APPENDIX C
Two-Phase Heat Transfer and Pressure Drop Correlations

Although the focus in this book is on single-phase flow heat exchanger design and analysis, there are situations when phase-change (condensation or vaporizing) fluid having negligible thermal resistance is on one fluid side of a two-fluid heat exchanger; the design and analysis for such an exchanger can be done using the slightly modified single-phase theory outlined in this book. However, we need to compute the heat transfer coefficient on the phase-change side even for this situation. Additionally, if one would like to estimate approximately the performance or size of the phase-change exchanger, it can be treated as a single-phase exchanger once the average heat transfer coefficient on the phase-change side is determined. Hence, in this appendix we provide some correlations for condensation and convective boiling. For the detailed information on the phase-change correlations and related phenomena, a comprehensive source is the handbook by Kandilkar et al. (1999). For completeness, we also provide a method to compute the pressure drop on the phase-change side and present it before the heat transfer correlations. Of course, many important topics of phase-change exchangers, such as the phase-change side not having the negligible thermal resistance, rating and sizing of the exchanger when phase change occurs on both fluid sides, flow maldistribution, and so on, are beyond the scope of this appendix and the book.

C.1 TWO-PHASE PRESSURE DROP CORRELATIONS

Due to the phase change during condensation or vaporization, the pressure gradient within the fluid changes along the flow path or axial length. The pressure drop in the phase-change fluid can then be computed by integrating the nonlinear pressure gradient along the flow path. In contrast, the pressure gradient is linear along the flow length (axial direction) in many single-phase flow applications, and hence we generally work directly with the pressure drop since there is no need to compute the pressure gradient in single-phase flow.

The total local pressure gradient in two-phase flow through a one-dimensional duct can be calculated as follows[†]:

$$\frac{dp}{dz} = \frac{dp_{\mathrm{fr}}}{dz} + \frac{dp_{\mathrm{mo}}}{dz} + \frac{dp_{\mathrm{gr}}}{dz} \tag{C.1}$$

[†] Additional symbols used in this appendix are all defined here and are not included in the main nomenclature section.

where the three terms on the right-hand side correspond to the contributions by friction, momentum rate change, and gravity denoted by the subscripts fr, mo, and gr, respectively. The analysis that follows is based on a homogeneous model. The entrance and exit pressure loss terms of single-phase flow [see Eq. (6.28)] are lumped into the Δp_{fr} term since the information about these contributions is not available, due to the difficulty in measurements. The in-tube two-phase frictional pressure drop is computed from the corresponding pressure drop for single-phase flow as follows using the two-phase friction multiplier denoted as φ^2:

$$\left(\frac{dp}{dz}\right)_{\text{fr}} = f_{lo}\,\frac{4}{D_h}\,\frac{G^2}{2g_c\,\rho_l}\,\varphi_{lo}^2 \qquad \text{where} \quad \varphi_{lo}^2 = \frac{(dp/dz)_{\text{fr}}}{(dp/dz)_{\text{fr},lo}} \tag{C.2}$$

where f_{lo} is the single-phase Fanning friction factor (see Tables 7.3 through 7.8) based on the total mass flow rate as liquid and G is also based on the total mass flow rate as liquid; this means that the subscript "lo" indicates the two-phase flow considered as all liquid flow. The subscripts l and g in Eqs. (C.2) and (C.3) denote liquid and gas/vapor phases, respectively, and the subscript lo stands for entire two-phase flow as liquid flow.

Alternatively, $(dp/dz)_{\text{fr}}$ is determined using the liquid or vapor-phase pressure drop multiplier as follows.

$$\left(\frac{dp}{dz}\right)_{\text{fr}} = \left(\frac{dp}{dz}\right)_{\text{fr},l}\varphi_l^2 = \left(\frac{dp}{dz}\right)_{\text{fr},g}\varphi_g^2 \tag{C.3}$$

where

$$\varphi_l^2 = \frac{(dp/dz)_{\text{fr}}}{(dp/dz)_{\text{fr},l}} \quad \varphi_g^2 = \frac{(dp/dz)_{\text{fr}}}{(dp/dz)_{\text{fr},g}} \quad \left(\frac{dp}{dz}\right)_{\text{fr},l} = \frac{4f_l G^2}{2g_c\rho_l D_h} \quad \left(\frac{dp}{dz}\right)_{\text{fr},g} = \frac{4f_g G^2}{2g_c\rho_g D_h} \tag{C.4}$$

where the subscripts l and g denote liquid and gas/vapor phases. φ_{lo}^2 and φ_l^2 or φ_g^2 are functions of the parameter X (*Martinelli parameter*). φ_{go}^2 [defined similar to φ_{lo}^2 of Eq. (C.2), with the subscript lo replaced by go] is a function of Y (*Chisholm parameter*). The X and Y are defined as follows:

$$X^2 = \frac{(dp/dz)_{\text{fr},l}}{(dp/dz)_{\text{fr},g}} \qquad Y^2 = \frac{(dp/dz)_{\text{fr},go}}{(dp/dz)_{\text{fr},lo}} \tag{C.5}$$

Here the subscript go means the total two-phase flow considered as all gas flow. The correlations to determine the two-phase frictional pressure gradient are presented in Table C.1 for various ranges of G and μ_l/μ_g (Kandlikar et al., 1999, p. 228).

The momentum pressure gradient can be calculated integrating the momentum balance equation (Collier and Thome, 1994), thus obtaining

$$\left(\frac{dp}{dz}\right)_{mo} = \frac{d}{dz}\left[\frac{G^2}{g_c}\left(\frac{x^2}{\alpha\rho_g} + \frac{(1-x)^2}{(1-\alpha)\rho_l}\right)\right] \tag{C.6}$$

where α represents the void fraction of the gas (vapor) phase (a ratio of volumetric flow rate of the gas/vapor phase divided by the total volumetric flow rate of the two-phase mixture), and x is the mass quality (a ratio of the mass flow rate of the vapor/gas phase

TABLE C.1 Frictional Multiplier Correlations Used for Determining the Two-Phase Frictional Pressure Gradient in Eq. (C.2)

Correlation	Parameters
Friedel correlation (1979) for $\mu_l/\mu_g > 1000$ and all values of G: $\varphi_{lo}^2 = E + \frac{3.24FH}{\mathrm{Fr}^{0.045} \cdot \mathrm{We}^{0.035}}$ Accuracy for annular flow: ±21% (Ould Dide et al., 2002	$E = (1-x)^2 + x^2 \frac{\rho_l}{\rho_g}\frac{f_{go}}{f_{lo}}$ $F = x^{0.78}(1-x)^{0.24}$ $H = \left(\frac{\rho_l}{\rho_g}\right)^{0.91}\left(\frac{\mu_g}{\mu_l}\right)^{0.19}\left(1-\frac{\mu_g}{\mu_l}\right)^{0.7}$ $\mathrm{Fr} = \frac{G^2}{gd_i\rho_{\mathrm{hom}}^2}$ $\mathrm{We} = \frac{G^2 d_i}{\rho_{\mathrm{hom}}\sigma}$ $\frac{1}{\rho_{\mathrm{hom}}} = \frac{x}{\rho_g} + \frac{1-x}{\rho_l}$ σ = surface tension (N/m)
Chisholm correlation (1973) for $\mu_l/\mu_g > 1000$ and $G > 100\,\mathrm{kg}/m^2 \cdot \mathrm{s}$: $\varphi_{lo}^2 = 1 + (Y^2-1)[Bx^{n^*}(1-x)^{n^*} + x^{1-n}]$ $n^* = \frac{2-n}{2}$ Accuracy for annular flow: ±38% (Ould Didi et al., 2002)	Y defined in Eq. (C.4); $n = \frac{1}{4}$ (exponent in $f = C\,\mathrm{Re}^n$) G = total mass velocity, kg/m^2 s $B = \begin{cases} 4.8 & G < 500 \\ 2400/G & 500 \le G \le 1900 \\ 55/G^{1/2} & G \ge 1900 \end{cases}$ for $0 < Y \le 9.5$ $B = \begin{cases} 520/(YG^{1/2}) & G \le 600 \\ 21/G & G > 600 \end{cases}$ for $9.5 < Y \le 28$ $B = 15{,}000/(Y^2G^{1/2})$ for $Y > 28$
Lockhart-Martinelli correlation (1949) for $\mu_l\mu_g > 1000$ and $G < 100\,\mathrm{kg}/m^2 \cdot \mathrm{s}$: $\varphi_1^2 = \frac{d\rho/dz)_{fr}}{(dp/dz)_l} = 1 + \frac{c}{X} + \frac{1}{X^2}$ $\varphi_g^2 = \frac{(dp/dz)_{fr}}{(dp/dz)_g} = 1 + cX + X^2$ Accuracy for annular flow: ±29% (Ould Didi et al., 2002)	Correlation constant by Chisholm (1967): $c = 20$ for liquid and vapor both turbulent; $c = 10$ for liquid-turbulent, vapor-laminar; $c = 12$ for liquid-laminar, vapor-turbulent; $c = 5$ for liquid and vapor both laminar

divided by the total mass flow rate of the two-phase mixture). Equation (C.6) is valid for constant cross-sectional (flow) area along the flow length. For the homogeneous model, the two-phase flow behaves like a single phase and the vapor and liquid velocities are equal. A number of correlations for the void fraction α are given by Carey (1992) and Kandlikar et al. (1999). An empirical correlation for the void fraction whose general form is valid for several frequently used models is given by Butterworth (Carey, 1992) as

$$\alpha = \left[1 + A\left(\frac{1-x}{x}\right)^p \left(\frac{\rho_g}{\rho_l}\right)^q \left(\frac{\mu_l}{\mu_g}\right)^r\right]^{-1} \tag{C.7}$$

where the constants A, p, q, and r depend on the two-phase model and/or empirical data chosen. These constants for a nonhomogeneous model, based on steam–water data, are $A = 1$, $p = 1$, $q = 0.89$, and $r = 0.18$. For the homogeneous model, $A = p = q = 1$ and $r = 0$. For the Lockhart and Martinelli model, $A = 0.28$, $p = 0.64$, $q = 0.36$, and $r = 0.07$. For engineering design calculations, the homogeneous model yields the best results when the slip velocity between the gas and liquid phases is small (for bubbly or mist flows).

Finally, the pressure gradient due to the gravity (hydrostatic) effect is

$$\left(\frac{dp}{dz}\right)_{\mathrm{gr}} = \pm \frac{g}{g_c} \sin\theta[\alpha\rho_g + (1-\alpha)\rho_l] \tag{C.8}$$

Note that the negative sign (i.e., the pressure recovery) stands for downward flow in inclined or vertical tubes/channels, and the positive sign (i.e., pressure drop) represents upward flow in inclined or vertical tubes/channels. And θ represents the angle of tube/channel inclination measured from the horizontal axis.

C.2 HEAT TRANSFER CORRELATIONS FOR CONDENSATION

Condensation represents a vapor–liquid phase-change phenomenon that usually takes place when vapor is cooled below its saturation temperature at a given pressure. The heat transfer rate per unit heat transfer surface area from the pure condensing fluid to the wall is given by

$$q'' = h_{\mathrm{con}}(T_{\mathrm{sat}} - T_w) \tag{C.9}$$

where h_{con} is the condensation heat transfer coefficient, T_{sat} is the saturation temperature of the condensing fluid at a given pressure, and T_w is the wall temperature. We summarize here the correlations for *filmwise* in-tube condensation, a common condensation mode in

TABLE C.2 Heat Transfer Correlations for Internal Condensation in Horizontal Tubes

Stratification Conditions	Correlation
Annular flow[a] (film condensation) (Shah, 1977), accuracy $\pm$ 14.4% (Kandlikar et al., 1999)	$h_{\mathrm{loc}} = 0.023\dfrac{k_l}{d_i} \cdot \mathrm{Re}_l^{0.8} \cdot \mathrm{Pr}_l^{0.4}\left[(1-x)^{0.8} + \dfrac{3.8x^{0.76}(1-x)^{0.04}}{(p_{\mathrm{sat}}/p_{cr})^{0.38}}\right]$ $\mathrm{Re}_l = \dfrac{Gd_i}{\mu_l}$, G = total mass velocity (kg/m$^2\cdot$s) $0.002 \le p_{\mathrm{sat}}/p_{cr} \le 0.44$ $11 \le G \le 1599\ \mathrm{kg/m^2\cdot s}$ $21 \le T_{\mathrm{sat}} \le 310°\mathrm{C}$, $0 \le x \le 1$, $\mathrm{Pr}_l > 0.5$ $3 \le u_{\mathrm{vap}} \le 300\ \mathrm{m/s}$, no limit on q $7 \le d_i \le 40\ \mathrm{mm}$ $\mathrm{Re}_l > 350$ for circular tubes
Stratified flow (Carey, 1992), accuracy: $\pm$ 18% (Ould Didi et al., 2002)	$h_m = 0.728\left[1 + \dfrac{1-x}{x}\left(\dfrac{\rho_g}{\rho_l}\right)^{2/3}\right]^{-3/4}\left[\dfrac{k_l^3\rho_l(\rho_l-\rho_g)g\mathbf{h}'_{lg}}{\mu_1(T_{\mathrm{sat}}-T_w)d_i}\right]^{1/4}$ where $\mathbf{h}'_{lg} = \mathbf{h}_{lg} + 0.68c_{p,l}(T_{\mathrm{sat}} - T_w)$

[a] Valid for horizontal, vertical, or inclined tubes.

most industrial applications. The two most common flow patterns for convective condensation are annular film flow in horizontal and vertical tubes and stratified flow in horizontal tubes. For annular film flow, the correlation for the local heat transfer coefficient h_{loc} [$h_{\text{con}} = h_{\text{loc}}$ in Eq. (C.9)] is given in Table C.2; and also for stratified flow, the correlation for mean condensation heat transfer coefficient $h_{\text{con}} = h_m$ is given in Table C.2. Shah et al. (1999) provide condensation correlations for a number of noncircular flow passage geometries.

C.3 HEAT TRANSFER CORRELATIONS FOR BOILING

Vaporization (boiling and evaporation) phenomena have been investigated and reported extensively in the literature. In this case, the heat transfer rate per unit heat transfer surface area from the wall to the pure vaporizing fluid is given by

$$q'' = h_{\text{tp}}(T_w - T_{\text{sat}}) \tag{C.10}$$

where h_{tp} is the two-phase heat transfer coefficient during the vaporization process. We present here a most general intube forced convective boiling correlation proposed by Kandlikar (1991). It is based on empirical data for water, refrigerants and cryogens. The correlation consists of two parts, the convective and nucleate boiling terms, and utilizes a fluid–surface parameter. The Kandlikar correlation for the two-phase heat transfer coefficient is as follows:

$$\frac{h_{\text{tp}}}{h_{lo}} = \text{larger of} \begin{cases} [0.6683\text{Co}^{-0.2} \cdot f_2(\text{Fr}_{lo}) + 1058\,\text{Bo}^{0.7} \cdot F_{\text{fl}}](1-x)^{0.8} \\ [1.136\text{Co}^{-0.9} \cdot f_2(\text{Fr}_{lo}) + 667.2\text{Bo}^{0.7} \cdot \text{F}_{\text{fl}}](1-x)^{0.8} \end{cases} \tag{C.11}$$

where

$$h_{lo} = \begin{cases} \dfrac{\text{Re}_{lo} \cdot \text{Pr}_l (f/2)(k_l/d_i)}{1.07 + 12.7(\text{Pr}^{2/3} - 1)(f/2)^{0.5}} & 10^4 \le \text{Re}_{lo} \le 5 \times 10^6 \\[2ex] \dfrac{\text{Re}_{lo} \cdot \text{Pr}_l (f/2)(k_l/d_i)}{1.07 + 12.7(\text{Pr}^{2/3} - 1)(f/2)^{0.5}} & 2300 \le \text{Re}_{lo} \le 10^4 \end{cases} \tag{C.12}$$

$$f_2(\text{Fr}_{lo}) = \begin{cases} (25\text{Fr}_{lo})^{0.3} & \text{for Fr}_{lo} < 0.04 \text{ in horizontal tubes} \\ 1 & \text{for vertical tubes and for Fr}_{lo} \ge 0.04 \text{ in horizontal tubes} \end{cases} \tag{C.13}$$

$$f = \frac{1}{[1.58\ln(\text{Re}_{lo}) - 3.28]^2} \tag{C.14}$$

Here h_{lo} is the single-phase heat transfer coefficient for the entire flow as liquid flow. Also, the convection number Co, the nucleate boiling number Bo, and the Froude number Fr for the entire flow as liquid are defined as follows:

$$\text{Co} = \left(\frac{\rho_g}{\rho_l}\right)^{0.5} \left(\frac{1-x}{x}\right)^{0.8} \qquad \text{Bo} = \frac{q''}{G\mathbf{h}_{\ell g}} \qquad \text{Fr} = \frac{G^2}{\rho_l^2 g d_i} \tag{C.15}$$

TABLE C.3 F_{fl} Recommended by Kandlikar (1991)

Fluid	F_{fl}	Fluid	F_{fl}
Water	1.00	R-114	1.24
R-11	1.30	R-134a	1.63
R-12	1.50	R-152a	1.10
R-13B1	1.31	R-32/R-132 (60%–40% wt.)	3.30
R-22	2.20	Kerosene	0.488
R-113	1.30		

F_{fl} is a fluid–surface parameter and depends on the fluid and the heat transfer surface. F_{fl} values for several fluids in copper tubes are presented in Table C.3. F_{fl} should be taken as 1.0 for stainless tubes. This correlation is valid for either vertical (upward and downward) or horizontal intube flow. A mean deviation of slightly less than 16% with water and 19% with refrigerants has been reported by Kandlikar (1991).

Note that being fluid specific, F_{fl} cannot be used for other fluids (new refrigerants) and mixtures. It is also not accurate for stratified wavy flows and at high vapor qualities since it is not based on the onset of dryout. The Thome model (Kattan et al., 1998; Zrcher et al., 1999), based on a flow pattern map, is recommended for those cases.

REFERENCES

Carey, V. P., 1992, *Liquid-Vapor Phase Change Phenomena*, Taylor & Francis, Bristol, PA.

Chisholm, D., 1967, A theoretical basis for the Lockhart–Martinelli correlation for two-phase flow, *Int. J. Heat Mass Transfer*, Vol. 10, pp. 1767–1778.

Chisholm, D., 1973, Pressure gradients due to friction during the flow of evaporating two-phase mixtures in smooth tubes and channels, *Int. J. Heat Mass Transfer*, Vol. 16, pp. 347–358.

Collier, J. G., and J. R. Thome, 1994, *Convective Boiling and Condensation*, 3rd ed., McGraw-Hill, New York.

Friedel, L., 1979, Improved friction pressure drop correlations for horizontal and vertical two-phase pipe flow, European Two-Phase Flow Group Meeting, Ispra, Italy, Paper E2.

Hewitt, G. F., 1998, Gas–liquid flow, in *Handbook of Heat Exchanger Design*, G. F. Hewitt, ed., Begell House, New York, Sect. 2.3.2.

Kandlikar, S. G., 1991, Development of a flow boiling map for subcooled and saturated flow boiling of different fluids in circular tubes, *ASME J. Heat Transfer*, Vol. 113, pp. 190–200.

Kandlikar, S. G., M. Shoji, and V. K. Dhir, eds., 1999, *Handbook of Phase Change: Boiling and Condensation*, Taylor & Francis, New York.

Kattan, N., J. R. Thome, and D. Favrat, 1998, Flow boiling in horizontal tubes, Part 1; Development of a diabatic two-phase flow pattern map, *ASME J. Heat Transfer*, Vol. 120, pp. 140–147; Part 2; New heat transfer data for five refrigerants, *ASME J. Heat Transfer*, Vol. 120, pp. 148–155; Part 3; Development of a new heat transfer model based on flow patterns, *ASME J. Heat Transfer*, Vol. 120, pp. 156–165.

Ould Didi, M. B., N. Kattan, and J. R. Thome, 2002, Prediction of two-phase pressure gradients of refrigerants in horizontal tubes, *Int. J. Refrig.*, Vol. 25, pp. 935–947.

Shah, M. M., 1977, A general correlation for heat transfer during subcooled boiling in pipes and annuli, *ASHRAE Trans.*, Vol. 83, No. 1, pp. 205–215; also, M. M. Shah, 1982, Chart correlation for saturated boiling heat transfer: equations and further study, *ASHRAE Trans.*, Vol. 88, No. 1, pp. 185–196.

Shah, R. K., S. Q. Zhou, and K. Tagavi, 1999, The role of surface tension in film condensation in extended surface passages, *J. Enhanced Heat Transfer*, Vol. 6, pp. 179–216.

Zrcher, O., J. R. Thome, and D. Favrat, 1999, Evaporation of ammonia in a smooth horizontal tube: heat transfer measurements and predictions, *ASME J. Heat Transfer*, Vol. 121, pp. 89–101.