.493)} = 0.5656 \text{ in} \tag{16}$$

giving for the Reynolds number

$$Re = \frac{(1.42 \times 10^6)(0.5656/12)}{0.23} = 290,997 \tag{17}$$

From the Moody Chart, $f \cong 0.0145$. The non boiling height is given by

$$\begin{aligned}
H_o &= \frac{\dot{m}(h_f - h(0))}{q'' \pi D} \\
&= \frac{(1.42 \times 10^6)(0.219/144)(547.85 - 526.8)}{(144,032)(\pi)(0.493/12)} \\
&= 2.45 \text{ ft}
\end{aligned} \tag{18}$$

Assuming the Martinelli-Nelson form of the two-phase multiplier, gives $\overline{\Phi}_{\ell o}^2 \cong 3.5$, such that the frictional pressure drop is

$$\begin{aligned}
\Delta P_{friction} &\cong \frac{f_f}{D_e} \frac{G^2}{2\rho_f g_c} \left(H_o + H_B \overline{\Phi_{lo}^2} \right) \\
&= \frac{(0.0145)}{(0.5656/12)} \frac{(1.42 \times 10^6)^2}{(2)(46.05)(4.17 \times 10^8)} (2.45 + 12.22 \times 3.5) \\
&= 730.4 \text{ lbf/ft}^2 = 5.07 \text{ lbf/in}^2
\end{aligned} \tag{19}$$

Local Losses

The local or forms losses are due to the grid spacers as well as the core inlet and exit losses. To be strictly correct, we should evaluate the two-phase multiplier at the specific locations of the grids. For this example, assume we can use an average two-phase multiplier, similar to the approach taken for the frictional losses.

$$\begin{aligned}
\Delta P_{local} &\cong \frac{G^2}{2\rho_f g_c} \left(\sum_{j \in H_B} K_j \overline{\Psi} + \sum_{j \in H_o} K_j \right) \\
&= \frac{(1.42 \times 10^6)^2}{(2)(45.06)(4.17 \times 10^8)} (1 \times 6 \times 3.5 + 3.5 \times 3.5 + 3.5 + 1) \\
&= 2,035.5 \text{ lbf/ft}^2 = 14.1 \text{ lbf/in}^2
\end{aligned} \tag{20}$$

Elevation Pressure Drop

The elevation pressure drop is the sum of the single-phase and two-phase terms. Assuming the void fraction varies linearly over the boiling height, and the subcooling is sufficiently small, the elevation pressure drop can be written as

$$\begin{aligned}
\Delta P_{elev} &= \rho_f \frac{g}{g_c} H_o + \left(\overline{\alpha_t} \rho_f + \overline{\alpha_g} \rho_g \right) \frac{g}{g_c} H_B \\
&= (45.06)(1)(2.45) + (0.672 \times 45.06 + 0.328 \times 2.329)(1)(12.22) \\
&= 489.7 \text{ lbf/ft}^2 = 3.4 \text{ lbf/in}^2
\end{aligned} \tag{21}$$

The total pressure drop is then

$$\begin{aligned}
\Delta P_{core} &= \Delta P_{acc} + \Delta P_{friction} + \Delta P_{local} + \Delta P_{elev} \\
&= 1.27 + 5.07 + 14.1 + 3.41 \\
&= 23.85 \text{ lbf/in}^2
\end{aligned} \tag{22}$$

It should be noted, that even though the core mass flux is approximately half that in a Pressurized Water Reactor, the total core pressure drop is similar. This is due primarily to the enhanced friction and local losses in two-phase systems. In addition, the acceleration drop while not dominant, is still a significant contributor to the total loss. This is in contrast to the acceleration drop in single phase systems.