4

Fundamentals of Heat Exchanger Design

4.1 Definition and Requirements

Heat exchangers are devices that facilitate energy transfer between two fluids at different temperatures while keeping them from mixing with each other. Fundamental knowledge of heat conduction and heat convection are required for heat exchanger design and/or selection. Examples of heat exchangers include (i) car radiator; (ii) hydronic baseboard heaters; (iii) condensers; (iv) superheaters; (v) boilers; and (vi) regenerators/recurperators.

4.2 Types of Heat Exchangers

4.2.1 Double-Pipe Heat Exchangers

In this type of heat exchanger, one fluid flows through a pipe and the other fluid flows through an annular space that encloses the pipe. There are two types of flows in double-pipe heat exchangers: parallel flow and counter flow. Figure 4.1 shows schematics of double-pipe heat exchangers:

Parallel flow: Both fluids enter and exit at the same point (ie. fluids flow in the same direction). For very long systems, the temperatures of the two fluids will eventually become equal at the exit.
Counter flow: The fluids enter and leave at opposite ends (ie. fluids flow in opposite directions). The flows are in opposite directions. A larger temperature difference exists over the length of this heat exchanger compared to the parallel flow heat exchanger.

Introduction to Thermo-Fluids Systems Design, First Edition. André G. McDonald and Hugh L. Magande.

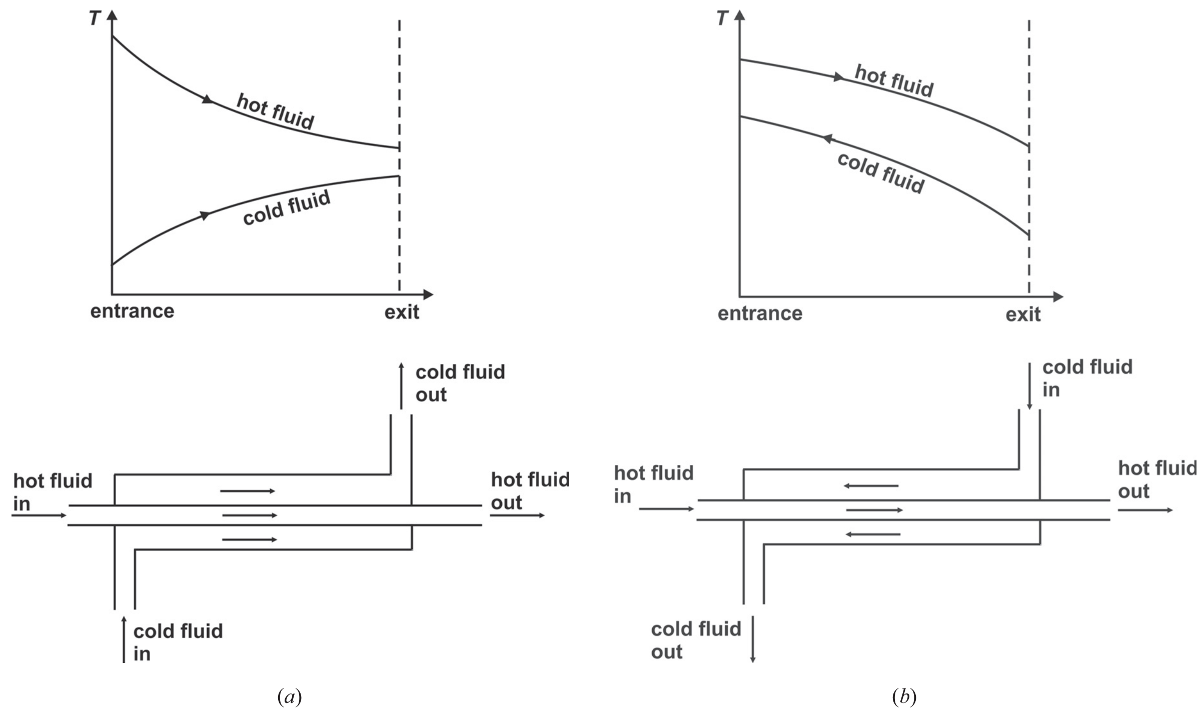


Figure 4.1 Temperature profiles and schematics of (a) parallel and (b) counter flow double-pipe heat exchangers

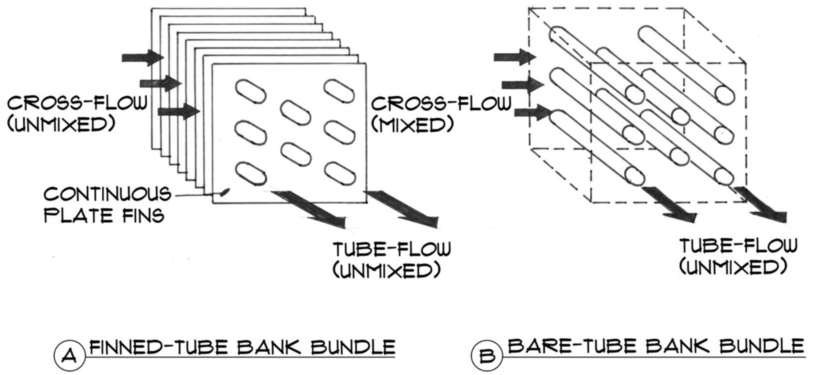


Figure 4.2 Cross-flow heat exchangers

4.2.2 Compact Heat Exchangers

Compact heat exchangers were developed to provide large surface areas for heat transfer (per unit volume). A heat exchanger is considered compact if the **area density** (**β**) is large. Area density is

$$\beta = \frac{A_s}{V}, \tag{4.1}$$

where A_s is the surface area and V is the volume.
A heat exchanger is considered compact if: $\beta > 200\ \mathrm{ft}^2/\mathrm{ft}^3$.

Cross-flow heat exchangers are excellent examples of compact heat exchangers. The addition of fins to extend the heat transfer surface area will make the heat exchanger more compact. Figure 4.2 shows some cross-flow heat exchangers with fins (finned-tube) and without fins (bare). The fins and tubes separate the working fluids into distinct sections, producing **unmixed flow**. **Mixed flow** occurs when the working fluid is not separated into smaller subsections (see Figure 4.2b). Figure 4.3 shows a picture of a continuous plate-fin-tube type cross-flow heat exchanger.

4.2.3 Shell-and-Tube Heat Exchangers

Shell-and-tube heat exchangers have a large number of tubes (**tube bank**) packed in a shell casing. Heat transfer occurs as the fluid (at temperature A) in the shell flows across the tubes. Fluid flows through the tubes (at temperature B). **Baffles** are typically used to force the fluid to flow across the tubes in the shell in a serpentine fashion, thus enhancing the heat transfer. The fluid in the tubes collects in **headers** before it enters or leaves the system. Figure 4.4 shows schematics of several shell-and-tube heat exchangers. While outside the scope of this textbook, shell-and-tube heat exchanger design has been treated extensively by Lee [1] and Kays and London [2].

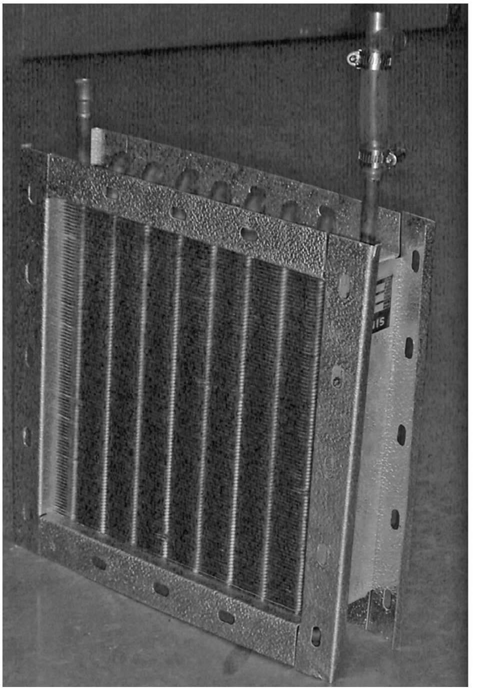
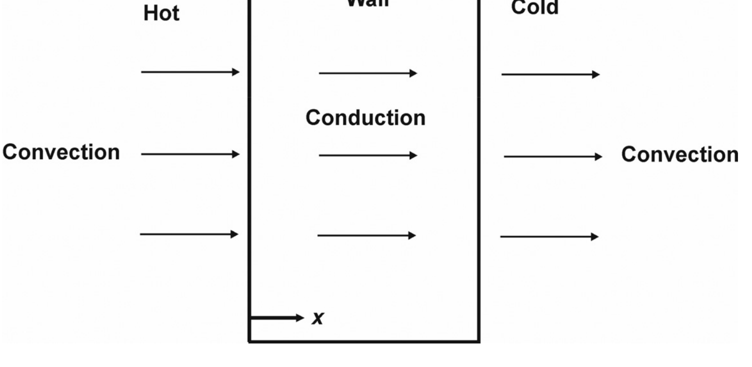


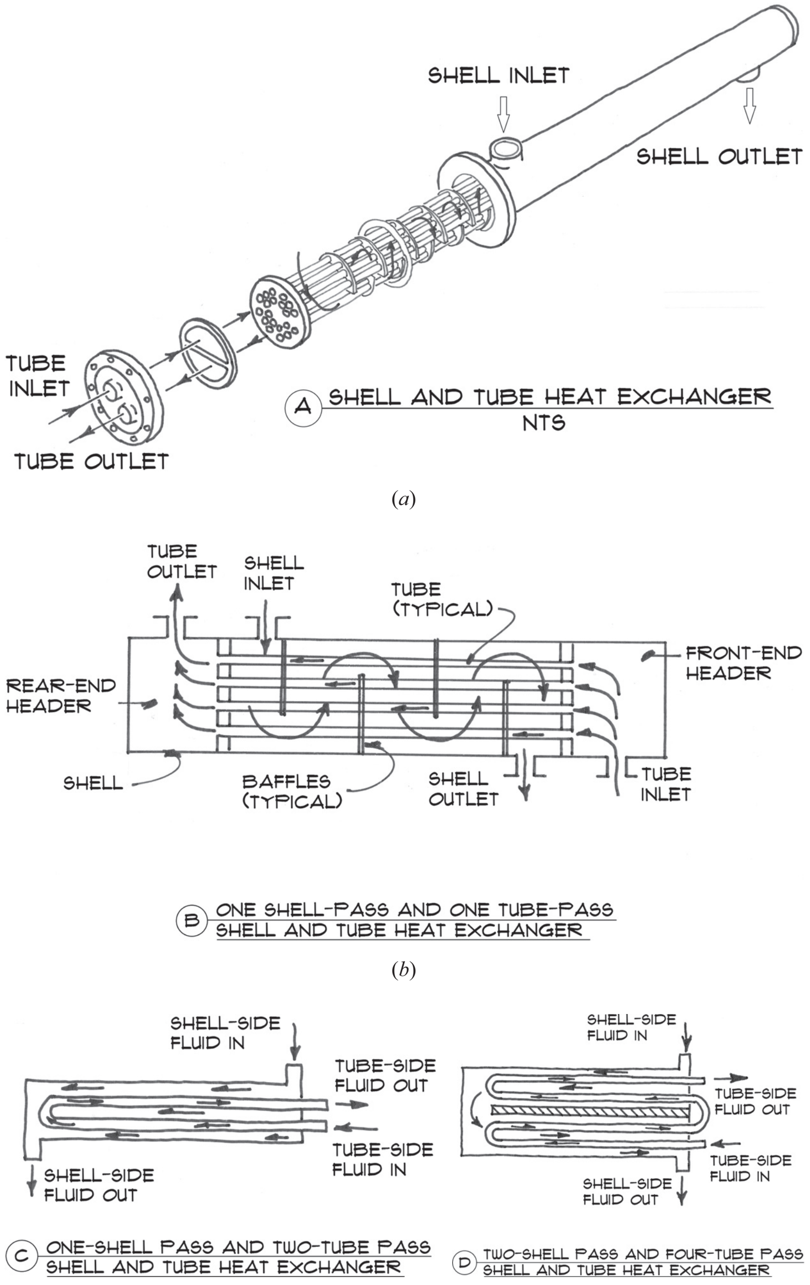
Figure 4.3 Picture of a continuous plate-fin-tube type cross-flow heat exchanger

4.3 The Overall Heat Transfer Coefficient

The rate of heat transfer across the surfaces in a heat exchanger will be governed, in part, by the **thermal resistance** to heat transfer across those surfaces. Determination of the thermal resistance will require simplified heat conduction and heat convection analyses.

In heat exchangers, heat transfer occurs primarily by conduction and convection. The schematic drawing below shows these modes of heat transfer through an ideal plane wall:





(*a*)

(*b*)

(*c*) (*d*)

Figure 4.4 Schematics of shell-and-tube heat exchangers

Note the following assumptions:

(a) Heat transfer is one-dimensional.
(b) Steady state exists.
(c) Radiation effects are negligible or are included in the convection terms through the heat transfer coefficients.
(d) Isotropic material.

4.3.1 The Thermal Resistance Network for Plane Walls—Brief Review

Figure 4.5 shows a schematic drawing that presents the temperature distribution around and through a one-dimensional plane wall that experiences convection on both sides.

The schematic diagram shows that the temperature decreases through the wall from the hotter to the colder fluid. Note that $h_{\text{fluid},1}$ and $h_{\text{fluid},2}$ are the convective heat transfer coefficients of the two fluids. Of interest will be the heat transferred across the wall between the two fluids.

The heat flux (heat transfer rate per unit area) through the wall is given by Fourier's law:

$$q''_x = -k\frac{\mathrm{d}T}{\mathrm{d}x}, \tag{4.2}$$

where q''_x is the heat flux, k is the constant thermal conductivity of the wall, and T is the temperature distribution of the wall.

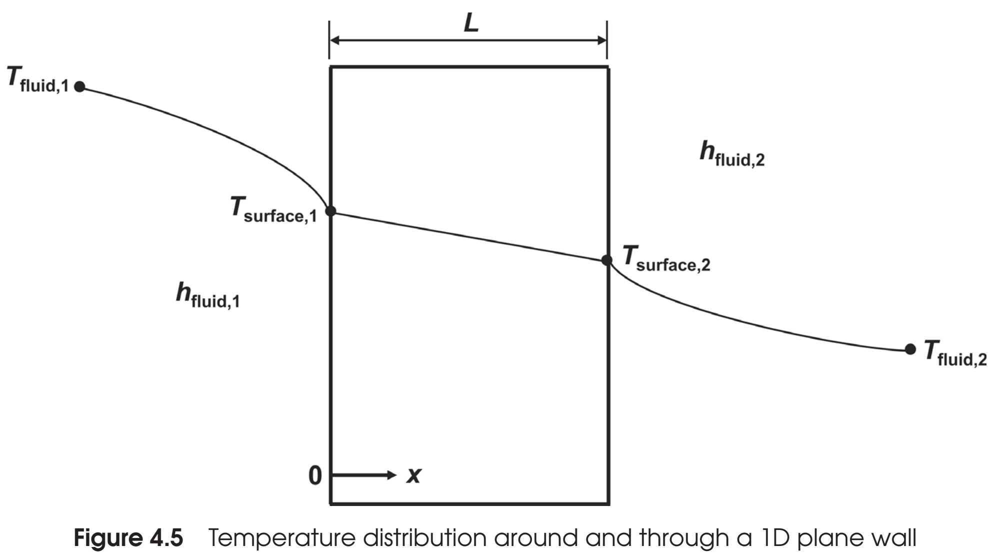


Figure 4.5 Temperature distribution around and through a 1D plane wall

The heat transfer rate is

$$q''_{\mathrm{x}} = \frac{q_{\mathrm{x}}}{A} = -k\frac{\mathrm{d}T}{\mathrm{d}x} \tag{4.3}$$

$$q_{\mathrm{x}} = -kA\frac{\mathrm{d}T}{\mathrm{d}x}, \tag{4.4}$$

where q_{x} is the heat transfer rate.

To determine the heat transfer rate through the wall, the temperature distribution in the wall (T) must be known. For one-dimensional, steady heat transfer through an isotropic wall, the governing equation for the temperature distribution is

$$\frac{\mathrm{d}^2T}{\mathrm{d}x^2} = 0. \tag{4.5}$$

Solution of this ordinary, one-dimensional differential equation gives

$$T(x) = Cx + D, \tag{4.6}$$

where C and D are constants of integration.

Two boundary conditions are needed to determine the constants of integration:

$$\text{(i)} \quad -k\frac{\mathrm{d}T\,(0)}{\mathrm{d}x} = h_{\text{fluid},1}\,[T_{\text{fluid},1} - T_{\text{surface},1}] \tag{4.7}$$

$$\text{(ii)} \quad -k\frac{\mathrm{d}T(L)}{\mathrm{d}x} = h_{\text{fluid},2}\,[T_{\text{surface},2} - T_{\text{fluid},2}]\,. \tag{4.8}$$

At this point, make a note of the following points:

$$T_{\text{surface},1} = T(0) \text{ at } x = 0 \tag{4.9}$$

$$T_{\text{surface},2} = T(L) \text{ at } x = 0. \tag{4.10}$$

Therefore, the boundary conditions become

$$\text{(i)} \quad -k\frac{\mathrm{d}T\,(0)}{\mathrm{d}x} = h_{\text{fluid},1}\,[T_{\text{fluid},1} - T\,(0)] \tag{4.11}$$

$$\text{(ii)} \quad -k\frac{\mathrm{d}T(L)}{\mathrm{d}x} = h_{\text{fluid},2}\,[T(L) - T_{\text{fluid},2}]\,. \tag{4.12}$$

Use the boundary conditions and the temperature distribution equation to find the constants of integration:

$$-kC = h_{\text{fluid},1}\,[T_{\text{fluid},1} - D] \tag{4.13}$$

$$-kC = h_{\text{fluid},2}\,[CL + D - T_{\text{fluid},2}]\,. \tag{4.14}$$

Solve the equations for the C constant of integration:

$$C = \frac{T_{\text{fluid},2} - T_{\text{fluid},1}}{k\left[\dfrac{1}{h_{\text{fluid},1}} + \dfrac{L}{k} + \dfrac{1}{h_{\text{fluid},2}}\right]}. \tag{4.15}$$

Thus,

$$T(x) = \frac{T_{\text{fluid},2} - T_{\text{fluid},1}}{k\left[\dfrac{1}{h_{\text{fluid},1}} + \dfrac{L}{k} + \dfrac{1}{h_{\text{fluid},2}}\right]} x + D, \tag{4.16}$$

and

$$\frac{\mathrm{d}T}{\mathrm{d}x} = \frac{T_{\text{fluid},2} - T_{\text{fluid},1}}{k\left[\dfrac{1}{h_{\text{fluid},1}} + \dfrac{L}{k} + \dfrac{1}{h_{\text{fluid},2}}\right]}. \tag{4.17}$$

The heat transfer rate $\left(q_{\mathrm{x}} = -kA\frac{\mathrm{d}T}{\mathrm{d}x}\right)$ becomes

$$q_{\mathrm{x}} = \frac{T_{\text{fluid},1} - T_{\text{fluid},2}}{\dfrac{1}{Ah_{\text{fluid},1}} + \dfrac{L}{Ak} + \dfrac{1}{Ah_{\text{fluid},2}}}. \tag{4.18}$$

The terms in the denominator of the expression for the heat transfer rate represent the resistances to heat transfer. Figure 4.6 shows this **resistance network** around the wall.

Therefore,

$$\text{Convective Resistance 1:} \quad R_{\text{cv},1} = \frac{1}{Ah_{\text{fluid},1}} \tag{4.19}$$

$$\text{Conductive Resistance:} \quad R_{\text{cd}} = \frac{L}{Ak} = R_{\text{wall}} \tag{4.20}$$

$$\text{Convective Resistance 2:} \quad R_{\text{cv},2} = \frac{1}{Ah_{\text{fluid},2}}. \tag{4.21}$$

Thus,

$$q_{\mathrm{x}} = \frac{T_{\text{fluid},1} - T_{\text{fluid},2}}{\frac{1}{Ah_{\text{fluid},1}} + R_{\text{wall}} + \frac{1}{Ah_{\text{fluid},2}}}. \tag{4.22}$$

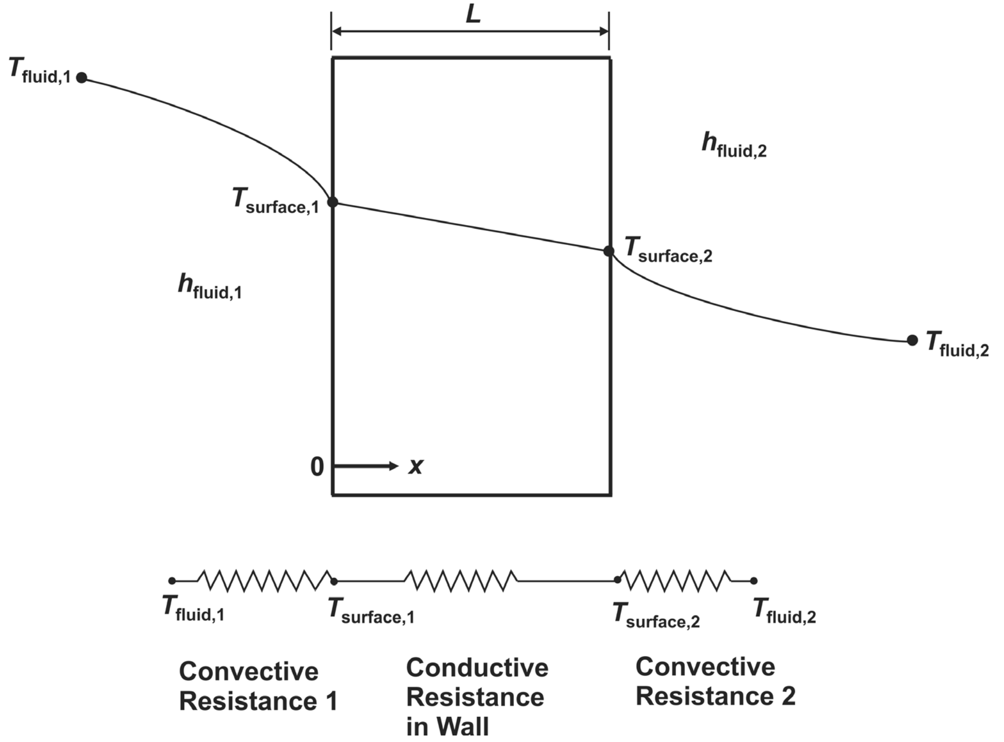


Figure 4.6 Thermal resistance network around a plane wall

The heat transfer rate may be rewritten as

$$q_x = \frac{\Delta T_{fluid}}{R_T}, \tag{4.23}$$

where $\Delta T_{fluid} = T_{fluid,1} - T_{fluid,2}$ and $R_T = \frac{1}{Ah_{fluid,1}} + R_{wall} + \frac{1}{Ah_{fluid,2}}$ = total resistance to heat transfer.

It is typical to see the heat transfer rate through the wall written in a form similar to that of Newton's law of cooling:

$$q_x = \frac{\Delta T_{fluid}}{R_T} = UA\Delta T_{fluid}, \tag{4.24}$$

where ***U* is the overall heat transfer coefficient**.

Therefore,

$$R_T = \frac{1}{UA} = \frac{1}{Ah_{fluid,1}} + R_{wall} + \frac{1}{Ah_{fluid,2}} \tag{4.25}$$

and

$$\frac{1}{U} = \frac{1}{h_{\text{fluid},1}} + R_{\text{wall}}A + \frac{1}{h_{\text{fluid},2}}. \tag{4.26}$$

It should be noted at this point that the area, A is the **surface area** of the wall. It should also be noted that the expression for the wall resistance (R_{wall}) will be different for plane walls and cylindrical walls (pipes, tubes):

$$\text{For plane walls:} \quad R_{\text{wall}} = \frac{L}{Ak}. \tag{4.27}$$

$$\text{For cylindrical walls:} \quad R_{\text{wall}} = \frac{1}{2\pi kL}\ln\left(\frac{r_o}{r_i}\right). \tag{4.28}$$

For cylindrical walls, r_o is outer wall thickness, r_i is the inner wall thickness, and $A = 2\pi rL$.

Later in this chapter, it will be shown that determination of the overall heat transfer coefficient is essential to the design of heat exchangers. Tables 4.1 and 4.2 provide some representative values for some types of heat exchangers. Note that **heat exchangers are defined based on the working fluids**.

4.3.2 Thermal Resistance from Fouling—The Fouling Factor

An additional resistance may be encountered on heat exchanger tubes or plates that have been in operation for prolonged periods of time. Materials or other contaminants may deposit on the surface, restricting heat transfer. This deposition of material

Table 4.1 Values of the overall heat transfer coefficient (US)

Type of Heat Exchanger	U, Btu/(h ft^2 °F)
Water-to-water	150–300
Water-to-oil	18–60
Water-to-gasoline or kerosene	55–180
Feedwater heaters	180–1500
Steam-to-light fuel oil	35–70
Steam-to-heavy fuel oil	10–35
Steam condenser	180–1060
Freon condenser (water cooled)	55–180
Ammonia condenser (water cooled)	140–250
Alcohol condensers (water cooled)	45–125
Gas-to-gas	2–7
Water-to-air in finned tubes (water in tubes)	5–10 (air); 70–150 (water)
Steam-to-air in finned tubes (steam in tubes)	5–50 (air); 70–705 (water)

Table 4.2 Values of the overall heat transfer coefficient (SI)

Type of Heat Exchanger	U (W/(m^2 °C))
Water-to-water	850–1700
Water-to-oil	100–350
Water-to-gasoline or kerosene	300–1000
Feedwater heaters	1000–8500
Steam-to-light fuel oil	200–400
Steam-to-heavy fuel oil	50–200
Steam condenser	1000–6000
Freon condenser (water cooled)	300–1000
Ammonia condenser (water cooled)	800–1400
Alcohol condensers (water cooled)	250–700
Gas-to-gas	10–40
Water-to-air in finned tubes (water in tubes)	30–60 (air); 400–850 (water)
Steam-to-air in finned tubes (steam in tubes)	30–300 (air); 400–4000 (water)

Source: Çengel [3].

or contaminants on the heat exchanger surfaces is known as **fouling**. Fouling will deteriorate the performance of the heat exchanger by adding an additional resistance.

Some sources of fouling include

(i) precipitation of salts from hard water;
(ii) chemical reactions which form solid deposits;
(iii) biological species growing on surfaces;
(iv) sedimentation of solid deposits;
(v) corrosion, which forms low thermal conductivity metal oxides on the heat exchanger surfaces.

The design engineer must make allowances to account for fouling, where applicable. In this case, the designer must consider the fouling factor, R_f, or the additional resistance due to fouling in the calculation of the overall heat transfer coefficient.

Therefore, with fouling, the overall heat transfer coefficient becomes

$$\frac{1}{U} = \frac{1}{h_{\text{fluid},1}} + R_{f,1} + R_{\text{wall}} A + R_{f,2} + \frac{1}{h_{\text{fluid},2}}, \tag{4.29}$$

where $R_{f,1}$ is the fouling resistance on the side of the wall in contact with fluid 1 and $R_{f,2}$ is the fouling resistance on the side of the wall in contact with fluid 2.

Table 4.3 provides some representative values of fouling factors for some working fluids.

Table 4.3 Representative fouling factors in heat exchangers

Fluid	R_f (ft^2 h °F)/Btu
Gas oil	0.00051
Transformer oil	0.00102
Lubrication oil	0.00102
Heat transfer oil	0.00102
Hydraulic oil	0.00102
Fuel oil	0.0051
Hydrogen	0.00999
Engine exhaust	0.00999
Steam (oil-free)	0.00051
Steam with oil traces	0.0010
Cooling fluid vapors with oil traces	0.00199
Organic solvent vapors	0.0010
Alcohol vapors	0.00057
Refrigerants (vapor)	0.0023
Compressed air	0.00199
Natural gas	0.0010
Distilled water, seawater, river water, boiler feedwater: below 122°F	0.00057
Distilled water, seawater, river water, boiler feedwater: above 122°F	0.0011
Refrigerants (liquid)	0.0011
Cooling fluid	0.0010
Organic heat transfer fluids	0.0010
Salts	0.00051
Liquefied petroleum gas (LPG), liquefied natural gas (LNG)	0.0010
MEA and DEA (Amines) solutions	0.00199
DEG and TEG (glycols) solutions	0.00199
Vegetable oils	0.0030

4.4 The Convection Heat Transfer Coefficients—Forced Convection

Determination of the convective heat transfer coefficients, $h_{\text{fluid},1}$ and $h_{\text{fluid},2}$, is needed to calculate values of the overall heat transfer coefficient. The heat transfer coefficient is a parameter (not a property) that depends on the surface geometry, fluid velocity, fluid properties, and temperatures of the solid surface and fluid.

Heat transfer coefficients are typically nondimensionalized as the **Nusselt number**. The Nusselt number is defined as

$$\text{Nu} = \frac{h\delta}{k_{\text{fluid}}}, \tag{4.30}$$

where δ is a characteristic length.

For heat exchangers with circular tubes, $\delta = D$. For noncircular tubes, the hydraulic diameter is used. The hydraulic diameter is $D_\text{h} = \frac{4A}{P}$.

Therefore,

$$\mathrm{Nu} = \frac{hD}{k_{\mathrm{fluid}}}. \tag{4.31}$$

Fundamental Note: The thermal conductivity (k) in the Nusselt number is that of the fluid at a **mean temperature (T_m)**. This mean temperature is typically the average of the inlet and outlet temperatures of the fluid in the tube.

4.4.1 Nusselt Number—Fully Developed Internal Laminar Flows

For fully-developed (hydrodynamic and thermal boundary layers have merged) laminar flows in channels (circular tubes, square ducts, and pipes), the Nusselt numbers will be different for constant surface temperature ($T_s =$ constant) and constant surface heat flux ($q''_s =$ constant) conditions. Table 4.4 shows values of the Nusselt number for a variety of channel geometries.

4.4.2 Nusselt Number—Developing Internal Laminar Flows—Correlation Equation

A correlation equation exists for the case of internal laminar flows with fully developed velocity profiles (hydrodynamic boundary layers have merged) and developing temperature profiles (thermal boundary layers have not merged). Note that a correlation equation is an equation that was partially developed analytically, and experimental data was used to develop the final form of the equation.

For developing temperature profiles,

$$L_{\mathrm{tube}} \le L_{\mathrm{t,laminar}},$$

where $L_{\mathrm{t,laminar}} \approx 0.05\,\mathrm{Re\,Pr}D =$ thermal entry length and $\mathrm{Pr} = \frac{c_p \mu}{k_{\mathrm{fluid}}} =$ Prandtl number.

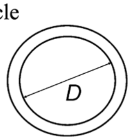
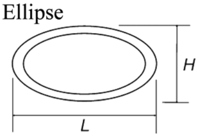
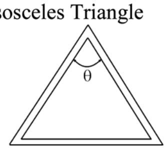
Therefore, the correlation equation is [4]

$$\mathrm{Nu} = 3.66 + \frac{0.065\left(\frac{D}{L}\right)\mathrm{Re\,Pr}}{1 + 0.04\left[\left(\frac{D}{L}\right)\mathrm{Re\,Pr}\right]^{2/3}}. \tag{4.32}$$

This correlation equation is applied to the following:

(i) Entrance region of the tube.
(ii) Uniform surface temperature.
(iii) Fully developed velocity profile in laminar flows. This correlation equation will give good approximations for hydrodynamically developing laminar flows.
(iv) Developing temperature profiles in laminar flow.
(v) Fluid properties that are at the mean temperature of the bulk fluid.

Table 4.4 Nusselt numbers and friction factors for fully developed laminar flow in tubes of various cross sections: constant surface temperature and surface heat flux [3]

Pipe Geometry	L/H ratio or θ	Nusselt Number, Nu_{D_h}: $T_s =$ constant	Nusselt Number, Nu_{D_h}: $q''_s =$ constant	Friction Factor, f
Circle 	—	3.66	4.36	64.00/Re
Square/Rectangle 	L/H ratio			
	1	2.98	3.61	56.92/Re
	2	3.39	4.12	62.20/Re
	3	3.96	4.79	68.36/Re
	4	4.44	5.33	72.92/Re
	6	5.14	6.05	78.80/Re
	8	5.60	6.49	82.32/Re
	∞	7.54	8.24	96.00/Re
Ellipse 	L/H ratio			
	1	3.66	4.36	64.00/Re
	2	3.74	4.56	67.28/Re
	4	3.79	4.88	72.96/Re
	8	3.72	5.09	76.60/Re
	16	3.65	5.18	78.16/Re
Isosceles Triangle 	Θ			
	10°	1.61	2.45	50.80/Re
	30°	2.26	2.91	52.28/Re
	60°	2.47	3.11	53.32/Re
	90°	2.34	2.98	52.60/Re
	120°	2.00	2.68	50.96/Re

In some cases, the temperature of the fluid in contact with the surface of the tube may differ greatly from that of the bulk fluid in the rest of the tube. In that case, viscosity differences in the fluid will affect the Nusselt number. The appropriate correlation equation to use is [5]:

$$\mathrm{Nu} = 1.86\left(\frac{\mathrm{Re}\,\mathrm{Pr}\,D}{L}\right)^{1/3}\left(\frac{\mu_{\text{bulk fluid}}}{\mu_{\text{surface fluid}}}\right)^{0.14}, \tag{4.33}$$

where $\mu_{\text{surface fluid}}$ is the viscosity of the fluid at the surface temperature.

This correlation equation is applied to

(i) $0.48 < \mathrm{Pr} < 16700$;
(ii) $0.0044 < \frac{\mu_{\text{bulk fluid}}}{\mu_{\text{surface fluid}}} < 9.75$.

4.4.3 Nusselt Number—Turbulent Flows in Smooth Tubes: Dittus–Boelter Equation

For fully developed turbulent flows in smooth tubes, the Dittus–Boelter correlation equation [6] may be used to determine the Nusselt number.

Therefore,

$$\mathrm{Nu} = 0.023\mathrm{Re}^{0.8}\mathrm{Pr}^{n}. \tag{4.34}$$

Note: $n = 0.4$ for heating the fluid flowing in the tube.
$n = 0.3$ for cooling the fluid flowing in the tube.

This correlation equation is applied to

(i) fully developed turbulent flow;
(ii) smooth tubes;
(iii) $\mathrm{Re_D} > 10000$;
(iv) $0.7 < \mathrm{Pr} < 160$;
(v) $\frac{L}{D} > 60$.

In some cases, the temperature of the fluid in contact with the surface of the tube may differ greatly from that of the bulk fluid in the rest of the tube. In that case, viscosity differences in the fluid will affect the Nusselt number. The appropriate correlation equation to use is [5]:

$$\mathrm{Nu} = 0.027\mathrm{Re}^{0.8}\mathrm{Pr}^{1/3}\left(\frac{\mu_{\text{bulk fluid}}}{\mu_{\text{surface fluid}}}\right)^{0.14}, \tag{4.35}$$

where $\mu_{\text{surface fluid}}$ is the viscosity of the fluid at the surface temperature.

This correlation equation is applied to

(i) $0.7 < \mathrm{Pr} < 17600$;
(ii) $\mathrm{Re_D} > 10000$.

4.4.4 Nusselt Number—Turbulent Flows in Smooth Tubes: Gnielinski's Equation

For fully developed turbulent flows in smooth tubes, Gnielinski's correlation equation [7] may be used as an alternative to the Dittus–Boelter correlation equation to determine the Nusselt number. Though more complex, Gnielinski's correlation equation will give more accurate values of the Nusselt number, especially at lower Reynolds number flows ($\mathrm{Re}_D > 2300$).

Therefore,

$$\mathrm{Nu} = \frac{\left(\frac{f}{8}\right)(\mathrm{Re}_D - 1000)\,\mathrm{Pr}}{1.0 + 12.7\left(\frac{f}{8}\right)^{0.5}\left(\mathrm{Pr}^{2/3} - 1\right)}\left[1 + \left(\frac{D}{L}\right)^{2/3}\right], \tag{4.36}$$

where f is the Darcy friction factor. Consult Chapter 2 for correlation equations for the friction factor.

Note: For fully developed flow, $\frac{D}{L} \approx 0$.

This correlation equation is applied to

(i) developing or fully developed turbulent flow;
(ii) smooth tubes;
(iii) $2300 < \mathrm{Re}_D < 5 \times 10^6$;
(iv) $0.5 < \mathrm{Pr} < 2000$;
(v) $0 < \frac{D}{L} < 1$.

Practical Note 4.1 Industrial Flows

For most industrial applications involving flow through tubes in heat exchangers, the flow is turbulent.

Practical Note 4.2 Flow in Rough Pipes

In practice, the Dittus–Boelter and Gnielinski's correlation equations are also used with rough surfaces. This is due to a lack of appropriate, simple equations for rough tubes.

4.5 Heat Exchanger Analysis

4.5.1 Preliminary Considerations

Heat exchanger analysis involves the determination of the size (area, volume, length of tubes, where applicable) of the heat exchanger required to transfer a specified amount of heat and the quantification of the performance of the heat exchanger.

Some fundamental assumptions are as follows:

(i) Heat exchangers are steady-flow devices. That is, the system is in steady state.
(ii) All fluid and thermal properties are constant.

(iii) The overall heat transfer coefficient (U) is constant.
(iv) There is no heat exchange between the heat exchanger and the surroundings.
(v) All heat exchange occurs between the fluids through the solid surfaces.

On the basis of these assumptions, the first law of thermodynamics may be written as

$$\dot{Q}_{\text{hot}} = \dot{Q}_{\text{cold}}, \tag{4.37}$$

where $\dot{Q}_{\text{hot}}$ is the heat transfer rate from the hot fluid and $\dot{Q}_{\text{cold}}$ is the heat transfer rate to the cold fluid.

For flowing fluids,

$$\dot{Q}_{\text{hot}} = \dot{m}_{\text{h}} c_{\text{ph}} (T_{\text{h,in}} - T_{\text{h,out}}) \tag{4.38}$$

$$\dot{Q}_{\text{cold}} = \dot{m}_{\text{c}} c_{\text{pc}} (T_{\text{c,out}} - T_{\text{c,in}}), \tag{4.39}$$

where the subscripts "in" and "out" represent the fluid entering and leaving the heat exchanger system, respectively.

Therefore,

$$\dot{Q}_{\text{hot}} = \dot{Q}_{\text{cold}} = \dot{m}_{\text{h}} c_{\text{ph}} (T_{\text{h,in}} - T_{\text{h,out}}) = \dot{m}_{\text{c}} c_{\text{pc}} (T_{\text{c,out}} - T_{\text{c,in}}). \tag{4.40}$$

In heat exchanger analysis, the heat capacity rates are used.

$$C_{\text{h}} = \dot{m}_{\text{h}} c_{\text{ph}} \text{ and } C_{\text{c}} = \dot{m}_{\text{c}} c_{\text{pc}}. \tag{4.41}$$

Thus,

$$C_{\text{h}} (T_{\text{h,in}} - T_{\text{h,out}}) = C_{\text{c}} (T_{\text{c,out}} - T_{\text{c,in}}). \tag{4.42}$$

4.5.2 Axial Temperature Variation in the Working Fluids—Single Phase Flow

As heat is transferred along the length of the heat exchanger, the temperature of the working fluids will change along the length of heat exchanger (axial direction). So, the temperature of the hot fluid may decrease from the inlet to the outlet of the heat exchanger while the temperature of the cold fluid may increase from the inlet to the outlet as heat is transferred.

Single-pass parallel flow heat exchangers

In this case, the working fluids enter and exit at the same end. At the exit point, the temperatures of the hot and cold fluids approach equality, since the temperature difference (ΔT) becomes smaller (see Figure 4.7).

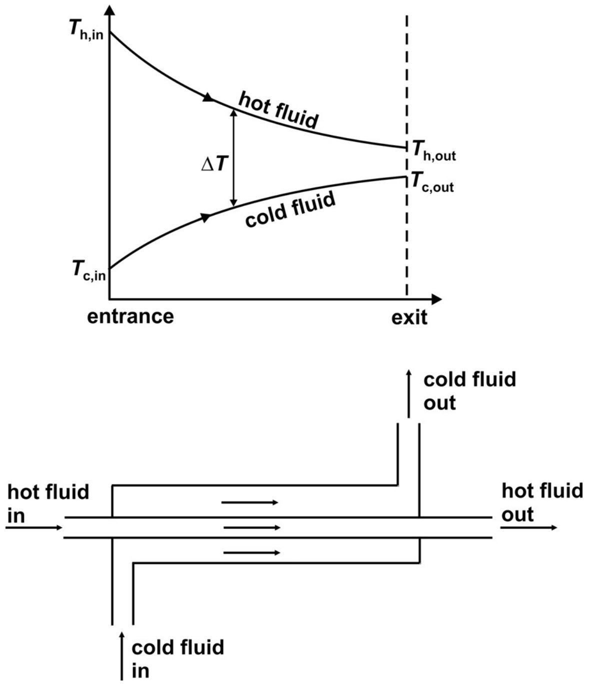


Figure 4.7 Axial temperature variation in parallel flow heat exchanger

Single-pass counter flow heat exchanger

In this case, the working fluids enter and leave at opposite ends. It requires longer heat exchangers to achieve equality of the temperatures at the exit points of any of the fluids. This improves the ability of counter flow heat exchangers to transfer more heat compared to parallel flow heat exchangers since ΔT remains larger (see Figure 4.8).

Single-pass counter flow heat exchanger with ΔT = constant

In this single-pass counter flow heat exchanger case, the temperature difference between the hot and cold fluids are maintained constant along the length of the heat exchanger (see Figure 4.9). For this to occur, the specific heat rates must be equal.

Thus,

$$C_{\mathrm{h}} = C_{\mathrm{c}} = \dot{m}_{\mathrm{h}} c_{\mathrm{ph}} = \dot{m}_{\mathrm{c}} c_{\mathrm{pc}} \text{ (``balanced'' heat exchanger).} \tag{4.43}$$

Then,

$$(T_{\mathrm{h,in}} - T_{\mathrm{h,out}}) = (T_{\mathrm{c,out}} - T_{\mathrm{c,in}}) \tag{4.44}$$

$$(T_{\mathrm{h,in}} - T_{\mathrm{c,out}}) = (T_{\mathrm{h,out}} - T_{\mathrm{c,in}}) = \Delta T = \text{constant.} \tag{4.45}$$

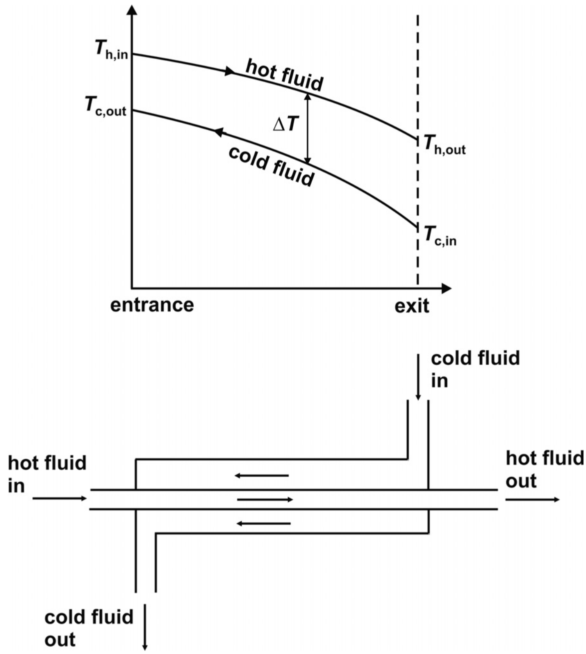


Figure 4.8 Axial temperature variation in counter flow heat exchanger

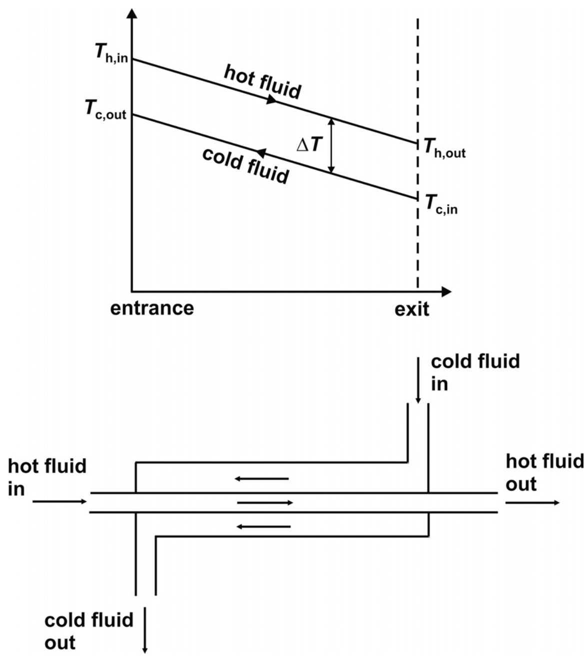


Figure 4.9 Axial temperature variation in a balanced heat exchanger

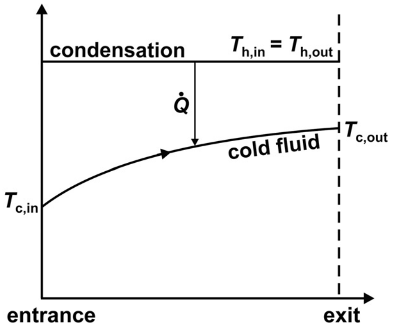


Figure 4.10 Axial temperature variation in a heat exchanger with condensation

Fundamental Note: The temperatures of the hot fluid (including $T_{h,in}$ and $T_{h,out}$) must be greater than the temperatures of the cold fluid (including $T_{c,in}$ and $T_{c,out}$) for heat transfer in accordance with the second law of thermodynamics.

Condensation in counter flow or parallel heat exchangers

In this case, a **phase change** occurs in one of the working fluids. The hot fluid will condense by transferring heat to the cold fluid. Since this is a phase change situation, the temperature of the hot fluid will remain constant. As the cold fluid becomes hotter, its temperature will approach that of the hot fluid (see Figure 4.10).

Boiling in counter flow or parallel heat exchangers

In this case, a phase change occurs in one of the working fluids. The cold fluid will boil due to the transfer of heat from the hot fluid. Since this is a phase change situation, the temperature of the cold fluid will remain constant. As the hot fluid becomes colder, its temperature will approach that of the cold fluid (see Figure 4.11).

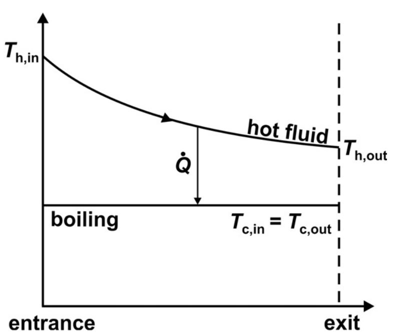


Figure 4.11 Axial temperature variation in a heat exchanger with boiling

Practical Note 4.3 Condensers and Boilers

Condensers and **boilers** are heat exchangers in which condensation and boiling phase changes occur, respectively. **Pinch point analysis** and other Nusselt number correlation equations for phase changes should be used for heat exchangers with phase changes.

4.6 Heat Exchanger Design and Performance Analysis: Part 1

Heat exchanger **design or sizing** refers to the specification of the construction and flow arrangement of the heat exchanger.

This may involve determination of the following:

(i) Heat transfer surface area (A)
(ii) Tube diameters (D)
(iii) Tube lengths (L)
(iv) Number of tubes (n)

The objective of the design will be to provide a heat exchanger that will provide a specified outlet temperature of the working fluids or transfer a specified amount of heat.

Heat exchanger **performance** analysis serves to determine the outlet temperatures (T_{out}) of the working fluids and/or the amount of heat ($\dot{Q}$) exchanged by the heat exchanger. As a design engineer, it may be necessary to analyze the performance of a newly designed or existing heat exchanger within a system.

4.6.1 The Log-Mean Temperature Difference Method

This method may be used to determine the size (heat transfer surface area), the outlet temperatures of the working fluid, or the heat transfer rate of/within the heat exchanger. This method can be used to find the amount of heat exchanged in the heat exchanger by using Newton's law of cooling:

$$\dot{Q} = UA\Delta T_{lm}, \tag{4.46}$$

where U is the overall heat transfer coefficient, A is the heat exchanger surface area, and ΔT_{lm} is the log-mean temperature difference (LMTD).

This method works best in heat exchanger design and sizing problems when **all** the inlet and outlet temperatures of the working fluids are known.

In heat exchanger performance analysis where all the temperatures of the working fluids are not known and the outlet temperatures must be determined, the LMTD method becomes iterative, tedious, and long. In this course, emphasis will be placed on the more powerful **effectiveness-number of transfer units (ε-NTU) method** for both heat exchanger design and performance analysis.

4.6.2 The Effectiveness-Number of Transfer Units Method: Introduction

This method was developed by Kays and London [2] to allow for easier design and performance analysis of heat exchangers without exhaustive iterations. The method is more straightforward than the LMTD method.

The dimensionless heat transfer effectiveness (ε) is defined as

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\text{max}}} = \frac{\text{actual heat transfer rate}}{\text{maximum possible heat transfer rate}}. \tag{4.47}$$

This expression may be considered as the efficiency of the heat exchanger to exchange energy between the fluid streams.

The maximum heat transfer rate ($\dot{Q}_{\text{max}}$) will occur over the maximum possible temperature difference. The maximum possible temperature difference is

$$\Delta T_{\text{max}} = T_{\text{h,in}} - T_{\text{c,in}}. \tag{4.48}$$

In real heat exchangers, the maximum heat transfer and maximum temperature difference will occur in the working fluid with smaller heat capacity. The fluid with the smaller heat capacity will experience faster temperature changes, will store less energy, and will transfer heat faster.

Therefore,

$$\dot{Q}_{\text{max}} = \dot{m}c_{\text{p,min}}\left(T_{\text{h,in}} - T_{\text{c,in}}\right) = C_{\text{min}}\left(T_{\text{h,in}} - T_{\text{c,in}}\right). \tag{4.49}$$

The working fluid with the lower heat capacity ($c_{\text{p,min}}$) or lower heat capacity rate (C_{min}) could be either hot or cold.

Hence,

$$C_{\text{min}} = C_{\text{c}} \text{ or } C_{\text{min}} = C_{\text{h}}. \tag{4.50}$$

The effectiveness becomes

$$\varepsilon = \frac{C_{\text{c}}}{C_{\text{min}}} \frac{\left(T_{\text{c,out}} - T_{\text{c,in}}\right)}{\left(T_{\text{h,in}} - T_{\text{c,in}}\right)}$$

or

$$\varepsilon = \frac{C_{\text{h}}}{C_{\text{min}}} \frac{\left(T_{\text{h,in}} - T_{\text{h,out}}\right)}{\left(T_{\text{h,in}} - T_{\text{c,in}}\right)}. \tag{4.51}$$

The actual heat transfer rate is

$$\dot{Q} = \varepsilon \dot{Q}_{\text{max}} = \varepsilon C_{\text{min}}\left(T_{\text{h,in}} - T_{\text{c,in}}\right). \tag{4.52}$$

Practical Note 4.4 Real Heat Exchangers

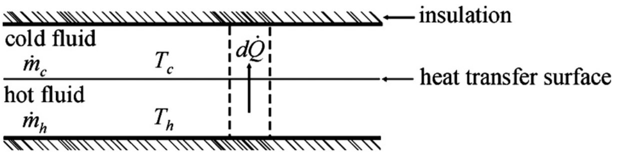
Typically, the inlet temperatures of the working fluids, the mass or volume flow rates, and the heat capacities are known in real heat exchanger applications and design. The heat transfer rate and the outlet temperatures of the working fluids are usually unknown.

4.6.3 The Effectiveness-Number of Transfer Units Method: ε-NTU Relations

A variety of ε-NTU relations have been developed to facilitate heat exchanger design and performance analysis. The derivation of the ε-NTU relation for a simple double-pipe parallel flow heat exchanger is shown below. For other types of heat exchangers, the derivations become complicated. As a result, designers rely on referenced relation equations or charts.

Derivation of the ε-NTU relation for a simple double-pipe parallel flow heat exchanger

Consider a single-pass parallel-flow heat exchanger with a schematic drawing shown below:



Take an infinitesimal section of the system for analysis

$$\mathrm{d}\dot{Q} = -\dot{m}_\mathrm{h} c_\mathrm{ph} \mathrm{d}T_\mathrm{h} \tag{4.53}$$

$$\mathrm{d}\dot{Q} = +\dot{m}_\mathrm{c} c_\mathrm{pc} \mathrm{d}T_\mathrm{c} \tag{4.54}$$

Rearrangement gives

$$\mathrm{d}T_\mathrm{h} = -\frac{\mathrm{d}\dot{Q}}{\dot{m}_\mathrm{h} c_\mathrm{ph}}; \quad \mathrm{d}T_\mathrm{c} = +\frac{\mathrm{d}\dot{Q}}{\dot{m}_\mathrm{c} c_\mathrm{pc}} \tag{4.55}$$

Substract the differential temperatures:

$$\mathrm{d}T_\mathrm{h} - \mathrm{d}T_\mathrm{c} = -\mathrm{d}\dot{Q}\left[\frac{1}{\dot{m}_\mathrm{h} c_\mathrm{ph}} + \frac{1}{\dot{m}_\mathrm{c} c_\mathrm{pc}}\right]. \tag{4.56}$$

The differential heat transfer rate is governed by Newton's Law of Cooling:

$$\mathrm{d}\dot{Q} = U(T_\mathrm{h} - T_\mathrm{c})\mathrm{d}A \tag{4.57}$$

Substitution:

$$\mathrm{d}T_{\mathrm{h}} - \mathrm{d}T_{\mathrm{c}} = -U\left(T_{\mathrm{h}} - T_{\mathrm{c}}\right)\mathrm{d}A\left[\frac{1}{\dot{m}_{\mathrm{h}}c_{\mathrm{ph}}} + \frac{1}{\dot{m}_{\mathrm{c}}c_{\mathrm{pc}}}\right] \tag{4.58}$$

Or

$$\frac{\mathrm{d}\left(T_{\mathrm{h}} - T_{\mathrm{c}}\right)}{T_{\mathrm{h}} - T_{\mathrm{c}}} = -U\mathrm{d}A\left[\frac{1}{\dot{m}_{\mathrm{h}}c_{\mathrm{ph}}} + \frac{1}{\dot{m}_{\mathrm{c}}c_{\mathrm{pc}}}\right] \tag{4.59}$$

Integrate between the inlet and outlet of the heat exchanger:

$$\ln\left(T_{\mathrm{h,out}} - T_{\mathrm{c,out}}\right) - \ln\left(T_{\mathrm{h,in}} - T_{\mathrm{c,in}}\right) = -UA\left[\frac{1}{\dot{m}_{\mathrm{h}}c_{\mathrm{ph}}} + \frac{1}{\dot{m}_{\mathrm{c}}c_{\mathrm{pc}}}\right] \tag{4.60}$$

$$\ln\left[\frac{T_{\mathrm{h,out}} - T_{\mathrm{c,out}}}{T_{\mathrm{h,in}} - T_{\mathrm{c,in}}}\right] = -UA\left[\frac{1}{\dot{m}_{\mathrm{h}}c_{\mathrm{ph}}} + \frac{1}{\dot{m}_{\mathrm{c}}c_{\mathrm{pc}}}\right]. \tag{4.61}$$

Remember: $C_{\mathrm{h}} = \dot{m}_{\mathrm{h}}c_{\mathrm{ph}}$ and $C_{\mathrm{c}} = \dot{m}_{\mathrm{c}}c_{\mathrm{pc}}$.
Therefore,

$$\ln\left[\frac{T_{\mathrm{h,out}} - T_{\mathrm{c,out}}}{T_{\mathrm{h,in}} - T_{\mathrm{c,in}}}\right] = -\frac{UA}{C_{\mathrm{c}}}\left[1 + \frac{C_{\mathrm{c}}}{C_{\mathrm{h}}}\right]. \tag{4.62}$$

From the expressions for heat transfer rate,

$$C_{\mathrm{h}}\left(T_{\mathrm{h,in}} - T_{\mathrm{h,out}}\right) = C_{\mathrm{c}}\left(T_{\mathrm{c,out}} - T_{\mathrm{c,in}}\right)$$

$$T_{\mathrm{h,out}} = T_{\mathrm{h,in}} - \frac{C_{\mathrm{c}}}{C_{\mathrm{h}}}\left(T_{\mathrm{c,out}} - T_{\mathrm{c,in}}\right). \tag{4.63}$$

Substitution:

$$\ln\left[\frac{T_{\mathrm{h,in}} - \frac{C_{\mathrm{c}}}{C_{\mathrm{h}}}\left(T_{\mathrm{c,out}} - T_{\mathrm{c,in}}\right) - T_{\mathrm{c,out}}}{T_{\mathrm{h,in}} - T_{\mathrm{c,in}}}\right] = -\frac{UA}{C_{\mathrm{c}}}\left[1 + \frac{C_{\mathrm{c}}}{C_{\mathrm{h}}}\right] \tag{4.64}$$

$$\ln\left[\frac{T_{\mathrm{h,in}} - T_{\mathrm{c,in}} - \frac{C_{\mathrm{c}}}{C_{\mathrm{h}}}\left(T_{\mathrm{c,out}} - T_{\mathrm{c,in}}\right) + T_{\mathrm{c,in}} - T_{\mathrm{c,out}}}{T_{\mathrm{h,in}} - T_{\mathrm{c,in}}}\right] = -\frac{UA}{C_{\mathrm{c}}}\left[1 + \frac{C_{\mathrm{c}}}{C_{\mathrm{h}}}\right] \tag{4.65}$$

$$\ln\left[1 - \left(1 + \frac{C_{\mathrm{c}}}{C_{\mathrm{h}}}\right)\frac{T_{\mathrm{c,out}} - T_{\mathrm{c,in}}}{T_{\mathrm{h,in}} - T_{\mathrm{c,in}}}\right] = -\frac{UA}{C_{\mathrm{c}}}\left[1 + \frac{C_{\mathrm{c}}}{C_{\mathrm{h}}}\right]. \tag{4.66}$$

Remember that the effectiveness is defined as

$$\varepsilon = \frac{C_{\mathrm{c}}}{C_{\mathrm{min}}}\frac{T_{\mathrm{c,out}} - T_{\mathrm{c,in}}}{T_{\mathrm{h,in}} - T_{\mathrm{c,in}}}.$$

Therefore,

$$\frac{T_{\mathrm{c,out}} - T_{\mathrm{c,in}}}{T_{\mathrm{h,in}} - T_{\mathrm{c,in}}} = \varepsilon \frac{C_{\min}}{C_{\mathrm{c}}}. \tag{4.67}$$

Then,

$$\ln\left[1 - \left(1 + \frac{C_{\mathrm{c}}}{C_{\mathrm{h}}}\right)\varepsilon\frac{C_{\min}}{C_{\mathrm{c}}}\right] = -\frac{UA}{C_{\mathrm{c}}}\left[1 + \frac{C_{\mathrm{c}}}{C_{\mathrm{h}}}\right]. \tag{4.68}$$

Solve for ε:

$$\varepsilon\frac{C_{\min}}{C_{\mathrm{c}}} = \frac{1 - \exp\left[-\frac{UA}{C_{\mathrm{c}}}\left(1 + \frac{C_{\mathrm{c}}}{C_{\mathrm{h}}}\right)\right]}{\left(1 + \frac{C_{\mathrm{c}}}{C_{\mathrm{h}}}\right)}. \tag{4.69}$$

$C_{\min}$ can be either C_{c} or C_{h}. Let $C_{\min}$ be C_{c} and $C_{\max}$ be C_{h}.

Thus, effectiveness is

$$\varepsilon = \frac{1 - \exp\left[-\frac{UA}{C_{\min}}\left(1 + \frac{C_{\min}}{C_{\max}}\right)\right]}{\left(1 + \frac{C_{\min}}{C_{\max}}\right)}. \tag{4.70}$$

Let the **number of transfer units** and the **capacity ratio** be

$$\mathrm{NTU} = \frac{UA}{C_{\min}} \text{ and } c = \frac{C_{\min}}{C_{\max}}. \tag{4.71}$$

Therefore,

$$\varepsilon = \frac{1 - \exp\left[-\mathrm{NTU}\,(1 + c)\right]}{(1 + c)} \text{ for double-pipe, parallel flow heat exchangers.} \tag{4.72}$$

Table 4.5 and the charts of Figure 4.12 provide additional ε-NTU relations for other types of heat exchangers.

4.6.4 Comments on the Number of Transfer Units and the Capacity Ratio (c)

(i) The following are some general points regarding the number of transfer units and the capacity ratio (c).

Table 4.5 Effectiveness relations for heat exchangers

Heat Exchanger Type	Effectiveness Relation
Double pipe: parallel flow	$\varepsilon = \dfrac{1-\exp\left[-\mathrm{NTU}\,(1+c)\right]}{1+c}$
Double pipe: counter flow	$\varepsilon = \dfrac{1-\exp\left[-\mathrm{NTU}\,(1-c)\right]}{1-c\,\exp\left[-\mathrm{NTU}\,(1-c)\right]}$ for $c<1$ $\varepsilon = \dfrac{\mathrm{NTU}}{1+\mathrm{NTU}}$ for $c=1$
Shell-and-tube: one-shell pass and an even number of tube passes	$\varepsilon_1 = 2\left[1+c+\sqrt{1+c^2}\,\dfrac{1+\exp\left[-\mathrm{NTU}_1\sqrt{1+c^2}\right]}{1-\exp\left[-\mathrm{NTU}_1\sqrt{1+c^2}\right]}\right]^{-1}$
Shell-and-tube: N shell passes and $2N, 4N\ldots$ tube passes	$\varepsilon = \dfrac{\left(\dfrac{1-\varepsilon_1 c}{1-\varepsilon_1}\right)^N - 1}{\left(\dfrac{1-\varepsilon_1 c}{1-\varepsilon_1}\right)^N - c}$ ε_1 and NTU_1 are for one-shell pass
Cross-flow (single-pass): both fluids unmixed	$\varepsilon = 1-\exp\left[\dfrac{\mathrm{NTU}^{0.22}}{c}\left[\exp\left(-c\mathrm{NTU}^{0.78}\right)-1\right]\right]$
Cross-flow (single-pass): both mixed	$\varepsilon = \left[\dfrac{1}{1-\exp\left(-\mathrm{NTU}\right)} + \dfrac{c}{1-\exp\left(-c\mathrm{NTU}\right)} - \dfrac{1}{\mathrm{NTU}}\right]^{-1}$
Cross-flow (single-pass): $C_{\max}$ mixed; $C_{\min}$ unmixed	$\varepsilon = \dfrac{1}{c}\left(1-\exp\left[1-c\left[1-\exp\left(-\mathrm{NTU}\right)\right]\right]\right)$
Cross-flow (single-pass): $C_{\min}$ mixed; $C_{\max}$ unmixed	$\varepsilon = 1-\exp\left[-\dfrac{1}{c}\left[1-\exp\left(-c\mathrm{NTU}\right)\right]\right]$
All heat exchangers with $c=0$ (boiling or condensation)	$\varepsilon = 1-\exp(-\mathrm{NTU})$

(ii) The $\mathrm{NTU} = \frac{UA}{C_{\min}}$ is a measure of the heat transfer surface area of the heat exchanger. It is directly proportional to the surface area. Larger NTU values indicate larger heat exchanger surface areas. With larger surface areas, fabrication costs will increase. For NTU values larger than 10, economic factors should drive the fabrication of such heat exchangers.

(iii) The ε-NTU relation charts shown in Section 4.6.3 show that the effectiveness increases rapidly for smaller values of NTU. For NTU greater than 1.5, increases in ε are small. The design engineer may be hard pressed to justify larger heat exchanger and larger NTU values. Therefore, **NTU values are typically lower than 3 to 5** (depending on the heat exchanger configuration).

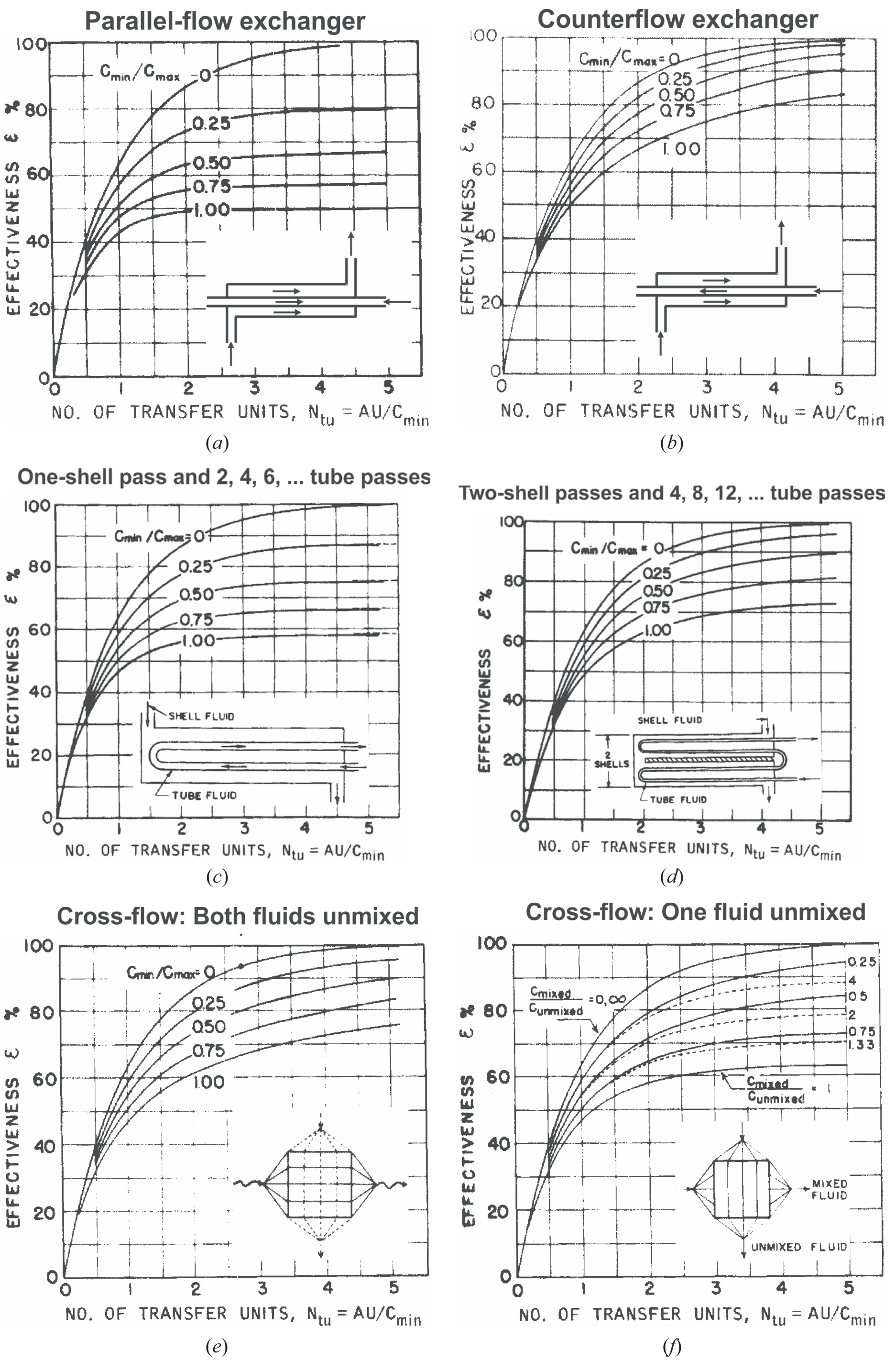


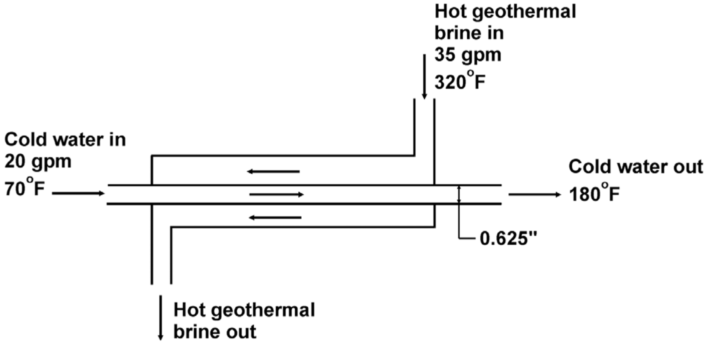
Figure 4.12 Effectiveness charts for some heat exchangers (Kays and London (2))

(iv) NTU is also a measure of the ability of the heat exchanger to transfer energy. Higher NTU values indicate higher heat transfer rates.

(v) For the special case of $c = \frac{C_{\min}}{C_{\max}} = 0, C_{\max} \to \infty$. This means that one of the working fluids absorbs or rejects heat without a temperature change. This is most typical of a phase change situation (condensation or boiling). This case produces the maximum possible ε for a given NTU value and heat exchanger configuration. Boilers, condensers, and refrigeration systems are examples of equipment/systems in which this will occur.

Example 4.1 Heating Water in a Counter Flow Heat Exchanger

A counter flow double-pipe heat exchanger is used to heat water from 70°F to 180°F for use in a specialized self-service laundromat. The water will flow at a rate of 20 gpm. The heating is to be accomplished by hot, compressed geothermal brine available at 320°F at a flow rate of 35 gpm. The inner tube is thin-walled and has a diameter of 5/8 in. If the overall heat transfer coefficient of the heat exchanger is 110 Btu/(h ft^2 °F), determine the length of the heat exchanger required to achieve the desired heating.



Solution. Several assumptions are relevant and will serve to guide the analysis:

(i) Steady state operation of the heat exchanger.
(ii) Constant fluid and thermal properties.
(iii) There is no fouling. So, the fouling resistance is zero.
(iv) The heat exchanger is well insulated. There is no heat loss from the device.
(v) The brine solution is sufficiently dilute to assume that its properties are equal to those of pure water.

The total length of the heat exchanger is found from the surface area:

$$A = npD_{\text{tube}}L.$$

For this single-pipe heat exchanger, $n = 1$.
Therefore,

$$L = \frac{A}{\pi D_{\text{tube}}}.$$

The area is needed. The ε-NTU method can be used in the analysis of the heat exchanger. The NTU is

$$\mathrm{NTU} = \frac{UA}{C_{\min}}.$$

The area is

$$A = \frac{C_{\min}\mathrm{NTU}}{U}.$$

Consider each term in the area equation. To find $C_{\min}$, the average temperatures will be used. In this case, the average cold fluid temperature is

$$T_{\mathrm{c}} = \frac{T_{\mathrm{c,in}} + T_{\mathrm{c,out}}}{2} = \frac{70^{\circ}\mathrm{F} + 180^{\circ}\mathrm{F}}{2} = 125^{\circ}\mathrm{F}.$$

Find the cold-water properties at 125°F and the hot water properties at 320°F. The heat capacity rates are:

$$\begin{aligned} C_{\mathrm{c}} &= \dot{m}_{\mathrm{c}}c_{\mathrm{pc}} = \rho_{\mathrm{c}}\dot{V}_{\mathrm{c}}c_{\mathrm{pc}} = \left(61.63\ \mathrm{lb/ft^3}\right)\left(20\ \mathrm{gpm}\right)\left(0.999\ \mathrm{Btu/(lb\ R)}\right)\left(\frac{35.315\ \mathrm{ft^3/s}}{15850\ \mathrm{gpm}}\right) \\ &= 2.74\ \mathrm{Btu/(s\,R)} = C_{\min} \\ C_{\mathrm{h}} &= \dot{m}_{\mathrm{h}}c_{\mathrm{ph}} = \rho_{\mathrm{h}}\dot{V}_{\mathrm{h}}c_{\mathrm{ph}} = \left(56.65\ \mathrm{lb/ft^3}\right)\left(35\ \mathrm{gpm}\right)\left(1.036\ \mathrm{Btu/(lb\ R)}\right)\left(\frac{35.315\ \mathrm{ft^3/s}}{15850\ \mathrm{gpm}}\right) \\ &= 4.58\ \mathrm{Btu/(s\ R)} = C_{\max} \end{aligned}$$

Find the NTU. The capacity ratio is

$$c = \frac{C_{\min}}{C_{\max}} = \frac{C_{\mathrm{c}}}{C_{\mathrm{h}}} = \frac{2.74\ \mathrm{Btu/(s\ R)}}{4.58\ \mathrm{Btu/(s\ R)}} = 0.599.$$

The effectiveness is

$$\varepsilon = \frac{C_{\mathrm{c}}}{C_{\min}}\frac{T_{\mathrm{c,out}} - T_{\mathrm{c,in}}}{T_{\mathrm{h,in}} - T_{\mathrm{c,in}}} = \frac{T_{\mathrm{c,out}} - T_{\mathrm{c,in}}}{T_{\mathrm{h,in}} - T_{\mathrm{c,in}}} = \frac{(180 - 70)\ {}^{\circ}\mathrm{F}}{(320 - 70)\ {}^{\circ}\mathrm{F}} = 0.44 = 44\%.$$

This is a counter flow heat exchanger problem. So, with c and ε known, the charts of Figure 4.12 or the equations in Table 4.5 can be used to find the NTU. From the charts,

$$\mathrm{NTU} \approx 0.70.$$

The effectiveness equation for a double-pipe counter flow heat exchanger with $c < 1$ gives

$$\varepsilon = \frac{1 - \exp\left[-\mathrm{NTU}\,(1 - c)\right]}{1 - c\,\exp\left[-\mathrm{NTU}\,(1 - c)\right]} = \frac{1 - \exp\left[-0.401\ \mathrm{NTU}\right]}{1 - 0.599\exp\left[-0.401\ \mathrm{NTU}\right]} = 0.44.$$

Therefore,

$$\mathrm{NTU} = 0.68.$$

With the NTU known, the area can be determined.

$$A = \frac{(2.74\ \mathrm{Btu/(s\ R)})\,(0.68)}{110\ \mathrm{Btu/(h\ ft^2\ R)}} \times \frac{3600\ \mathrm{s}}{1\ \mathrm{h}} = 60.98\ \mathrm{ft}^2$$

Thus,

$$L = \frac{60.98\ \mathrm{ft}^2}{\pi\ (0.625\ \mathrm{in.})} \times \frac{12\ \mathrm{in.}}{1\ \mathrm{ft}}$$

$$\boldsymbol{L = 373\ \mathrm{ft}}.$$

4.6.5 Procedures for the ε-NTU Method

The ε-NTU method can be used in both heat exchanger design and performance analysis problems in a direct, straightforward manner, without the need for iterations/trial-and-error.

(A) Consider the design of a heat exchanger to find the heat transfer surface area, A. In a design problem, the following may/will be known:

$$T_{\mathrm{h,in}}; T_{\mathrm{h,out}}; T_{\mathrm{c,in}}; \dot{m}_{\mathrm{c}}; \dot{m}_{\mathrm{h}}.$$

The ε-NTU solution method approach is as follows:
(i) Compute ε and c from available data.
(ii) Determine the NTU from an appropriate relation or chart.
(iii) Compute A from $\mathrm{NTU} = \frac{UA}{C_{\mathrm{min}}}$.
(iv) For circular tubes, the total tube length is found from $A = n\pi D_{\mathrm{tube}} L$.

(B) Consider a performance analysis of a heat exchanger to find $T_{\mathrm{c,out}}; T_{\mathrm{h,out}}; \dot{Q}$. In a performance analysis, the following may/will be known:

$$A; U; \dot{m}_{\mathrm{c}}; \dot{m}_h; T_{\mathrm{h,in}}; T_{\mathrm{c,in}}.$$

The ε-NTU solution method approach is as follows:
(i) Calculate the NTU from given data.
(ii) Use relations or charts to find ε using the NTU and c.
(iii) Calculate one of the outlet temperatures with $\varepsilon = \frac{C_{\mathrm{c}}}{C_{\mathrm{min}}} \frac{(T_{\mathrm{c,out}} - T_{\mathrm{c,in}})}{(T_{\mathrm{h,in}} - T_{\mathrm{c,in}})}$ or $\varepsilon = \frac{C_{\mathrm{h}}}{C_{\mathrm{min}}} \frac{(T_{\mathrm{h,in}} - T_{\mathrm{h,out}})}{(T_{\mathrm{h,in}} - T_{\mathrm{c,in}})}$.
(iv) Calculate $\dot{Q}$ and the other outlet temperature.

4.6.6 Heat Exchanger Design Considerations

During the design of heat exchangers, consider the following:

(i) Properties of the working fluids must be determined at their mean temperatures. That is,

$$\overline{T}_{\mathrm{c}} = \frac{T_{\mathrm{c,in}} + T_{\mathrm{c,out}}}{2} \tag{4.73}$$

$$\overline{T}_{\mathrm{h}} = \frac{T_{\mathrm{h,in}} + T_{\mathrm{h,out}}}{2}. \tag{4.74}$$

In some cases (performance analysis), it may be necessary to assume some temperature values.

(ii) Correlation equations may be needed for the heat transfer coefficients to find the overall heat transfer coefficient. Be prepared to use correlation equations for the Nusselt numbers for internal and external flows.

(iii) The design engineer is responsible for the selection of an appropriate heat exchanger construction, tube material, and flow arrangement. That is, one-shell-pass, two-tube-passes-copper tube, etc.

(iv) It may be necessary to use concepts from fundamental fluid mechanics or charts to size the tubes, if the diameter is not given.

(v) Sizing and selection of an appropriate pump may be necessary. Calculation of the pressure drops in the system would be needed.

4.7 Heat Exchanger Design and Performance Analysis: Part 2

4.7.1 External Flow over Bare Tubes in Cross Flow—Equations and Charts

External flow over tubes is of great importance in heat exchanger design and performance analysis. For example, in shell-and-tube heat exchangers, fluid in the shell flows over the tubes in a similar fashion to **external flow over a tube bank**. In some cases, the tubes in the tube bank will be devoid of **fins—extended heat transfer surfaces—**and they will be **bare**. Figure 4.13 shows finned-tube and bare tube bank bundles.

Heat transfer coefficients (h_{o}) for external flow over bare tube banks are found through the ***j*-factor** developed by Chilton and Colburn:

$$j = \frac{\overline{h}_{\mathrm{o}}}{G_{\mathrm{m}} c_{\mathrm{p}}} \mathrm{Pr}^{2/3}, \tag{4.75}$$

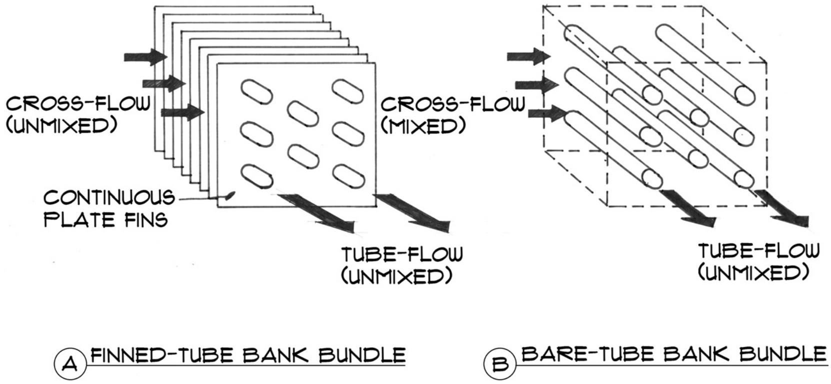


Figure 4.13 (a) Finned tube and (b) bare tube bank bundles

where

$\frac{\bar{h}_o}{G_m c_p}$ = St = Stanton number, which is a dimension-less heat transfer coefficient;
$\bar{h}_o$ = average heat transfer coefficient for external flow over the tube bank;
G_m = mass flow rate per unit minimum flow area between the tubes in the bank;
j = j-factor. This is found from charts showing j versus Re_{G_m}.

The mass flow rate per unit minimum flow area through the tube bank passages (G_m) is

$$G_m = \frac{\rho_f V_f A_T}{A_m} = \frac{\dot{m}}{A_m}, \tag{4.76}$$

where "f" refers to the free-stream working fluid flowing towards the front face of the tube bank, A_T is the total front face area of the coil normal to the direction of flow, and A_m is the minimum flow area between the tubes.

Typically, G_m is written as

$$G_m = \frac{\rho_f V_f}{\frac{A_m}{A_T}} = \frac{\rho_f V_f}{\sigma}, \tag{4.77}$$

where σ is the ratio of the minimum flow area between the tubes to the total area of the heat exchanger. σ will be presented with the j versus Re_{G_m} charts for different arrangements of tubes in the bank.

The Reynolds number (Re_{G_m}) is based on G_m, and is

$$\mathrm{Re}_{G_m} = \frac{G_m D_h}{\mu}, \tag{4.78}$$

Figure 4.14 Flow pattern for an in-line tube bank (Çengel (3), reprinted with permission)

where D_h is the hydraulic diameter of the flow passage space between the tubes in the bank, and is found from j versus $\mathbf{Re}_{G_m}$ charts.

Charts of j versus Re_{G_m} for In-line Tube Bank Arrangements

Figure 4.14 shows the pattern of fluid flow through a bank of tubes arranged in line.

Charts of j versus Re_{G_m} must be prepared experimentally. Figure 4.15 shows such a chart for flow through an in-line tube bank. The chart and those presented in Appendix C were developed with experimental data from Kays and London [2], and are approximate. The design engineer is encouraged to consult Reference [2] for more detailed information.

The following observations are made regarding Figure 4.15:

(i) This chart was developed from transient tests. So, the number of rows of tubes is variable, and must be **determined through the design process**. In addition, curves of this type have wider application.
(ii) A curve of the friction factor (f) versus Re_{G_m} is provided at the top of the chart.
(iii) A curve of the j-factor versus Re_{G_m} is provided at the bottom of the chart.
(iv) D is the tube outer diameter, and is given as 0.375 in.
(v) X_t is the traverse tube-pitch ratio, $\frac{x_t}{D}$.

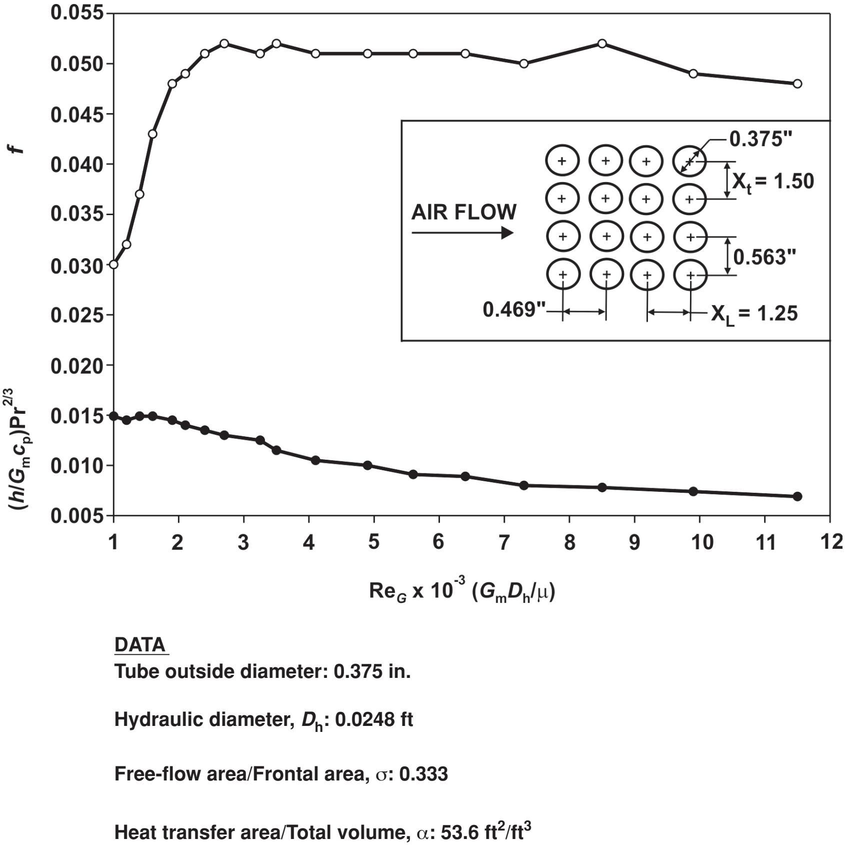


Figure 4.15 Data for flow normal to an in-line tube bank (Kays and London (2))

(vi) X_L is the longitudinal tube-pitch ratio, $\frac{x_L}{D}$.
(vii) σ is given.
(viii) α is the ratio of the total heat transfer area on the external side of the tubes (outer surface area) to the total volume of the exchanger.
(ix) D_h is given.

Additional charts are provided in Appendix C. Refer to the appendix for a more extensive listing of charts.

Charts of *j* versus Re_{G_m} for Staggered Tube Bank Arrangements

Figure 4.16 shows the pattern of fluid flow through a bank of tubes arranged in a staggered fashion.

Figure 4.17 shows a chart for flow through staggered tube banks.

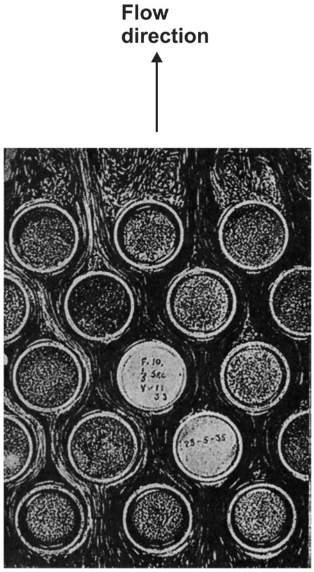


Figure 4.16 Flow pattern for a staggered tube bank (Çengel (3), reprinted with permission)

The following observations are made regarding Figure 4.17:

(i) This chart was developed from transient tests. So, the number of rows of tubes is variable, and must be **determined through the design process**. In addition, curves of this type have wider application.

Additional charts are provided in Appendix C. Refer to the appendix for a more extensive listing of charts for staggered tube banks.

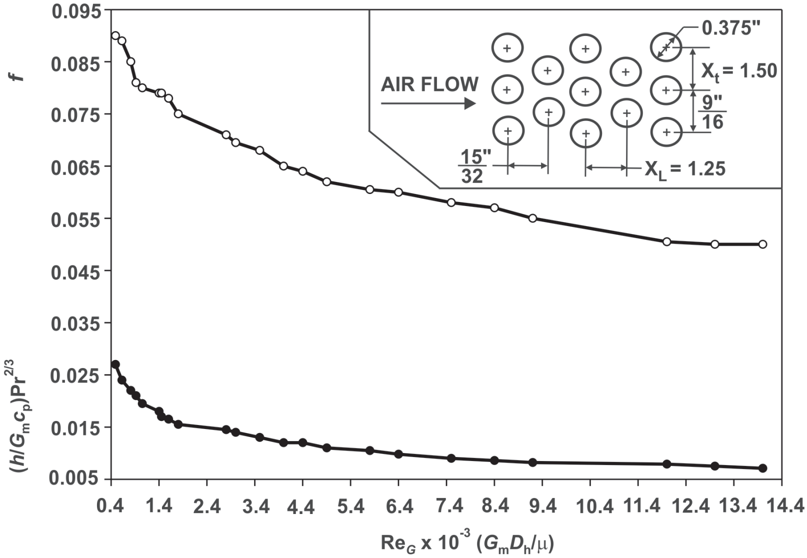
Practical Note 4.5 Heat Transfer from Staggered Tube Banks

Observation of the staggered tube banks show that the heat transfer coefficient is larger for a given value of Re_{G_m} compared to the in-line tube banks, especially for lower values of Re_{G_m}. Therefore, the rate of heat transfer will be higher with staggered tube banks.

Fundamental Note: Churchill and Bernstein [8] have presented a correlation equation for external flow over **single cylinders**. The equation applies when RePr greater than 0.2 and the film temperature must be used to find the fluid properties.

Therefore,

$$\mathrm{Nu_{cyl}} = 0.3 + \frac{0.62\,\mathrm{Re}^{1/2}\mathrm{Pr}^{1/3}}{\left[1 + (0.4/\mathrm{Pr})^{2/3}\right]^{1/4}}\left[1 + \left(\frac{\mathrm{Re}}{282000}\right)^{5/8}\right]^{4/5}. \tag{4.79}$$



DATA
Tube outside diameter: 0.375 in.

Hydraulic diameter, D_h: 0.0249 ft

Free-flow area/Frontal area, σ: 0.333

Heat transfer area/Total volume, α: 53.6 ft^2/ft^3

Note: Minimum free-flow area is in the spaces transverse to the flow

Figure 4.17 Data for flow normal to a staggered tube bank (Kays and London (2))

This correlation equation cannot be used for a bank of tubes since the fluid flows over the tubes will have an influence on each other. However, this equation may apply to the case of a single row of circular tubes oriented such that the flow over them will not be affected by the presence of others.

4.7.2 External Flow over Tube Banks—Pressure Drop

Pressure drop for **external flow over bare and finned-tubes in tube banks** is given empirically as [2]

$$\Delta P_{\text{bank}} = \frac{G_{\text{m}}^2}{2\rho_{\text{i}}}\left[\left(K_{\text{i}} + 1 - \sigma^2\right) + 2\left(\frac{\rho_{\text{i}}}{\rho_{\text{e}}} - 1\right) + f\,\frac{A_{\text{T}}}{A_{\text{c}}}\,\frac{\rho_{\text{i}}}{\rho_{\text{mean}}} - \left(1 - \sigma^2 - K_{\text{e}}\right)\frac{\rho_{\text{i}}}{\rho_{\text{e}}}\right], \tag{4.80}$$

where

"i" = inlet to the heat exchanger tube bank;
"e" = outlet of the heat exchanger tube bank;
K_i = loss coefficient at the inlet to the tube bank;
K_e = loss coefficient at the outlet of the tube bank;
f = friction factor found from the charts presented in Figures 4.15, 4.17, and Appendix C;
ρ_{mean} = average fluid density, $\frac{\rho_i + \rho_e}{2}$;
$\frac{A_T}{A_c} = \frac{\text{total heat transfer area}}{\text{flow cross – sectional area}}$.

Note that

$$\frac{A_T}{A_c} = \frac{4L_{total}}{D_h}, \tag{4.81}$$

where L_{total} is the total length of the heat exchanger in the direction of flow. Figure 4.18 shows a schematic drawing of this total length.

For most industrial applications, the inlet and exit losses are small compared to the other losses in the core of the heat exchanger. So, $K_i = K_e \approx 0$.

Therefore,

$$\Delta P_{bank} = \frac{G_m^2}{2\rho_i}\left[\left(1+\sigma^2\right)\left(\frac{\rho_i}{\rho_e} - 1\right) + f\frac{A_T}{A_c}\frac{\rho_i}{\rho_{mean}}\right]. \tag{4.82}$$

In terms of the head loss in units of length ($l_{h,bank}$), remember: $\Delta P_{bank} = \rho_{mean} g l_{h,bank}$. Thus,

$$l_{h,bank} = \frac{G_m^2}{2\rho_{mean} g \rho_i}\left[\left(1+\sigma^2\right)\left(\frac{\rho_i}{\rho_e} - 1\right) + f\frac{A_T}{A_c}\frac{\rho_i}{\rho_{mean}}\right]. \tag{4.83}$$

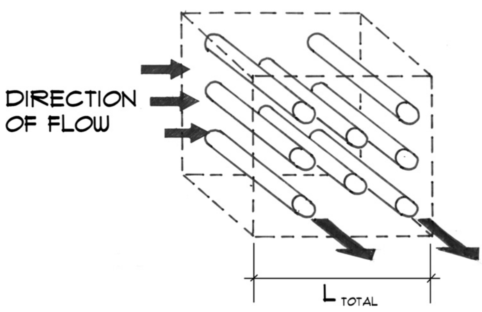


Figure 4.18 Schematic drawing of tube bank showing the total length, L_{total}

Practical Note 4.6 Coil Arrangement in Air-to-Water Heat Exchangers

For most small applications that involve air-to-water heat exchangers, where airflows over a tube bank and water flows in the tubes, the **number of rows of tubes is usually 4 to 6**. While oversizing the heat exchanger may be necessary due to equipment availability or design requirements, the design engineer should be aware that increased number of tubes in a tube bank will require greater powers from a fan (for air) or a pump (for liquids) to move the fluid through the bank. If the number of rows become larger than 6, the design engineer may wish to increase the center-to-center spacing (longitudinal tube pitch) between the tube rows to reduce pressure losses. An even number of rows is usually preferred to ensure that the tube entrance and exit points are on the same side and level of the heat exchanger. This will approach a counter flow heat exchanger system.

Practical Note 4.7 Pressure Drop Over Tube Banks

The equations presented earlier for the determination of the pressure drop and head loss for external flow over the tube bank apply to both bare tube and finned-tube heat exchangers. **Finned-tube heat exchangers are more widely used in industry**.

Example 4.2 Design of a Heating Coil Heat Exchanger

Mr. Chin has hired a junior design engineer to design a small heating coil for his fresh air intake duct line. Mr. Chin is a frugal entrepreneur, and has requested a bare-tube type heating coil arrangement. The HVAC engineer has presented the following **duty** and specifications for the coil:

(i) Heat fresh air from 50°F to about 100°F.
(ii) Duct airflow rate is 2000 cfm.
(iii) Entering hot water temperature is about 150°F.
(iv) This is a low-velocity duct section.
(v) Water connections must be on the same end of the coil system to facilitate easy installation and to approach a counter flow heat exchanger system.
(vi) One large inlet and exit header is required. The tubes will be attached to the headers.

Further Information: This design problem should address and present the following points:

(i) Exit water temperature
(ii) Heating coil configuration and number of coils
(iii) Final dimensions of the heat exchanger (length, width, and depth)
(iv) Pressure loss for the airflow through the coil bank
(v) Head loss in the waterside of the tube

Possible Solution

Definition

Design a heating coil to heat fresh air in a duct line.

Preliminary Specifications and Constraints

(i) Heat fresh in-take outdoor air.
(ii) Heat air to 100°F.
(iii) Bare-tube heating coils are required by the client.
(iv) The airflow rate is constrained to 2000 cfm.
(v) This is a low-velocity duct section.
(vi) The water connections on the coiled heat exchanger must be on the same end of the system.

Detailed Design

Objective

To determine the coil configuration, number of coils, size, and performance of a heat exchanger.

Data Given or Known

(i) Heat fresh air from 50 to 100°F.
(ii) The airflow rate is 2000 cfm.
(iii) The entering hot water temperature is 150°F.

Assumptions/Limitations/Constraints

(i) Let the air velocity over the tubes be 1000 fpm. This is lower than 1200 fpm, which meets the requirement for a low-velocity duct section.
(ii) Let the flow velocity of water in the tubes be about 4 fps. This is acceptable for general building service or potable water. In addition, this velocity does not exceed the erosion limits of any general pipe material.
(iii) The tubes are arranged in a staggered fashion. This will enhance heat transfer.
(iv) Let the pipe material be Type L copper. Copper has high heat transfer properties and availability.
(v) The 180° return bends (regular) will be soldered to make the tube (hairpin) connections. Soldering or brazing is typically done for heating/cooling coils.
(vi) The tube (hairpin) wall thickness is small compared to its outer diameter.
(vii) Let the tube (hairpin) outer diameter be $^3\!/_8$ in. This is a reasonable tube size for heating coils used for this application.
(viii) Negligible elevation head. Assume that all components are on the same level.
(ix) This will be a counter flow arrangement.
(x) Entrance/exit losses of the air over the coils will be neglected. Other losses will be much larger.

Sketch

A complete drawing will be provided after the design to show the tube flow circuitry in the heating coil.

Analysis

Determine the overall heat transfer coefficient (U)

The ε-NTU method will be used in this design. To determine the effectiveness and NTU, the overall heat transfer coefficient (U) must be determined first. The overall heat transfer coefficient is

$$\frac{1}{U} = \frac{1}{h_\mathrm{i}} + R_\mathrm{fi} + R_\mathrm{wall}A + R_\mathrm{fo} + \frac{1}{h_\mathrm{o}}.$$

The wall resistance is

$$R_\mathrm{wall} = \frac{1}{2\pi kL}\ln\left(\frac{r_\mathrm{o}}{r_\mathrm{i}}\right).$$

It was assumed that the wall thickness is small. Therefore, $r_\mathrm{o} \approx r_\mathrm{i}$. So, $R_\mathrm{wall} \approx 0$. Thus,

$$\frac{1}{U} = \frac{1}{h_\mathrm{i}} + R_\mathrm{fi} + R_\mathrm{fo} + \frac{1}{h_\mathrm{o}}.$$

The heat transfer coefficient for flow in the tube (hairpin) is found from the Dittus–Boelter correlation equation. Note that, though more complex, the Gnielinski correlation could have been used:

$$\mathrm{Nu} = \frac{\overline{h}_\mathrm{i} D_\mathrm{i}}{k} = 0.023\mathrm{Re}_{D_\mathrm{i}}^{0.8}\mathrm{Pr}^{0.3}.$$

The inlet and outlet temperatures of the working fluids will needed to find appropriate properties for the determination of Re_D and Pr.

Given: $T_\mathrm{a,i} = 50^\circ\mathrm{F}$, $T_\mathrm{w,i} = 150^\circ\mathrm{F}$
$T_\mathrm{a,o} = 100^\circ\mathrm{F}$, $T_\mathrm{w,o} =$ unknown.

An educated guess of $T_\mathrm{w,o}$ will be needed. Water has a very high specific heat capacity compared to air. Therefore, the water temperature changes slowly as heat is transferred. In general,

$$\frac{c_\mathrm{p,w}}{c_\mathrm{p,a}} \approx 4.$$

Therefore, if $\Delta T_\mathrm{air} = (100 - 50)^\circ\mathrm{F} = 50^\circ\mathrm{F}$, then $\Delta T_\mathrm{water} \approx 10^\circ\mathrm{F}$. Let $\boldsymbol{T_\mathrm{w,o} = (150 - 10)^\circ\mathrm{F} = 140^\circ\mathrm{F}}$.

The average temperatures are:

$$\overline{T}_{\mathrm{a}} = \frac{(50 + 100)^{\circ}\mathrm{F}}{2} = 75^{\circ}\mathrm{F}$$

$$\overline{T}_{\mathrm{w}} = \frac{(150 + 140)^{\circ}\mathrm{F}}{2} = 145^{\circ}\mathrm{F}.$$

For water at 145°F, the Reynolds number is

$$\mathrm{Re}_{D_{\mathrm{i}}} = \frac{\rho \overline{V} D_{\mathrm{i}}}{\mu} = \frac{\left(61\ \mathrm{lb/ft^3}\right)(4\ \mathrm{ft/s})\,(0.315\ \mathrm{in.})}{2.9 \times 10^{-4}\ \mathrm{lb/(fts)}} \times \frac{1\ \mathrm{ft}}{12\ \mathrm{in.}} = 22086.$$

Note that for $^3/_8$ in. outer diameter Type L copper, the inside diameter is 0.315 in. (Table A.6).

The Prandtl number is Pr = 2.73.

The flow in the tubes is turbulent and the Pr is between 0.7 and 160. Therefore, the Dittus–Boelter equation is valid for this analysis.

Thus,

$$\overline{h}_{\mathrm{i}} = \frac{0.023k}{D_{\mathrm{i}}} \mathrm{Re}_{D_{\mathrm{i}}}^{0.8} \mathrm{Pr}^{0.3}$$

$$\overline{h}_{\mathrm{i}} = \frac{0.023\,(0.38\ \mathrm{Btu/(h\ ft\ R)})}{0.315\ \mathrm{in.}} \times \frac{12\ \mathrm{in.}}{1\ \mathrm{ft}} \times (22086)^{0.8}\,(2.73)^{0.3}$$

$$\overline{h}_{\mathrm{i}} = 1344\ \mathrm{Btu/(h\ ft^2\ R)} = 1344\ \mathrm{Btu/(h\ ft^2\ ^{\circ}F)}.$$

The fouling resistances can be found from Table C.3.

Assume temperature of distilled water to be above 122°F: $R_{\mathrm{fi}} = 0.00114\ (\mathrm{h\ ft^2\ ^{\circ}F})/\mathrm{Btu}$.
Assume compressed air: $R_{\mathrm{fo}} = 0.00199\ (\mathrm{h\ ft^2\ ^{\circ}F})/\mathrm{Btu}$.

The average heat transfer coefficient for airflow over the bare tube bank is found from charts and appropriate correlation equations:

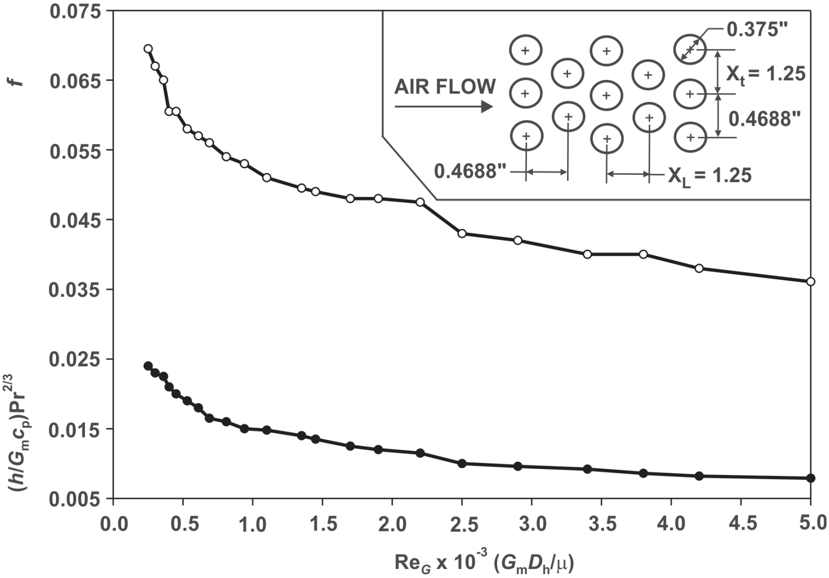
$$\overline{h}_{\mathrm{o}} = \frac{j G_{\mathrm{m}} c_{\mathrm{p}}}{\mathrm{Pr}^{2/3}},$$

and the Chilton–Colburn *j*-factor is

$$j = \frac{\overline{h}_{\mathrm{o}}}{G_{\mathrm{m}} c_{\mathrm{p}}} \mathrm{Pr}^{2/3}.$$

The following *j*-factor versus $\mathrm{Re}_{G_{\mathrm{m}}}$ chart (Figure C.2b) will be used since the tube outer diameter is $^3/_8$ in. In addition, the ratio of the free-flow area to the frontal area (σ) is small. This implies that the mass flow rate per unit minimum flow area between the tubes (G_{m})

will be large, resulting in a higher $\overline{h}_o$ and improved heat transfer. Consult Figure C.2 for additional charts.



DATA
Tube outside diameter: 0.375 in.

Hydraulic diameter, D_h: 0.0125 ft

Free-flow area/Frontal area, σ: 0.200

Heat transfer area/Total volume, α: 64.4 ft^2/ft^3

Note: Minimum free-flow area is in the spaces transverse to the flow

Remember that air properties are found at $\overline{T}_a = 75°F$. So, G_m is

$$G_m = \frac{\rho_a \overline{V}_a}{\sigma} = \frac{\left(0.074\ \text{lb/ft}^3\right)(1000\ \text{ft/min})}{0.200} \times \frac{1\ \text{min}}{60\ \text{s}} = 6.17\ \text{lb/(s ft}^2\text{)}.$$

The Reynolds number based on G_m is

$$\text{Re}_{G_m} = \frac{G_m D_h}{\mu_a} = \frac{\left(6.17\ \text{lb/(s ft}^2\text{)}\right)(0.0125\ \text{ft})}{1.25 \times 10^{-5}\ \text{lb/(ft s)}} = 6167 \approx 6.0 \times 10^3.$$

From the chart, $j \approx 0.007$. Note that the j-factor changes only slightly beyond $\text{Re}_{G_m} = 4.0 \times 10^3$.

Therefore,

$$\bar{h}_o = \frac{(0.007)\left(6.17\ \text{lb/(s ft}^2)\right)(0.24\ \text{Btu/(lb °F))}}{(0.73)^{2/3}} \times \frac{3600\ \text{s}}{1\ \text{h}}$$

$$\bar{h}_o = 46.0\ \text{Btu/(h ft}^2\ \text{°F)}.$$

As expected, the heat transfer coefficient of the liquid water is higher than that of gaseous air.

The overall heat transfer coefficient is

$$\frac{1}{U} = \frac{1}{1344\ \text{Btu/(h ft}^2\ \text{°F)}} + 0.00114\ \text{(h ft}^2\ \text{°F)/Btu} + 0.00199\ \text{(h ft}^2\ \text{°F)/Btu} + \frac{1}{46.0\ \text{(h ft}^2\ \text{°F)/Btu}}$$

$$\frac{1}{U} = 0.0256\ \text{(h ft}^2\ \text{°F)/Btu}$$

$$\boldsymbol{U = 39\ \textbf{Btu/(h ft}^2\ \text{°}\textbf{F)}}.$$

Determine the configuration, number of coils, and dimensions of the heat exchanger. Find the total surface area of the tubes by using the ε-NTU method.

$$\text{NTU} = \frac{UA}{C_{min}}$$

$$A = \frac{C_{min}\text{NTU}}{U}$$

The minimum heat capacity is needed. For air,

$$C_a = \dot{m}_a c_{pa} = \rho_a \dot{V}_a c_{pa} = (0.074\ \text{lb/ft}^3)(2000\ \text{ft}^3\text{/min})\ (0.24\ \text{Btu/(lb °F))} = 35.5\ \text{Btu/(min °F)} = C_c.$$

For water, the law of conservation of energy can be used:

$$C_w = C_h = C_c \frac{T_{a,o} - T_{a,i}}{T_{w,i} - T_{w,o}} = (35.5\ \text{Btu/(min °F))} \left[\frac{100\text{°F} - 50\text{°F}}{150\text{°F} - 140\text{°F}}\right] = 177.5\ \text{Btu/(min °F)}.$$

Therefore, $C_a = C_c = C_{min}$ and $C_w = C_h = C_{max}$.

The capacity ratio is

$$c = \frac{C_{min}}{C_{max}} = \frac{35.5\ \text{Btu/(min °F)}}{177.5\ \text{Btu/(min °F)}}$$

$$\boldsymbol{c = 0.20}.$$

The effectiveness is

$$\varepsilon = \frac{C_c}{C_{min}} \frac{(T_{a,o} - T_{a,i})}{(T_{w,i} - T_{a,i})} = \frac{(T_{a,o} - T_{a,i})}{(T_{w,i} - T_{a,i})} = \frac{100\text{°F} - 50\text{°F}}{150\text{°F} - 50\text{°F}}$$

$$\boldsymbol{\varepsilon = 0.50}.$$

At this point the thermal capacity or heat exchanged in the heat exchanger can be determined:

$$\dot{Q} = \varepsilon C_{\min}\,(T_{\mathrm{w,i}} - T_{\mathrm{a,i}}) = 0.50\,(35.5\ \mathrm{Btu/(min\ ^\circ F)})\,(150^\circ\mathrm{F} - 50^\circ\mathrm{F})$$

$$\boldsymbol{\dot{Q} = 1775\ \mathrm{Btu/min} = 106500\ \mathrm{Btu/h}}.$$

The system will be designed to approach a counter flow heat exchanger. With c and ε known, the NTU value can be read directly from Figure 4.12. Hence,

$$\mathbf{NTU \approx 0.75}.$$

The total heat transfer surface area of the tubes is

$$A = \frac{(35.5\ \mathrm{Btu/(min\ ^\circ F)})\,(0.75)}{39\ \mathrm{Btu/(h\ ft^2\ ^\circ F)}} \times \frac{60\ \mathrm{min}}{1\ \mathrm{h}}$$

$$A = 41\ \mathrm{ft^2}.$$

The total heat transfer volume is found from the ratio of the heat transfer area to the total volume:

$$\alpha = \frac{A}{\Omega}.$$

Therefore,

$$\Omega = \frac{A}{\alpha} = \frac{41\ \mathrm{ft^2}}{64.4\ \mathrm{ft^2/ft^3}} = 0.64\ \mathrm{ft^3}.$$

The depth of the heat exchanger system (dimension in the direction of airflow) is

$$W = \frac{\Omega}{A_{\mathrm{f}}}.$$

A_{f} is the face area of the heat exchanger box normal to the airflow direction.

$$\text{Let: } A_{\mathrm{f}} = \frac{\dot{V}_{\mathrm{a}}}{V_{\mathrm{a}}} = \frac{2000\ \mathrm{ft^3/min}}{1000\ \mathrm{ft/min}} = 2\ \mathrm{ft^2}.$$

Thus,

$$W = \frac{0.64\ \mathrm{ft^3}}{2\ \mathrm{ft^2}} \times \frac{12\ \mathrm{in.}}{1\ \mathrm{ft}}$$

$$\boldsymbol{W = 3.8\ \mathrm{in.} \approx 4\ \mathrm{in.}}$$

From the j-factor versus Re_{G_m} chart, the longitudinal tube pitch is 0.4688 in. The number of tube rows is

$$N_r = \frac{W}{x_L} = \frac{3.8\text{ in.}}{0.4688\text{ in.}}$$

$$\mathbf{N_r = 8.1 \approx 8\ tube\ rows}.$$

Determine the number of tubes per row. The cross-sectional area for the tube can be found from the mass flow rate equation (for water):

$$\dot{m}_w = \rho_w \overline{V}_w A_{\text{tube}}.$$

The tube area is $A_{\text{tube}} = N_{\text{tube}} \frac{\pi D_i^2}{4}$. The mass flow rate can be determined from the definition of the heat capacity rate, $C_h = \dot{m}_w c_{p,w}$.

Therefore,

$$\frac{C_h}{c_{p,w}} = \rho_w \overline{V}_w N_{\text{tube}} \frac{\pi D_i^2}{4}$$

$$N_{\text{tube}} = \frac{4C_h}{c_{p,w}\rho_w \overline{V}_w \pi D_i^2}$$

$$N_{\text{tube}} = \frac{4\,(177.5\text{ Btu/(min °F)})}{(1.0\text{ Btu/(lb °F)})\left(61\text{ lb/ft}^3\right)(4\text{ ft/s})\,\pi\,(0.315\text{ in.})^2} \times \left(\frac{12\text{ in.}}{1\text{ ft}}\right)^2 \times \frac{1\text{ min}}{60\text{ s}}$$

$$\mathbf{N_{tube} = 22.4\ tubes\ per\ row \approx 22\ tubes\ per\ row}.$$

The total number of tubes along the length of the heat exchanger will be 176 tubes (8 rows × 22 tubes per row). If each row has 22 tubes, the height of the heating coil is

$$H = N_{\text{tube}} x_t = (22)(0.4688\text{ in.})$$

$$\mathbf{H = 10.3\ in. \approx 11\ in.}$$

The length of the heat exchanger is found by considering an alternate definition of the face area normal to the direction of airflow:

$$A_f = LH.$$

The heat exchanger length is

$$L = \frac{A_f}{H} = \frac{2\text{ ft}^2}{10.3\text{ in.}} \times \left(\frac{12\text{ in.}}{1\text{ ft}}\right)^2$$

$$\mathbf{L = 27.96\ in. \approx 28\ in.}$$

The total length of the tubes in the heat exchanger is 410 ft (28 in. per tube × 176 tubes).

Pressure loss of air across the tube coils

The pressure drop of the air across the tube coil bank is given by

$$\Delta P_{\text{bank}} = \frac{G_{\text{m}}^2}{2\rho_{\text{a,i}}}\left[\left(1+\sigma^2\right)\left(\frac{\rho_{\text{a,i}}}{\rho_{\text{a,o}}}-1\right)+f\,\frac{A_{\text{T}}}{A_{\text{c}}}\frac{\rho_{\text{a,i}}}{\rho_{\text{mean}}}\right].$$

Remember: $G_{\text{m}} = 6.17\ \text{lb}/(\text{s ft}^2)$ and $\text{Re}_{G_{\text{m}}} = 6167 \approx 6.0\times 10^3$.
From the j-factor versus $\text{Re}_{G_{\text{m}}}$ chart, the friction factor (f) is

$$f \approx 0.035.$$

The area ratio is

$$\frac{A_{\text{T}}}{A_{\text{c}}} = \frac{4W}{D_{\text{h}}} = \frac{4\,(4\ \text{in.})}{0.0125\ \text{ft}}\times\frac{1\ \text{ft}}{12\ \text{in.}} = 106.7.$$

The mean density (ρ_{mean}) is

$$\rho_{\text{mean}} = \frac{\rho_{\text{a,i}}+\rho_{\text{a,o}}}{2} = \frac{(0.078+0.071)\ \text{lb/ft}^3}{2} = 0.075\ \text{lb/ft}^3.$$

Therefore,

$$\Delta P_{\text{bank}} = \frac{\left(6.17\ \text{lb}/(\text{s ft}^2)\right)^2}{2\left(0.078\ \text{lb/ft}^3\right)}\left[\left(1+0.20^2\right)\left(\frac{0.078\ \text{lb/ft}^3}{0.071\ \text{lb/ft}^3}-1\right)+(0.035)\,(106.7)\,\frac{0.078\ \text{lb/ft}^3}{0.075\ \text{lb/ft}^3}\right]$$

$$\Delta P_{\text{bank}} = 972.8\ \text{lb}/(\text{ft s}^2)\times\frac{1\ \text{lbf}}{32.2\ (\text{lb ft})/\text{s}^2} = 30.2\ \text{lbf/ft}^2 = 30.2\ \text{psf}.$$

In practice, the pressure drop is reported in inches of water. Thus,

$$\Delta P_{\text{bank}} = 972.8\ \text{lb}/(\text{ft s}^2)\times\frac{1}{0.075\ \text{lb/ft}^3}\times\frac{1}{32.2\ \text{ft/s}^2} = 402.8\ \text{ft of air}$$

$$\Delta P_{\text{bank}} = 402.8\ \text{ft of air}\times SG_{\text{air},75^\circ\text{F}} = 402.8\ \text{ft of air}\times\frac{0.075\ \text{lb/ft}^3}{62\ \text{lb/ft}^3}\times\frac{12\ \text{in.}}{1\ \text{ft}}$$

$$\boldsymbol{\Delta P_{\textbf{bank}} = 5.9\ \textbf{in. of water} = 5.9\ \textbf{in. wg}.}$$

Note: This is a very large pressure drop across the tube coils. This is due to the large airflow rate and velocity, large value of G_{m}, and small values of x_{L} and σ.

Pressure loss of water in the tube coils

Determination of the pressure loss of the water in the tubes is needed to find the pump power required to move the fluid through the heat exchanger system. The total head loss is

$$H_{\text{lT}} = \left(f\,\frac{L_{\text{tube}}}{D_{\text{i}}}+K\right)\frac{\overline{V}_{\text{w}}^2}{2g}.$$

From the Moody chart, for $\mathrm{Re}_{D_i} = 22086$ and for copper tubes with relative roughness, $\frac{\varepsilon}{D_i} = 0.00019$, the friction factor is $f \approx 0.0285$.

For the primary piping circuitry through the heat exchanger, there are 8 tube rows, including one supply run and one return run, each 28 in. long.

Therefore,

$$L_{\text{tube}} = (8 \text{ rows})(28 \text{ in. per row}) = 224 \text{ in.} = 18.7 \text{ ft.}$$

For the minor losses, $K = 2.0$ for the soldered/brazed 180° regular return bends. Note that the K value for soldered/brazed 180° regular return bends is probably lower than 2.0, and more on the order of the value for flanged 180° regular return bends.

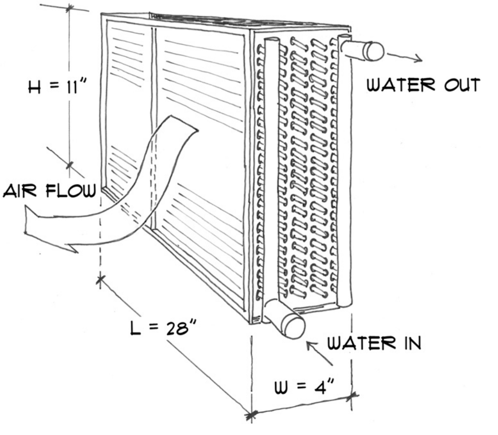
Thus,

$$H_{\text{IT}} = \left((0.0285)\frac{224 \text{ in.}}{0.315 \text{ in.}} + (7)(2.0)\right)\frac{(4 \text{ ft/s})^2}{2\left(32.2 \text{ ft/s}^2\right)}$$

$$\boldsymbol{H_{\text{IT}} = 8.5 \text{ ft wg}}.$$

It should be noted that only the longest run of piping is needed to determine the total head loss that will be used to find the total pump power required. In this case, the use of inlet and outlet headers in which each row of tubes has its inlet and outlet attached to a header will give the longest run of pipe with a length of 224 in. If this were not the case, and only one tube inlet and one tube outlet were available for the entire heating coil unit, the longest run of pipe would be the total length of all the tubes. In that case, the length of piping would be 4928 in., the total number of 180° return bends would be 154, and the total head loss would be 187 ft wg. The total head loss would be more than 20 times larger than that of the design with headers.

Drawings



Conclusions

A heating coil heat exchanger with a bare-tube bank has been designed. The following points should be noted:

(i) The Re_{G_m} value that was used with the *j*-factor versus Re_{G_m} chart was slightly off the curve. The error incurred was small.
(ii) The pressure drop of the air across the tube bank and the number of rows of tubes were large. The tubes were too close, since $\sigma = 0.200$. $\sigma = 0.333$ may have been a better choice for improvement of the design.
(iii) A pressure drop of 5.9 in. wg on the air side is too large. This pressure drop should be limited to an order of 1 in. wg across the tube bank, if possible. Increasing x_L and σ would reduce the pressure drop.

A heat exchanger design data sheet is shown below.

Heat Exchanger Design Data Sheet	
Type	Counter Flow
Section: Tube Bank	
Working fluid	Air
Volume flow rate	2000 cfm
Inlet temperature	50°F
Outlet temperature	100°F
Pressure drop	5.9 in. wg
Section: Tube	
Tube material	Copper
Working fluid	Water
Velocity	4 ft/s
Tube inner diameter	0.315 in.
Tube outer diameter	0.375 in.
Number of tube rows	8
Number of tubes per row	22
Tube spacing ($x_t \times x_L$)	0.4688 in. × 0.4688 in.
Total tube length	410 ft
Inlet temperature	150°F
Outlet temperature	140°F
Head loss	8.5 ft wg
Heat Exchanger Parameters	
Thermal capacity	106500 Btu/h
Effectiveness	0.50
Capacity ratio	0.20
Overall heat transfer coefficient	39 Btu/(h ft^2 °F)
Number of transfer units (NTU)	0.75
Heat exchanger dimensions ($L \times H \times W$)	28 in. × 11 in. × 4 in.

4.7.3 External Flow over Finned-Tubes in Cross Flow—Equations and Charts

External flow over finned-tube tube banks is more prevalent in industrial applications. The addition of **fins** extends the heat transfer surface area of the bare tubes in the bank.

A staggered arrangement of the tubes, coupled with continuous fins across the length of the tube bank and heat exchanger will increase the heat transfer capability of the exchanger. Figure 4.19 shows some examples of finned heat exchangers that may be encountered by the design engineer in industry.

Of importance in the design and performance analysis of heat exchangers is the determination of the overall heat transfer coefficient (U). To determine the overall heat transfer coefficient for finned-tubes, consider the following assumptions and observations:

(i) Constant area, straight fins (e.g., continuous fins, plate-fin-tube geometry).
(ii) The base of the fin transfers heat also. Figure 4.20 shows a schematic of general constant area, straight fins attached to a surface (the base).
(iii) So, the combined fin/base surface effectiveness, η_s, is used in heat exchanger calculations, instead of the fin efficiency, η.
(iv) The fins are very thin. So, $l \gg t$. This is typical of fins used in industrial heat exchangers.
(v) The fins are rigidly and perfectly attached to the base. So, there is no contact resistance to heat transfer between the fin and the base.
(vi) The heat transfer coefficient, fluid properties, and thermal properties are constant.
(vii) The temperature distribution is uniform and steady (steady state) through the fin. So, the Biot number is small compared to unity $\left(\mathrm{Bi} = \frac{h\delta}{k} \ll 1\right)$. This assumption will hold for very thin fins ($\delta = t$ or $\delta = t/2$) and/or high fin material thermal conductivities (k), which can be easily incorporated into any design. Low heat transfer coefficients (h) reduce Bi also. This is less desirable from a fin design/performance perspective.

Without derivation, the overall heat transfer coefficient for finned-tube heat exchangers is

$$\frac{1}{U_{\mathrm{FT}}} = \frac{1}{h_o \eta_{so}} + R_{fo} + \frac{R_{fi}}{\left(A_i/A_o\right)} + \frac{1}{h_i \eta_{si}\left(A_i/A_o\right)}, \tag{4.84}$$

where “o” refers to the working fluid outside the tube in the tube bank, and “i” refers to the working fluid inside the tubes.

For most industrial applications, the tubes are completely filled with fluid. So, $\eta_{si} = 1$.

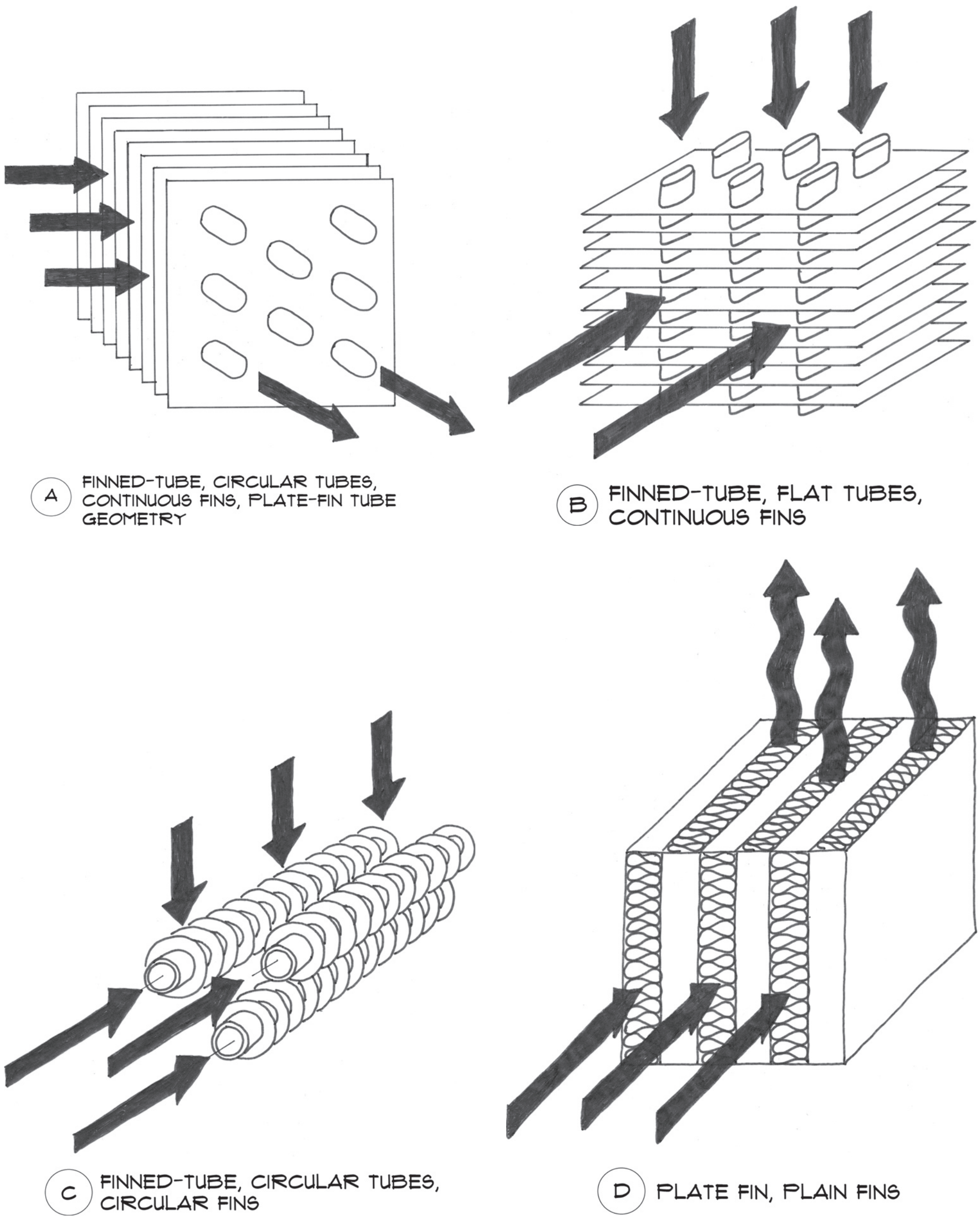


Figure 4.19 Examples of finned heat exchangers

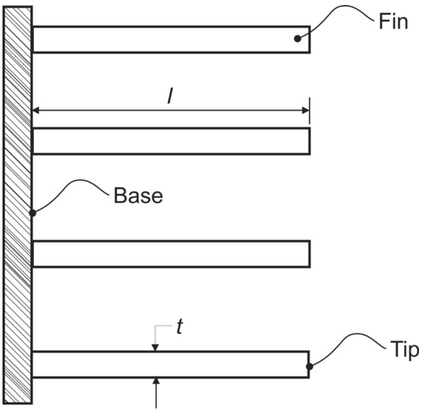


Figure 4.20 General constant area, straight fins attached to a surface

Therefore,

$$\frac{1}{U_{\mathrm{FT}}} = \frac{1}{h_{\mathrm{o}}\eta_{\mathrm{so}}} + R_{\mathrm{fo}} + \frac{R_{\mathrm{fi}}}{\left(A_{\mathrm{i}}/A_{\mathrm{o}}\right)} + \frac{1}{h_{\mathrm{i}}\left(A_{\mathrm{i}}/A_{\mathrm{o}}\right)}. \tag{4.85}$$

Fin Surface Effectiveness, η_s

The fin surface effectiveness is defined as

$$\begin{aligned}\eta_{\mathrm{s}} &= \frac{\text{Actual heat transfer from fin and base}}{\text{Heat transfer from fin and base when fin is at the base temperature}} \\ &= 1 - \frac{A_{\mathrm{f}}}{A}\left(1-\eta\right),\end{aligned} \tag{4.86}$$

where

$A_{\mathrm{f}} =$ area of the fin, only;
$A =$ the total area of the fin and base;
$\frac{A_{\mathrm{f}}}{A} =$ is determined from available and appropriate charts;
$\eta =$ fin efficiency.

For continuous fins that connect tubes in a staggered tube bank, it may be assumed that the tubes are arranged in a hexangular tube array, with a tube centered in each array. It is difficult to determine the fin efficiency (η) for a hexangular tube array. So, **empirical expressions** for η for circular fins are used to find an equivalent η for continuous plate hexangular finned-tube arrays. The empirical relations were developed by Schmidt [9].

Figure 4.21 shows a schematic drawing of a staggered tube bank with a hexangular finned-tube array. Note that the traverse tube pitch (x_{t}) and the longitudinal tube pitch (x_{L}) are shown.

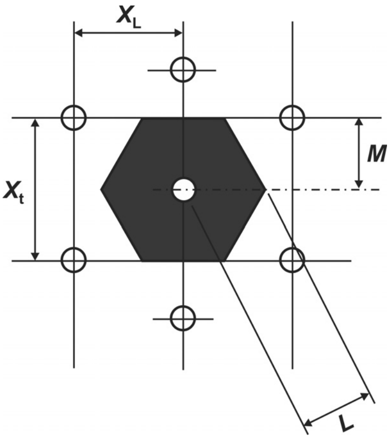


Figure 4.21 Staggered tube bank with a hexangular finned-tube array

The fin efficiency for this situation is defined as

$$\eta = \frac{\tanh(mr\phi)}{mr\phi}, \tag{4.87}$$

where

$$r = \text{tube outer radius},$$

$$m = \left(\frac{2h_o}{k_{fin}t}\right)^{1/2}, \tag{4.88}$$

$$k_{fin} = \text{thermal conductivity of the fin material},$$

$$\phi = \left(\frac{R_{EQ}}{r} - 1\right)\left[1 + 0.35\ln\left(\frac{R_{EQ}}{r}\right)\right], \tag{4.89}$$

R_{EQ} = the equivalent fin radius that will give an equivalent η for continuous plate hexangular fins.

$\frac{R_{EQ}}{r}$ is given empirically as

$$\frac{R_{EQ}}{r} = 1.27\psi\,(\beta - 0.3)^{1/2}, \tag{4.90}$$

where

$$\psi = \frac{M}{r} = \frac{x_t}{2} \times \frac{1}{r} = \frac{x_t}{D} = X_t, \tag{4.91}$$

$$\beta = \frac{L}{M}. \tag{4.92}$$

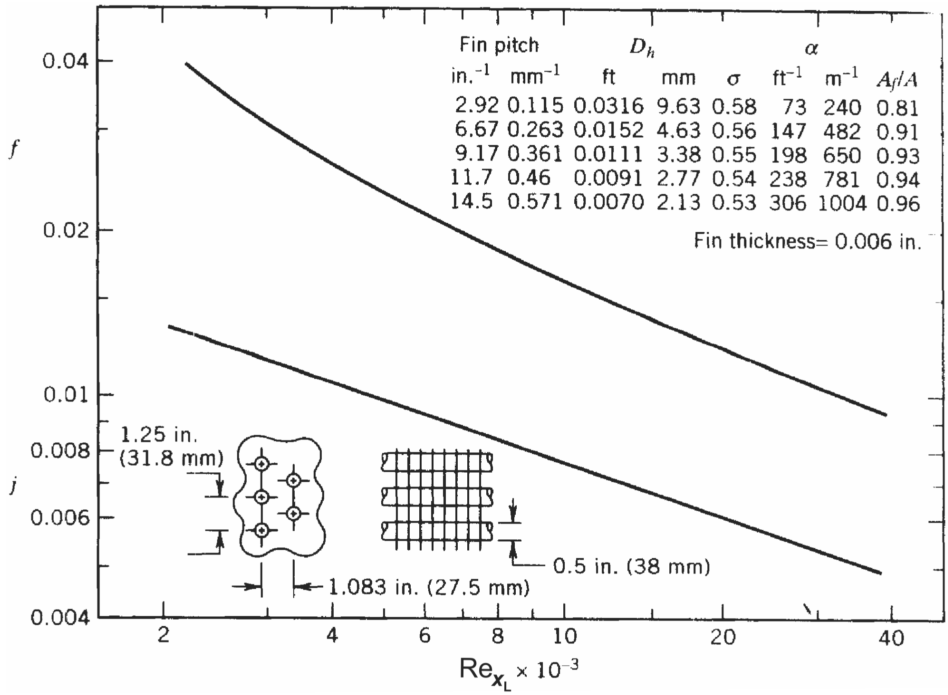


Figure 4.22 Data for flow normal to a finned staggered tube bank (*ASHRAE Transactions*, Vol. 79, Part II, 1973; reprinted with permission)

Analysis of the geometry shown in Figure 4.22 will give

$$L = \frac{\left[\left(\frac{x_t}{2}\right)^2 + x_L^2\right]^{1/2}}{2}. \tag{4.93}$$

The efficiency and the surface effectiveness of the fin can be determined.

> **Practical Note 4.8 L and M values**
>
> In practice, it is typical that $L \geq M$ and $\beta \geq 1$. The traverse tube pitch (x_t) and the longitudinal tube pitch (x_L) that are used to calculate L and M will be available from charts. The design engineer should verify that $L \geq M$ and $\beta \geq 1$.

Heat Transfer Coefficient, h_o

Heat transfer coefficients for flow through finned tube banks are also found by using the j-factor and appropriate charts. Remember: $j = \frac{\bar{h}_o}{G_m c_p} \mathrm{Pr}^{2/3}$.

Two types of charts may be presented to find the j-factor: j versus $\mathrm{Re}_{G_\mathrm{m}}$ and j versus $\mathrm{Re}_{x_\mathrm{L}}$, where $\mathrm{Re}_{G_\mathrm{m}} = \frac{G_\mathrm{m} D_\mathrm{h}}{\mu}$ and $\mathrm{Re}_{x_\mathrm{L}} = \frac{G_\mathrm{m} x_\mathrm{L}}{\mu}$.

Charts of j versus $\mathrm{Re}_{x_\mathrm{L}}$ for Staggered Tube Bank Arrangements (Finned Tubes)

Figure 4.22 shows a j versus $\mathrm{Re}_{x_\mathrm{L}}$ chart for a staggered tube bank of finned tubes (plate-fin coils).

The following observations are made regarding Figure 4.22:

(i) This chart applies to a finned-tube bank with five rows.
(ii) The fin pitch is the number of fins per in. The inverse of the fin pitch will give the space between each fin.
(iii) The hydraulic diameter, D_h, varies with the fin pitch.
(iv) σ, α and $\frac{A_\mathrm{f}}{A}$ (fin area-to-total area ratio) are all provided.
(v) The chart is based on $\mathrm{Re}_{x_\mathrm{L}}$.
(vi) The friction factor (f) curve is given.

More general charts may be used, which show varying numbers of tube rows or fin pitch. Figure 4.23 shows a group of j versus $\mathrm{Re}_{x_\mathrm{L}}$ curves on one chart. Each curve represents a different number of rows of tubes.

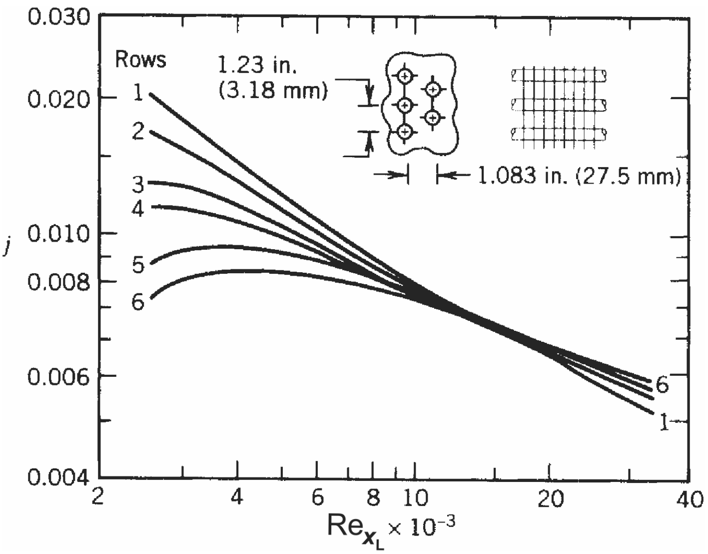


Figure 4.23 Data for flow normal to staggered tube banks: multiple tube rows (*ASHRAE Transactions*, Vol. 81, Part I, 1975; reprinted with permission)

The following observations are made regarding Figure 4.23. Additional charts are available in Appendix C:

(i) This chart applies to various numbers of tube rows.
(ii) There are large deviations in the j-factor for low values of Re_{x_L}.
(iii) At about $Re_{x_L} \approx 8000$, the j-factor becomes approximately independent of the number of rows.
(iv) Hence, for $Re_{x_L} > 8000$, any chart or curve may be used independently of the number of rows for other specified data such as tube diameter, fin pitch, σ, α, and $\frac{A_f}{A}$.

Example 4.3 Design of a Heating Coil Heat Exchanger c/w Finned Tubes

Mr. Chin is concerned about the large pressure drops across the bare tube bank, and has decided that he would prefer a small heating coil c/w finned tubes for his fresh air in-take duct line. A HVAC engineer had presented the following **duty** for the coil:

(i) Heat fresh air from 50 to about 100°F.
(ii) Duct airflow rate is 2000 cfm.
(iii) Entering hot water temperature is about 150°F.
(iv) This is a low-velocity duct section.
(v) Water connections must be on the same end of the coil system to facilitate easy installation and to approach a counter flow heat exchanger system.

Further Information: This design problem should address and present the following points:

(i) Exit water temperature
(ii) Heating coil configuration, number of coils, fin pitch
(iii) Final dimensions of the heat exchanger (length, width, and depth)
(iv) Pressure loss for the airflow through the coil bank
(v) Head loss in the waterside of the tube

Possible Solution

Definition

Design a heating coil complete with finned tubes to heat fresh air in a duct line.

Preliminary Specifications and Constraints

Same as Example 4.2, in addition to

(i) finned-tube heating coils required by the client.

Detailed Design

Objective

To determine the coil configuration, number of coils, size, and performance of a heat exchanger that has finned tubes.

Data Given or Known

(i) Same as Example 4.2.

Assumptions/Limitations/Constraints

(i) Same as Example 4.2, in addition to/except.
(ii) Let the tube outside diameter be 0.676 in. After studying the data in Table A.6, assume that the tube thickness is 0.04 in.
(iii) The tubes longitudinal pitch is 1.75 in. A large pitch is chosen (close to 1 in.) to reduce possible pressure losses on the air side.
(iv) Let the fin material be aluminum. This material is typically used in industry for the fabrication of fins for heating or cooling coils. Copper could also be used.
(v) Assume 7.75 fins per inch.

Sketch

A sketch has been provided with the problem preamble. The sketch will be modified to show the tube flow circuitry after the design.

Analysis

Determine the overall heat transfer coefficient (U_{FT})

The ε-NTU method will be used in this design. To determine the effectiveness and NTU, the overall heat transfer coefficient (U_{FT}) must be determined first. The overall heat transfer coefficient for finned tubes is

$$\frac{1}{U_{FT}} = \frac{1}{h_i\,(A_i/A_o)} + \frac{R_{fi}}{(A_i/A_o)} + R_{fo} + \frac{1}{h_o \eta_{so}}.$$

The heat transfer coefficient for flow in the tube is found from the Dittus–Boelter correlation equation:

$$\text{Nu} = \frac{\overline{h}_i D_i}{k} = 0.023 \text{Re}_{D_i}^{0.8} \text{Pr}^{0.3}.$$

Appropriate properties for the determination of Re_D and Pr will be found at 145°F (see Example 4.2). The inner diameter of the copper tube is approximately 0.596 in.

Therefore, the Reynolds number is

$$\mathrm{Re}_{D_\mathrm{i}} = \frac{\rho \overline{V} D_\mathrm{i}}{\mu} = \frac{\left(61\ \mathrm{lb/ft^3}\right)(4\ \mathrm{ft/s})(0.596\ \mathrm{in.})}{2.9 \times 10^{-4}\ \mathrm{lb/(ft\ s)}} \times \frac{1\ \mathrm{ft}}{12\ \mathrm{in.}} = 41789.$$

$$\mathrm{Pr} = 2.73.$$

The flow in the tubes is turbulent and the Pr is between 0.7 and 160. Thus, the Dittus–Boelter equation is valid for this analysis.

Therefore,

$$\overline{h}_\mathrm{i} = \frac{0.023k}{D_\mathrm{i}} \mathrm{Re}_{D_\mathrm{i}}^{0.8} \mathrm{Pr}^{0.3}$$

$$\overline{h}_\mathrm{i} = \frac{0.023\,(0.38\ \mathrm{Btu/(h\ ft\ R)})}{0.596\ \mathrm{in.}} \times \frac{12\ \mathrm{in.}}{1\ \mathrm{ft}} \times (41789)^{0.8}\,(2.73)^{0.3}$$

$$\overline{h}_\mathrm{i} = 1183\ \mathrm{Btu/(h\ ft^2\ ^\circ F)}.$$

The fouling resistances can be found from Table C.3.

Assume distilled water above 122°F: $R_\mathrm{fi} = 0.00114\ (\mathrm{h\ ft^2\ ^\circ F})/\mathrm{Btu}$.
Assume compressed air: $R_\mathrm{fo} = 0.00199\ (\mathrm{h\ ft^2\ ^\circ F})/\mathrm{Btu}$.

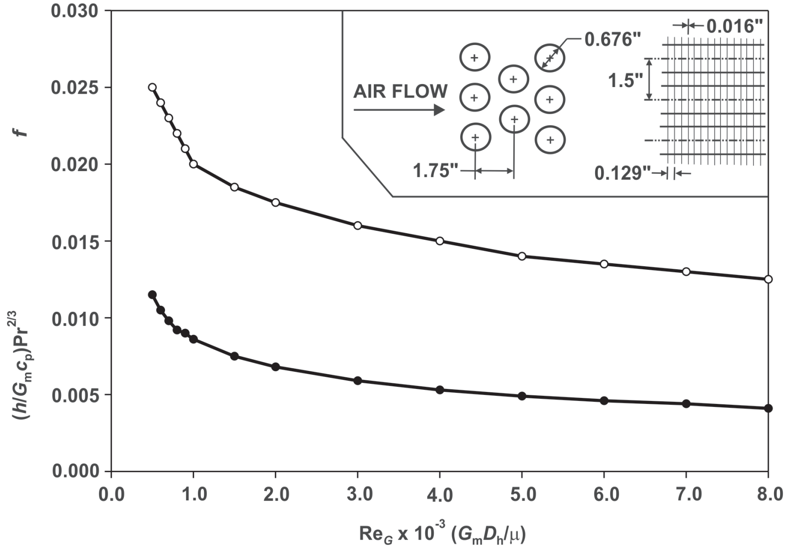
The average heat transfer coefficient for airflow over the finned-tube bank is found from charts and appropriate correlation equations:

$$\overline{h}_\mathrm{o} = \frac{j G_\mathrm{m} c_\mathrm{p}}{\mathrm{Pr}^{2/3}} \text{ for air at } 75^\circ\mathrm{F},$$

and the Chilton–Colburn j-factor is

$$j = \frac{\overline{h}_\mathrm{o}}{G_\mathrm{m} c_\mathrm{p}} \mathrm{Pr}^{2/3}.$$

The following j-factor versus $\mathrm{Re}_{G_\mathrm{m}}$ chart (Figure C.4b) will be used. Note that the longitudinal pitch is 1.75 in. and the traverse pitch is 1.50 in. Consult Figure C.4 for additional charts.



DATA

Tube outside diameter: 0.676 in.

Fin pitch: 7.75 fins per in.

Fin thickness: 0.016 in.

Hydraulic diameter, D_h: 0.0114 ft

Free-flow area/Frontal area, σ: 0.481

Heat transfer area/Total volume, α: 169 ft^2/ft^3

Fin area/Total area: 0.950

Note: Minimum free-flow area is in the spaces transverse to the flow

Remember that air properties are found at $\overline{T}_a = 75°F$. So, G_m is

$$G_m = \frac{\rho_a \overline{V}_a}{\sigma} = \frac{\left(0.074\ \text{lb/ft}^3\right)(1000\ \text{ft/min})}{0.481} \times \frac{1\ \text{min}}{60\ \text{s}} = 2.56\ \text{lb/s ft}^2.$$

The Reynolds number based on G_m is

$$\text{Re}_{G_m} = \frac{G_m D_h}{\mu_a} = \frac{\left(2.56\ \text{lb/(s ft}^2)\right)(0.0114\ \text{ft})}{1.25 \times 10^{-5}\ \text{lb/(ft s)}} = 2335 \approx 2.3 \times 10^3.$$

From the chart, $j \approx 0.0065$.

Therefore,

$$\bar{h}_o = \frac{(0.0065)\left(2.56\ \text{lb}/(\text{s ft}^2)\right)(0.24\ \text{Btu}/(\text{lb}\ ^\circ\text{F}))}{(0.73)^{2/3}} \times \frac{3600\ \text{s}}{1\ \text{h}}$$

$$\bar{h}_o = 17.7\ \text{Btu}/(\text{h ft}^2\ ^\circ\text{F}).$$

As expected, the heat transfer coefficient of the liquid water is higher than that of gaseous air.

The fin surface effectiveness is

$$\eta_{so} = 1 - \frac{A_{fin}}{A}(1 - \eta_o).$$

The fin efficiency is

$$\eta_o = \frac{\tan h\,(mr\phi)}{mr\phi}.$$

For aluminum, $k_{fin} = 100$ Btu/(h ft °F). For the fins, the thickness is $t = 0.016$ in., as obtained from the j-factor versus Re_{G_m} chart. Thus,

$$m = \frac{(2\bar{h}_o)^{1/2}}{(k_{fin}t)^{1/2}} = \left[\frac{2\left(17.7\ \text{Btu}/(\text{h ft}^2\ ^\circ\text{F})\right)}{(100\ \text{Btu}/(\text{h ft}\ ^\circ\text{F}))\,(0.016\ \text{in.})\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)}\right]^{1/2} = 16.3\ \text{ft}^{-1}$$

The tube outer radius is

$$r = \frac{0.676\ \text{in.}}{2} \times \frac{1\ \text{ft}}{12\ \text{in.}} = 0.0282\ \text{ft}.$$

Find ϕ.

$$\phi = \left(\frac{R_{EQ}}{r} - 1\right)\left[1 + 0.35\ln\left(\frac{R_{EQ}}{r}\right)\right]$$

$$\frac{R_{EQ}}{r} = 1.27\psi\,(\beta - 0.3)^{1/2}$$

$$\psi = \frac{M}{r} = \frac{x_t}{2} \times \frac{1}{r} = \frac{x_t}{D_o} = X_t = \frac{1.50\ \text{in.}}{0.676\ \text{in.}} = 2.22$$

$$\beta = \frac{L}{M}$$

$$M = \frac{x_t}{2} = \frac{1.50\ \text{in.}}{2} = 0.75\ \text{in.}$$

$$L = \frac{\left[\left(\frac{x_t}{2}\right)^2 + x_L^2\right]^{1/2}}{2} = \frac{\left[\left(\frac{1.50\ \text{in.}}{2}\right)^2 + (1.75\ \text{in.})^2\right]^{1/2}}{2} = 0.952\ \text{in.}$$

Thus,

$$\beta = \frac{0.952 \text{ in.}}{0.750 \text{ in.}} = 1.27.$$

Note that $L \geq M$ and $\beta \geq 1$, as observed in practice.
Therefore,

$$\frac{R_{EQ}}{r} = 1.27\,(2.22)\,(1.27 - 0.3)^{1/2} = 2.78.$$
$$\phi = (2.78 - 1)\,[1 + 0.35 \ln (2.78)] = 2.42.$$

The fin efficiency is

$$\eta_o = \frac{\tanh\left(16.3 \text{ ft}^{-1} \times 0.0282 \text{ ft} \times 2.42\right)}{16.3 \text{ ft}^{-1} \times 0.0282 \text{ ft} \times 2.42} = 0.724.$$

Thus, the fin effectiveness is

$$\eta_{so} = 1 - 0.950\,(1 - 0.724) = 0.737.$$

Find the $\frac{A_i}{A_o}$ ratio. Remember that $\alpha = \frac{A_o}{\Omega}$. From the j-factor versus Re_{G_m} chart, $\alpha = 169 \text{ ft}^2/\text{ft}^3$. Assume that the tubes fill the volume bounded by $x_t x_L L_{\text{tube}}$. Hence,

$$\frac{A_i}{\Omega} \approx \frac{\pi D_i L_{\text{tube}}}{x_t x_L L_{\text{tube}}} = \frac{\pi D_i}{x_t x_L}$$
$$\frac{A_i}{A_o} \approx \frac{\left(\frac{A_i}{\Omega}\right)}{\left(\frac{A_o}{\Omega}\right)} \approx \frac{\frac{\pi D_i}{x_t x_L}}{\alpha} = \frac{\pi D_i}{\alpha x_t x_L}$$
$$\frac{A_i}{A_o} \approx \frac{\pi\,(0.596 \text{ in.})}{\left(169 \text{ ft}^2/\text{ft}^3\right)(1.50 \text{ in.})\,(1.75 \text{ in.})} \times \frac{12 \text{ in.}}{1 \text{ ft}} = 0.0507.$$

The overall heat transfer coefficient is

$$\frac{1}{U_{FT}} = \frac{1}{\left(1183 \text{ Btu}/(\text{h ft}^2\,{}^\circ\text{F})\right)(0.0507)} + \left(\frac{0.00114}{0.0507} + 0.00199\right)(\text{h ft}^2\,{}^\circ\text{F})/\text{Btu}$$
$$+ \frac{1}{\left(17.7\,(\text{h ft}^2\,{}^\circ\text{F})/\text{Btu}\right)(0.737)}$$
$$\frac{1}{U} = 0.118\,(\text{h ft}^2\,{}^\circ\text{F})/\text{Btu}$$
$$\boldsymbol{U = 8.49\ \textbf{Btu}/(\textbf{h ft}^2\ {}^\circ\textbf{F})}$$

Determine the configuration, number of coils, and dimensions of the heat exchanger. Find the total surface area of the tubes by using the ε-NTU method:

$$A_o = \frac{C_{min}\text{NTU}}{U}.$$

From Example 4.2,

$$C_a = C_c = C_{min} = 35.5 \text{ Btu/(min °F)}$$
$$C_w = C_h = C_{max} = 177.5 \text{ Btu/(min °F)}$$
$$\boldsymbol{c = 0.20}$$
$$\boldsymbol{\varepsilon = 0.50}$$
$$\mathbf{\dot{Q} = 1775\ Btu/min = 106500\ Btu/h}$$
$$\mathbf{NTU \approx 0.75}.$$

The heat transfer surface area is

$$A_o = \frac{(35.5 \text{ Btu/(min °F)})\,(0.75)}{8.49 \text{ Btu/(h ft}^2\text{ °F)}} \times \frac{60 \text{ min}}{1 \text{ h}}$$
$$A_o = 188 \text{ ft}^2.$$

This value for the heat transfer surface area includes the extended surface area of the fins.

The total heat transfer volume is found from the ratio of the heat transfer area to the total volume:

$$\alpha = \frac{A_o}{\Omega}.$$

Therefore,

$$\Omega = \frac{A_o}{\alpha} = \frac{188 \text{ ft}^2}{169 \text{ ft}^2/\text{ft}^3} = 1.11 \text{ ft}^3.$$

The depth of the heat exchanger system (dimension in the direction of airflow) is

$$W = \frac{\Omega}{A_f}.$$

A_f is the face area normal to the airflow direction, and is equal to 2 ft^2.

Therefore,

$$W = \frac{1.11 \text{ ft}^3}{2 \text{ ft}^2} \times \frac{12 \text{ in.}}{1 \text{ ft}}$$
$$\mathbf{W = 6.7\ in. \approx 7\ in.}$$

From the *j*-factor versus Re_{G_m} chart, the longitudinal tube pitch is 1.75 in. The number of tube rows is

$$N_r = \frac{W}{x_L} = \frac{6.7\text{ in.}}{1.75\text{ in.}}$$

$$\mathbf{N_r = 3.8 \approx 4\ tube\ rows}.$$

Determine the number of tubes per row. From Example 4.2,

$$N_{\text{tube}} = \frac{4C_h}{c_{p,w}\rho_w \overline{V}_w \pi D_i^2}$$

$$N_{\text{tube}} = \frac{4\,(177.5\text{ Btu/(min °F)})}{(1.0\text{ Btu/(lb °F)})\left(61\text{ lb/ft}^3\right)(4\text{ ft/s})\,\pi\,(0.596\text{ in.})^2} \times \left(\frac{12\text{ in.}}{1\text{ ft}}\right)^2 \times \frac{1\text{ min}}{60\text{ s}}$$

$$\mathbf{N_{tube} = 6.2 \approx 6\ tubes\ per\ row}.$$

The total number of tubes along the length of the heat exchanger will be 24 tubes (4 rows × 6 tubes per row). If each row has six tubes, the height of the heating coil is

$$H = N_{\text{tube}} x_t = (6.2)(1.50\text{ in.})$$

$$\mathbf{H = 9.3\ in. \approx 10\ in.}$$

The length of the heat exchanger is

$$L = \frac{A_f}{H} = \frac{2\text{ ft}^2}{9.3\text{ in.}} \times \left(\frac{12\text{ in.}}{1\text{ ft}}\right)^2$$

$$\mathbf{L = 30.96\ in. \approx 31\ in.}$$

The total length of the tubes in the heat exchanger is 62 ft (31 in. per tube × 24 tubes).

Pressure loss of air across the tube coils

The pressure drop of the air across the tube coil bank is given by

$$\Delta P_{\text{bank}} = \frac{G_m^2}{2\rho_{a,i}}\left[\left(1+\sigma^2\right)\left(\frac{\rho_{a,i}}{\rho_{a,o}} - 1\right) + f\,\frac{A_T}{A_c}\frac{\rho_{a,i}}{\rho_{\text{mean}}}\right].$$

Remember: $G_m = 2.56$ lb/s ft^2 and $\mathrm{Re}_{G_m} \approx 2.3 \times 10^3$.
From the *j*-factor versus Re_{G_m} chart, the friction factor (*f*) is

$$f \approx 0.016.$$

The area ratio is

$$\frac{A_T}{A_c} = \frac{4W}{D_h} = \frac{4\,(6.7\text{ in.})}{0.0114\text{ ft}} \times \frac{1\text{ ft}}{12\text{ in.}} = 195.9.$$

From Example 4.2, the mean density (ρ_{mean}) is $\rho_{mean} = 0.075\ \text{lb/ft}^3$. From the j-factor versus Re_{G_m} chart, $\sigma = 0.481$.

Therefore,

$$\Delta P_{bank} = \frac{(2.56\ \text{lb/(s ft}^2))^2}{2(0.078\ \text{lb/ft}^3)}\left[(1+0.481^2)\left(\frac{0.078\ \text{lb/ft}^3}{0.071\ \text{lb/ft}^3}-1\right)+(0.016)\,(195.9)\,\frac{0.078\ \text{lb/ft}^3}{0.075\ \text{lb/ft}^3}\right]$$

$$\Delta P_{bank} = 142\ \text{lb/ft s}^2 \times \frac{1\ \text{lbf}}{32.2\ \text{lb ft/s}^2} = 4.4\ \text{lbf/ft}^2 = 4.4\ \text{psf}.$$

In practice, the pressure drop is reported in inches of water. Therefore,

$$\Delta P_{bank} = 142\ \text{lb/ft s}^2 \times \frac{1}{0.075\ \text{lb/ft}^3} \times \frac{1}{32.2\ \text{ft/s}^2} = 58.8\ \text{ft of air}$$

$$\Delta P_{bank} = 58.8\ \text{ft of air} \times SG_{air,75^\circ F} = 58.8\ \text{ft of air} \times \frac{0.075\ \text{lb/ft}^3}{62\ \text{lb/ft}^3} \times \frac{12\ \text{in.}}{1\ \text{ft}}$$

$$\boldsymbol{\Delta P_{bank} = 0.85\ \textbf{in. of water} = 0.85\ \textbf{in. wg}}.$$

Pressure loss of water in the tube coils

Determination of the pressure loss of the water in the tubes is needed to find the pump power required to move the fluid through the heat exchanger system. The total head loss is

$$H_{lT} = \left(f\,\frac{L_{tube}}{D_i} + K\right)\frac{\overline{V}_w^2}{2g}.$$

From the Moody chart, for $\text{Re}_{D_i} = 41789$ and for copper tubes with relative roughness, $\frac{\varepsilon}{D_i} = 0.0001$, the friction factor is $f \approx 0.022$.

For the primary piping circuitry through the heat exchanger, there are four tube rows, including 1 supply run and 1 return run, each 31 in. long.

Therefore,

$$L_{tube} = (4\ \text{rows})(31\ \text{in. per row}) = 124\ \text{in.} = 10.3\ \text{ft}.$$

For the minor losses, $K = 2.0$ for the soldered/brazed 180° return bends.

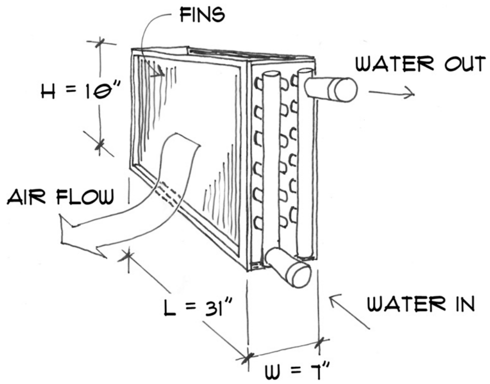
Hence,

$$H_{lT} = \left((0.022)\,\frac{124\ \text{in.}}{0.596\ \text{in.}} + (3)\,(2.0)\right)\frac{(4\ \text{ft/s})^2}{2\left(32.2\ \text{ft/s}^2\right)}$$

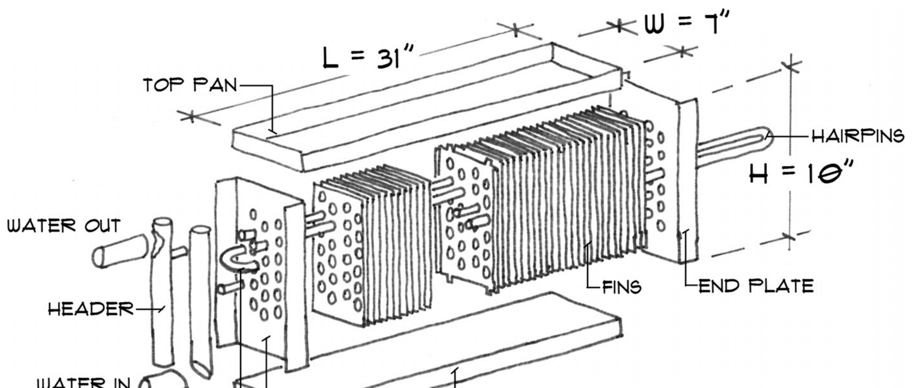
$$\boldsymbol{H_{lT} = 2.6\ \textbf{ft wg}}.$$

Drawings

A general schematic drawing of the heating coil is shown.



A detailed schematic drawing that shows an exploded view of the heat exchanger is also shown. It is not typical to show, in routine design problems, this type of schematic drawing with an exploded view. However, it will be required for fabrication of the coil. Additional information on dimensions would also be required.



Conclusions

A heating coil heat exchanger complete with finned tubes has been designed. Improvements in the performance have been achieved by using finned tubes. The following points should be noted:

(i) G_m was lowered by choosing larger tube spacings, given the high airflow rate.
(ii) The number of rows and the number of tubes per row decreased compared with the heat exchanger with bare tubes. In addition, this heat exchanger is more compact with $\alpha = 169\ \text{ft}^2/\text{ft}^3$ compared with $64.4\ \text{ft}^2/\text{ft}^3$ for the heat exchanger with bare tubes. The

total length of tube required decreased from 410 ft in the bare tube heat exchanger design to 62 ft in the finned-tube heat exchanger.

(iii) The calculated value of N_r was 3.8. It was assumed that N_r was equal to 4. However, the calculated value of N_{tube} was 6.2. It was assumed that N_{tube} was equal to 6. While it is typical to increase to the next whole number for more conservative designs, the compromise between N_r and N_{tube} should mitigate any significant error.

(iv) Lower pressure drop of the air across the tube bank was achieved. In particular, the pressure drop was on the order of 1 in. wg.

(v) The use of headers in this design resulted in low total head loss in the heating coil piping.

A heat exchanger design data sheet is shown in the following table.

Heat Exchanger Design Data Sheet

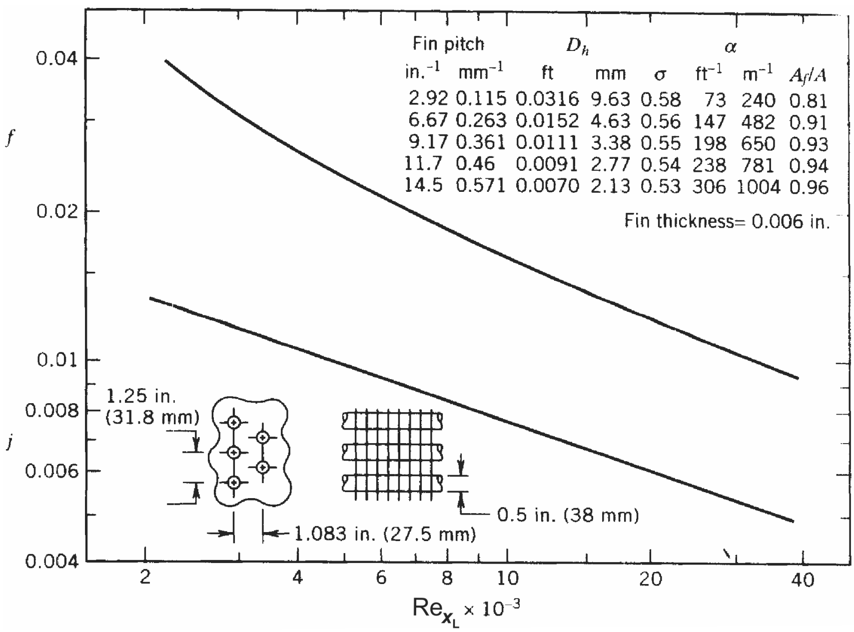
Type	Counter Flow
Section: Tube Bank	
Working fluid	Air
Volume flow rate	2000 cfm
Inlet temperature	50°F
Outlet temperature	100°F
Pressure drop	0.85 in. wg
Section: Tube	
Tube material	Copper
Working fluid	Water
Velocity	4 ft/s
Tube inner diameter	0.596 in.
Tube outer diameter	0.676 in.
Number of tube rows	4
Number of tubes per row	6
Tube spacing ($x_t \times x_L$)	1.50 in. × 1.75 in.
Total tube length	62 ft
Inlet temperature	150°F
Outlet temperature	140°F
Head loss	2.6 ft wg
Fin material	Aluminum
Fin pitch	7.75 fins per in.
Fin thickness	0.016 in.
Heat Exchanger Parameters	
Thermal capacity	106500 Btu/h
Effectiveness	0.50
Capacity ratio	0.20
Overall heat transfer coefficient	8.49 Btu/(h ft^2 °F)
Number of transfer units (NTU)	0.75
Heat exchanger dimensions ($L \times H \times W$)	31 in. × 10 in. × 7 in.

Example 4.4 Performance of an Oil Cooler

An oil cooler has been designed for use in a factory. The cooler has one tube inlet and one tube outlet into and out of a tube bank, respectively, to form an oil recycling system. Clean, unused engine oil is circulated through ½-in. tubes that are connected with continuous-plate fins with a fin pitch of 8 fins per in. Cool air is forced over the tubes. The system operates under the following conditions:

(i) Four rows of tubes
(ii) Six tubes per row
(iii) Width is 26 in.
(iv) Fin thickness is 0.006 in.
(v) Entering air temperature is 65°F
(vi) Coil face velocity is 650 ft/min
(vii) Entering engine oil temperature is 150°F

The design engineer who designed the original system used the following chart to aid in the design:



Source: ASHRAE Transactions, Vol. 79, Part II, 1973 (Reprinted with permission)

Analyze the performance of the oil cooler and comment on the design.

Possible Solution

Definition

Analyze the performance of a finned-tube oil cooler (heat exchanger).

Preliminary Specifications and Constraints

(i) The oil cooler is a heat exchanger with one tube inlet and one tube outlet.
(ii) The working fluid in the tubes is clean, unused engine oil.
(iii) The working fluid outside the tubes is cool air.
(iv) The tubes have a nominal diameter of $^1/_2$ in.
(v) This is a finned-tube heat exchanger with a fin pitch of 8 fins per in.
(vi) Additional specifications are given in the problem preamble.

Detailed Design

Objective

To determine the thermal capacity, outlet temperatures, and pressure drops in a finned-tube oil cooler.

Data Given or Known

(i) The working fluid in the tubes is clean, unused engine oil.
(ii) The working fluid outside the tubes is cool air.
(iii) The tubes have a nominal diameter of $^1/_2$ in.
(iv) This is a finned-tube heat exchanger with a fin pitch of 8 fins per in. and fin thickness of 0.006 in.
(v) There are four rows of tubes with six tubes per row.
(vi) The width of the oil cooler is 26 in.
(vii) The coil velocity (of air) is 650 fpm.
(viii) The entering air temperature is 65°F.
(ix) The entering engine oil temperature is 150°F.
(x) A *j*-factor vs Re_{x_L} chart was provided.

Assumptions/Limitations/Constraints

(i) Let the tube material be Type L copper. Copper has high heat transfer properties.
(ii) Let the fin material be aluminum. This material is typically for the fabrication of fins for heating or cooling coils.
(iii) Let the flow velocity of engine oil in the copper tubes be 3 fps. This velocity is lower than the erosion limit of copper with unused engine oil (≈7 fps calculated for "Other Liquids" from Table A.13). A low tube velocity is desired since engine oil is highly viscous at lower temperatures. As the engine oil cools, its viscosity will increase, and it will require higher pump power to move through the coils.
(iv) The tube thickness is approximately 0.04 in (see Table A.6). It will be assumed that the tube wall thickness is negligible compared to the inner and outer diameters.
(v) The return bends will be soldered 180° return bends (regular).
(vi) There is negligible elevation head. Assume that all the components are on the same level.
(vii) Entrance and exit losses of the air over the coils will be negligible compared to other losses across the coils.
(viii) This is a counter flow arrangement. The inlet and exit of the coils are on the same side of the cooler.

Sketch

A sketch is not needed for this performance analysis.

Performance Analysis

Determine the overall heat transfer coefficient (UFT)

The ε-NTU method will be used in this performance problem. To determine the effectiveness and NTU, the overall heat transfer coefficient (U_{FT}) must be determined first. The overall heat transfer coefficient for finned tubes is

$$\frac{1}{U_{FT}} = \frac{1}{h_i\,(A_i/A_o)} + \frac{R_{fi}}{(A_i/A_o)} + R_{fo} + \frac{1}{h_o \eta_{so}}.$$

Appropriate engine oil properties for the determination of Re_D and Pr will be found at 150°F. Note that the average temperature should be used, rather than the entering temperature. This will introduce a small error in the analysis. This will be verified. The inner diameter of the copper tube is approximately 0.545 in. (Table A.6).

The Reynolds number is

$$Re_{D_i} = \frac{\rho \overline{V} D_i}{\mu} = \frac{\left(53.73\ \text{lb/ft}^3\right)(3\ \text{ft/s})(0.545\ \text{in.})}{3.833 \times 10^{-2}\ \text{lb/(fts)}} \times \frac{1\ \text{ft}}{12\ \text{in.}} = 191.$$

The Prandtl number is

$$Pr = 848.3.$$

Assuming a constant surface heat flux from the tubes, and since the flow is laminar, the Nusselt number and heat transfer coefficient for flow in the tube is

$$\begin{aligned}
Nu &= \frac{\overline{h}_i D_i}{k} = 4.36 \\
\overline{h}_i &= \frac{4.36k}{D_i} \\
\overline{h}_i &= \frac{4.36\,(0.08046\ \text{Btu/(h ft R)})}{0.545\ \text{in.}} \times \frac{12\ \text{in.}}{1\ \text{ft}} = 7.724\ \text{Btu/(h ft R)} = 7.724\ \text{Btu/(h ft °F)}.
\end{aligned}$$

The fouling resistances can be found from Table C.3.

For clean, unused engine oil: $R_{fi} = 0$.
Assume compressed air: $R_{fo} = 0.00199\ (\text{h ft}^2\ °\text{F})/\text{Btu}$.

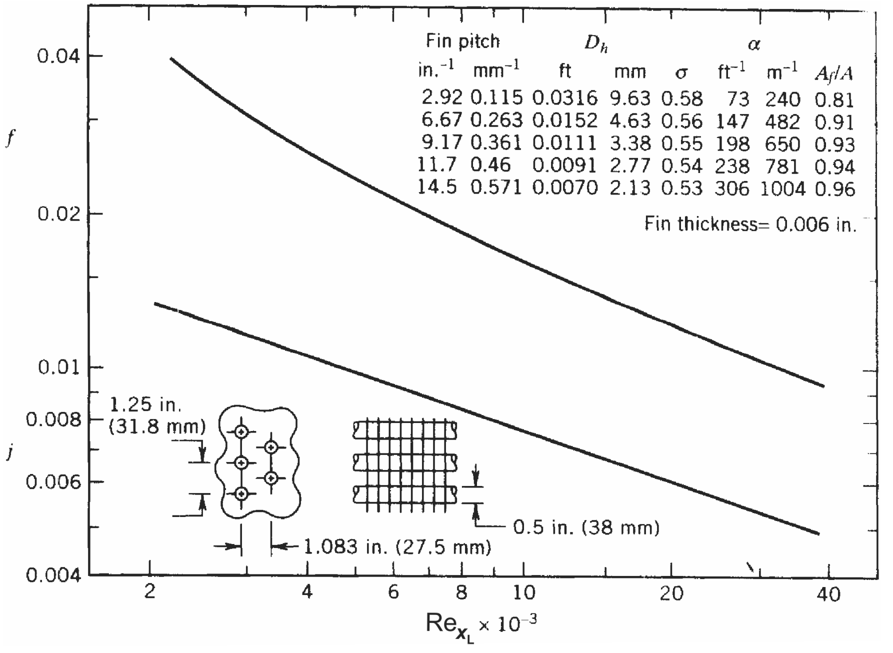
The average heat transfer coefficient for airflow over the finned-tube bank is found from charts and appropriate correlation equations:

$$\overline{h}_o = \frac{j G_m c_p}{Pr^{2/3}} \text{ for air at 65°F,}$$

and the Chilton–Colburn j-factor is

$$j = \frac{\overline{h}_o}{G_m c_p} \mathrm{Pr}^{2/3}.$$

The j-factor versus Re_{G_m} chart provided by the design engineer will be used. Note that the longitudinal pitch is 1.083 in. and the traverse pitch is 1.25 in.



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Remember that air properties are found at 65°F. So, G_m is

$$G_m = \frac{\rho_a \overline{V}_a}{\sigma} = \frac{\left(0.07561\ \mathrm{lb/ft^3}\right)(650\ \mathrm{ft/min})}{0.555} \times \frac{1\ \mathrm{min}}{60\ \mathrm{s}} = 1.48\ \mathrm{lb/(s\ ft^2)}.$$

The Reynolds number based on G_m and x_L is

$$\mathrm{Re}_{x_L} = \frac{G_m x_L}{\mu_a} = \frac{\left(1.48\ \mathrm{lb/(s\ ft^2)}\right)(0.09025\ \mathrm{ft})}{1.221 \times 10^{-5}\ \mathrm{lb/(ft\ s)}} = 10939 \approx 11 \times 10^3.$$

Note that the values of ρ_a, σ, D_h, and μ were determined through interpolation. From the chart, $j \approx 0.0073$.

Therefore,

$$\overline{h}_{\mathrm{o}} = \frac{jG_{\mathrm{m}}c_{\mathrm{p}}}{\mathrm{Pr}^{2/3}}$$

$$\overline{h}_{\mathrm{o}} = \frac{(0.0073)\left(1.48\ \mathrm{lb/(s\ ft^2)}\right)(0.2404\ \mathrm{Btu/(lb\ ^\circ F)})}{(0.73)^{2/3}} \times \frac{3600\ \mathrm{s}}{1\ \mathrm{h}}$$

$$\overline{h}_{\mathrm{o}} = 11.5\ \mathrm{Btu/(h\ ft^2\ ^\circ F)}.$$

The fin surface effectiveness is

$$\eta_{\mathrm{so}} = 1 - \frac{A_{\mathrm{fin}}}{A}(1-\eta_{\mathrm{o}}).$$

The fin efficiency is

$$\eta_{\mathrm{o}} = \frac{\tan h\,(mr\phi)}{mr\phi}.$$

For aluminum, $k_{\mathrm{fin}} = 100$ Btu/(hr ft °F). For the fins, the thickness is $t = 0.006$ in., as obtained from the j-factor versus $\mathrm{Re}_{x_{\mathrm{L}}}$ chart. Therefore,

$$m = \frac{(2\overline{h}_{\mathrm{o}})^{1/2}}{(k_{\mathrm{fin}}t)^{1/2}} = \left[\frac{2\left(22.1\ \mathrm{Btu/(h\ ft^2\ ^\circ F)}\right)}{\left(100\ \mathrm{Btu/(h\ ft\ ^oF)}\right)(0.006\ \mathrm{in.})\left(\dfrac{1\ \mathrm{ft}}{12\ \mathrm{in.}}\right)}\right]^{1/2} = 29.7\ \mathrm{ft}^{-1}$$

The tube outer radius is

$$r = \frac{0.625\ \mathrm{in.}}{2} \times \frac{1\ \mathrm{ft}}{12\ \mathrm{in.}} = 0.0260\ \mathrm{ft}.$$

Find ϕ.

$$\phi = \left(\frac{R_{\mathrm{EQ}}}{r} - 1\right)\left[1 + 0.35\ln\left(\frac{R_{\mathrm{EQ}}}{r}\right)\right]$$

$$\frac{R_{\mathrm{EQ}}}{r} = 1.27\psi\,(\beta - 0.3)^{1/2}$$

$$\psi = \frac{M}{r} = \frac{x_{\mathrm{t}}}{2} * \frac{1}{r} = \frac{x_{\mathrm{t}}}{D} = X_{\mathrm{t}} = \frac{1.25\ \mathrm{in.}}{0.625\ \mathrm{in.}} = 2.0$$

$$\beta = \frac{L}{M}$$

$$M = \frac{x_{\mathrm{t}}}{2} = \frac{1.25\ \mathrm{in.}}{2} = 0.625\ \mathrm{in.}$$

$$L = \frac{\left[\left(\dfrac{x_{\mathrm{t}}}{2}\right)^2 + x_{\mathrm{L}}^2\right]^{1/2}}{2} = \frac{\left[\left(\dfrac{1.25\ \mathrm{in.}}{2}\right)^2 + (1.083\ \mathrm{in.})^2\right]^{1/2}}{2} = 0.625\ \mathrm{in.}$$

Therefore,

$$\beta = \frac{0.625 \text{ in.}}{0.625 \text{ in.}} = 1.0.$$

Note that $L \geq M$ and $\beta \geq 1$, as observed in practice.
Thus,

$$\frac{R_{\mathrm{EQ}}}{r} = 1.27\,(2.0)\,(1.0 - 0.3)^{1/2} = 2.13.$$

$$\phi = (2.13 - 1)\,[1 + 0.35 \ln(2.13)] = 1.43.$$

The fin efficiency is

$$\eta_{\mathrm{o}} = \frac{\tanh\left(29.7 \text{ ft}^{-1} \times 0.0260 \text{ ft} \times 1.43\right)}{29.7 \text{ ft}^{-1} \times 0.0260 \text{ ft} \times 1.43} = 0.727.$$

Therefore, the fin effectiveness is

$$\eta_{\mathrm{so}} = 1 - 0.921\,(1 - 0.727) = 0.749.$$

Note that $\frac{A_{\mathrm{fin}}}{A}$ was interpolated from data shown in the chart provided by the design engineer.

Find the $\frac{A_{\mathrm{i}}}{A_{\mathrm{o}}}$ ratio. Remember that $\alpha = \frac{A_{\mathrm{o}}}{\Omega}$. From the j-factor versus $\mathrm{Re}_{x_{\mathrm{L}}}$ chart and interpolation, $\alpha = 174 \text{ ft}^2/\text{ft}^3$. Assume that the tubes fill the volume bounded by $x_{\mathrm{t}} x_{\mathrm{L}} L_{\mathrm{tube}}$.

Therefore,

$$\frac{A_{\mathrm{i}}}{\Omega} \approx \frac{\pi D_{\mathrm{i}} L_{\mathrm{tube}}}{x_{\mathrm{t}} x_{\mathrm{L}} L_{\mathrm{tube}}} = \frac{\pi D_{\mathrm{i}}}{x_{\mathrm{t}} x_{\mathrm{L}}}$$

$$\frac{A_{\mathrm{i}}}{A_{\mathrm{o}}} \approx \frac{\left(\dfrac{A_{\mathrm{i}}}{\Omega}\right)}{\left(\dfrac{A_{\mathrm{o}}}{\Omega}\right)} \approx \frac{\dfrac{\pi D_{\mathrm{i}}}{x_{\mathrm{t}} x_{\mathrm{L}}}}{\alpha} = \frac{\pi D_{\mathrm{i}}}{\alpha x_{\mathrm{t}} x_{\mathrm{L}}}$$

$$\frac{A_{\mathrm{i}}}{A_{\mathrm{o}}} \approx \frac{\pi\,(0.545 \text{ in.})}{\left(174 \text{ ft}^2/\text{ft}^3\right)(1.25 \text{ in.})\,(1.083 \text{ in.})} \times \frac{12 \text{ in.}}{1 \text{ ft}} = 0.0872.$$

The overall heat transfer coefficient is

$$\frac{1}{U_{\mathrm{FT}}} = \frac{1}{h_{\mathrm{i}}\,(A_{\mathrm{i}}/A_{\mathrm{o}})} + \frac{R_{\mathrm{fi}}}{(A_{\mathrm{i}}/A_{\mathrm{o}})} + R_{\mathrm{fo}} + \frac{1}{h_{\mathrm{o}} \eta_{\mathrm{so}}}$$

$$\frac{1}{U_{\mathrm{FT}}} = \frac{1}{(7.724 \text{ Btu}/(\text{h ft}^2\,{}^\circ\text{F}))\,(0.0872)} + 0.00199 \text{ (h ft}^2\,{}^\circ\text{F)/Btu} + \frac{1}{(11.5(\text{h ft}^2\,{}^\circ\text{F})/\text{Btu})\,(0.749)}$$

$$\frac{1}{U} = 1.60 \text{ (h ft}^2\,{}^\circ\text{F)/Btu}$$

$$\boldsymbol{U = 0.62 \textbf{ Btu}/(\textbf{h ft}^2\,{}^\circ\textbf{F})}.$$

Determine the NTU of the heat exchanger.

The NTU is

$$\mathrm{NTU} = \frac{UA_\mathrm{o}}{C_{\min}}.$$

The heat transfer surface area (A) is needed.

$$\alpha = \frac{A_\mathrm{o}}{\Omega} = 174\ \mathrm{ft}^2/\mathrm{ft}^3$$
$$A_\mathrm{o} = \left(174\ \mathrm{ft}^2/\mathrm{ft}^3\right)\Omega,$$

where Ω is the heat exchanger volume.

The depth of the oil cooler is

$$W = \frac{\Omega}{A_\mathrm{f}},\ \text{and}$$
$$N_\mathrm{r} = \frac{W}{x_\mathrm{L}},\ H = N_{\mathrm{tub}e}\, x_\mathrm{t},\ \ A_\mathrm{f} = LH.$$

Hence,

$$W = N_\mathrm{r} x_\mathrm{L} = (4)\,(1.083\ \mathrm{in.}) = 4.332\ \mathrm{in.} \approx 5\ \mathrm{in.}$$
$$H = N_{\mathrm{tube}} x_\mathrm{t} = (6)(1.25\ \mathrm{in.}) = 7.5\ \mathrm{in.} \approx 8\ \mathrm{in.}$$
$$A_\mathrm{f} = LH = (26\ \mathrm{in.})(7.5\ \mathrm{in.}) = 195\ \mathrm{in.}^2 = 1.35\ \mathrm{ft}^2$$
$$\Omega = WA_\mathrm{f} = (4.332\ \mathrm{in.})\left(195\ \mathrm{in.}^2\right)\ \mathrm{x}\left(\frac{1\ \mathrm{ft}}{12\ \mathrm{in.}}\right)^3 = 0.49\ \mathrm{ft}^3.$$

Thus,

$$A_\mathrm{o} = \left(174\ \mathrm{ft}^2/\mathrm{ft}^3\right)\left(0.49\ \mathrm{ft}^3\right) = 85\ \mathrm{ft}^2.$$

Determine the heat capacities. For air,

$$C_\mathrm{a} = \dot{m}_\mathrm{a} c_{\mathrm{pa}} = \rho_\mathrm{a} \dot{V}_\mathrm{a} c_{\mathrm{pa}} = \rho_\mathrm{a} V_\mathrm{a} A_\mathrm{f} c_{\mathrm{pa}}$$
$$C_\mathrm{a} = \left(0.07561\ \mathrm{lb/ft}^3\right)(650\ \mathrm{ft/min})\left(1.35\ \mathrm{ft}^2\right)(0.2404\ \mathrm{Btu/lb\,^\circ F}) = 16.0\ \mathrm{Btu/(min\,^\circ F)}.$$

Since the outlet temperatures are unknown, the law of conservation energy cannot be used to find the heat capacity of the hot engine oil. The definition of the heat capacity for the engine oil is

$$C_\mathrm{e} = \dot{m}_\mathrm{e} c_{\mathrm{pe}} = \rho_\mathrm{e} \overline{V}_\mathrm{e} c_{\mathrm{pe}} A_{\mathrm{tube}} = \rho_\mathrm{e} \overline{V}_\mathrm{e} c_{\mathrm{pe}} \frac{\pi D_\mathrm{i}^2}{4}$$
$$C_\mathrm{e} = \left(53.73\ \mathrm{lb/ft}^3\right)(3\ \mathrm{ft/s})\,(0.4946\ \mathrm{Btu/(lb\,^\circ F)})\,\frac{\pi\,(0.545\ \mathrm{in.})^2}{4} \times \left(\frac{1\ \mathrm{ft}}{12\ \mathrm{in.}}\right)^2 \times \frac{60\ \mathrm{s}}{1\ \mathrm{min}}$$
$$= 7.75\ \mathrm{Btu/(min\,^\circ F)}.$$

Therefore, $C_a = C_c = C_{max}$ and $C_e = C_h = C_{min}$.

It should be noted that since there is only one inlet tube connection in this heat exchanger, it was not necessary to include N_{tube} in the calculation of A_{tube}.

The NTU is

$$\text{NTU} = \frac{\left(0.62\ \text{Btu/(h ft}^2\ ^\circ\text{F)}\right)\left(85\ \text{ft}^2\right)}{7.75\ \text{Btu/(min}\ ^\circ\text{F)}} \times \frac{1\ \text{h}}{60\ \text{min}}$$

$$\mathbf{NTU = 0.113.}$$

Determine the thermal capacity and outlet temperatures

The thermal capacity and the outlet temperatures can be determined after determination of the effectiveness of the oil cooler.

The capacity ratio is

$$c = \frac{C_{min}}{C_{max}} = \frac{7.75\ \text{Btu/(min}\ ^\circ\text{F)}}{16.0\ \text{Btu/(min}\ ^\circ\text{F)}} = 0.484$$

$$\boldsymbol{c = 0.484.}$$

Therefore,

$$\varepsilon = \frac{1 - \exp\left[-\text{NTU}\,(1-c)\right]}{1 - c\,\exp\left[-\text{NTU}\,(1-c)\right]}$$

$$\varepsilon = \frac{1 - \exp\left[-\,(0.113)\,(1-0.484)\right]}{1 - (0.484)\exp\left[-\,(0.113)\,(1-0.484)\right]}$$

$$\boldsymbol{\varepsilon = 0.104 = 10.4\%}$$

The outlet temperature of the air can be determined:

$$\varepsilon = \frac{C_a}{C_{min}}\frac{(T_{a,o} - T_{a,i})}{(T_{e,i} - T_{a,i})}$$

$$T_{a,o} = T_{a,i} + \varepsilon\frac{C_{min}}{C_a}(T_{e,i} - T_{a,i}) = 65^\circ\text{F} + (0.104)\,\frac{7.75\ \text{Btu/(min}\ ^\circ\text{F)}}{16\ \text{Btu/(min}\ ^\circ\text{F)}}\,(150-65)\ ^\circ\text{F}$$

$$\boldsymbol{T_{a,o} = 69.3^\circ\mathbf{F} = 70^\circ\mathbf{F}.}$$

The thermal capacity of the oil cooler is

$$\dot{Q} = \dot{m}_a c_{pa}\,(T_{a,o} - T_{a,i}) = \rho_a V_a A_f c_{pa}\,(T_{a,o} - T_{a,i})$$

$$\dot{Q} = (0.07561\ \text{lb/ft}^3)(650\ \text{ft/min})(1.35\ \text{ft}^2)\,(0.2404\ \text{Btu/(lb}\ ^\circ\text{F)})\,(70-65)^\circ\text{F} \times \frac{60\ \text{min}}{1\ \text{h}}$$

$$\dot{Q} = 4785\ \text{Btu/h}.$$

The outlet temperature of the engine oil can be determined:

$$\dot{Q} = \dot{m}_e c_{pe}\,(T_{e,i} - T_{e,o}) = C_e\,(T_{e,i} - T_{e,o})$$

$$T_{e,o} = T_{e,i} - \frac{\dot{Q}}{C_e} = 150^\circ\text{F} - \frac{4785\ \text{Btu/h}}{7.75\ \text{Btu/(min}\ ^\circ\text{F)}} \times \frac{1\ \text{h}}{60\ \text{min}}$$

$$\boldsymbol{T_{e,o} = 139.7^\circ\mathbf{F} = 140^\circ\mathbf{F}.}$$

The following will be needed to complete a heat exchanger design data sheet to show the results of the performance analysis:

Airflow rate: $\dot{V}_a = V_a A_f = (650\ \text{ft/min})(1.35\ \text{ft}^2) = 878\ \text{cfm}$;
Total tube length: $L = (4\ \text{rows})(6\ \text{tubes per row})(26\ \text{in. per tube}) = 624\ \text{in.} = 52\ \text{ft}$.

Pressure loss of air across the tube coils

The pressure drop of the air across the tube coil bank is given by

$$\Delta P_{\text{bank}} = \frac{G_m^2}{2\rho_{a,i}}\left[(1+\sigma^2)\left(\frac{\rho_{a,i}}{\rho_{a,o}} - 1\right) + f\frac{A_T}{A_c}\frac{\rho_{a,i}}{\rho_{\text{mean}}}\right].$$

Remember: $G_m = 1.48\ \text{lb/(s ft}^2)$ and $\text{Re}_{x_L} \approx 11 \times 10^3$.
From the *j*-factor versus Re_{x_L} chart, the friction factor (f) is

$$f \approx 0.016.$$

The area ratio is

$$\frac{A_T}{A_c} = \frac{4W}{D_h} = \frac{4\,(4.332\ \text{in.})}{0.0127\ \text{ft}} \times \frac{1\ \text{ft}}{12\ \text{in.}} = 113.7.$$

The mean density (ρ_{mean}) is

$$\rho_{\text{mean}} = \frac{\rho_{a,i} + \rho_{a,o}}{2} = \frac{(0.07561 + 0.07489)\ \text{lb/ft}^3}{2} = 0.07525\ \text{lb/ft}^3.$$

From the *j*-factor versus Re_{x_L} chart, $\sigma = 0.555$.
Therefore,

$$\Delta P_{\text{bank}} = \frac{\left(1.48\ \text{lb/(s ft}^2)\right)^2}{2\left(0.07561\ \text{lb/ft}^3\right)}\left[(1+0.555^2)\left(\frac{0.07561\ \text{lb/ft}^3}{0.07489\ \text{lb/ft}^3} - 1\right) + (0.016)(113.7)\frac{0.07561\ \text{lb/ft}^3}{0.07525\ \text{lb/ft}^3}\right]$$

$$\Delta P_{\text{bank}} = 26.7\ \text{lb/(ft s}^2) \times \frac{1\ \text{lbf}}{32.2\ \text{(lb ft)/s}^2} = 0.828\ \text{lbf/ft}^2 = 0.828\ \text{psf}.$$

In practice, the pressure drop is reported in inches of water. Thus,

$$\Delta P_{\text{bank}} = 26.7\ \text{lb/(ft s}^2) \times \frac{1}{0.07525\ \text{lb/ft}^3} \times \frac{1}{32.2\ \text{ft/s}^2} = 11.1\ \text{ft of air}$$

$$\Delta P_{\text{bank}} = 11.1\ \text{ft of air} \times SG_{\text{air},68^\circ\text{F}} = 11.1\ \text{ft of air} \times \frac{0.07525\ \text{lb/ft}^3}{62\ \text{lb/ft}^3} \times \frac{12\ \text{in.}}{1\ \text{ft}}$$

$$\boldsymbol{\Delta P_{\text{bank}} = 0.16\ \textbf{in. of water} = 0.16\ \textbf{in. wg}.}$$

Pressure loss of water in the tube coils

Determination of the pressure loss of the engine oil in the tubes is needed to find the pump power required to move the fluid through the oil cooler system. The total head loss is

$$H_{\mathrm{lT}} = \left(f\frac{L_{\mathrm{tube}}}{D_{\mathrm{i}}} + K\right)\frac{\overline{V}_{\mathrm{w}}^2}{2g}.$$

For laminar flow in tubes,

$$f = \frac{64}{\mathrm{Re}_{D_{\mathrm{i}}}} = \frac{64}{191} = 0.335.$$

For the piping circuitry through the heat exchanger (excluding the hairpins), there are four tube rows, including one supply run and one return run, each 26 in. long.

Therefore,

$$L_{\mathrm{tube}} = (4\text{ rows})(6\text{ tubes per row})(26\text{ in. per tube}) = 624\text{ in.} = 52\text{ ft.}$$

For the minor losses, $K = 2.0$ for the soldered/brazed 180° return bends.

Thus,

$$H_{\mathrm{lT}} = \left((0.335)\frac{624\text{ in.}}{0.545\text{ in.}} + (3)(6)(2.0)\right)\frac{(3\text{ ft/s})^2}{2\left(32.2\text{ ft/s}^2\right)}$$

$$\boldsymbol{H_{\mathrm{lT}} = 58.6\text{ ft wg}}.$$

Drawings

No drawings are required for this performance analysis.

Conclusions

(i) A data sheet for this heat exchanger (oil cooler) is shown in the following table. This was a performance analysis problem. The following points should be noted:

(ii) The design engineer has designed an oil cooler that may not be capable of sufficiently cooling the engine oil. The temperature drop of the oil was approximately 10°F. The engineer does not have control over the entering air temperature. Therefore, they should increase the size of the heat exchanger to increase the NTU, effectiveness, and the thermal capacity of the cooler.

(iii) Due to the viscosity of the engine oil and the assumed flow velocity (3 fps), the major head loss in the tube is very high. A large pump may be required, probably on the order of 1-½ hp. It would be recommended to decrease the flow velocity to reduce the pump size. However, the tube-side heat transfer coefficient would decrease further, resulting in lower heat transfer performance. A compromise may be needed. Another alternative could be to use inlet and outlet headers on the tubes, rather than having one tube inlet and one tube outlet. Use of the headers would reduce the longest run of piping in the heat exchanger to 104 in. (4 rows at 26 in. per tube), instead of 624 in. and the number of 180° return bends to 3, in lieu of 18. This reduction in length and number of bends would reduce the major head loss to 9.8 ft wg from 58.6 ft wg.

(iv) The pressure drop of the air over the oil cooler coils is low (less than 1 in. wg).

(v) The entering temperatures were used to find the fluid properties throughout the analysis. Since the temperature changes of the air and engine oil were small (less than 8%), the error introduced would be small.

A heat exchanger design data sheet is shown in the following table.

Heat Exchanger Design Data Sheet	
Type	Counter Flow
Section: Tube Bank	
Working fluid	Air
Volume flow rate	878 cfm
Inlet temperature	65°F
Outlet temperature	70°F
Pressure drop	0.16 in. wg
Section: Tube	
Tube material	Copper
Working fluid	Engine oil
Velocity	3 ft/s
Tube inner diameter	0.545 in.
Tube outer diameter	0.625 in.
Number of tube rows	4
Number of tubes per row	6
Tube spacing ($x_t \times x_L$)	1.083 in. × 1.25 in.
Total tube length	52 ft
Inlet temperature	150°F
Outlet temperature	140°F
Head loss	58.6 ft wg
Fin material	Aluminum
Fin pitch	8 fins per in.
Fin thickness	0.006 in.
Heat Exchanger Parameters	
Thermal capacity	4785 Btu/h
Effectiveness	0.104
Capacity ratio	0.484
Overall heat transfer coefficient	0.62 Btu/(h ft^2 °F)
Number of transfer units (NTU)	0.113
Heat exchanger dimensions ($L \times H \times W$)	26 in. × 8 in. × 5 in.

4.8 Manufacturer's Catalog Sheets for Heat Exchanger Selection

The material covered in most of this chapter focuses on developing an in-depth understanding of the fundamental design of heat exchangers. Given that the design of heat exchangers is time-consuming and laborious, in practice, only a select group of engineers will design heat exchangers as per the procedures outlined in this chapter.

Bulletin 20-20.4 / June 2007

The Unico System®

ENGINEERING SPECIFICATIONS

M SERIES HEATING MODULE

Packing List

Carton contains:
- 1 – Cabinet
- 1 – Hook Flange
- 2 – Latch keepers
- 2 – Latches
- 1 – Hot water heating coil *(Optional)*

Gasket as needed.

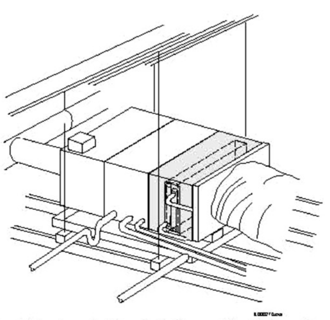
Applications

Unico System designed and built heating units can be easily installed with the matching blower and cooling modules. For matchups see table below. The heating module can be matched to a blower module for a heating only system or it can be matched with both a blower and a cooling module for a system that heats and cools. The slide-in hot water/glycol heating coil is supplied separately. If potable water is used, refer to Technote 112 for disinfection procedures.

Note: The MH2430 replaces the MH2436 and the MH3660 replaces the MH4260. Add HW to Part Number to include the coil, Example MH2430HW (coil included).

Table 1. Compatible Modules

Heating Module	Matching Unit	
	Blower Module	Cooling Module
MH2430	MB2430L	MC2430(C,H,W)
MH3660	MB3642L MB4860L	MC3642(C,H,W) MC4860(C,H,W)



Typical Horizontal Installation with Unico System Blower Module and Cooling Module

Figure 1. Heating Module

Cabinet Construction

The cabinet is constructed of 22 gauge galvanized steel. It has a removable panel to insert a hot water coil. The cabinet is fully lined with closed cell insulation. Easy snap latches are included for quick field assembly with the matching modules.

Coil Construction

Unico designed and fabricated hot water coils are constructed of evenly spaced corrugated aluminum fins mechanically bonded to copper tubes. The tubes are ½-in. diameter on staggered centers. The fins have full collars to provide greater tube-fin contact for excellent heat transfer. The coil is pressure tested at the factory. Bleed and drain valves are provided on the headers outside the cabinet. The coil is sold separately or with the cabinet.

Certified to UL Standard 1995
Conforms to CAN/CSA Standard C22.2 NO. 236

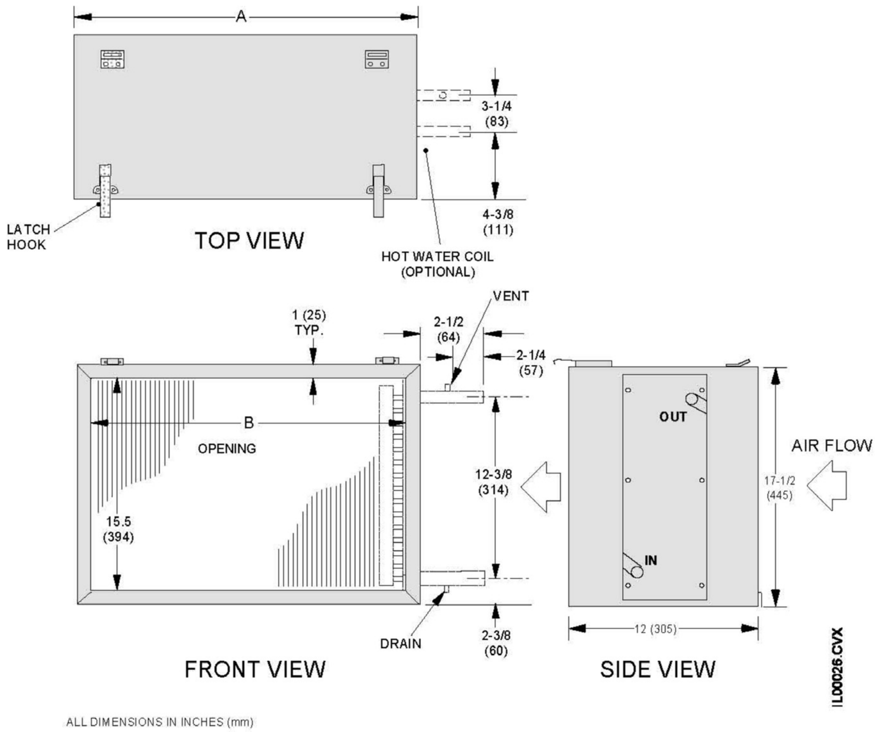
Unico products comply with the European regulations that guarantee the safety of the product.

NOTE — Specifications, Ratings, and Dimensions are subject to change without notice.

Figure 4.24 M series heating coil from Unico, Inc.: (a) page 1 of the M series heating coil from Unico, Inc. (Unico, Inc., reprinted with permission); (b) page 2 of the M series heating coil from Unico, Inc. (Unico, Inc.; reprinted with permission); (c) page 3 of the M series heating coil from Unico, Inc. (Unico, Inc., reprinted with permission); (d) page 4 of the M series heating coil from Unico, Inc. (Unico, Inc.; reprinted with permission)

Model No.		MH2430	MH3660
Heating Coil	Coil Model	HW-2430	HW-3660
	Net Face Area, sq. ft. (m^2)	2.08 (0.20)	3.43 (0.32)
	Tube Diameter, in. (mm)	1/2 (12.7)	1/2 (12.7)
	Number of Rows	4	4
	Fins per inch (m)	12 (472)	12 (472)
	Connection Size, in. (mm) sweat	7/8 (22.2)	7/8 (22.2)
	Coil-only Shipping weight, lb. (kg)	33 (15)	47 (21)
Design Pressure, psi (kPa)		150 (1034)	150 (1034)
Dimensions, in. (mm)	A	25 (635)	38 (965)
	B	23 (584)	36 (914)
Shipping weight (without coil), lb. (kg)		20 (9)	28 (13)
Coil Water Volume, gal. (liters)		0.4 (1.8)	0.7 (3.2)

Module Dimensions



ALL DIMENSIONS IN INCHES (mm)

Figure 4.24 (*Continued*)

Hot Water Coil Performance

Capacity*, MBH (kW)

HW-2430

600 CFM (0.28 m^3/s) — 18 Outlets minimum

Water Flow, GPM (L/s)		Entering Water Temperature, °F (°C)							
		120	(49)	140	(60)	160	(71)	180	(82)
4	(0.25)	26.7	(7.8)	37.5	(11.0)	48.4	(14.2)	59.4	(17.4)
6	(0.38)	27.7	(8.1)	38.9	(11.4)	50.1	(14.7)	61.4	(18.0)
8	(0.50)	28.2	(8.3)	39.5	(11.6)	50.9	(14.9)	62.4	(18.3)

500 CFM (0.24 m^3/s) — 15 Outlets minimum

Water Flow, GPM (L/s)		Entering Water Temperature, °F (°C)							
		120	(49)	140	(60)	160	(71)	180	(82)
4	(0.25)	23.0	(6.7)	32.4	(9.5)	41.8	(12.2)	51.2	(15.0)
6	(0.38)	23.8	(7.0)	33.4	(9.8)	43.0	(12.6)	52.7	(15.4)
8	(0.50)	24.1	(7.1)	33.8	(9.9)	43.6	(12.8)	53.4	(15.6)

400 CFM (0.19 m^3/s) — 12 Outlets minimum

Water Flow, GPM (L/s)		Entering Water Temperature, °F (°C)							
		120	(49)	140	(60)	160	(71)	180	(82)
4	(0.25)	19.3	(5.7)	27.1	(7.9)	34.9	(10.2)	42.8	(12.5)
6	(0.38)	19.7	(5.8)	27.7	(8.1)	35.6	(10.4)	43.6	(12.8)
8	(0.50)	19.9	(5.8)	27.9	(8.2)	36.0	(10.5)	44.0	(12.9)

WARNING
To prevent injury or damage from high temperatures, do not install floor outlets when operating in the shaded area. Discharge temperatures in this range can exceed 160°F (71°C)

EQUATIONS

The general equation of the sensible heat capacity, q, is:

$$q = \rho \dot{Q} c_p (\Delta T) \qquad (1)$$

where ρ is density,

$\dot{Q}$ is the volumetric flow rate,

c_p is the specific heat capacity constant,

and ΔT is temperature difference through the coil.

The temperature difference is expressed differently depending on whether the fluid is being heated or cooled. It is expressed in the following way

$$\text{Heated fluid: } \Delta T = T_{out} - T_{in} \qquad (2)$$

$$\text{Cooled fluid: } \Delta T = T_{in} - T_{out} \qquad (3)$$

where T_{in} is the inlet temperature of the fluid,
and T_{out} is the outlet temperature of the fluid.
The fluid is either air or water.

HW-3660

1250 CFM (0.59 m^3/s) — 37 Outlets minimum

Water Flow, GPM (l/s)		Entering Water Temperature, °F (°C)							
		120	(49)	140	(60)	160	(71)	180	(82)
4	(0.25)	45.0	13.2	63.4	18.6	81.8	24.0	100.4	29.4
6	(0.38)	49.9	14.6	70.2	20.6	90.5	26.5	111	32.6
8	(0.50)	52.5	15.4	73.8	21.6	95.2	27.9	117	34.3
10	(0.63)	54.1	15.9	76.0	22.3	98.0	28.7	120	35.2

1100 CFM (0.52 m^3/s) — 33 Outlets minimum

Water Flow, GPM (l/s)		Entering Water Temperature, °F (°C)							
		120	(49)	140	(60)	160	(71)	180	(82)
4	(0.25)	42.1	12.3	59.2	17.4	76.5	22.4	93.8	27.5
6	(0.38)	46.1	13.5	64.8	19.0	83.6	24.5	102.5	30.1
8	(0.50)	48.2	14.1	67.7	19.9	87.3	25.6	107	31.4
10	(0.63)	49.5	14.5	69.5	20.4	90.0	26.4	110	32.3

1000 CFM (0.47 m^3/s) — 30 Outlets minimum

Water Flow, GPM (l/s)		Entering Water Temperature, °F (°C)							
		120	(49)	140	(60)	160	(71)	180	(82)
4	(0.25)	39.9	11.7	56.2	16.5	72.5	21.3	88.9	26.1
6	(0.38)	43.4	12.7	61	17.9	78.6	23.1	96.3	28.2
8	(0.50)	45.2	13.3	63.4	18.6	81.8	24.0	100	29.3
10	(0.63)	46.2	13.6	64.9	19.0	83.6	24.5	102.3	30.0

900 CFM (0.42 m^3/s) — 27 Outlets minimum

Water Flow, GPM (l/s)		Entering Water Temperature, °F (°C)							
		120	(49)	140	(60)	160	(71)	180	(82)
4	(0.25)	37.6	11.0	52.9	15.5	68.3	20.0	83.7	24.5
6	(0.38)	40.5	11.9	56.9	16.7	73.4	21.5	90	26.4
8	(0.50)	42	12.3	58.9	17.3	76	22.3	93	27.3
10	(0.63)	42.8	12.6	60.1	17.6	77.5	22.7	94.8	27.8

800 CFM (0.38 m^3/s) — 24 Outlets minimum

Water Flow, GPM (l/s)		Entering Water Temperature, °F (°C)							
		120	(49)	140	(60)	160	(71)	180	(82)
4	(0.25)	35	10.3	49.3	14.5	63.5	18.6	77.9	22.8
6	(0.38)	37.3	10.9	52.5	15.4	67.6	19.8	82.8	24.3
8	(0.50)	38.5	11.3	54.1	15.9	69.6	20.4	85.3	25.0
10	(0.63)	39.2	11.5	55.0	16.1	70.8	20.8	86.7	25.4

700 CFM (0.33 m^3/s) — 21 Outlets minimum

Water Flow, GPM (l/s)		Entering Water Temperature, °F (°C)							
		120	(49)	140	(60)	160	(71)	180	(82)
4	(0.25)	32.1	9.4	45.1	13.2	58.2	17.1	71.3	20.9
6	(0.38)	33.9	9.9	48.8	14.3	61.3	18.0	75.1	22.0
8	(0.50)	34.8	10.2	49.5	14.5	62.8	18.4	76.9	22.6
10	(0.63)	35.3	10.4	45.1	13.2	63.7	18.7	77.9	22.8

600 CFM (0.28 m^3/s) — 18 Outlets minimum

Water Flow, GPM (l/s)		Entering Water Temperature, °F (°C)							
		120	(49)	140	(60)	160	(71)	180	(82)
4	(0.25)	28.8	8.4	40.5	11.9	52.2	15.3	63.9	18.7
6	(0.38)	30.1	8.8	42.2	12.4	54.4	16.0	66.6	19.5
8	(0.50)	30.7	9.0	43.1	12.6	55.4	16.2	67.8	19.9
10	(0.63)	31	9.1	43.5	12.8	56	16.4	68.5	20.1

* Capacity is based on 70°F (21°C) return air temperature (T_{in})
Conversion Factors: MBH = 1000 Btu/hr, 1 kW = 3413 Btu/hr

Figure 4.24 (*Continued*)

Bulletin 20-20.4 — Page 4

Equation 1 can be simplified by using standard density and specific heat. If you are at a high altitude please refer to Tech Note 103, *High Altitude Applications*, for more detailed information about effects of air density. Otherwise, use the following equations to find the leaving fluid temperature.

For air:

$q = 1.08\,(\text{CFM})\,\Delta T$ Btu/hr (ΔT is in °F) (4)

$q = 1.21\,(\text{L/s})\,\Delta T$ Watts (ΔT is in °C) (5)

For water:

$q = 500\,(\text{GPM})\,\Delta T$ Btu/hr (ΔT is in °F) (4)

$q = 4.15\,(\text{L/s})\,\Delta T$ kW (ΔT is in °C) (5)

Example. Consider a MH2430 with 6 GPM (38 L/s) at 140 °F (60 °C) and 600 CFM (280 L/s). The capacity from the table is 38.9 MBH (11.6 kW). Therefore, the leaving air temperature (LAT) is as follows:

$$\text{LAT} = 70 + \frac{38.9 \times 1000}{1.08 \times 600} = 130 \quad °F$$

$$\text{LAT} = 21 + \frac{11.4 \times 1000}{1.21 \times 280} = 54.6 \quad °C$$

Likewise, determine the Leaving Water Temperature (LWT) by using one of the following equations:

$$\text{LWT} = 140 - \frac{38.9 \times 1000}{500 \times 6} = 127 \quad °F$$

$$\text{LWT} = 60 - \frac{11.4}{4.15 \times .38} = 52.8 \quad °C$$

Coil Pressure Drop

Air Pressure Drop

Air Flow rate, CFM (m^3/s)	ΔP, in. water (kPa) HW2430		ΔP, in. water (kPa) HW3660	
400 (0.19)	0.07	(0.017)	-	
500 (0.24)	0.10	(0.025)	-	
600 (0.28)	0.12	(0.030)	0.06	(0.015)
700 (0.33)	-		0.08	(0.020)
800 (0.38)	-		0.09	(0.022)
900 (0.42)	-		0.11	(0.027)
1000 (0.47)	-		0.13	(0.032)
1100 (0.52)	-		0.15	(0.037)
1250 (0.59)	-		0.19	(0.047)

Water Pressure Drop

Water Flow rate, GPM (L/s)	ΔP_w, ft. water (kPa) HW2430		ΔP_w, ft. water (kPa) HW3660	
4 (0.25)	4.3	(12.9)	2.6	(7.8)
6 (0.38)	9.3	(27.8)	5.5	(16.4)
8 (0.50)	16.1	(48.1)	9.6	(28.7)
10 (0.63)	-	-	14.6	(43.6)

Entering Water Temperature, ºF (ºC)	120 (49)	140 (60)	160 (71)	180 (82)	200 (93)
F1	1.046	1.000	0.959	0.921	0.888

Water Pressure drop = ΔP_w x F1

Figure 4.24 (*Continued*)

Most design engineers will use **manufacturer's catalog sheets** to select a heat exchanger. The designer would calculate the required thermal capacity, and with other parameters, an appropriate heat exchanger for the application would be selected. In practice, the size (physical dimensions) of the heat exchanger is typically a major factor that drives the final selection of a unit.

Figure 4.24 shows four sheets of a heat exchanger selection bulletin for a heating coil model used in high-velocity duct systems. In the bulletin, the manufacturer presents an overview of their M series heating coil heat exchanger (Figure 4.24a). In this overview, the manufacturer describes the construction of the unit, provides a list of the parts, and shows a drawing of the unit. Detailed drawings and specifications on the heating coil and the construction are also provided (Figure 4.24b). The manufacturer understands that each design will require different thermal capacities, and has provided three different models of hot water heating coils, HW-2430 and HW-3660 (Figure 4.24c). Each model is subdivided based on the flow rate of air across the heating coils and the minimum number of outlets from the duct system. The design engineer should note that to determine the heat transfer performance (i.e., thermal capacity) of the unit, the water flow rate and entering water temperature will need to be known. The final page of the bulletin (Figure 4.24d) shows pressure drops in and across the coils for different pipe velocities and airflow rates, respectively.

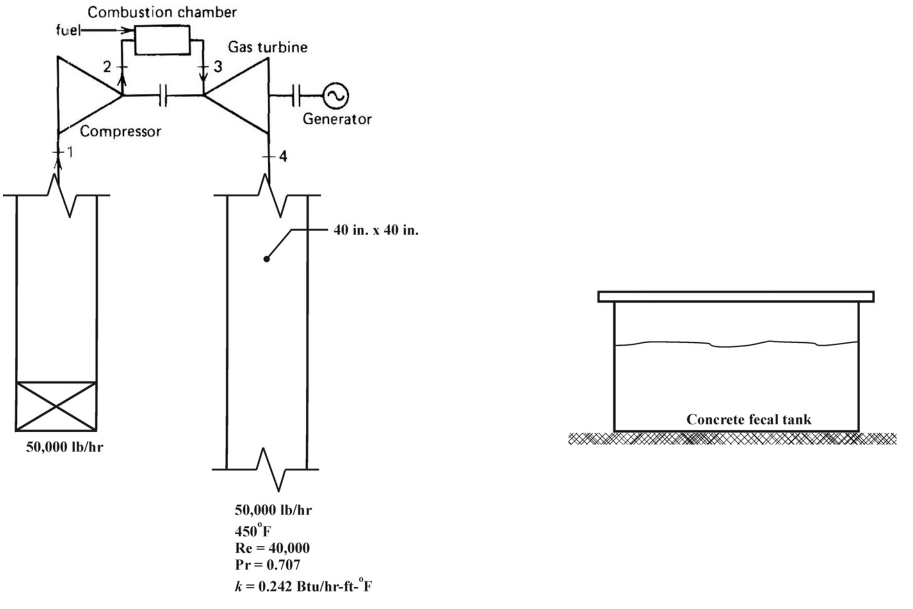
Problems

4.1. The design of a simple refrigerant condenser has been proposed for development. The condenser will accept hot, high-pressure, saturated refrigerant type R-134a from a compressor to reject its heat to ambient air at standard atmospheric conditions. The cooled refrigerant is drained back to an evaporator through an expansion valve to complete a standard refrigeration cycle. The design concept will be to use bare $^3/_8$-in. outer diameter tubes arranged in a staggered tube bank. To simplify the design, there will be only one tube inlet and one tube outlet into and out of the bank, respectively. No front end or rear end headers will be considered. It is hoped that the pressure drop across the tube bank will be kept as low as possible to ensure the smallest possible fan size. The system will operate under the following conditions:

(i) Four rows of tubes
(ii) Sixteen tubes per row
(iii) Width is 32 in.
(iv) Entering air temperature is 95°F
(v) Coil face velocity is not to exceed 700 ft/min
(vi) R-134a enters at a saturation pressure of 200 psia (assume liquid phase)

Determine the heat rejected by the condenser and the temperature of the exiting air.

4.2. The Canadian Biosolids Partnership is considering the use of a small gas turbine with air as the working fluid in a cogeneration system to produce electricity for one of their infrastructures and heat for the bacterial decomposition of fecal matter in a water-based mixture. The pH of the water-based mixture is 7 (neutral; neither acidic or basic). The mixture is moved gently to maintain homogeneity of properties. Of interest is the use of fecal coliform bacteria to decompose human fecal matter in a large uninsulated concrete tank covered and located outside. After decomposition of the fecal matter, the solution must be heated to 170°F. The concrete tank has sides that are 10 ft long and 5 ft high. The construction is 8 in. thick concrete blocks. The cover has similar construction, and the tank is not pressurized. A large mechanical scrubber unit ensures that solid fecal matter never adheres to the concrete wall surface. A high-level switch ensures that the water level in the tank never exceeds $3^1/_2$ ft. The option exists to run a coiled piping system through the exhaust duct of the gas-turbine subsystem. The system would be directly connected to the concrete tank. High efficiency, fine mesh filters could be used at the concrete wall-to-pipe connection points. High-temperature resistant, high-efficiency HEPA filters will be installed near the exhaust of the gas turbine. The total distance between the concrete tank and the gas turbine exhaust duct is limited to 20 ft. Use of this tank will be restricted to the autumn months where the average outdoor temperature is approximately 55°F and wind speeds are low. Ground temperatures are between 40 and 48°F. The given schematic drawing provides additional information. Not all accessories are shown. Design an appropriate heat exchanger subsystem in this cogeneration system to provide enough energy to heat the fecal water to the required temperature.



4.3. Rinnai US has recently developed a new hydronic air handling unit (AHU Series 37AHA) to heat air in residential buildings. It is expected that hot water from a tankless water heater will be pumped through a finned-tube heating coil installed in the main branch (20 in. × 18 in.) of a ductwork system. A homeowner has calculated that 96000 Btu/h will be needed to heat their home as desired. Air will enter the duct at approximately 70°F and should leave at 150°F. Water will enter the coil system at 180°F. Design the hot water heating coil for this application.

4.4. A *superheater* is a counter flow heat exchanger used in power plant systems to transfer heat from hot exhaust flue gases to saturated steam to increase its temperature before entrance to a steam turbine. The tubes containing saturated steam are usually arranged in-line and may be fabricated from low carbon steel, chrome-moly, stainless steel, super-alloys or other types of heat-resistant alloys. The tubes tend to be devoid of extended surfaces. A superheater will be required to produce steam at a rate of 1000000 lb/h at 1900 psia and 1000°F. The hot exhaust flue gas enters the superheater at 2000°F at a rate of 1230000 lb/h. Estimate the superheater size. In other words, estimate the heat transfer surface area, the number of tubes, and the number of tube rows for the following design conditions:

Tube nominal diameter	$2^1/_2$ in.
Tube center to center spacing	7 in.
Typical superheater length	12 ft
Typical superheater width	Variable
Estimated overall heat Transfer coefficient	8.8 Btu/(h ft^2 °F)

Further Information: A consideration of enthalpies may be useful.

4.5. Public Service Enterprise Group (PSEG) is a publicly traded, diversified energy company headquartered in New Jersey. A junior mechanical engineer working with PSEG wishes to size a reheater for use in a plant in Irvington, NJ. The reheater will receive steam at 744 psia and 600°F and release it at 1000°F. Due to losses in the tubing, there is a 7% pressure drop in the steam. The steam flow rate is 4000000 lb/h. Hot flue gases from a burner enter the reheater at 1600°F and 5250000 lb/h to flow over the steam tubes that are arranged in-line. The overall heat transfer coefficient is approximately 8.5 Btu/(h ft^2 °F). Of interest are the estimation of the reheater surface area and the temperature of the flue gas leaving the reheater.

Further Information: A consideration of enthalpies may be useful.

4.6. Fuel oils such as #2 fuel oil may be used in boiler burners to provide energy to the working fluid in the boiler tubes. Due to the high viscosity of the fuel oil, a heater is usually needed in the oil storage tank to warm the oil to facilitate pumping. Heating the oil to at least 150°F will maintain good combustion in the burner. A design engineer wishes to reduce the total power required by the oil storage tank heater. They have devised a system in which hot exhaust gases from the boiler burner will be directed through an 18 in. × 20 in. rectangular ductwork to a heat exchanger for the purposes of providing additional energy to heat the oil. The average flue gas temperature from the burner is on the order of 650°F, and to avoid metal corrosion and proper operation of pollution control equipment, the temperature can never be lower than 480°F. The oil pumping system available to the engineer will fail if the oil temperature is lower than 50°F. Design a heat exchanger that could be used in the engineer's system.

Properties of #2 fuel oil are: $T_{\text{freezing}} = -22°F$; $T_{\text{boiling}} = 374$–$689°F$; $SG = 0.86$; $\nu = 3.66 \times 10^{-5}$ ft^2/s; $c_p = 0.44$ Btu/lb-°F; $k = 0.0797$ Btu/(h ft °F).

4.7. A counter flow evaporator based on R-134a refrigerant has been designed with a capacity of 12 tons of refrigeration. The evaporator is used to cool

50 vol% ethylene glycol solution ($c_p = 0.775$ Btu/(lb °F) that enters the heat exchanger at 55°F with a mass flow rate of 20000 lb/h. The R-134a enters the heat exchanger at 35°F with a quality of 0.1 and leaves with a quality of 0.65. Estimate the exit temperature of the ethylene glycol solution, the mass flow rate of the refrigerant, and the increase in efficiency of the heat exchanger if the surface area of the evaporator were doubled.

Further Information: The R-134a refrigerant has experienced a phase change.

4.8. A low-velocity residential air duct is to be equipped with a manufacturer's heating module. It is expected that the module will temper dry air by heating it from 40°F to 80°F. Hot water from a boiler package is available. Due to inefficiencies in the boiler, the exit water temperatures will range from 140°F to 180°F. Select a manufacturer's heating module to heat the air and prepare an equipment schedule for the client's contract documents. The air duct size will depend on the size of the module unit that is selected by the mechanical engineer.

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