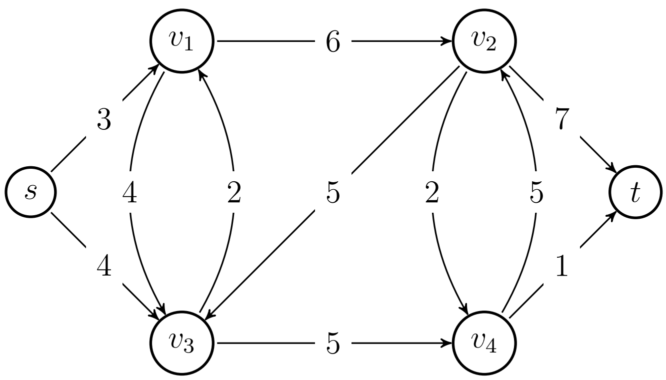
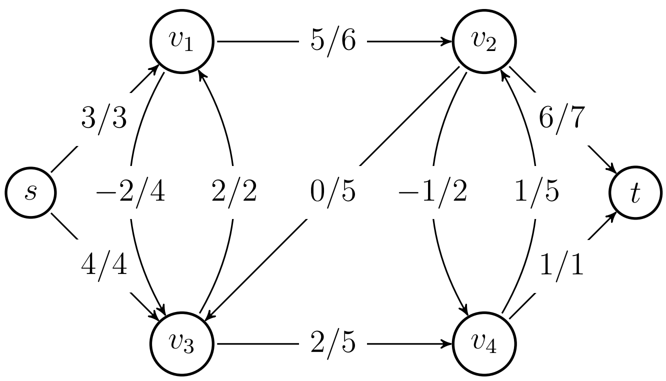
3. **Flow Conservation**: If $u \in V$ and $u \neq s$, $u \neq t$, then $\sum_{v \in V} f(u, v) = 0$.

We say that $f(u, v)$ is the flow from vertex u to vertex v. For two sets of vertices X and Y and a flow function f, we define $f(X, Y) = \sum_{x \in X} \sum_{y \in Y} f(x, y)$ and $c(X, Y) = \sum_{x \in X} \sum_{y \in Y} c(x, y)$. The *value* of a flow f, not to be confused with absolute value or norm, is denoted $|f|$ and defined as $|f| = \sum_{v \in V} f(s, v)$; i.e. the total flow out of the source.

The following is an example of a flow network that illustrates these concepts:



(a) A flow network $G = (V, E)$ where each edge $(u, v) \in E$ is labeled with $c(u, v)$.



(b) The same flow network G, where each edge $(u, v) \in E$ is labeled with $f(u, v)/c(u, v)$ for a flow function f.

In the first figure, we have constructed a flow network $G = (V, E)$ with vertices $V = \{s, v_1, v_2, v_3, v_4, t\}$ and edges E as indicated, where the capacity $c(u, v)$ of each edge (u, v) is labeled. In the second figure, we have labeled the flow across each edge of a flow function f to the left of each edge's capacity. Though not pictured for every edge, we set $f(v, u) = -f(u, v)$ for all $(u, v) \in E$. The reader may quickly verify that f satisfies the Capacity Constraint and the Skew Symmetry properties, and by summing the flows into and out of each vertex, that f also satisfies the Flow Conservation property of flow functions. The value of the flow f, in this case, is $|f| = 7$, which is the total flow out of the source s and into the sink t.

Additionally, for a flow network $G = (V, E)$ and a flow f, we define the *residual capacity* of