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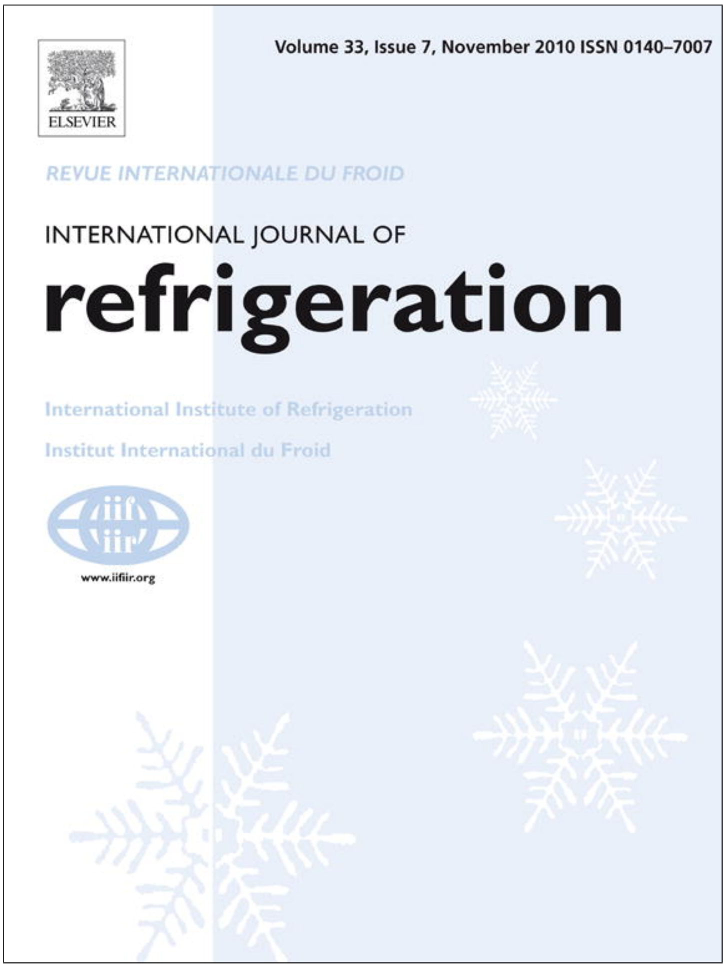

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Analytical expressions for optimum flow rates in evaporators and condensers of heat pumping systems

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Dedicated to Professor Dr.-Ing. Dr.h.c.mult. Karl Stephan on the occasion of his 80th birthday.

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ABSTRACT

The flow velocities on the air or liquid side of evaporators and condensers in refrigerating or heat pump systems affect the system performance considerably. Furthermore the velocity can often be chosen rather freely without obvious first cost implications. The purpose of the paper is to show analytical relations indicating possible optimum operating conditions.

Considering a base case where the design data are known, simple analytical relations are deduced for optimum flow rates that will result in highest overall COP of the system when energy demand for the compressor as well as pumps or fans are included. This optimum is equivalent to the solution for minimum total energy demand of the system for a given cooling load. It is also shown that a different (and higher) flow rate will result in maximum net cooling capacity for a refrigerating system with fixed compressor speed.

The expressions can be used for design purposes as well as for checking suitable flow velocities in existing plants. The relations may also be incorporated in algorithms for optimal operation of systems with variable speed compressors.

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Expressions analytiques pour déterminer les écoulements optimaux dans les évaporateurs et les condenseurs des systèmes à pompe à chaleur

Mots clés : Système frigorifique ; Pompe à chaleur ; Optimisation ; Débit ; Évaporateur ; Condenseur ; COP ; Ventilateur

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Nomenclature

A	analogy number $= f/(8 \cdot J)$
A_w	area for fluid flow, m^2
A_Q	heat transfer area, m^2
C_T	temperature dependence factor introduced in eq. 22, K^{-1}
$C_{T1} = \frac{1-(T_{10}-T_{20})\cdot k_E}{T_{10}-T_{20}\cdot(1-\eta_{Ct})}$	applies to condenser side, defined in eq. 22b
$C_{T2} = \frac{T_{10}}{\eta_{Ct}\cdot T_{20}^2}$	applies to evaporator side, defined in eq. 22a
COP	coefficient of performance
d	hydraulic diameter, m
E	power demand, W
f	friction factor
h	heat transfer coefficient, $W\ m^{-2}\ K^{-1}$
J	Colburn factor $= St \cdot Pr^{2/3}$
$k_q = \partial(Q_2/Q_{20})/\partial T_2$	= cooling capacity dependence of evaporating temperature, t_2, K^{-1}
$k_E = \partial(\eta_{Ct}/\eta_{Ct}0)/\partial T_1$	Carnot efficiency dependence of condensing temperature, t_1, K^{-1}
n	exponent
Pr	Prandtl number
Q	cooling capacity, W
St	Stanton number $= h/(w \cdot \rho \cdot c_p)$
T	temperature, K
t	temperature, °C
U	overall heat transfer coefficient, $W\ m^{-2}\ K^{-1}$
w	velocity, $m\ s^{-1}$
$\eta_{Ct} =$	$COP_2/(T_2/(T_1-T_2))$ = total Carnot efficiency
ρ	density, $kg\ m^{-3}$
θ	temperature difference, K

Index:

0	refers to base case
1	refers to condenser side
2	refers to evaporator side
h	heat transfer (on air or brine side)
c	compressor
COP_{max}	conditions for maximum overall COP
f	friction factor
in	inlet (to heat exchanger)
Is	isentropic
N	net
p	pump or fan
pm	pump or fan including motor
Q_{max}	conditions for maximum net cooling capacity
Si	sink
So	source
tot	total
TM	average temperature (in heat exchangers logarithmic average)
Tin	inlet temperature

1. Introduction

Fig. 1a depicts a simple general air to air refrigerating system which is in focus for the present paper. The temperature differences in the evaporator and condenser have a significant influence on the performance of the system. For a system with given compressor and given designs of heat exchangers, the operating temperature differences are strongly affected by the air velocity, which often can be chosen rather freely. For instance, *increasing the velocity on the evaporator side* will increase the evaporating temperature, thus *increasing the capacity and the COP of the compressor* but the cost of this is a *higher power consumption of the evaporator fan*. It is obvious that there must be a velocity which gives the most favorable operation, representing an optimum velocity. The purpose of the paper is to derive relations for optimum velocity (or fan power), simple enough to be used in practice. Also in indirect systems, as shown in Fig. 1b, similar questions related to the influence of the flow rates arise, and similar relations can be derived.

The influence of the fans (and pumps) on the total operating energy of refrigerating systems has been treated by Granryd (1974, 1998) with similar methodology as here. Results from second law approach minimizing total exergy loss in heat exchangers are given for instance by Bejan, 1996, DeJong et al., 1997 and Kuenth, 1986. Examples of experimental results are given by Waller (1988).

The choice of fan speed (or flow rate) does only marginally influence the first cost of the system. Thus the question of optimal flow rates is mainly of a *technical* character and the solutions are not influenced by investment strategies, interest rates or energy prices. This simplifies the problem to derive *general relations for optimum flow velocity* for applications in refrigerating, air conditioning or heat pump systems.

2. An example

For the following treatment it is assumed that we have a given system design (comprising evaporator, compressor and condenser). In a "*base case*" the fluid velocity is chosen to a certain value (arbitrarily or by experience) and for a given application the operating temperatures depends on the performance of the evaporator and condenser.

Figs. 2 and 3 gives an example of temperature difference and the pressure drop versus air velocity based on relations for a fin coil evaporator with geometries as indicated (estimated by relations given by Granryd, 1964). Fig. 2 shows temperature differences that can be expected for different frontal air velocity if this evaporator is used in a system with given capacity, Q_2. Curves are shown for θ_M (the logarithmic *mean* air temperature difference) and θ_{in} (the difference between *incoming* air temperature and refrigerant saturation temperature at the exit of the coil). As expected, the inlet temperature difference, θ_{in}, is much more influenced by the air velocity than the logarithmic average, θ_M. Which temperature difference that is of interest depends on the application.

In Fig. 3 the estimated air side pressure drop versus air speed is shown for the same fin coil. In most cases the fan flow

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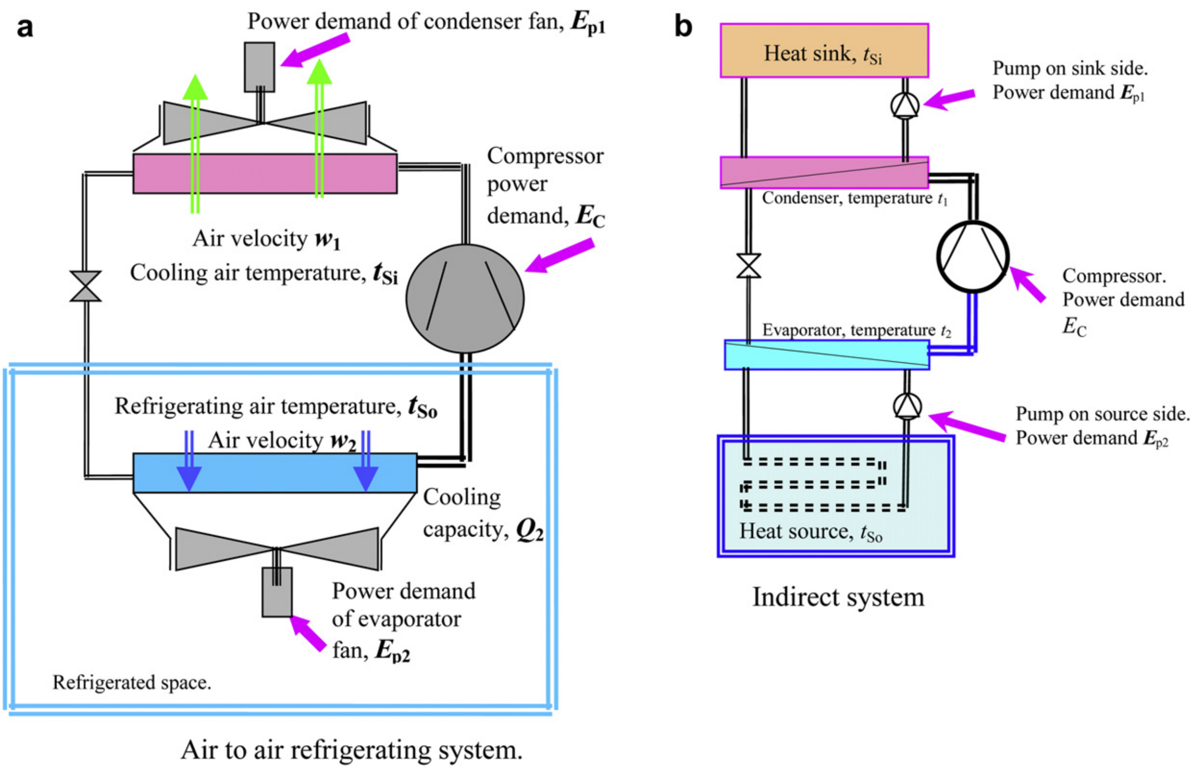

Fig. 1 – a Air to air refrigerating system. b Indirect system.

area itself is much smaller than the front area of the coil. The total work the fan has to overcome is not only that caused by the pressure drop in the fin coil but also the work to accelerate the air through the fan opening. In the figure curves are given for different ratios of fan area/coil frontal area = 1/2, 1/3 and 1/4. It is clear that this area ratio has a large influence on the pressure drop and the necessary fan power.

Fig. 4 shows results from simulations of a refrigerating system equipped with fan coils as in Figs. 2 and 3 in which the air velocity on the evaporator side is varied. Results are derived for an example with refrigerant R134a and an efficient hermetic reciprocating compressor operating at constant speed. As seen the velocity has a relatively large influence on the *net refrigerating capacity* Q_{2N} (the compressor refrigerating capacity minus the fan power which adds to the load) as well as on the overall Coefficient of Performance, COP_{2tot}, of the system. Notice that the optimal velocity to save energy (max COP_{2tot}) is almost half that corresponding to maximum net

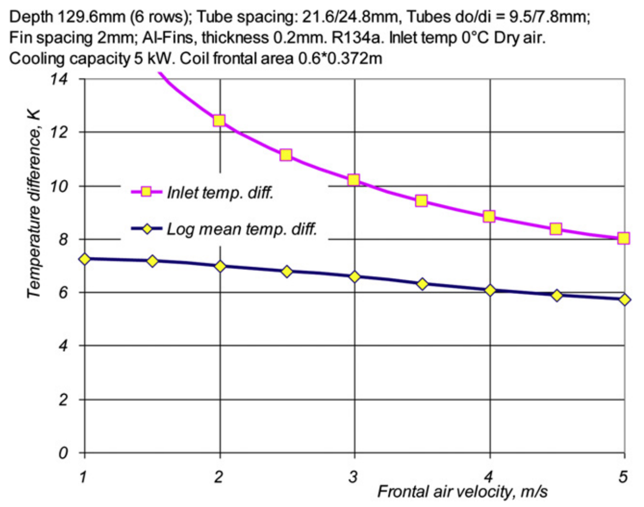

Fig. 2 – Inlet and log mean temperature difference.

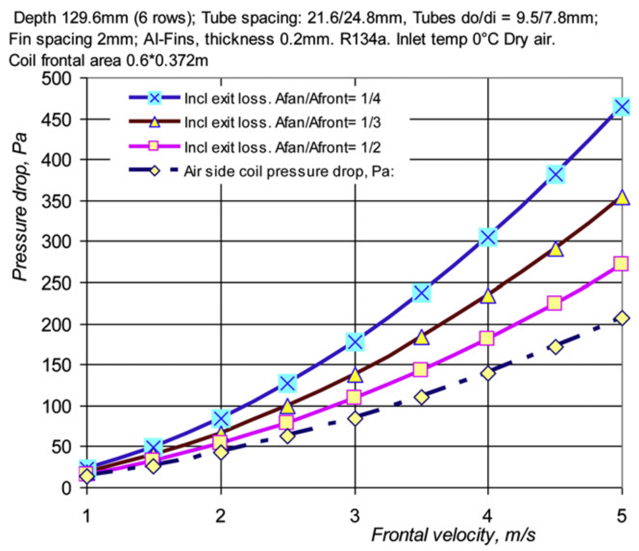

Fig. 3 – Pressure drop.

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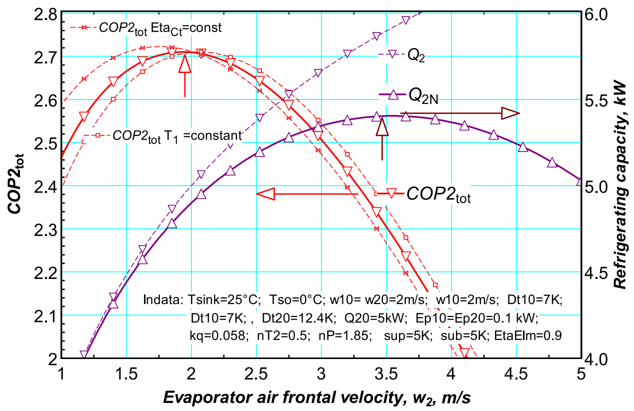

Fig. 4 – Results from simulation of a refrigerating system. Curves are shown for a system with a reciprocating compressor (swept volume 0.005 m^3/s) with isentropic and volumetric efficiencies typical for a "good" compressor. Inlet temperature difference at base case ($w_0 = 2\ m\ s^{-1}$) is used as given in Fig. 2 and pressure drop as in Fig. 3 with free-blowing fan with area ratio $A_{fan}/A_{front} = 1/4$. Three curves are shown for the COP: – The solid line is the correctly simulated value. For the dotted lines different simplifications are made as discussed in the text: For the curve indicated "T_1 = constant" the condensation temperature is assumed unaffected by evaporator air velocity; for the one indicated "Eta_{Ct} = constant" the overall Carnot efficiency is assumed constant.

capacity, Q_{2N}. If the refrigerating capacity of the system permits, it is obviously favorable to use the lower velocity in order to save energy. The purpose of the paper is to derive useful and simple tools to find optimum flow conditions in the heat exchangers of refrigeration systems.

3. General relations

The temperature differences in heat exchangers (θ) influence the system operating temperatures, t_1 and t_2. In an application where *the mean temperature of the heat source* (t_{SoM}) is of interest we can write the *evaporating* temperature:

$$t_2 = t_{SoM} - \theta_{2M}. \tag{1}$$

Similarly if the *mean temperature of the heat sink* is of interest then the *condensing* temperature is

$$t_1 = t_{SiM} + \theta_{1M}. \tag{2}$$

In many cases, however, *instead the inlet temperature difference* is of interest (not the mean difference). For instance in an *air source heat pump* the ambient air temperature is given, and the outlet temperature from the evaporator is of no interest. We have there:

$$t_2 = t_{Soin} - \theta_{2in} \tag{3}$$

In an *air cooled condenser* the ambient air temperature is given, and the outlet temperature is of no interest. We have there obviously

$$t_1 = t_{Siin} + \theta_{1in}, \tag{4}$$

The basis for the treatment is the following simplified (but quite general) assumptions for the temperature differences in the heat exchangers:

The *heat transfer coefficient* on the air side can often be set proportional to the air side velocity raised to an exponent, n_h, generally about 0.3–0.5 for laminar and 0.8 for turbulent convective heat transfer. The *overall* heat transfer coefficient is similarly affected, but with an exponent n_{TM}, lower that the value of n_h due to the resistance on the refrigerant side of the heat exchanger. *For given capacity of a heat exchanger the mean temperature difference will be proportional to* $w^{-n_{TM}}$. Referring to a "base case" (index "$_o$") and with the heat load (Q) given, we can express the mean temperature difference:

$$\theta_M = \theta_{0M} \cdot (w/w_0)^{-n_{TM}} \tag{5a}$$

For situations where the *mean* temperature difference is of interest Fig. 5 illustrates values of the exponent n_{TM} for heat transfer cases with different n_h. The exponent n_{TM} is often about 0.2–0.4.

If, instead, the *inlet* temperature difference is of interest we can set

$$\theta_{in} = \theta_0 \cdot (w/w_0)^{-n_{Tin}} \tag{5b}$$

Fig. 6 illustrates how the inlet difference θ_{in} is affected by flow rates w/w_0 (the same cases as in Fig. 5). The flow rate has a much stronger influence here than for the previous case. The exponent n_{Tin} is often around 0.5 to 0.6, even for cases with quite a small influence of the flow rates on the air side heat transfer as seen in Fig. 6. (The temperature efficiency of the heat exchanger affects the result to some degree.)

The *pressure drop*, Δp, on the air side can in a similar fashion be set proportional to the velocity raised to an exponent, n_p. For example classical relations in turbulent flow give pressure drops proportional to $w^{1,8}$, while in pure laminar flow the exponent is = 1 and for Boda-Carnot type losses = 2. Thus the pressure drop can be written:

$$\Delta p = \Delta p_0 \cdot (w/w_0)^{np} \tag{6}$$

The exponent is often in the range 1.5–2. The associated fan power needed for operation, if the fan efficiency is assumed

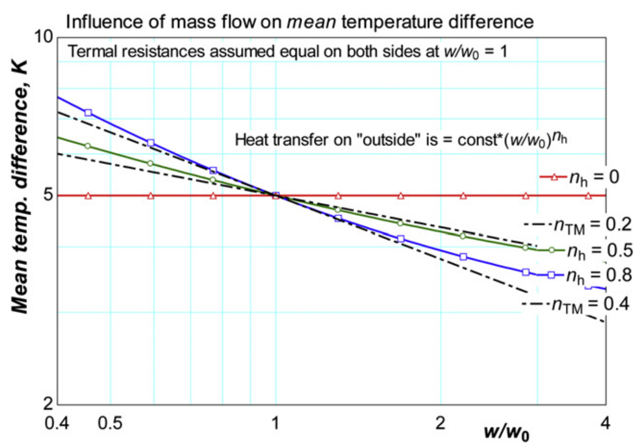

Fig. 5 – Influence of flow rates on mean temperature difference.

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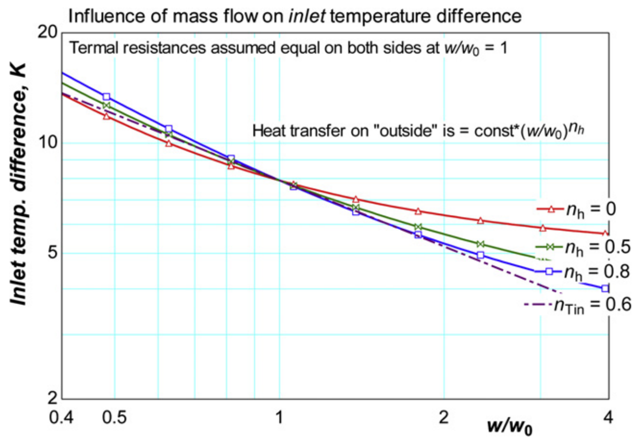

Fig. 6 – Influence of flow rates on and inlet temperature difference. Comments to Figs. 5 and 6: Average temperature difference θ_M is set to 5 K at $(w/w_0) = 1$. Curves are estimated for heat exchangers with constant temperature on one side (typical for evaporators or condensers) and with different values of the exponent describing the influence of velocity on air side heat transfer (n_h). For $w = w_0$ the $U\cdot A/(m\cdot c_p)$ is set = 1 (at that point the temperature efficiency is 0,63 for $(w/w_0) = 1$). Values of $0.4 < (w/w_0) < 4$ covers cases with temperature efficiencies about 0.2–0.9. Straight lines show exponents n_{TM} (in Eq. (5a)) and n_{Tin} (in Eq. (5b)). The exponent n_{Tin} is considerably larger than n_{TM}. Exponent n_{Tin} is about 0.5–0.6 almost regardless of the exponent for n_h.

independent of the air velocity within the range of interest[1], is thus:

$$E_p = E_{p0}\cdot(w/w_0)^{(n_p+1)} \tag{7}$$

We can apply similar relations to e.g. indirect systems where a liquid is used instead of air.

4. Net refrigeration capacity – evaporator side

4.1. *Net refrigerating capacity*

In *refrigeration* the fan power in the evaporator is added to the load (also fan motor losses will increase the load provided that the motor in located in cooled space). The *net cooling capacity* is thus:

$$Q_{2N} = Q_2 - E_{p2} \tag{8}$$

The cooling capacity Q_2 of a system with given compressor speed is affected by primarily the evaporating temperature, t_2, (and to a minor degree by the condenser temperature). We can set:

$$Q_2 = Q_{20}\cdot(1 + k_q\cdot(t_2 - t_{20})) \tag{9}$$

where k_q is a factor indicating how the cooling capacity of a system depends on the evaporating temperature, t_2. The factor is influenced by refrigerant properties and also by the type of compressor (the volumetric efficiency is influenced by the pressure ratio). Often the refrigerating capacity increases 4 to 6% per °C of increasing evaporator temperature[2]. This means $k_q = 0.04$ to 0.06 1/°C

By Eqs 8, 9 and 5 we can write.

$$Q_2 = Q_{20}\cdot\left[1 + k_q\cdot\theta_{20}\cdot\left(1 - \left(\frac{w_2}{w_0}\right)^{-n_T}\right)\right] \tag{10}$$

The exponent n_T is used to denote either n_{TM} or n_{Tin} depending on which temperature is considered to be important for the application in question (as discussed in relation to Figs. 2,5 and 6).

Eqs (8), (10) and (7) gives:

$$Q_{2N} = Q_2 - E_{p2} = Q_{20}\cdot\left[1 + k_q\cdot\theta_{20}\cdot\left(1 - \left(\frac{w_2}{w_0}\right)^{-n_T}\right)\right] - E_{p20}\cdot\left(\frac{w_2}{w_0}\right)^{(n_p+1)} \tag{11}$$

(For an application as a heat pump the last term will not appear.)

4.2. *Optimum velocity on evaporator side for maximum net capacity*

An optimum velocity, equivalent to maximum net capacity, is found by $\partial(Q_{2N}/Q_{20})/\partial(w/w_{20}) = 0$, and with Eq. (11) the resulting relation for maximum refrigerating capacity is

$$w_{Q\max} = w_0\cdot\left(\frac{n_T}{1+n_p}\cdot k_q\cdot\frac{\theta_{20}}{E_{p20}/Q_{20}}\right)^{1/(n_T+n_p+1)} \tag{12}$$

Example: Assume a system with evaporator coil data related to the example in Figs. 2 and 3. Source temperature is $t_{So} = 0$ °C. At the "base case" with $w_0 = 2$ m s^{-1} the system has the following data: Compressor cooling capacity is $Q_{20} = 5$ kW. *Inlet* temperature is of importance in the application, and $\theta_{20in} = 12.4$ K. The evaporator coil has a free-blowing fan with $A_{fan}/A_{front} = ¼$ and the overall pressure drop at 2 m s^{-1} is 84 Pa including exit loss with free-blowing fan. The front area of the evaporator coil is 0.22 m^2 and the fan including motor has $\eta_{pm} = 37\%$ overall efficiency. Thus the fan power at 2 m s^{-1} is $E_{p0} = w_0\cdot A_{fr}\cdot\Delta p/0.37 \cong 100$ W. Hence $E_{p0}/Q_{20} = 0.02$. Simulation of data for the system give $k_q = 5.8\%$/K. From evaporator data in Figs. 2 and 3 one can estimate the exponent $n_T = n_{TIn}$ to about 0.5 and the exponent $n_p = 1.85$. Eq. (12) with these data inserted suggests that the maximum net cooling capacity would be reached at an air velocity of:

[1] In order to take into account variations of the fan efficiency by flow rate the value of the exponent may be adjusted (in a limited range).

[2] A simple general relation for the value of factor k_q for refrigerants – disregarding the influence of the volumetric efficiency of the compressor – can be derived by assuming the vapor as an ideal gas and by combining Clapeyrons equation and Troutons rule. The result is $k_q \cong C_C/R_M\cdot 1/T_2 = 88000/8314\cdot 1/T_2 = 10,6/T_2$. This gives for $T_2 = 263$ K a value of $k_q \cong 0.040$. The relation is only valid approximately, but forms a good rule of thumb for all refrigerants. In practice the value of k_q is larger than this due to the influence of volumetric efficiency of the compressor.

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$$w_{Q\max} = 2\cdot\left(\frac{0.5}{1+1.85}\cdot 0.058\cdot\frac{12.5}{100/5000}\right)^{11/(0.5+1.85+1)} = 3.47\ \mathrm{m\cdot s^{-1}}$$

The arrow on the Q_{2N}-curve in Fig. 4 indicates this result, in good agreement with the simulation.

Notice that a large temperature difference (θ_{20}) increases the optimum velocity. (If instead the mean air temperature had been important in this application we would have $\theta_{20M} = 7K$ (see Fig. 2) and $n_T = n_{TM} \cong 0.2$. Optimum velocity for max capacity instead becomes 2.25 m s^{-1})

5. System COP_2 and total operating energy

5.1. *Overall total coefficient of performance*

The overall energy (or total power) to operate the system is comprised of the energy used for the compressor and for the fans (or pumps). This can be expressed:

$$E_{tot} = E_C + E_{p2} + E_{p1} = Q_2/COP_2 + E_{p2} + E_{p1} \tag{13}$$

The Coefficient of Performance, COP_2 is influenced by the evaporating and the condensing temperatures, t_2 and t_1. Using the concept of an overall Carnot efficiency, η_{Ct}, we can write:

$$COP_2 = \frac{Q_2}{E_C} = \eta_{Ct}\cdot\frac{T_2}{T_1 - T_2} \tag{14}$$

The overall total COP_{2tot}, defined as Q_{2N}/E_{tot} is of interest in *refrigerating* applications. The overall COP_{2tot} including the two auxiliaries can thus be expressed:

$$\frac{1}{COP_{2tot}} = \frac{E_{tot}}{Q_{2N}} = \frac{Q_2}{Q_{2N}}\bullet\left[\frac{1}{COP_2} + \frac{E_{p2}}{Q_2} + \frac{Q_1}{Q_2}\bullet\frac{E_{p1}}{Q_1}\right] \tag{15}$$

5.2. *Optimum velocity on evaporator side for maximum overall system* COP_{2tot}

Let us first *focus on the evaporator, considering the condenser side given*. Inserting previous relations into Eq. (15) give the following expression:

$$\frac{1}{COP_{2tot}} = \frac{Q_2}{Q_{2N}}\cdot\left[\frac{1}{\eta_{Ct}}\cdot\left(\frac{T_1}{T_2} - 1\right) + \frac{E_{p20}}{Q_2}\cdot\left(\frac{w_2}{w_{20}}\right)^{(n_p+1)} + \frac{Q_1}{Q_2}\cdot\frac{E_{p1}}{Q_1}\right] \tag{16}$$

where $T_2 = (T_{So} - \theta_{20}\cdot(w_2/w_{20})^{-n_T})$. To simplify expressions let us here omit the variation of Q_2 with velocity, and set $E_p << Q_2$, (by which $Q_2/Q_{2N} \cong 1$), consider η_{Ct} constant, and the condenser side unaffected by the conditions on the evaporator side (thus T_1 is $= T_{10}$). These simplifications give the derivative:

$$\frac{\partial(1/COP_{2tot})}{\partial(w/w_{20})} \cong \frac{-n_T}{\eta_{Ct}}\cdot\frac{T_{10}}{T_{20}^2}\cdot\theta_{20}\cdot\left(\frac{w_2}{w_{20}}\right)^{-n_T-1} + (n_p+1)\cdot\frac{E_{p0}}{Q_2}\cdot\left(\frac{w_2}{w_{20}}\right)^{n_p} \tag{17}$$

By setting $\partial(1/COP_{2tot})/\partial(w_2/w_{20}) = 0$ an optimum value for the (w_2/w_{20}) will be found corresponding to an extreme value for the COP_{2tot}. The result is:

$$w_{2COP_{\max}} \cong w_{20}\cdot\left(\frac{n_T}{n_p+1}\cdot\frac{T_{10}}{\eta_{Ct}\cdot T_{20}^2}\cdot\frac{\theta_{20}}{E_{p20}/Q_{20}}\right)^{1/(n_T+n_p+1)} \tag{18}$$

Example: Assume the same data as the previous example. In addition to the data given under 4.2 for the base case we need in addition the source temperature $t_{Soin} = 0$ °C giving $T_{20} \cong 273-12.4 = 260.6$ K and the condensing temperature which is $t_{10} = +32$ °C ($T_{10} = 305$ K) related to the data in Figs. 2–4. As before the compressor cooling capacity at base case is $Q_{20} = 5$ kW, $E_{p0} = 100$ W. Exponents $n_T = 0.5$; $n_p = 1.85$. Overall Carnot efficiency η_{Ct} is 0.52. With these data inserted in Eq. (18) the maximum COP_2 should be reached with an air velocity of

$$w_{2COP_{\max}} = 2\cdot\left(\frac{0.5}{1+1.85}\cdot\frac{305}{0.52\cdot 260.5^2}\cdot\frac{12.5}{100/5000}\right)^{1/(0.5+1.85+1)} = 1.97\ \mathrm{m\cdot s^{-1}}$$

In Fig. 4 the result of this expression is shown by the solid arrow on the COP-curve. As seen it is a reasonably good agreement with the optimum from the simulation.

In the diagram there are three curves for the COP_{tot}, one full and two (dotted) additional curves simulated for the system to check the influence of simplifications made in the derivation; one where the condensing temperature T_1 is assumed constant and one where the Carnot efficiency (η_{Ct}) of the system is constant. The full curve in the figure includes variations of T_1 and η_{Ct} as in a real system. The errors of the simplifications made in the derivation of Eq. (18) seem to more or less cancel each other out. The optimum is quite flat and the simplified result may be justified.

5.3. *Condenser side*

Let us now instead *focus on the condenser, considering the evaporator side given*. In a similar fashion as for Eq. (16) we can derive the following expression:

$$\frac{1}{COP_{2tot}} = \frac{Q_2}{Q_{2N}}\cdot\left[\frac{\left(T_{Si} + \theta_{10}\cdot(w_1/w_{10})^{-n_T}\right) - T_{20}}{\eta_{Ct}\cdot T_{20}} + \frac{E_{p20}}{Q_{20}} + \frac{Q_1}{Q_2}\cdot\frac{E_{p10}}{Q_1}\cdot\left(\frac{w_1}{w_{10}}\right)^{(n_p+1)}\right] \tag{19}$$

The condenser temperature will only marginally influence the evaporator temperature, t_2, but it does influence the Carnot efficiency (η_{Ct}). To take the variation of the Carnot efficiency by the condenser temperature (with given sub cooling), into account a parameter is introduced, defined as

$$k_E = \frac{\partial(\eta_{Ct}/\eta_{Ct0})}{\partial T_1} \tag{20}$$

The decrease in the cycle efficiency is in the order of 1%/K, equivalent to $k_E = -0.01$, but varies with cycle and compressor performance.

In a similar way as for the evaporator we can find optimum velocity on the condenser side for maximum COP_{2tot}. Neglecting second order derivatives, the following expression can be derived (noticing that $Q_1/Q_2 \cong Q_{10}/Q_{20} = 1 + 1/COP_{20} = 1 + (T_{10} - T_{20})/(\eta_{Ct}\cdot T_{20}) = (T_{10} - T_{20}\cdot(1-\eta_{Ct}))/(\eta_{Ct}\cdot T_{20})$):

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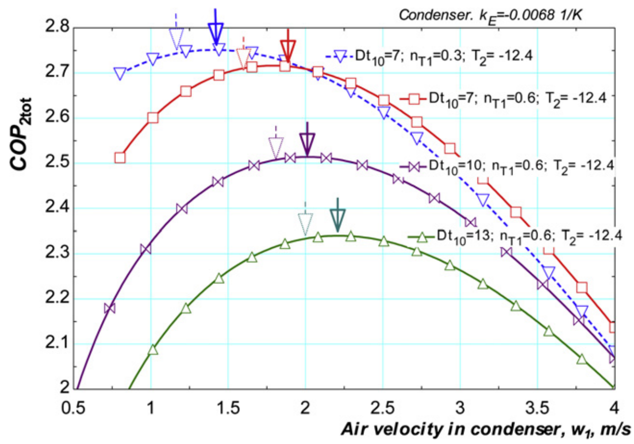

Fig. 7 – COP-variation with condenser side velocity. Three different base case temperature differences with $n_T = 0{,}6$ and one of the cases also with $n_T = 0{,}3$ for illustration.

$$w_{1COP_{max}} = w_{10}\cdot\left[\frac{n_T}{n_p+1}\cdot\frac{1-(T_{10}-T_{20})\cdot k_E}{T_{10}-T_{20}\cdot(1-\eta_{Ct})}\cdot\frac{\theta_{10}}{E_{p10}/Q_{10}}\right]^{1/(n_T+n_p+1)} \tag{21}$$

In Fig. 7 results are exemplified for a few different combinations of base case temperature differences and exponents. The solid arrows indicate the optimum values given by Eq. (21) with $k_E = -0.0068$ 1/K, estimated for the working conditions. (Also shown are dotted arrows, estimated with $k_E = 0$ and the differences between solid and dotted arrows thus indicate the influence of η_{Ct}-variations. Compare the related discussion for the evaporator.)

5.4. *Comments*

The relations for optimum velocity on evaporator and condenser side differ only by a temperature dependence factor. Introducing a temperature factor C_T in the expressions for the evaporator as well as condenser optimum velocity can be written.

$$w_{COP_{max}} = w_0\cdot\left[\frac{n_T}{n_p+1}\cdot C_T\cdot\frac{\theta_0}{E_{p0}/Q_0}\right]^{1/(n_T+n_p+1)} \tag{22}$$

Where C_T for the evaporator is:

$$C_{T2} = \frac{T_{10}}{\eta_{Ct}\bullet T_{20}^2} \tag{22a}$$

and for the condenser:

$$C_{T1} = \frac{1-(T_{10}-T_{20})\bullet kE}{T_{10}-T_{20}\bullet(1-\eta_{Ct})} \tag{22b}$$

All the symbols in Eq. (22) apply to evaporator or condenser conditions, respectively.

The numerical values of the temperature factors C_{T1} and C_{T2} are relatively similar. Fig. 8 is an illustration. The curves are generated with estimated values of η_{Ct} and k_E for cycles

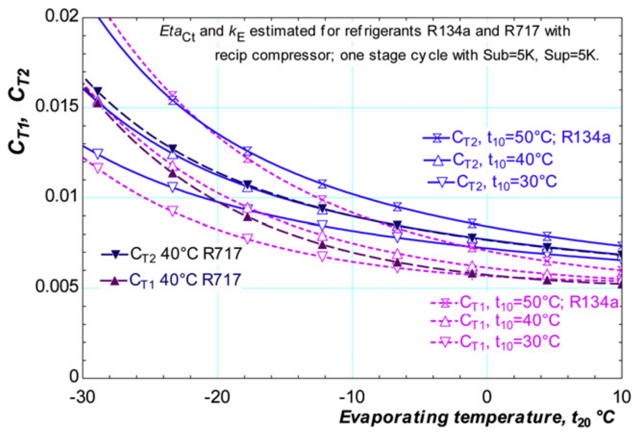

Fig. 8 – Temperature dependence factors for evaporators (C_{T2}) and condensers (C_{T1}) as of equations 22a and 22b. The factors η_{Ct} and k_E are estimated for real cycle with R134a, sub cooling in condenser = 5K; superheating in evaporator = 5K, advanced recip. compressor. Other refrigerants give quite similar results for normal operation.

with R134a and with efficiency characteristics of a reciprocating compressor. The type of refrigerant has a limited influence. Values of C_{T1} and C_{T2} are for evaporating temperatures typical for air conditioning or cooling applications in the range 0.006–0.008; for lower evaporating temperatures 0.01 to 0.015.

6. Expressions for optimal fan power

We can also use the expression for optimal velocity to establish simple expressions for the fan power that will give maximum capacity, Q_{2N}, or maximum COP_{2tot} by the following observation:

Assume that the base case (index 0) were chosen exactly equivalent to the desired optimum. Then obviously the optimal velocity would be equal to the base case velocity w_{20} and the expression in the bracket of Eqs (12, 18 and 21) would be equal to 1. The fan power in the expressions would correspond to the optimum.

Using Eq. 12 this reasoning gives for the evaporator side the following simple expression of fan power corresponding to *maximum net capacity* in a refrigeration system:

$$\left(\frac{E_{p2}}{Q_2}\right)_{Q_{max}} = \frac{n_T}{n_p+1}\cdot k_q\cdot\theta_{2Q_{max}} \tag{23}$$

Notice that the temperature difference $\theta_{2Q_{max}}$ is the difference that should prevail *at the optimum velocity corresponding to the solution for maximum net capacity*.

Similarly, for saving energy we can, by using Eq. (22), find the fan (pump) power on the evaporator or the condenser side for to *maximum overall* COP_{tot} given by:

$$\left(\frac{E_p}{Q}\right)_{COP_{max}} = \frac{n_T}{n_p+1}\cdot C_T\cdot\theta_{COP_{max}} \tag{24}$$

Notice, as previously mentioned that $\theta_{COP_{max}}$ denotes the *temperature difference in the evaporator or condenser, respectively,*

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which will prevail at the solutions for COP$_{max}$. (The temperature differences in the expressions of Eq. (23) and (24) are obviously not the same even for a given system. It also means that an iterative process often is required for an accurate result.)

Example: Use data from previous cases and search for max COP_{2tot}:

For the evaporator we have in this case $\theta_{2COP_{max}} \cong 12.4$ K(close to θ_{20}), $n_T = 0.5$; $n_p \cong 1.85$; $T_1 \cong 305$ (33 °C); $T_2 \cong 260$ K(−13 °C). Fig. 8 gives $C_{T2} = 0.0084$. Inserted in Eq. (24) gives:

$(E_{p2}/Q_2)_{COP_{max}} = 0.5/(1+1.85)\cdot 0.0084\cdot\theta_{2COP_{max}} \cong 0.018$ or 1.8%.

For the condenser Eq. (24) gives with the same data except $\theta_{1COP_{max}} \cong 7$ K and $C_{T1} = 0.007$, 1/K:

$(E_{p1}/Q_1)_{COP_{max}} = 0.5/(1+1.85)\cdot 0.007\cdot\theta_{1COP_{max}} = 0.009$ or 0.9%.

Adding the fan powers on the evaporator and the condenser side together (with $COP_2 \cong 2.7$), the numbers in this example will in total correspond to about $1.8 + 0.9\cdot(1 + 1/2.7) \cong 3.0\%$ of the refrigerating capacity Q_2, equivalent to about $3.0\cdot 2.7 \cong 8\%$ of the compressor power, E_C.

7. Heat pump applications

Fig. 9 shows results from simulation with the aim to illustrate a *heat pump* application. For heat pumps $COP_{1tot} = (Q_1 + E_{p1})/(E_C + E_{p2} + E_{p1})$ is of more interest than COP_{2tot}. If there are no heat losses then $Q_1 = Q_2 + E_C$ where Q_2 defines the low temperature energy absorbed from the surrounding.[3]

Slightly different base case temperature differences are chosen ($\theta_{20} = \theta_{10} = 10$ K) for the heat pump system in Fig. 9 compared to previous examples. Two sets of curves are given for $COP1_{tot}$ and $Q1_{tot}$: The solid curves are derived for the case that the evaporator velocity w_2 varies while condenser speed is unchanged ($w_1 = w_{10}$). Dotted curves give the result if both velocities vary so that $w_1 = w_2$. The latter strategy makes the optimum more pronounced. (The same observation is true also for all previous examples.)

The curves in Fig. 9 can also help us to draw some conclusions specific for heat pump applications. In certain situations the heat demand is higher than the capacity of the heat pump. Increasing evaporator fan speed gives a possibility to increase the capacity. The price of this is however a decreasing coefficient of performance.

An example: Consider to increase the evaporator fan speed from w_2 about 2 m s^{-1} to a value, say $w_2 = 3.5$ m s^{-1}. The COP_{1tot} decreases from about 3.5 to 3.2, while the capacity Q_{1tot} increases from 6.8 to 8 kW. The overall *operating power* has thus increased from 6.8/3.5 = 1.94 kW to 8/3.2 = 2.5 kW. We have gained $\Delta Q_1 = 8-6.8 = 1.2$ kW extra heating capacity by adding $\Delta E = 2.5-1.94 = 0.56$ kW for the operation. The "incremental COP" for the additional heating capacity is about 1.2/0.56 = 2.14. Obviously the strategy of increasing w_2 is more favorable than adding supplemental heating (at least if based on electricity).

[3] No relation for optimum COP_{1tot} has been derived, but in Fig. 8 a curve is shown for COP_{2tot} and as can be seen the optimum of COP_{1tot} calls for only marginally larger velocity, w_2, than the COP_{2tot} −curve. The difference between the optima can in all practical cases be ignored.

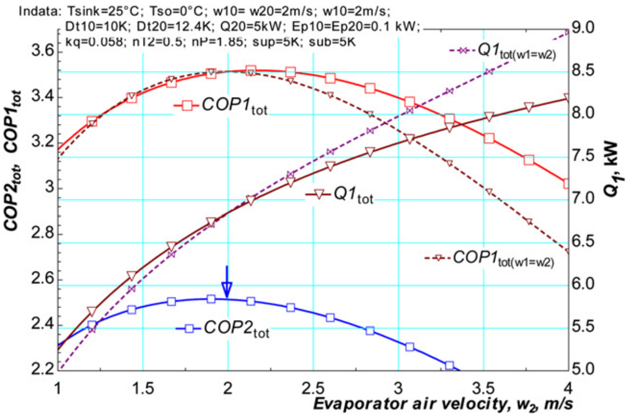

Fig. 9 – Heat pump system performance.

Variations on the condenser side (so that $w_1 = w_2$) are illustrated by the dotted curves in Fig. 9. A similar reasoning as above shows that it is unfavorable to increase the condenser fan speed.

8. General design considerations for low pump power ratio

The previous treatment has been focused on a system with given design of heat exchanger, for which the requirements of fans (or pumps) are known. But: *How to design heat exchangers for low pump power ratio?* To finalize the treatment it might be interesting to discuss this topic briefly.

The fan (or pump) power E_p and heat load Q in heat exchangers can be expressed in a general way by well known relations for fluid friction and heat transfer found in textbooks (such as Baehr and Stephan, 1998) as follows:

The pressure drop is usually expressed by means of a friction factor, *f*:

$$\Delta p = f\cdot\frac{\rho\cdot w^2}{2}\cdot\frac{L}{d} \tag{25}$$

and the associated ideal (or isentropic) pumping power is:

$$E_{pIs} = \Delta p\cdot A_w\cdot w \tag{26}$$

where A_w is the total cross sectional flow area.

The heat load transferred in the heat exchanger can be written:

$$Q = A_Q\cdot h\cdot\theta_M \tag{27}$$

where A_Q is the heat transfer area. The heat transfer coefficient, h, can be expressed by means of a Stanton number:

$$St = \frac{h}{w\cdot\rho\cdot c_p} \tag{28}$$

Combining the Eqs. (25)–(28) gives the *"pump power ratio"*:

$$\frac{E_{pIs}}{Q} = f\cdot\frac{\rho\cdot w^2}{2}\cdot\frac{L}{d}\cdot\frac{A_w}{A_Q\cdot St\cdot\rho\cdot c_p}\cdot\frac{1}{\theta_M} \tag{29}$$

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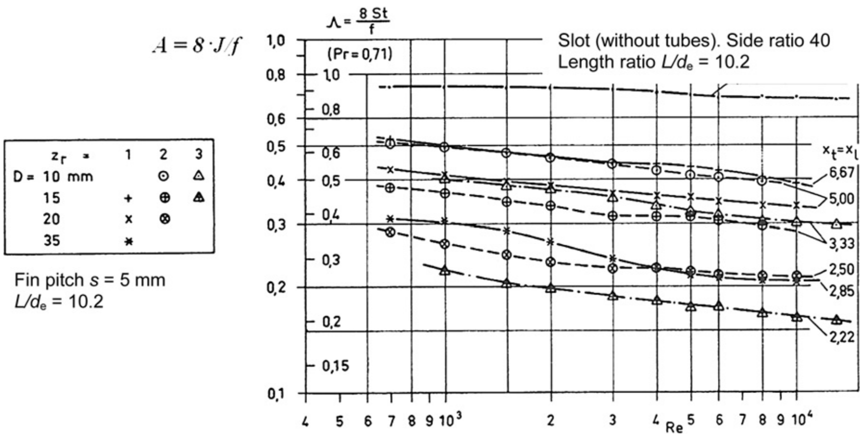

Fig. 10 – Test results of the analogy number A for fin coil geometries at Re = 5000. (x_t and x_l denotes tube transverse and longitudinal pitch/tube diameter, s = fin spacing, l = fin length in flow direction, d_e = equivalent diameter = 2 s). Source: Granryd, 1964.

Introducing a definition (as e.g. Kays and London, 1964) of hydraulic diameter for compact heat exchangers: $d = 4 \cdot A_w/(A_Q \cdot L)$ into Eq. (29) it simplifies to

$$\frac{E_{pls}}{Q} = \frac{w^2}{cp} \cdot \frac{f}{8 \cdot \mathrm{St}} \cdot \frac{1}{\theta_M} \tag{30}$$

The result of the Reynolds analogy between heat transfer and fluid friction in turbulent flow can be expressed as St = f/8. To correct for fluids with Pr ≠ 1 we can introduce a Colburn *J*-factor $J = \mathrm{St} \cdot Pr^{2/3} = \mathrm{Nu}/(\mathrm{Re} \cdot \mathrm{Pr}^{1/3})$. For flow along a surface or in a tube without additional pressure losses than pure friction we have thus $J \cong f/8$. Introducing an analogy parameter "A" defined as

$$A = \frac{8 \cdot J}{f} = \frac{8 \cdot \mathrm{St} \cdot \mathrm{Pr}^{2/3}}{f} \tag{31}$$

the Eq. 30 becomes the following *simple relation*:

$$\frac{E_{pls}}{Q} = \frac{w^2}{\theta_M \cdot c_p \cdot A} \cdot \mathrm{Pr}^{2/3} \tag{32}$$

The equation indicates that the *"pump power ratio"* (E_{pls}/Q) ideally is only influenced by w^2/θ_M, the analogy number *A* and the fluid properties Pr and c_p.

The analogy number *A* is seldom = 1 for compact heat exchangers in practice. For most heat exchanger designs (like finned coils) the overall pressure drop is higher than that caused by pure friction. The additional pressure drop caused by restrictions, tubes etc appears not to be fully effective from a heat transfer point of view and the effect of this is that the analogy number *A* is <1. Examples of experimental results for finned coils are shown in Fig. 10 (from Granryd, 1964). As seen the analogy number decreases slightly by Reynolds number, and there is a strong dependence of the tube arrangements (especially the ratio minimum flow area/front area in the coil). Often A is in the order 0.4 to 0.15 or even lower. It appears that *"streamlined" designs have higher A-values*. However streamlined designs usually give lower heat transfer coefficients and can be less cost effective. The most economic design might not be the one with a high analogy number even if it results in lower pump lower ratio.

In practice we must also take into account the influence of:

- the fan (or pump) and the motor efficiency, η_{pm}
- the total pressure drop is greater than the pressure loss in the heat exchange surface itself, $\Delta p_{tot}/\Delta p_f$.
- the ratio of overall heat transfer coefficient and the surface coefficient, *U/h*

Introducing these parameters into Eq. (32) the result is:

$$\frac{E_p}{Q} = \frac{w^2}{c_p \cdot A \cdot \theta_M} \cdot \mathrm{Pr}^{2/3} \cdot \frac{\Delta p_{tot}/\Delta p_f}{U/h} \cdot \frac{1}{\eta_{pm}} \tag{33}$$

8.1. *Discussion*

It is desirable to have a design with a small value for the "pump power ratio". This ratio is strongly influenced by the operating parameters (*w* and θ_M) and of the fluid properties (c_p and Pr). The choice of velocity *w*, has been discussed in this paper. The choice of temperature difference, θ_M, is a classical *economical* optimum question, balancing first cost and cost of operating a refrigerating system.

To realize a low pump power ratio in a system one should obviously aim for:

- Heat exchanger geometries with a high analogy number *A*.
- High efficiency fan (pump) and motor.
- Heat exchanger designs with small additional flow restrictions in fan ducts (or pump piping).
- A heat exchanger with a high overall heat transfer coefficient, *U*.

9. Summary

Considering a base case of a refrigerating system where the design data for the evaporator and condenser are known, simple analytical relations are deduced for optimum flow rates that will result in highest overall COP of the system when energy demand

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for the compressor as well as fans (or pumps) are included. This solution is equivalent to the solution for minimum total energy demand for the operation of the system to achieve a given cooling load. It is also shown that a different (and *higher*) flow rate will result in maximum net cooling capacity for a refrigerating system with fixed compressor speed.

The expressions can be used for design purposes but also for checking flow velocities used in an existing plant. The relations may also be incorporated in algorithms for optimal operation of systems with variable speed compressors.

Finally a simple general relation is given for the "pump power ratio" for forced convection heat exchangers defined as the ratio between the necessary power for pumps or fans and the rate of heat transferred.

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