A phenomenological model for analyzing reciprocating compressors

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Abstract

A new model for hermetic reciprocating compressors is presented. This model is able to predict compressor efficiency and volumetric efficiency in terms of a certain number of parameters (10) representing the main sources of losses inside the compressor. The model provide users with helpful information about the way in which the compressor is designed and working.

A statistical fitting procedure based on the Monte Carlo method was developed for its adjustment. The model can predict compressor performance at most points with a maximum deviation of 3%.

A possible gas condensation on cold spots inside the cylinder during the last part of the compression stroke was also evaluated.

Key words: Model, Efficiency, Performance, Refrigerants, Reciprocating compressor.

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Nomenclature

η_k	Compressor efficiency
W_{ref}	Ideal isentropic work transferred to refrigerant by compresor
$\dot{E}_k$	Electric power consumption
η_s	Volumetric efficiency
$\dot{m}_{in}$	Mass flow rate
$\dot{V}_s$	Compressor swept volume flow
ρ_{in}	Density in compressor inlet
$\overline{\dot{m}}_{leaks}$	Mass flow rate leaked
$\overline{\dot{m}}_{pc}$	Mass flow rate in phase change
η_{sth}	Theoretical volumetric efficiency
ρ_i	Density in state i
V_d	Dead space
V_s	Swept volume
η_{el}	Electric efficiency
$\Delta h_{(4-5)}$	Enthalpy difference between states $4-5$
$\Delta h_{(1-8^*)}$	Enthalpy difference between states $1-8^*$
$\dot{E}_{mech}$	Energy lost in mechanical losses
Z_{elgas}	Fraction of electric motor losses going to the suction gas
Z_{mgas}	Fraction of mechanical losses going to the suction gas
c_{pi}	Specific heat at constant pressure in state i
$(UA)_{ht}$	Overall heat transfer coefficient per area for the heat transfer between the hot and the cold gas
h_{ht}	Heat transfer coefficient per area for the heat transfer between the hot and the cold gas
T_i	Temperature in state i

D	Cylinder diameter
S	Cylinder stroke
n	Compressor nominal speed
d_h	Effective hydraulic diameter
k	Specific heat at constant pressure
μ	Dynamic viscosity
k	Thermal diffusivity
P_i	Pressure in state i
ξ	Drag factor for compressor inlet and outlet valve
ζ	Equivalent flow resistance for leakages
w	Refrigerant velocity
R	Universal gas constant
A_L	Leakage effective area
h_{fg}	Heat of vaporization
A_v	Area for phase change inside the cylinder
h_{pc}	Heat transfer coefficient per area for phase change inside the cylinder
nz	Number of cylinders
K_i	Representative parameters for different model losses
ΔT_{sh}	Compressor inlet gas superheat
Rp	Pressure ratio

1 Introduction

Reciprocating hermetic compressors have been known since the 19th century and, due to their simplicity and flexibility when working in a wide range of conditions, they are still used nowadays in refrigeration and air conditioning systems.

A complete empirical characterization procedure for this type of compressor is described in [1]. However, a certain theoretical effort based on analysis and modelling may be useful at this point to estimate how a given compressor is likely to work under different operating conditions or with different refrigerants, but also in more general terms, to assess the proper operation of the compressor.

Two main categories of models are discussed in the literature:

- Models whose aim is to explain in a detailed and accurate manner, the behavior of specific parts or processes (mechanics of the valves, vibration, heat transfer) inside the compressor. In [2] one can find numerous examples of models of this type. Here the main objective is to assist the compressor optimization.
- Models whose objective is to describe the compressor globally. In this category, three main basic approaches to the problem can be established:

(1) Correlations from experimental data for some of the compressor significant variables such as COP, cooling capacity, [1], [3]. This is the methodology most commonly used by compressor manufacturers, but it does not give any valuable physically information about the processes inside the compressor. The correlations obtained can only be used for the range of conditions in which they were obtained.

(2) Other authors ([4], [5], [6], [7]) have attempted to model the most important physical compressor processes using numerical methods to solve the differential equations implied in the conservation laws of these processes. Although these kinds of models may deliver considerable information about the way in which the compressor is working, they usually require numerous data available only to the manufacturer. These models aims to optimize compressor design.

(3) The so–called semi–empirical models, for instance [8], [9], [10], seek to reproduce the main compressor performance variables like COP and cooling capacity using empirically adjusted, simple models retaining at least some portion of the physical background. Given their simplicity, these models do not need as much information as the detailed models described in approach (2. As a consequence, the information obtained is not as accurate, and there are usually problems in the physical interpretation of certain results.

This research describes a global analysis of a series of hermetic reciprocating compressors covering different strokes, piston numbers, sizes and refrigerants. The results of this research were divided in two papers. In this first paper, a new phenomenological model which aims to identify the most important phenomena occurring inside the compressor and the corresponding empirical coefficients required for their adjustment were described. In the second paper [11], the resulting adjustment of the model to experimental results from a vast experimental test campaign is discussed, and the empirical coefficients obtained are analyzed. Finally, a full discussion about the physical sense and interpretation of the obtained values, as well as the capabilities of the model to help in the analysis of the behavior and internal characteristics of the studied compressors is presented.

The proposed compressor model aims to reproduce the compressor efficiency ($\eta_k = \frac{W_{ref}}{\dot{E}_k}$) and the volumetric efficiency ($\eta_s = \frac{\dot{m}_{in}}{\dot{V}_s \rho_{in}}$) as a function of a set of parameters which may be obtained by correlations of standard characterization performance data. The philosophy is that these parameters have a physical background, so that once correlated, the model can be used to predict compressor performance under operating conditions which are not tested, for example, at extreme temperatures or at lower speeds. Furhter, the correlated model may estimate the compressor performance with other refrigerants for which there are no available data.

In the literature, one can find other models which follow this same approach (see [8], [10]). The present study attempts to move forward towards that objective, targeting the selection of the parameters in such a way that they retain the maximum physical significance. The values obtained in the correlations are expected to show a clear agreement with the reasonable order of magnitude of the compressor characteristics they represent.

Additionally, the fitting techniques employed have shown to have a considerable influence on the suitability of the obtained coefficients. The authors found that classical least square correlation methods are not useful in this kind of non linear systems. Monte Carlo based fitting methods, on the contrary, provide much greater freedom and stability in the definition of the model parameters, better final results and a better way of avoiding excess sensitivity problems.

This first paper is structured in three main parts. First, the model is presented; then the Monte Carlo fitting procedure is described and finally the model is validated for a compressor providing an initial assessment of the order of magnitude of the obtained parameters.

2 Compressor model for a reciprocating compressor

Examining the evolution of the refrigerant in the p-h diagram, the model will assume the evolution shown in fig. 1. The refrigerant enters the compressor at point 1 ("inlet conditions"). The reference for the compressor efficiency is given by an isentropic condition from the inlet to the outlet of the compressor: 1-8*. The real conditions at the outlet of the compressor are indicated by state 8 in fig. 1. The developed model assumes that the evolution of the refrigerant through the compressor can be divided in the following sequence of effects:

- 1–2: Vapor heating due to motor cooling and mechanical loss dissipation.
- 2–3: Vapor heating due to the heat transferred from the hot side of the compressor (discharge) to the inlet flow.
- 3–4: Isoenthalpic pressure lost at the suction valve.
- 4–5: Isentropic compression from real cylinder intake conditions (leaks and possible condensation also appear in this part of the process).
- 5–6: Isoenthalpic pressure lost at the discharge valve.
- 6–7: Vapor cooling due to the heat transferred to the suction side.

Regarding the evolution from 4 to 5, inside the cylinder, measurements from [12] show that the real compression from conditions at the bottom of the piston (beginning of the compression stroke) to the top dead centre are very close to isentropic. The main source of irreversibilities is the heat transfer to and from the wall during the compression stroke. However, the process is very fast and wall temperatures are quite close to fluid temperatures; thus, heat transfer effects per mass flow rate unit are slight. For this reason, a generalized polytropic compression was avoided since the polytropic exponent must be fitted for each refrigerant.

The internal leakage of refrigerant through the piston rings has a considerable effect on compressor and volumetric efficiencies. In order to simplify the treatment of this loss, an evaluation of the leak is made at state 5. It is considered as if the loss of the circulating mass flow rate ($\overline{\dot{m}}_{leak}$) takes place at state 5. Therefore, the compressor is assumed to consume the work of compression for the circulating mass flow rate plus leaks ($\dot{m}_{in}+\overline{\dot{m}}_{leak}$).

The possibility of condensation of a relatively significant fraction of refrigerant during compression was also evaluated throughout this study. The condensed mass does not flow towards the discharge, but it is later evaporated during the suction stroke. The effect of this possible condensation is similar to an increase of the dead space ratio.

In the model, this possible effect is treated as a loss of the circulating mass flow rate ($\overline{\dot{m}}_{pc}$).

Other effects that influence refrigerant temperature before leaving the compressor (7-8 in fig. 1) such as heat release to the environment or oil heating and electric motor heating of the vapor are not considered in this study. With these assumptions, a model describing compressor and volumetric efficiencies of the compressor is proposed.

The circulating mass flow rate can be calculated from the ideal flow rate by means of the expression:

$$\dot{m}_{in} = \eta_{Sth} \dot{V}_s \rho_4 - \overline{\dot{m}}_{leak} - \overline{\dot{m}}_{pc} \tag{1}$$

In expression (1), $\dot{V}_s$ is the swept volume flow given by $\dot{V}_s = n \cdot V_s$, being V_s the swept volume, n the nominal speed of the compressor and $\eta_{Sth} = 1 - \frac{V_d}{V_s}(\frac{\rho_{8*}}{\rho_1} - 1)$ the ideal volumetric efficiency [13]. It should be noted here that the compressor nominal speed was considered constant throughout the study. For the line frequency of 50 s^{-1}, the manufacturer estimates a value of 2900 rpm for these compressors.

Expression (1) implies that the total mass flow rate is given by the total cylinder refrigerant capacity at conditions corresponding to point 4 in fig. 1, subtracting the cylinder volume losses from the cylinder clearance, the vapor that is leaked during the compression process and the possible formation of small refrigerant droplets in some part of the cylinder surface that are not subsequently pumped out of the cylinder during the compression process.

Electric compressor power input $\dot{E}_k$ can be assumed as the energy that the refrigerant consumes to change from state 4 to state 5 in fig. 1 plus the energy that the compressor consumes in mechanical ($\dot{E}_{mech}$) and electrical losses. So $\dot{E}_k$ can be expressed as:

$$\dot{E}_k = \frac{1}{\eta_{el}}[\Delta h_{(4-5)}(\dot{m}_{in} + \overline{\dot{m}}_{leak}) + \dot{E}_{mech}] \tag{2}$$

From both equations, (1) and (2), the corresponding expressions for the volumetric and compressor efficiencies are obtained:

$$\eta_s = \frac{\dot{m}_{in}}{\dot{V}_s \rho_1} = \frac{\rho_4}{\rho_1}\eta_{Sth} - \frac{\overline{\dot{m}}_{leak}}{\dot{V}_s \rho_1} - \frac{\overline{\dot{m}}_{pc}}{\dot{V}_s \rho_1} \tag{3}$$

$$\eta_k = \frac{\dot{m}_{in}\Delta h_{(1-8^*)}}{\dot{E}_k} = \frac{\Delta h_{(1-8^*)}\eta_{el}}{\Delta h_{(4-5)}(1+\frac{\overline{\dot{m}}_{leak}}{\dot{m}_{in}})+\frac{\dot{E}_{mech}}{\dot{m}_{in}}} \quad (4)$$

The different processes considered in expressions (3) and (4) will now be described in further detail.

2.1 Vapor heating due to motor cooling and mechanical loss dissipation

The heating of the inlet refrigerant by motor cooling and mechanical loss dissipation are quantified by the following expression:

$$\dot{Q}_{1-2} = (1-\eta_{el})\dot{E}_k Z_{elvapor} + \dot{E}_{mech} Z_{mvapor}$$

where $Z_{elvapor}$ and Z_{mvapor} are the fractions of the losses transferred to the suction vapor as heat. It is assumed that the fraction of absorbed heat is the same for both losses, that is $Z_{elvapor} = Z_{mvapor}$, so these factors can be renamed as K_1. The remaining heat is released to the environment either through the outlet gases or the compressor surface.

The increase in the suction vapor temperature 1-2 is thus given by:

$$\Delta T_{1-2} = \frac{\dot{Q}_{1-2}}{\dot{m}_{in}c_{p1}} = K_1(\frac{(1-\eta_{el})}{\eta_k}\frac{\Delta h_{(1-8^*)}}{c_{p1}} + \frac{\dot{E}_{mech}}{\dot{V}_s\eta_s\rho_1 c_{p1}}) \quad (5)$$

2.2 Vapor heating due to heat transferred from the hot side of the compressor (discharge) to the inlet flow

Before leaving the compressor, the hot vapor flowing outside the cylinder heats the refrigerant at the low pressure side. As a first approximation, the heat transferred between both sides can be given by $\dot{Q}_{2-3} = (UA)_{ht}(T_{8^*} - T_1)$, where the temperature difference $(T_{8^*} - T_1)$, calculated from the isentropic – ideal process, is considered as an effective temperature difference, characteristic of the process.

The global heat transfer coefficient U_{ht} is related with a heat transfer coefficient h_{ht} using these approximations:

- The heat transfer between both sides takes place mainly in the suction and discharge pipes near the cylinder.
- U_{ht} is considered proportional to the heat transfer coefficient h_{ht}, $U_{ht} = C^{'} \cdot h_{ht}$.
- The heat transfer coefficient h_{ht} is considered as the one given for the turbulent internal flow in a pipe.

$$Nu = C \cdot Re^{0.8} Pr^{0.4} \rightarrow h_{ht} = \frac{k}{d_h} \cdot C \cdot \left(\frac{\dot{V}_s \cdot \eta_s \cdot \rho_1}{d_h \cdot \mu} \right)^{0.8} \left(\frac{\mu \cdot c_p}{k} \right)^{0.4} \tag{6}$$

Finally, considering these approximations, the temperature increase is expressed as follows:

$$\Delta T_{2-3} = K_2 \frac{(T_{8^*} - T_1)}{(\dot{V}_s \eta_s \rho_1)^{0.2}} \frac{k_2^{0.6}}{d_h^{1.8} c_{p2}^{0.6} \mu_2^{0.4}} = K_2^{'} \frac{(T_{8^*} - T_1)}{(\dot{V}_s \eta_s \rho_1)^{0.2}} \frac{k_2^{0.6}}{c_{p2}^{0.6} \mu_2^{0.4}} \tag{7}$$

where d_h is a hydraulic diameter characteristic of the narrow flow passages around the cylinder. If no reasonable estimation of these passages is available, the hydraulic diameter can be included within a new constant $K_2^{'} = \frac{K_2}{d_h^{1.8}}$.

2.3 Isoenthalpic pressure losses at the inlet valve

This pressure drop is estimated as $\Delta P_{3-4} = \xi_3 \rho_3 \frac{w_3^2}{2}$. Expressing the velocity as $w_3 = \frac{\dot{V}_s \eta_s}{nz A_{c3}}$ where nz is the number of cylinders and A_{c3} the effective inlet valve area per cylinder, the pressure drop is given by:

$$\Delta P_{3-4} = K_3 \rho_3 (\frac{\dot{V}_s \eta_s}{nz})^2 \tag{8}$$

where K_3 is given by $K_3 = \frac{\xi_3}{2A_{c3}^2}$.

From a practical point of view, it may be useful to express the inlet valve area A_{c3} in terms of the cylinder diameter $A_{c3} = \widetilde{K_3} \cdot D^2$. Thus, if the value of K_3 is known for one compressor, an estimation can be made for other compressors of similar characteristics but different cylinder diameters.

2.4 Isoenthalpic pressure losses at the outlet valve

Using the same arguments as in the previous section, the pressure loss for the outlet valve is given as:

$$\Delta P_{5-6} = K_4 \rho_5 (\frac{\rho_0}{\rho_5} \frac{\dot{V}_s \eta_s}{nz})^2 \tag{9}$$

where K_4 is given by $K_4 = \frac{\xi_5}{2A_{c5}^2}$, and A_{c5} is the effective outlet valve area per cylinder.

2.5 Leakages

To describe leakages, it is assumed that they are a function of the overall pressure difference in the compressor and are not dependent on the vapor flow. The mass flow rate leaked can be deduced using the ideal vapor non compressible flow through an orifice as:

$$\overline{\dot{m}}_{leak} = nz \cdot A_L K_5 \sqrt{\frac{\Delta P_m}{\zeta \rho_m}} \rho_m \tag{10}$$

where ζ represents the equivalent flow resistance, and A_L is the effective leakage area per cylinder. A_L can be considered proportional to the cylinder area ($A_L \approx \widetilde{K}_5 D^2$), as done for the compressor inlet and outlet valve area. The pressure difference $\Delta P_m = K_5^2 (P_{8^*} - P_1)$ is assumed to be proportional to the overall pressure difference ($P_{8^*} - P_1$) and the mean density for leaked vapor $\rho_m \approx \sqrt{\rho_{8^*} \rho_1}$. With these assumptions $\overline{\dot{m}}_{leak}$ is given by:

$$\overline{\dot{m}}_{leak} = K_5^{'} \cdot nz \cdot D^2 \sqrt{\Delta P_m \rho_m} \tag{11}$$

where $K_5^{'} = K_5 \cdot \widetilde{K}_5 \cdot \zeta^{-\frac{1}{2}}$.

$\overline{\dot{m}}_{leak}$ influences the reduction of the total mass flow rate (see eq. (1)) and also affects the refrigerant temperature in the cylinder inlet, which is estimated as the corresponding mixing temperature:

$$\Delta T_{3-3'} = \frac{\overline{\dot{m}}_{leak}(T_{8^*} - T_1)}{\dot{m}_{in} + \overline{\dot{m}}_{leak}} \tag{12}$$

The use of more accurate expressions, like the compressible flow equation for unchoked flow -maybe more precise- would not significantly improve the results of the model,as commented in [14], and they would certainly lead to much longer computation times. The authors consider that the above approximation is a reasonable compromise between simplicity and physically relevant information regarding the internal process in the present model.

2.6 Refrigerant phase changes inside the cylinder

Some refrigerant liquid droplets can be formed on the colder areas of the cylinder. These droplets can be in continuous phase change, leading to a reduction in the total amount of vapor flowing through the compressor and thus affecting volumetric efficiency. The inlet valve is exposed to relatively cold vapor on the suction side; therefore, its temperature could fall below the dew point of the refrigerant during some fraction of time of the compression stroke. Thus, condensation may occur and some droplets could appear on the valve plate. These droplets would then evaporate during suction and, assuming that this evaporation takes place on the valve plate, a cooling of the valve plate would take place to allow again condensation in the subsequent cycle.

It is worth noting that the condensation part of this phase change process is quite a critical point because it has very little time to be produced (the pressure inside the cylinder is only higher than the refrigerant dew point for some fractions of a second).

Considering that the most important factor to facilite this process is the temperature difference between the cylinder inlet and outlet, the temperature difference $(T_{8^*} - T_1)$ may be considered as an effective temperature difference to characterize this process. The amount of condensing refrigerant for one compression cycle of the cylinder may be expressed as:

$$\dot{m}_{pc} = \frac{\tau \cdot nz \cdot h_{pc} \cdot A_v (T_{8^*} - T_1)}{h_{fg}(T_1)} \tag{13}$$

where τ is the time during the compression in which the condensation can be produced (time in which the pressure in the cylinder is higher than the dew point for the inlet temperature) and h_{pc} is an effective heat transfer coefficient, considered as approximately constant.

Expression (14) to be inserted into the model must be expressed in terms of the mass flow per time unit and not per cycle:

$$\overline{\dot{m}}_{pc} = \frac{\overline{\tau} \cdot nz \cdot h_{pc} \cdot A_v(T_{8^*} - T_1)}{h_{fg}(T_1)} = K_6 \frac{nz(T_{8^*} - T_1)}{h_{fg}(T_1)} \tag{14}$$

where $\overline{\tau}$ represents the percentage of the total cycle in which the pressure is high enough to produce the refrigerant condensation.

Regarding this phase change effect, previous researchs can be found in literature, for instance, in [15] for low speed piston compressors working in wet conditions. Nevertheless, there is no experimental information available regarding this phenomena in the present research and here, it is more intended as a proposal to explain some model results and to inspire further research.

2.7 *Mechanical loss influence on the compressor efficiency*

According to [16], the mechanical losses may be considered as a sum of two terms, one proportional to the energy consumption rate and the other dependent on the compressor speed:

$$\dot{E}_{mech} = K_7 \dot{E}_k + K_8 n^2 = K_7 \frac{\dot{V}_s \eta_s \rho_1 \Delta h_{(1-8^*)}}{\eta_k} + K_8 \cdot n^2 \tag{15}$$

2.8 *Final model structure*

To formulate the global model, the equations governing the different losses described in sections (2.1)-(2.6) are either introduced into eq. (3) and (4) by direct substitution of the obtained equations (eq. (11), (14) and (15)) or are used to calculate the refrigerant state 4 and 5 (eq. (5), (7), (8), (9) and (12)). This leads to a system of two implicit equations for the compressor and volumetric efficiencies:

$$f_1(\eta_k, \eta_s, \mathbf{G}, \mathbf{K}, \eta_{el}, \frac{V_0}{V_s}, P_1, P_{8^*}, \Delta T_{sh}) = 0 \tag{16}$$

$$f_2(\eta_k, \eta_s, \mathbf{G}, \mathbf{K}, \eta_{el}, \frac{V_0}{V_s}, P_1, P_{8^*}, \Delta T_{sh}) = 0 \tag{17}$$

where $\mathbf{K}(\mathbf{K} = (K_1, ..., K_8))$,$\eta_{el}$, $\frac{V_0}{V_s}$ represents compressor design parameters difficult to determine, whereas $\mathbf{G}$ ($\mathbf{G} = (G_1, ...G_n)$) stands for compressor design parameters that are easy to know like stroke (S), number of cylinders (nz), nominal speed (n), and the like. If for any reason some of these **G** parameters were not known, they can easily be regrouped inside the **K** parameters.

Once all the compressor design parameters **K** are known, the system of two equations can be solved for compressor and volumetric efficiencies, η_k and η_s, for a given working condition (P_1, P_{8^*}, ΔT_{sh}). Any solver for a system of non-linear equations can be employed. The results shown in this paper were obtained by the Gauss-Seidel procedure [17]. With this algorithm, given an initial value of 0.5 for compressor and volumetric efficiencies, the convergence to the solution is typically reached in less than fifteen iterations.

As commented above, the parameters $\mathbf{K}$, η_{el}, $\frac{V_0}{V_s}$ are difficult to estimate. A set of data for a number of working conditions obtained either from experiments or from manufacturer catalogs is required to obtain the proper value of **K** by a fitting procedure. The developed fitting procedure to find the best estimation of **K** is explained in section (3).

From the best obtained values of **K**, it is then possible to determine the value of compressor and volumetric efficiencies in conditions different from those tested. Besides, it is possible to obtain physically relevant information about the internal processes inside the compressor and to quantify the different losses.

A key assumption regarding **K** is that the different parameters are not significantly dependent on the test conditions or employed refrigerants. The obtained results indicate that this assumption is fairly reasonable.

3 Statistical fitting procedure

As explained in section 2, the last step to close the model is to estimate the compressor losses parameters **K** from experimental or catalog data. This is quite a critical issue in this kind of models because the dependency of the target functions on the parameters is non linear, and there is also some sort of indetermination, in the sense that several possible combinations of parameters could adjust the model properly with a final deviation in the predicted efficiency values which is smaller than the experimental uncertainties. In fact, several conventional non linear regression techniques (the standard routines in MsExcel, Origin, simplex algorithm [17]) were tested, yet they failed in the fitting process of the proposed model.

For this reason and to avoid possible problems arising from a step–by–step exploration of the parameter space (existence of a local minimum solution, too smooth dependence on certain parameters and the like), an heuristic algorithm based on a Monte Carlo type approach was designed. A review of the main trends in this field can be found in [18]. Although computationally not the most efficient these methods show great versatility and reliability.

A general scheme of the designed algorithm is shown in fig. 2. In this scheme, the program starts by assigning pseudo-random values (according to the uniform distribution) to the parameters and the "best" combination of them to fit the compressor and volumetric efficiencies data is sorted out (this is called the *first process*). The routine used to generate the pseudo-random numbers was the one proposed by Park and Miller [19]. This routine has a long enough period for this specific application. To select the "best" combination of parameters, an error function or residue (ϵ) must be defined. In this case, the Euclidean norm weighted by the standard deviation of each experimental point i was selected:

$$\epsilon = \sum_i \frac{\sqrt{\eta_k^2(x_i) - \eta_{kexp}^2(x_i)}}{\sigma(x_i)} + \frac{\sqrt{\eta_s^2(x_i) - \eta_{sexp}^2(x_i)}}{\sigma(x_i)}$$

The set of parameters with a lower value of ϵ is selected as a solution to the first process.

Nevertheless, as a consequence of experimental errors, intrinsical errors associated to the model and the nature of the mathematical functions involved, if the process is repeated, a different set of parameters is obtained as a solution which may also give a good value for the error function ϵ.

Therefore, this process is repeated until a representative map of the solutions in the parameter space is obtained (this is called the *second process*). This means that the probability distribution of the solutions for each parameter is obtained.

The result of this process is a set of probability distributions for each of the model parameters. From these distributions, the most probable value for each parameter is selected as the best fit value.

Some comments should be made regarding this scheme:

- An interval in which the value of the parameters must be found, must be defined. As a result of the intrinsical stability of the method, this is not a critical point in the model, yet a good selection of this interval reduces the number of iterations needed to find a suitable solution.
- Preliminary studies have been developed to determine the proper number of iterations in the first and the second process. For the first process, this number is reached only if, increasing the number of iterations, the order of magnitude of ϵ does not change. For the second process, this number is reached if, by increasing the number of iterations, the obtained parameter distribution function does not change.
- To reduce the high computational cost linked to the direct use of REFPROP [20] in the evaluation of the thermodynamical properties of the refrigerants, an approximation based on linear interpolations of bidimesional meshes from REFPROP was employed (see [21] for details).

4 Results from the analysis of a two–piston hermetic compressor

To validate the model and the fitting methodology, a two–cylinder reciprocating compressor working with propane was analyzed using the set of experimental data from [22] (compressor ST, in the aforementioned nomenclature). A comparison between the obtained results with and without the phase change term is also presented to understand the possible relevance of this term.

After an initial study of possible correlations among the different parameters $\mathbf{K}$ for several compressors, it was found that K_1 and K_2 showed some kind of coupling, which can only be solved with a very high number of trial points (in the first process of the Monte Carlo method). In order to keep the fitting time moderate and after finding that the value of K_1 was almost the same for several compressors, the criteria of assigning a constant value of 0.9 provided very good results.

Electric efficiency is a function of the operating conditions, yet this dependence is usually small in the nominal operation range of the compressor. For this study, information from the manufacturer about experimental electric motor efficiency was available. This information shows a maximum deviation of $\pm 1\%$ over the mean value of the electric efficiency in almost all the tested points, so that electric efficiency can be considered independent of the operation conditions throughout the whole study. Although this parameter was known, to check the model capacities, this parameter was left free during the fitting process because it is usually not a parameter available in catalogs.

Table 1 shows the value obtained for parameters **K** in both cases (including or not the phase change term), together with the interval in which the parameters were searched. The search interval was chosen to include all the possible values that the compressor parameters might have in an attempt to cover the widest possible range bounded only by the physical constraints.

Regarding the Monte Carlo methodology, 1,000,000 trial runs were required to obtain a stable value for the error function in the first process and 1,000 iterations were needed in the second process to obtain a representative distribution of probability of the space of parameters.

The values obtained for the parameters, despite the large number of assumptions involved in the model, provide a very good prediction of the compressor performance throughout the entire test sample. Further, they have values consistent with the compressor geometry and the physical process involved, as described in the following:

- K_2: As seen in table 1, the obtained value for $K_2^{'}$ is 2.80, if a value of 0.023 for the constant C in expression 6 is taken (dittus-boelter correlation [23]), and considering that $U \sim \frac{h}{2}$, the constant $K_2^{'}$ represents the relation between an effective heat transfer area and an effective hydraulic diameter and is given by the term $K_2^{'} = \frac{0.023 \cdot 4^{0.8}}{2 \cdot (\pi)^{0.8}} \frac{A_{ht}}{d_h^{1.8}}$. Therefore, the relation between both magnitudes is $\frac{A_{ht}}{d_h^{1.8}} \sim 215$. As d_h should be quite small, this value supports the hypothesis that the heat transfer area of this process should be small.

- K_3: The measured value for the inlet valve orifice section A_{c4} was 160 mm^2, assuming that it is approximately the value of the valve flow section. The value obtained for K_3 gives a value for the drag factor of the inlet valve of $\xi = 0.99$, which is quite reasonable considering the approximations involved in the process.

- K_4: The measured value for valve orifice section A_{c5} was 100 mm^2, assuming that it is approximately the value of the valve flow section. The value obtained for K_4 also gives a value of $\xi = 3.77$ for the drag factor of the outlet valve. This value seems too high, perhaps indicating that irreversibilities around the discharge valve are much higher.

- K_5: According to the estimations of [12], $A_L \sim 1 \cdot 10^{-6} m^2$. This value is consistent with the one obtained in the model for the constant $\frac{K_5 A_L}{\sqrt{\zeta}}$.

- K_6: In [22] a value of 0.037 was given for the dead space ratio of the compressor, the value obtained for the dead space ratio in the model with non phase change inside the cylinder was 0.068. This value was considerably larger than that expected (0.037) thus, the possible condensation of some fraction of refrigerant over a possible cold spot in the cylinder head was considered to explain this loss of mass flow. This assumption resulted in to an estimation of 0.039 for the dead space ratio. This value was quite near the expected.
  In the model with a phase-change inside the cylinder, the value obtained for the constant representative of this process was $K_6 = \overline{\tau} \cdot h_{pc} \cdot A_v = 1.73$. Considering, as a rough estimation, that:

(1) The moment in which the condensation can occur is $\overline{\tau} \sim 0.12\%$ of the total time of one revolution. This time is supposedly equal to the time of the process of delivering vapor through the outlet valve (period of time when the pressure inside the piston is higher).

(2) The area A_v in which this process can be produced is the inner surface of the inlet valve ($A_v \sim 7 \cdot 10^{-4}\ m^2$).

  The obtained heat transfer coefficient is $h_{pc} \sim 20000(W \cdot m^{-2} \cdot K^{-1})$; it is quite reasonable if, for example, a dropwise condensation phenomena is produced, since [24] and [25] reported heat transfer coefficients for dropwise condensation up to $300000(W \cdot m^{-2} \cdot K^{-1})$.

- K_7, K_8: The obtained values for the mechanical losses, approximately 10%, are in agreement with the expected value for this type of compressor.

- η_{el}: This is the most influential factor in compressor efficiency. The obtained value for η_{el} is 0.859, which is very close to the actual mean electric motor efficiency data $\eta_{el} = 0.862$.

Compressor and volumetric efficiencies obtained using both model versions and their relative errors are shown in figs. 3, 4, 5, 6. The differences between model results, using the best fit parameter values, and the experimental data are always lower than 5%. The correlation coefficients obtained between experimental and calculated compressor and volumetric efficiencies are shown in table 2.

The results show that both versions of the model (with and without phase change) reproduce the experimental results accurately enough. The values obtained for the different parameters in both models are in the same range. In brief, the effect of the phase change parameter seems to be equivalent to an increase in the dead space.

In fig. 7, the vapor temperatures at point 7 of the pressure–enthalpy diagram (fig. 1) are plotted against measured compressor outlet temperatures. The temperature given by the model in state 7 of this figure, in principle, can only be considered as a rough estimation of the real temperature at the exit of the compressor (point 8 in fig. 1). In any event, this estimation is actually quite good for low and medium pressure ratios where the temperature of the refrigerant is not high, and it is worse at high pressure ratios, where the temperature of the refrigerant is higher and some processes not considered in the model, like the heat transfer of the hot refrigerant to the crankcase, by radiation, could become more relevant.

5 Conclusions

A model for reciprocating compressors was developed. This model can reproduce the compressor and the volumetric efficiency with an error lower than 3% under a wide range of operating conditions. Although this model was developed for hermetic piston compressors, as a consequence of its general conception, it may be applied to analyze and describe any kind of positive displacement compressor.

Considering the main sources of losses, the model is based on an ideal evolution of the refrigerant throughout the compressor . This model has 10 empirical parameters, each with a direct physical interpretation. If these parameters are unknown, they must be fitted with some empirical data.

To this end, a statistical fitting methodology based on Monte Carlo techniques was designed. This methodology was tested on one compressor and the results with 16 experimental points are quite good. To apply the developed fitting methodology, only data commonly available in catalogs are required.

One drawback of the developed model is that the volumetric and compressor efficiencies are presented as a system of implicit equations. Fortunately, this is not a major drawback nowadays, given the existence of many numerical solvers adequate for this kind of system. Although the developed fitting methodology is very stable, the computational time involved in the fitting process is quite long (6 h. in a Pentium IV processor, 3.4 Ghz., 1 Gb RAM).

All model parameters have a direct physical interpretation and characterize the design and performance of the compressor. In general, the developed model could be quite useful in:

(1) Estimating compressor performance at operating points, different from the experimental points used on the fit. Furthermore, the model could be used to estimate the compressor performance with another refrigerant, other compressor speeds or with slight modifications in the cylinder geometry, for instance.
(2) Characterizing the compressor performance with 10 parameters and analyzing their adequacy from their absolute value or from comparison with a set of reference parameters. For example, unusual values of motor electric efficiency, mechanical losses or valve losses, can point to internal compressor malfunctions.

The value for the dead space ratio found for the studied compressor is too large (6.9%). Considering the possibility of a condensation phenomenon on a cold spot inside the cylinder, like the inner surface of the inlet valve, during the end of the compression stroke leads to a value of this parameter (3.9%) more similar to the real value (3.7%). A study of the order of magnitude of this process revealed that a dropwise condensation could explain this process. Together with similar results obtained from other compressors studied, this result seems to support the possible existence of such a phenomenon.

In the other article [11], the model proposed here will be used to analyze a set of hermetic reciprocating compressors working with propane and the capabilities of the model to predict compressor behavior using other refrigerants will be evaluated.

Acknowledgements

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Figure Index

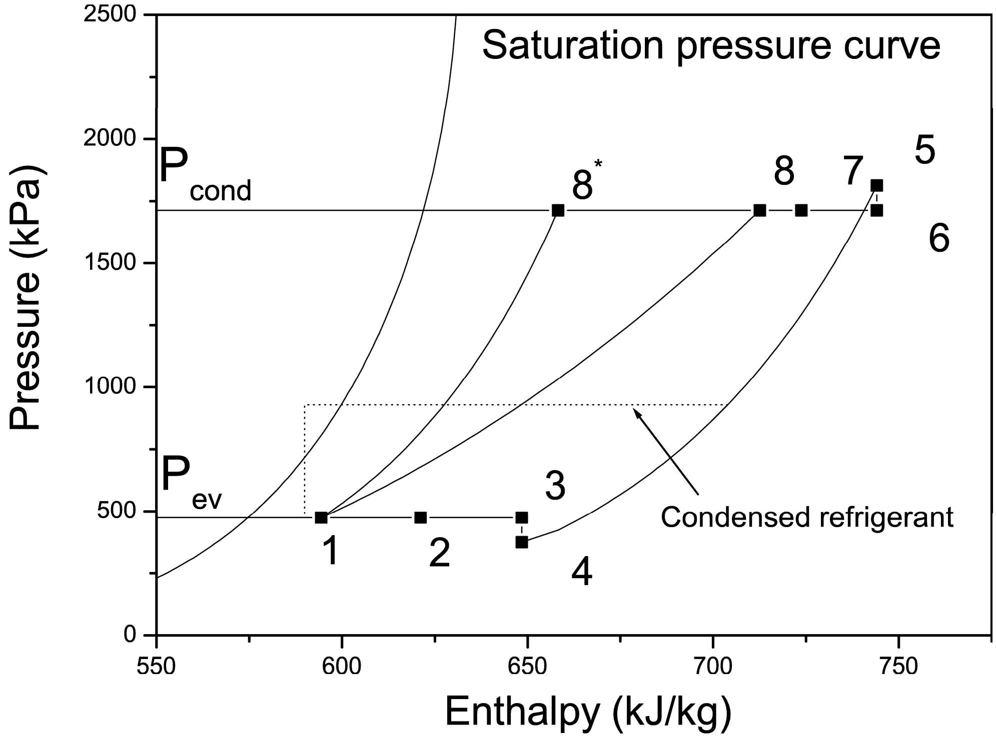

Saturation pressure curve
2500
2000
1500
1000
500
0
Pressure (kPa)
P cond
P ev
8*
8
7
5
6
3
1
2
4
Condensed refrigerant
550
600
650
700
750
Enthalpy (kJ/kg)

Figure 1.

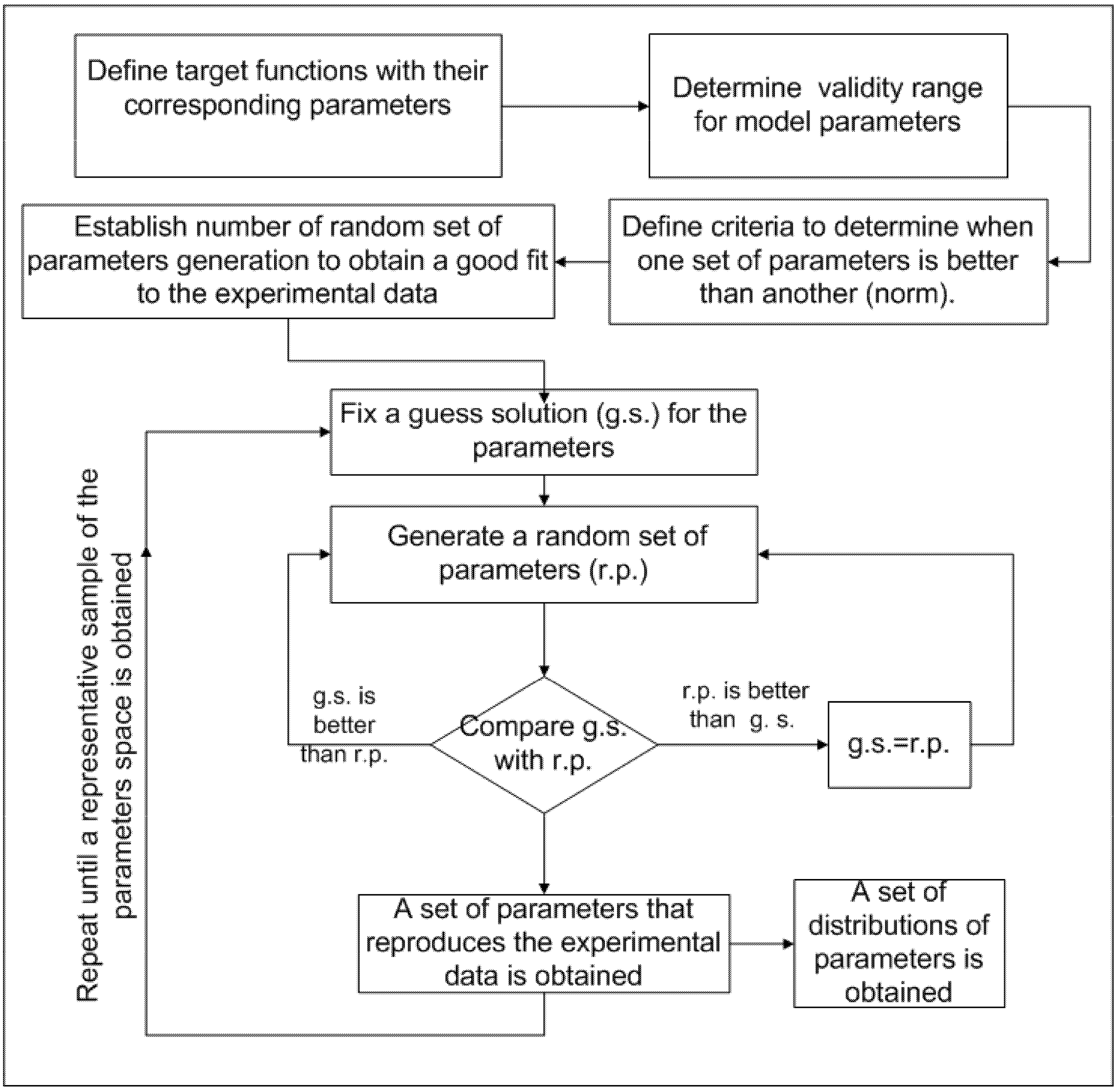

Define target functions with their corresponding parameters
Determine validity range for model parameters
Establish number of random set of parameters generation to obtain a good fit to the experimental data
Define criteria to determine when one set of parameters is better than another (norm).
Fix a guess solution (g.s.) for the parameters
Generate a random set of parameters (r.p.)
g.s. is better than r.p.
Compare g.s. with r.p.
r.p. is better than g. s.
g.s.=r.p.
A set of parameters that reproduces the experimental data is obtained
A set of distributions of parameters is obtained
Repeat until a representative sample of the parameters space is obtained

Figure 2.

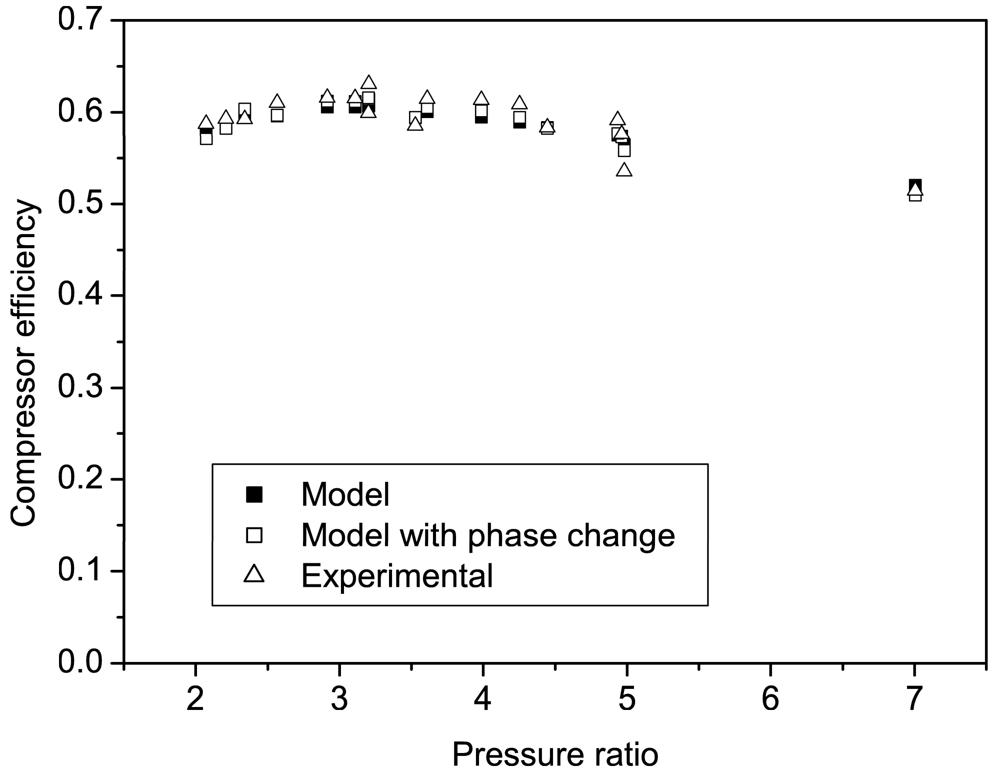

0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
Compressor efficiency
2
3
4
5
6
7
Pressure ratio
Model
Model with phase change
Experimental

Figure 3.

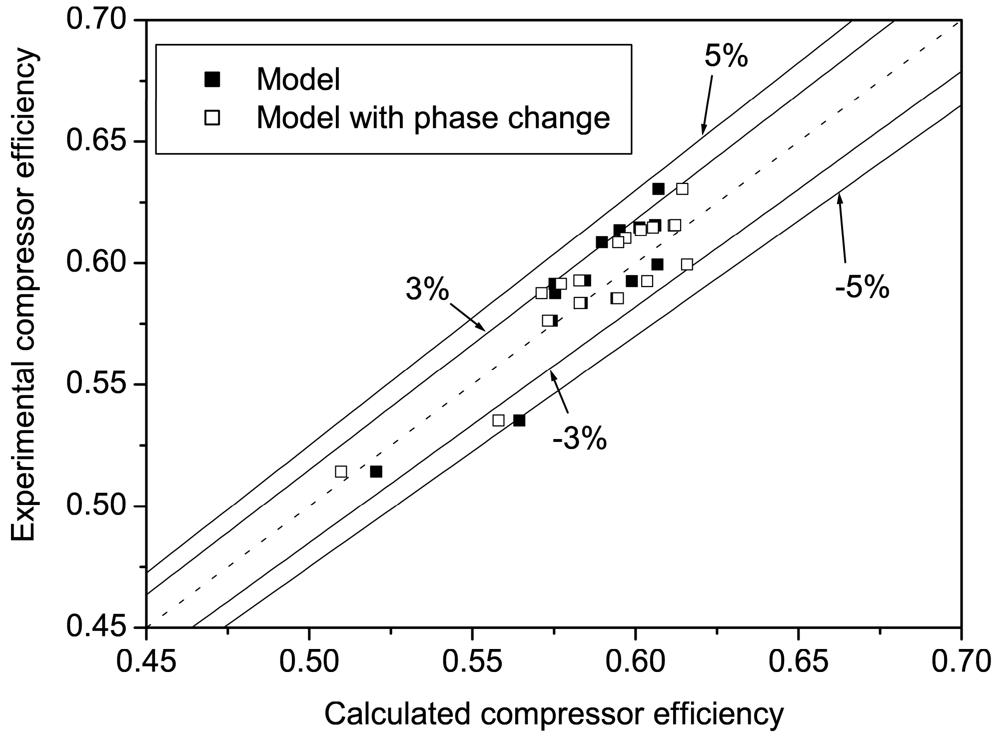
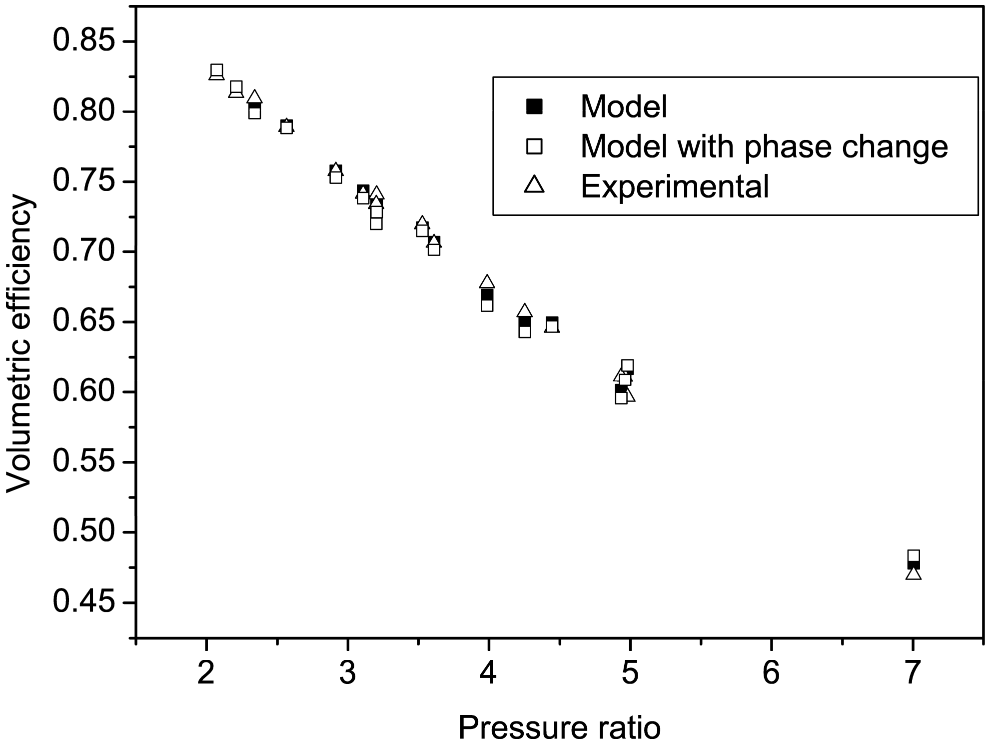

Model
Model with phase change
5%
3%
-5%
-3%
0.70
0.65
0.60
0.55
0.50
0.45
0.45
0.50
0.55
0.60
0.65
0.70
Experimental compressor efficiency
Calculated compressor efficiency

Figure 4.



0.85
0.80
0.75
0.70
0.65
0.60
0.55
0.50
0.45
Volumetric efficiency
2
3
4
5
6
7
Pressure ratio
Model
Model with phase change
Experimental

Figure 5.

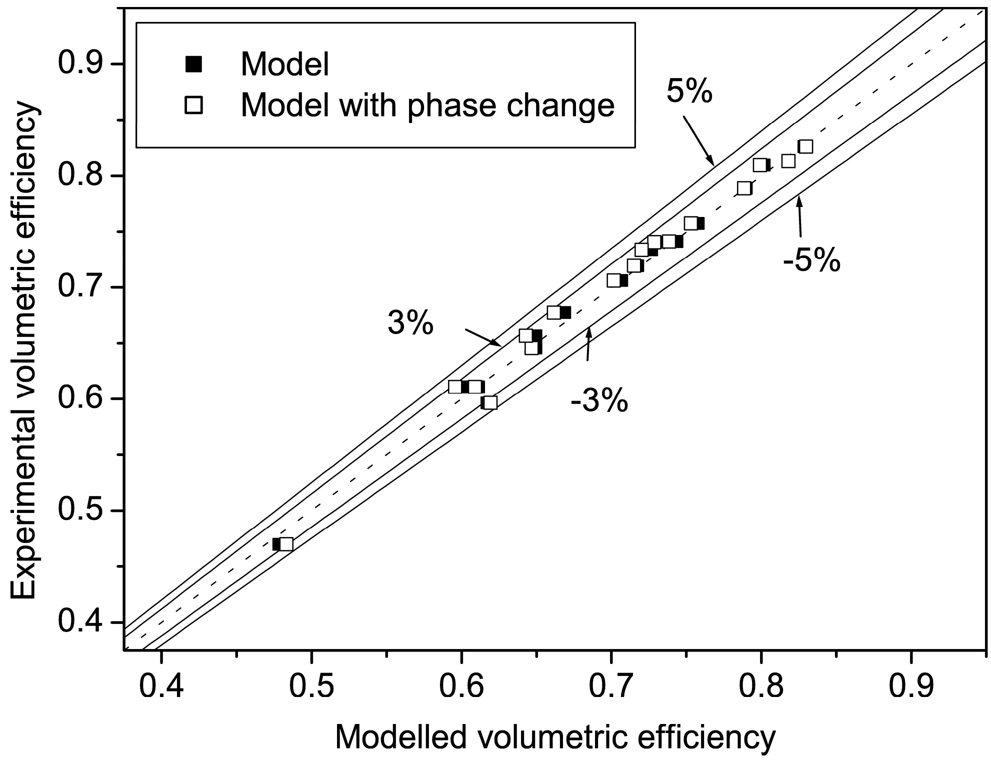
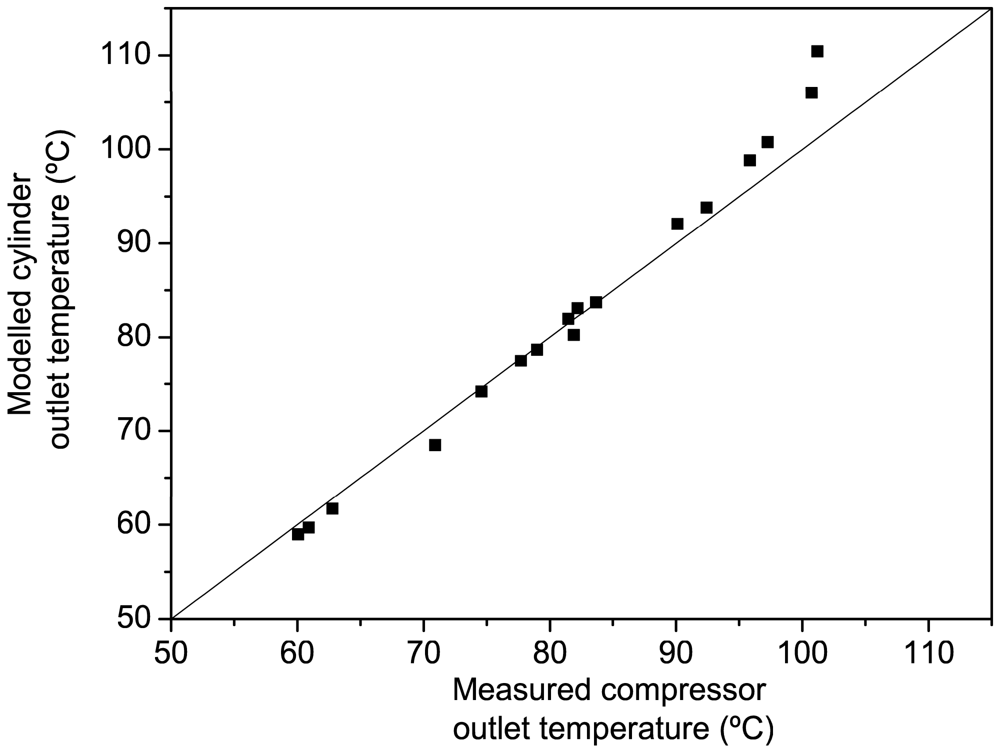

Model
Model with phase change
5%
-5%
3%
-3%
Experimental volumetric efficiency
Modelled volumetric efficiency
0.4
0.5
0.6
0.7
0.8
0.9

Figure 6.



110
100
90
80
70
60
50
50
60
70
80
90
100
110
Modelled cylinder
outlet temperature (ºC)
Measured compressor
outlet temperature (ºC)

Figure 7.

	Parameter definition	Model without phase change	Model with phase change inside the cylinder	Search interval
K_1	K_1	0.90	0.90	[—]
$K_2^{'}\ (m^{-5})$	$C^{'} \cdot C \cdot \frac{A_{ht}}{d_h^{1.8}}$	2.80	2.69	[0-18]
$K_3\ (m^{-2})$	$\frac{\xi_3}{2A_{c3}^2}$	$1.94 \cdot 10^7$	$1.89 \cdot 10^7$	$[0 - 8.74 \cdot 10^7]$
$K_4\ (m^{-2})$	$\frac{\xi_4}{2A_{c4}^2}$	$3.85 \cdot 10^8$	$3.4 \cdot 10^8$	$[0 - 2.5 \cdot 10^9]$
$K_5^{'} \cdot D^2\ (m^2)$	$\frac{A_L \cdot K_5}{\sqrt{\zeta}}$	$0.95 \cdot 10^{-6}$	$0.85 \cdot 10^{-6}$	$[4 \cdot 10^5]$
V_d / V_s	V_d / V_s	$6.77 \cdot 10^{-2}$	$3.90 \cdot 10^{-2}$	[0-0.4]
K_6	$\overline{\tau} \cdot h_{pc} \cdot A_v$	0.0	1.73	[0-18]
K_7	K_7	$5.11 \cdot 10^{-2}$	$4.64 \cdot 10^{-2}$	[0-1]
K_8 (Kw)	K_8	0.2052	0.2051	[0-3.73]
η_{el}	η_{el}	0.859	0.862	[0.5-1]

Table 1
Definition and obtained value of each model parameter for the compressor studied.

	Model without phase change	Model with phase change inside the cylinder
η_k	0.932	0.943
η_s	0.996	0.994

Table 2
Correlation coefficient obtained for the compressor and volumetric efficiencies for both versions of the model.