19th International Conference on Information Fusion
Heidelberg, Germany - July 5-8, 2016

Gamma Gaussian inverse-Wishart Poisson multi-Bernoulli Filter for Extended Target Tracking

Karl Granström, Maryam Fatemi and Lennart Svensson
Chalmers University of Technology, Gothenburg, Sweden. E-mail: firstname.lastsname@chalmers.se.

***Abstract*—This paper presents a gamma-Gaussian-inverse Wishart (GGIW) implementation of a Poisson multi-Bernoulli mixture (PMBM) filter for multiple extended target tracking. The GGIW density is the single extended target conjugate prior assuming a Poisson distributed number of Gaussian distributed measurements, and the PMBM density is the multi-object conjugate prior assuming Poisson target measurements, Poisson clutter, and Poisson target birth. Specifically, the Poisson part of the GGIW-PMBM multi-object density represents the distribution of targets that have not yet been detected, and the multi-Bernoulli mixture part of the GGIW-PMBM multi-object density represents the distribution of targets that have been detected at least once.**

The update and the prediction of the GGIW-PMBM density parameters are given, and the filter is evaluated in a simulation study. The results show that the GGIW-PMBM filter outperforms PHD and CPHD filters for extended target tracking.

I. Introduction

Multiple target tracking (MTT) is the processing of sets of measurements obtained from multiple sources in order to maintain estimates of targets' current states. In an MTT context an extended target is defined as a target that potentially gives rise to more than one measurement at each time step, where the set of measurements are spatially distributed around the extended target state [1]. Extended targets occur in scenarios where the resolution of the sensor, the size of the target, or the distance between target and sensor, are such that multiple resolution cells of the sensor are occupied by single targets. Examples of such extended target scenarios include vehicle tracking using automotive radars, tracking of sufficiently close airplanes or ships with ground or marine radar stations, and person tracking using laser range sensors.

A common extended target measurement model is the inhomogeneous Poisson Point Process (PPP), proposed in [2], [3]. At each time step, a Poisson distributed random number of measurements are generated, spatially distributed around the target. Several different alternative models for the spatial distribution have been presented, e.g., the random matrix model [4], [5] and the random hypersurface model [6]. The random matrix model, which is used in this paper, assumes that the measurements are Gaussian distributed around the target's center of mass. For a Poisson number of measurements with Gaussian spatial distribution the gamma-Gaussian-inverse Wishart (GGIW) distribution is conjugate prior for single extended target tracking [4], [5], [7].

Random Finite Sets (RFS) and Finite Set Statistics (FISST) [8], [9] is a theoretically elegant and appealing approach to the MTT problem where the targets and the measurements are modelled as sets of random variables. Computationally feasible RFS filters include the Probability Hypothesis Density (PHD) filters [10], the Cardinalized PHD (CPHD) filters [11], and the various multi-Bernoulli (MB) filters, see e.g. [12], [13]. For the PPP extended target model of [2], [3], a PHD filter was presented in [14], a CPHD filter was presented in [15], [16], and labelled multi-Bernoulli (LMB and GLMB) filters were presented in [17], [18]. GGIW implementations of the extended target PHD, CPHD and LMB filters can be found in [16]–[21]. Comparisons have shown that for multiple extended target tracking the (G)LMB filters outperform the CPHD filter, which in turn outperforms the PHD filter, see [16]–[18].

For extended target MTT the GLMB distribution is a conjugate prior for the multi-object probability density function [18]. For point target MTT, at least two types of RFS conjugate priors have been presented: a GLMB conjugate prior [13], and a PMBM conjugate prior [22]. The PMBM conjugate prior allows an elegant separation of the set of targets into two disjoint subsets: targets that have been detected, and targets that have not yet been detected; and the PMBM has already given rise to computationally efficient algorithms [22]. Recently, an extended target PMBM conjugate prior was presented [23], which is similar to the point target PMBM [22]. Note that while the GLMB and PMBM conjugate priors are theoretically exact, in practice approximations of the data association problem are required.

In this paper we present a GGIW implementation of the extended target PMBM filter [23]. The update and the prediction of all involved parameters are presented, and we also outline steps for the necessary complexity reduction. Further, the resulting GGIW-PMBM tracking filter is evaluated in a simulation study where its performance is compared to GGIW implementations of the PHD, CPHD, and LMB filters [16]–[21].

II. Problem formulation

Let ξ_k^i denote the state of the ith target at discrete time step k, and let the target set be denoted as

$$\mathbf{X}_k = \left\{\xi_k^i\right\}_{i=1}^{N_k^x}. \tag{1}$$

The target set cardinality $|\mathbf{X}_k| = N_k^x$ is a time-varying discrete random variable, and each target state ξ_k^i is a random variable.

The set of measurements obtained at time step k is denoted as

$$\mathbf{Z}_k = \left\{\mathbf{z}_k^j\right\}_{j=1}^{N_k^z}, \tag{2}$$

978-0-9964527-4-8©2016 ISIF

where $N_k^z = |\mathbf{Z}_k|$ is the cardinality of the measurement set at time k. There are two types of measurements: clutter measurements and target originated measurements, and the measurement origin is assumed unknown. Note that the sets above are without order and the set indexing is arbitrary; the particular choices $i = 1, \ldots, N_k^x$ and $j = 1, \ldots, N_k^z$ are only used for notational simplicity and convenience.

The ultimate objective is to approximate the multi-object distribution at time step k given the union of all measurement sets up to and including time step k, denoted $f_{k|k}(\mathbf{X}_k|\mathbf{Z}^k)$, where $\mathbf{Z}^k$ denotes all measurement sets $\mathbf{Z}_m$ from $m = 0$ up to, and including, $m = k$. In MTT the multi-object density $f_{k|k}(\mathbf{X}_k|\mathbf{Z}^k)$ is often approximated by a particle representation or a parametric density, and the involved single-object densities are in turn approximated by particle representations or parametric densities. In this paper the set density $f_{k|k}(\mathbf{X}_k|\mathbf{Z}^k)$ is approximated by a PMBM density, and the single-object densities are approximated by GGIW densities. The GGIW-PMBM is a conjugate prior, and thus it is possible to compute the posterior density exactly, given enough computational resources.

Given the PMBM and GGIW assumptions, estimating the multi-object distribution $f_{k|k}(\mathbf{X}_k|\mathbf{Z}^k)$ corresponds to the update and the prediction of the GGIW-PMBM parameters. The problem considered in this paper is to present the update and the prediction of the GGIW-PMBM parameters under standard modelling assumptions, and to find suitable approximations such that the GGIW-PMBM filter can be implemented using limited computational resources.

Notation is given in Table I, note the following important differences between a partition $\mathcal{P}$ of the set $\mathbf{Z}$, and a disjoint set of I subsets $\{\mathbf{V}_i\}_{i=1}^I$ whose union is the set $\mathbf{Z}$:

- The cells $\mathbf{C}$ in a partition $\mathcal{P}$ are always non-empty, whereas a subset $\mathbf{V}_i$ may be empty.
- The number of cells in a partition range from one (all measurements in one cell) to $|\mathbf{Z}|$ (each cell contains one unique measurement), whereas the number of subsets is determined by I, and we may have $I > |\mathbf{Z}|$.

III. Poisson random matrix single-target modeling

Each extended target is modeled using the PPP model [2], [3] and the random matrix model [4], [5]. In this model the target shape is approximated by an ellipse, and at each time step a Poisson distributed number of measurements are spatially distributed around the target. The Poisson process has unknown rate and the spatial distribution is a Gaussian with unknown mean and covariance.

The extended target state ξ_k is the combination of the scalar γ_k, the vector $\mathbf{x}_k$ and the matrix X_k. The random vector $\mathbf{x}_k \in \mathbb{R}^{n_x}$ is the kinematic state, which describes the target's position and its motion parameters (e.g., velocity, acceleration and turn-rate). The random matrix $X_k \in \mathbb{S}_{++}^d$ is the extent state and describes the target's size and shape, and d is the dimension of the extent (typically $d = 2$ or $d = 3$). Lastly, the random variable $\gamma_k > 0$ is the measurement model Poisson rate.

TABLE I
NOTATION

- $\mathbb{R}^n$ is the set of real n-vectors, and $\mathbb{S}_{++}^n$ is the set of symmetric positive definite $n \times n$-matrices.
- $|\mathbf{X}|$: set cardinality, i.e., number of elements in set $\mathbf{X}$.
- $\mathbf{Z}\backslash\mathbf{Y}$, where $\mathbf{Y} \subseteq \mathbf{Z}$, denotes set difference, i.e., $\mathbf{Z}\backslash\mathbf{Y}$ contains the elements in $\mathbf{Z}$ that are not in the subset $\mathbf{Y} \subseteq \mathbf{Z}$.
- $\mathbf{I}_m$: identity matrix of size $m \times m$.
- $\langle a; b\rangle = \int a(x)b(x)\mathrm{d}x$: inner product of $a(x)$ and $b(x)$
- Φ_m^n denotes the set of mappings

$$\alpha : \{1, \ldots, m\} \to \{0, \ldots, n\} \tag{3a}$$

subject to

$$\{1, \ldots, n\} \subseteq \alpha\left(\{1, \ldots, m\}\right) \tag{3b}$$
$$\alpha(i) > 0, i \neq \ell \Rightarrow \alpha(i) \neq \alpha(\ell) \tag{3c}$$

- $\mathcal{P}\angle\mathbf{Z}$ denotes that $\mathcal{P}$ partitions the set $\mathbf{Z}$ into non-empty subsets $\mathbf{C}$ (called cells), such that

$$\cup_{\mathbf{C}\in\mathcal{P}} \mathbf{C} = \mathbf{Z} \tag{4a}$$
$$\mathbf{C}_a \cap \mathbf{C}_b = \emptyset,\ \forall \mathbf{C}_a, \mathbf{C}_b \in \mathcal{P} : \mathbf{C}_a \neq \mathbf{C}_b \tag{4b}$$

- $\{\mathbf{V}_i\}_{i=1}^I : \uplus_{i=1}^I \mathbf{V}_i = \mathbf{Z}$ denotes a disjoint set of I possibly empty subsets $\mathbf{V}_i$, in other words

$$\cup_{i=1}^I \mathbf{V}_i = \mathbf{Z} \tag{5a}$$
$$\mathbf{V}_{i_1} \cap \mathbf{V}_{i_2} = \emptyset,\ \forall i_1 \neq i_2 \tag{5b}$$

The measurement likelihood for a single measurement $\mathbf{z}$ is

$$\phi(\mathbf{z}_k|\xi_k) = \mathcal{N}\left(\mathbf{z}_k\,;\, H_k\mathbf{x}_k, X_k\right), \tag{6}$$

where H_k is a known measurement model. The single-target conjugate prior for the Poisson random matrix model is a gamma-Gaussian-inverse Wishart (GGIW) distribution [5], [7],

$$\begin{aligned} f_{k|k}\left(\xi\right) =& \mathcal{G}\left(\gamma_k\,;\, \alpha_{k|k}, \beta_{k|k}\right) \mathcal{N}\left(\mathbf{x}_k\,;\, m_{k|k}, P_{k|k}\right) \\ & \times \mathcal{IW}_d\left(X_k\,;\, v_{k|k}, V_{k|k}\right), \end{aligned} \tag{7}$$

where $\alpha_{k|k}$ and $\beta_{k|k}$ are the Gamma distribution's shape and rate, $m_{k|k}$ and $P_{k|k}$ are the mean and covariance of the Gaussian distribution, and $v_{k|k}$ and $V_{k|k}$ are the degrees of freedom and shape matrix of the inverse Wishart distribution. The gamma distribution is the conjugate prior for the unknown Poisson rate, and the Gaussian-inverse Wishart distributions are the conjugate priors for Gaussian distributed detections with unknown mean and covariance. We also use the short hand notation

$$f_{k|k}\left(\xi_k\right) = \mathcal{GGIW}\left(\xi_k\,;\, \zeta_{k|k}\right), \tag{8}$$

where $\zeta_{k|k} = \left\{\alpha_{k|k}, \beta_{k|k}, m_{k|k}, P_{k|k}, v_{k|k}, V_{k|k}\right\}$ is the set of GGIW density parameters.

The updated parameters $\zeta_{k|k}$, and the corresponding predicted likelihood, for a GGIW distribution with prior parameters $\zeta_{k|k-1}$ that are updated with a set of detections $\mathbf{W}$ under the linear Gaussian model (6) are given in Table II. For further discussions about the measurement update within the random matrix extended target model see, e.g., [4], [5], [24].

TABLE II
GGIW UPDATE

Input: $\zeta_{k|k-1}$ and set of detections $\mathbf{W}$
Updated GGIW parameters

$$\zeta_{k|k} = \begin{cases} \alpha_{k|k} &= \alpha_{k|k-1} + |\mathbf{W}|, \\ \beta_{k|k} &= \beta_{k|k-1} + 1, \\ m_{k|k} &= m_{k|k-1} + K_{k|k-1}\varepsilon_{k|k-1}, \\ P_{k|k} &= P_{k|k-1} - K_{k|k-1}H_k P_{k|k-1}, \\ v_{k|k} &= v_{k|k-1} + |\mathbf{W}|, \\ V_{k|k} &= V_{k|k-1} + \hat{N}_{k|k-1} + Z_k \end{cases}$$

where

$$\begin{aligned} \bar{\mathbf{z}}_k &= \tfrac{1}{|\mathbf{W}|}\textstyle\sum_{\mathbf{z}_k^{(i)}\in\mathbf{W}} \mathbf{z}_k^{(i)}, \\ Z_k &= \textstyle\sum_{\mathbf{z}_k^{(i)}\in\mathbf{W}} \left(\mathbf{z}_k^{(i)} - \bar{\mathbf{z}}_k\right)\left(\mathbf{z}_k^{(i)} - \bar{\mathbf{z}}_k\right)^{\mathrm{T}} \\ \hat{X}_{k|k-1} &= V_{k|k-1}\left(v_{k|k-1} - 2d - 2\right)^{-1}, \\ \varepsilon_{k|k-1} &= \bar{\mathbf{z}}_k - H_k m_{k|k-1}, \\ N_{k|k-1} &= \varepsilon_{k|k-1}\left(\varepsilon_{k|k-1}\right)^{\mathrm{T}}, \\ S_{k|k-1} &= H_k P_{k|k-1} H_k^{\mathrm{T}} + \tfrac{\hat{X}_{k|k-1}}{|\mathbf{W}|}, \\ K_{k|k-1} &= P_{k|k-1} H_k^{\mathrm{T}} \left(S_{k|k-1}\right)^{-1}, \\ \hat{N}_{k|k-1} &= \left(\hat{X}_{k|k-1}\right)^{1/2}\left(S_{k|k-1}\right)^{-1/2} N_{k|k-1} \\ &\quad \times \left(S_{k|k-1}\right)^{-\mathrm{T}/2}\left(\hat{X}_{k|k-1}\right)^{\mathrm{T}/2} \end{aligned}$$

Predicted likelihood

$$\begin{aligned} \mathcal{L}_k &= \left(\pi^{|\mathbf{W}|}|\mathbf{W}|\right)^{-\frac{d}{2}} \frac{\left|V_{k|k-1}\right|^{\frac{v_{k|k-1}-d-1}{2}}}{\left|V_{k|k}\right|^{\frac{v_{k|k}-d-1}{2}}} \frac{\Gamma_d\left(\frac{v_{k|k}-d-1}{2}\right)}{\Gamma_d\left(\frac{v_{k|k-1}-d-1}{2}\right)} \\ &\quad \times \frac{\left|\hat{X}_{k|k-1}\right|^{\frac{1}{2}}}{\left|S_{k|k-1}\right|^{\frac{1}{2}}} \frac{\Gamma\left(\alpha_{k|k}\right)\left(\beta_{k|k-1}\right)^{\alpha_{k|k-1}}}{\Gamma\left(\alpha_{k|k-1}\right)\left(\beta_{k|k}\right)^{\alpha_{k|k}}} \end{aligned}$$

Output: $\zeta_{k|k}$ and likelihood $\mathcal{L}_k$

TABLE III
GGIW PREDICTION

Input: $\zeta_{k|k}$
Predicted GGIW parameters

$$\zeta_{k+1|k} = \begin{cases} \alpha_{k+1|k} &= \frac{\alpha_{k|k}}{\eta_k}, \\ \beta_{k+1|k} &= \frac{\beta_{k|k}}{\eta_k}, \\ m_{k+1|k} &= \mathbf{f}\left(m_{k|k}\right), \\ P_{k+1|k} &= \mathbf{F}_{k|k} P_{k|k} \left(\mathbf{F}_{k|k}\right)^{\mathrm{T}} + \mathbf{Q}, \\ v_{k+1|k} &= 2d + 2 + e^{-T_s/\tau}\left(v_{k|k} - 2d - 2\right), \\ V_{k+1|k} &= \left(v_{k+1|k} - 2d - 2\right)\left(v_{k|k} - 2d - 2\right)^{-1} \\ & \quad \times M\left(m_{k|k}\right) V_{k|k} M\left(m_{k|k}\right)^{\mathrm{T}} \end{cases}$$

where

$$\mathbf{F}_{k|k} = \nabla_{\mathbf{x}} \mathbf{f}(\mathbf{x})|_{\mathbf{x}=m_{k|k}}$$

Output: $\zeta_{k+1|k}$

For the kinematics state, the extent state, and the measurement rate, the motion models are

$$\mathbf{x}_{k+1} = \mathbf{f}\left(\mathbf{x}_k\right) + \mathbf{w}_k \tag{9}$$

$$X_{k+1} = M(\mathbf{x}_k) X_k M(\mathbf{x}_k)^{\mathrm{T}} \tag{10}$$

$$\gamma_{k+1} = \gamma_k. \tag{11}$$

where $\mathbf{w}_k$ is Gaussian process noise with zero mean and covariance $\mathbf{Q}$, and $M(\mathbf{x}_k)$ is a transformation matrix. For these motion models the predicted parameters $\zeta_{k+1|k}$ for a GGIW distribution with posterior parameters $\zeta_{k|k}$ are given in Table III. For longer discussions about prediction within the random matrix extended target model, see, e.g., [4], [5], [25].

IV. RFS MULTI-TARGET MODELING

This section first presents a review of random set theory; specifically the PPP and the MB process. Next the standard extended target measurement and motion models are presented.

A. Review of random set modeling

1) Poisson point process: A PPP is a type of RFS with pdf

$$f(\mathbf{X}) = e^{-\mu} \prod_{\xi\in\mathbf{X}} \mu f(\xi) \tag{12}$$

The cardinality is Poisson distributed with Poisson rate μ and each target is independent identically distributed (iid) with spatial distribution $f(\xi)$, and $D(\xi) = \mu f(\xi)$ is the PPP intensity function.

2) Multi Bernoulli process: A Bernoulli RFS $\mathbf{X}^i$ is a type of RFS that is empty with probability $1-r^i$ or, with probability r^i, contains a single element with distribution $f^i(\xi)$. The cardinality is Bernoulli distributed with parameter r^i and the pdf of $\mathbf{X}^i$ is

$$f(\mathbf{X}^i) = \begin{cases} 1 - r^i & \mathbf{X}^i = \emptyset \\ r^i \cdot f^i(\xi) & \mathbf{X}^i = \{\xi\} \\ 0 & |\mathbf{X}^j| \geq 2 \end{cases} \tag{13}$$

A typical assumption in MTT is that the targets are independent, see e.g. [26]. A MB RFS $\mathbf{X}$ is the union of a fixed number I of independent Bernoulli RFSs $\mathbf{X}^i$, $\mathbf{X} = \bigcup_{i=1}^{I} \mathbf{X}^i$ and is defined by the set of existence probabilities and distributions $\{r^i, f^i(\cdot)\}_{i=1}^{I}$. Here I is the maximum number of targets that the MB RFS can represent. The MB pdf for a set $\mathbf{X}$ can be expressed as

$$f(\mathbf{X}) = \begin{cases} \sum_{\alpha\in\Phi_I^{|\mathbf{X}|}} \prod_{i=1}^{I} f^i(\mathbf{X}^{\alpha(i)}) & \text{if} \quad |\mathbf{X}| \leq I \\ 0 & \text{if} \quad |\mathbf{X}| > I \end{cases} \tag{14}$$

$$\mathbf{X}^{\alpha(i)} = \begin{cases} \emptyset & \text{if} \quad \alpha(i) = 0 \\ \{\xi\} & \text{if} \quad \alpha(i) > 0 \end{cases} \tag{15}$$

where the mapping Φ, defined in Table I, describes all possible ways to assign exactly one Bernoulli estimate onto each target in $\mathbf{X}_k^d$, and (any) remaining Bernoulli estimates onto an empty set [22].

An MB mixture (MBM) is an RFS whose pdf is a normalized weighted sum of MB pdfs $f^j(\mathbf{X})$,

$$f^{mbm}(\mathbf{X}) = \sum_j \mathcal{W}^j f^j(\mathbf{X}) \ , \quad \sum_j \mathcal{W}^j = 1 \tag{16}$$

where the weights may correspond to, e.g., different data association sequences.

B. Standard multiple extended target measurement model

The set of measurements $\mathbf{Z}_k$ is the union of a set of clutter measurements and a set of target generated measurements; the sets are assumed independent. The clutter at time k is modelled as a PPP with rate λ and spatial distribution $c(\mathbf{z})$, and $\kappa(\mathbf{z}) = \lambda c(\mathbf{z})$ is the clutter PPP intensity function.

An extended target with state ξ is detected with state dependent probability of detection $p_{\mathrm{D}}(\xi)$, and, if it is detected, the target measurements are modelled as a PPP with state dependent Poisson rate γ and spatial distribution $\phi(\mathbf{z}|\xi)$. For a non-empty set of measurements ($|\mathbf{Z}| > 0$) the conditional extended target measurement set likelihood is denoted

$$\ell_{\mathbf{Z}}(\xi) = p_{\mathrm{D}}(\xi) p(\mathbf{Z}|\xi) = p_{\mathrm{D}}(\xi) e^{-\gamma} \prod_{\mathbf{z}\in\mathbf{Z}} \gamma\phi(\mathbf{z}|\xi) \tag{17}$$

Note that this is the product of the probability of detection and the PPP pdf. The extended targets are assumed to generate measurements independent of each other.

The effective probability of detection for an extended target with state ξ is $p_{\mathrm{D}}(\xi)(1 - e^{-\gamma})$ where $1 - e^{-\gamma}$ is the Poisson probability of generating at least one detection. Accordingly, the effective probability of missed detection, i.e., the probability that the target is not detected, is

$$q_{\mathrm{D}}(\xi) = 1 - p_{\mathrm{D}}(\xi) + p_{\mathrm{D}}(\xi) e^{-\gamma} \tag{18}$$

Note that $q_{\mathrm{D}}(\xi)$ is the conditional likelihood for an empty set of measurements, i.e., $\ell_{\emptyset}(\xi) = q_{\mathrm{D}}(\xi)$ (cf. (17)).

C. Standard multiple object dynamic model

The existing targets—both the detected and the undetected—survive from time step k to time step $k+1$ with state dependent probability of survival $p_{\mathrm{S}}(\xi_k)$. All targets that are present in the surveillance area are assumed to follow the same dynamic motion model. Further, it is assumed that each target evolves over time independently of all other targets. New targets appear independently of the targets that already exist. The target birth is assumed to be a PPP with Poisson rate μ^b_{k+1} and spatial density $f^b_{k+1}(\xi)$, i.e., the intensity is $D^b_{k+1}(\xi) = \mu^b_{k+1} f^b_{k+1}(\xi)$. In this work target spawning is not taken into account, for work on spawning in an extended target context see [27].

V. The GGIW-PMBM filter

In this section the main result of the paper is presented: a GGIW-PMBM filter. The PPP describes the distribution of the targets that are thus far undetected, while the MBM describes the distribution of the targets that have been detected at least once. Thus, the set of targets can be divided into two dis-joint subsets,

$$\mathbf{X}_k = \mathbf{X}^u_k \cup \mathbf{X}^d_k \tag{19}$$

corresponding to undetected targets $\mathbf{X}^u_k$ and detected targets $\mathbf{X}^d_k$. The PMBM set density at time k can be expressed as

$$f_{k|k}(\mathbf{X}_k|\mathbf{Z}^k) = \sum_{\mathbf{X}^d_k \subseteq \mathbf{X}_k} f^u_{k|k}(\mathbf{X}_k \backslash \mathbf{X}^d_k|\mathbf{Z}^k) f^d_{k|k}(\mathbf{X}^d_k|\mathbf{Z}^k) \tag{20a}$$

The PPP set density for undetected targets

$$f^u_{k|k}(\mathbf{X}_k \backslash \mathbf{X}^d_k|\mathbf{Z}^k) = e^{-\mu^u_{k|k}} \prod_{\xi\in\mathbf{X}_k\backslash\mathbf{X}^d_k} \mu^u_{k|k} f^u_{k|k}(\xi) \tag{20b}$$

has Poisson rate $\mu^u_{k|k}$ and spatial density $f^u_{k|k}(\xi)$. The MBM set density for detected targets

$$f^d_{k|k}(\mathbf{X}^d_k|\mathbf{Z}^k) = \sum_{j=1}^{J_{k|k}} \mathcal{W}^j_{k|k} f^{d,j}_{k|k}(\mathbf{X}^d_k|\mathbf{Z}^k) \tag{20c}$$

has $J_{k|k}$ MB set densities. The jth MB density is defined as

$$f^{d,j}_{k|k}(\mathbf{X}^d_k|\mathbf{Z}^k) = \sum_{\alpha^j_{k|k} \in \Phi^{|\mathbf{X}^d_k|}_{I^j_{k|k}}} \prod_{i=1}^{I^j_{k|k}} f^{j,i}_{k|k}\left(\mathbf{X}_k^{\alpha^j_{k|k}(i)}\right) \tag{20d}$$

where $\mathbf{X}_k^{\alpha^j_{k|k}(i)}$ is defined analogously to (15), $f^{j,i}_{k|k}(\cdot)$ are Bernoulli set densities, defined in (13), and the association mapping Φ is defined in Table I. There are $J_{k|k}$ MB components, the jth component has $I^j_{k|k}$ Bernoulli estimates, and the probability of the jth MB component is $\mathcal{W}^j_{k|k}$. All of the involved single-target densities, e.g., $f(\xi)$ in (12), $f^i(\xi)$ in (13), and $f^b_{k+1}(\xi)$, are assumed to be GGIW densities.

The GGIW-PMBM filter propagates in time the GGIW-PMBM density parameters, using a recursion that consists of an update and a prediction. The assumptions are listed in Table IV. The assumptions about the probabilities of detection and survival hold trivially if $p_{\mathrm{D}}(\cdot)$ and $p_{\mathrm{S}}(\cdot)$ are constants, and the assumptions are expected to hold when $p_{\mathrm{D}}(\cdot)$ and $p_{\mathrm{S}}(\cdot)$ are sufficiently smooth functions within the uncertainty area of the estimate. Note that the assumptions of GGIW mixture intensities for the birth PPP and the initial undetected PPP result in all single target densities in the PMBM filter being GGIW densities, due to the conjugacy property.

A. Update

Assuming that the predicted multi-object density at time k is a GGIW-PMBM of the form (20), the updated density is GGIW-PMBM. The updated MBM is given in (25) at the top of the next page, and contains an updated MB component for each predicted MB component and each possible data association. In (25), for the jth MB component, the data association is broken down into three parts

- A separation of the set of measurements $\mathbf{Z}$ into a set $\mathbf{Y}$ with measurements from clutter or previously undetected targets, and a set $\mathbf{Z}\backslash\mathbf{Y}$ with measurements from previously detected targets.
- Given $\mathbf{Y}$, a partition of $\mathcal{P}$ of $\mathbf{Y}$ into non-empty cells $\mathbf{C}$, where each cell contains measurements from one source (either clutter or a single target).
- Given $\mathbf{Z}\backslash\mathbf{Y}$, a set of subsets $\{\mathbf{V}_{j,i}\}_i$, where the j,ith subset contains measurements that are associated to the ith Bernoulli estimate in the jth MB component.

The updated Bernoulli parameters for the detected targets, and the updated PPP parameter for the undetected targets, are given in the following subsections.

$$f^d_{k|k}(\mathbf{X}^d_k|\mathbf{Z}^k) = \sum_{j=1}^{J_{k|k-1}} \sum_{\mathbf{Y}\subseteq\mathbf{Z}} \sum_{\mathcal{P}\angle\mathbf{Y}} \sum_{\substack{\{\mathbf{V}_{j,i}\}_i \\ \uplus_i \mathbf{V}_{j,i}=(\mathbf{Z}\setminus\mathbf{Y})}} \mathcal{W}^{j,\mathcal{P},\{\mathbf{V}_{j,i}\}_i}_{k|k} f^{d,j,\mathcal{P},\{\mathbf{V}_{j,i}\}_i}_{k|k}(\mathbf{X}^d_k|\mathbf{Z}^k) \tag{25a}$$

$$\mathcal{W}^{j,\mathcal{P},\{\mathbf{V}_{j,i}\}_i}_{k|k} = \frac{\mathcal{W}^j_{k|k-1}\mathcal{L}_{\mathcal{P}}\mathcal{L}_{\{\mathbf{V}_{j,i}\}_i}}{\sum_{j=1}^{J_{k|k-1}} \sum_{\mathbf{Y}\subseteq\mathbf{Z}} \sum_{\mathcal{P}\angle\mathbf{Y}} \sum_{\substack{\{\mathbf{V}_{j,i}\}_i \\ \uplus_i \mathbf{V}_{j,i}=(\mathbf{Z}\setminus\mathbf{Y})}} \mathcal{W}^j_{k|k-1}\mathcal{L}_{\mathcal{P}}\mathcal{L}_{\{\mathbf{V}_{j,i}\}_i}} \tag{25b}$$

1) Undetected targets: The PPP for undetected targets has updated Poisson rate

$$\mu^u_{k|k} = \mu^u_{k|k-1}\left\langle f^u_{k|k-1}; q_{\mathrm{D}}\right\rangle = \mu^u_{k|k-1} \sum_{j=1}^{N^u_{k|k-1}} q^{u,j}_{\mathrm{D}} \tag{26}$$

where $q_{\mathrm{D}}(\cdot)$ was defined in (18) and

$$q^{u,j}_{\mathrm{D}} = 1 - p_{\mathrm{D}}\left(\hat{\xi}^{(u,j)}_{k|k-1}\right) + p_{\mathrm{D}}\left(\hat{\xi}^{(u,j)}_{k|k-1}\right)\left(\frac{\beta^{(u,j)}_{k|k-1}}{\beta^{(u,j)}_{k|k-1}+1}\right)^{\alpha^{(u,j)}_{k|k-1}} \tag{27}$$

is the estimated effective probability of missed detection. The updated spatial density is

$$\begin{aligned} & f^u_{k|k}(\xi) \qquad (28) \\ =& \frac{\sum_{j=1}^{N^u_{k|k-1}}\left(1-p_{\mathrm{D}}\left(\hat{\xi}^{(u,j)}_{k|k-1}\right)\right)w^{u,j}_{k|k-1}}{\sum_{j'=1}^{N^u_{k|k-1}} q^{u,j'}_{\mathrm{D}} w^{u,j}_{k|k-1}} \mathcal{GGIW}\left(\xi_k\,;\,\zeta^{(u,j)}_{k|k-1}\right) \\ &+ \frac{\sum_{j=1}^{N^u_{k|k-1}} p_{\mathrm{D}}\left(\hat{\xi}^{(u,j)}_{k|k-1}\right)\left(\frac{\beta^{(u,j)}_{k|k-1}}{\beta^{(u,j)}_{k|k-1}+1}\right)^{\alpha^{(u,j)}_{k|k-1}} w^{u,j}_{k|k-1}}{\sum_{j'=1}^{N^u_{k|k-1}} q^{u,j'}_{\mathrm{D}} w^{u,j}_{k|k-1}} \\ &\times \mathcal{G}\left(\gamma_k\,;\,\alpha^{(u,j)}_{k|k-1}, \beta^{(u,j)}_{k|k-1}+1\right) \\ &\times \mathcal{N}\left(\mathbf{x}_k\,;\,m^{(u,j)}_{k|k-1}, P^{(u,j)}_{k|k-1}\right)\mathcal{IW}_d\left(X_k\,;\,v^{(u,j)}_{k|k-1}, V^{(u,j)}_{k|k-1}\right) \end{aligned}$$

Note that the updated weights sum to unity, and the updated density is indeed a proper density.

We see that for each GGIW component in the undetected mixture we get two new updated GGIW components; this is due to the fact that there are two ways for a target to results in an empty measurement set. The first corresponds to the detection process modeled by $p_{\mathrm{D}}(\cdot)$, which may result in a missed detection. The second corresponds to the Poisson number of detections governed by the parameter γ, i.e., the Poisson random number of detections is zero. Note that the Gaussian and inverse Wishart parameters are identical in both cases, it is only the gamma parameters that differ. Using gamma mixture reduction [7], the bi-modality of the γ_k estimate can be reduced to a single mode such that $f^u_{k|k}(\cdot)$ has $N^u_{k|k-1}$ GGIW components instead of $2N^u_{k|k-1}$ components.

TABLE IV
ASSUMPTIONS

- The birth PPP intensity is a GGIW mixture with known parameters,

$$D^b_{k+1} = \mu^b_{k+1} \sum_{j=1}^{N^b_{k+1}} w^{(b,j)}_{k+1} \mathcal{GGIW}\left(\xi_{k+1}\,;\,\zeta^{(b,j)}_{k+1}\right) \tag{29}$$

- The initial undetected PPP intensity is a GGIW mixture with known parameters,

$$D^u_0 = \mu^u_0 \sum_{j=1}^{N^u_0} w^{(u,j)}_0 \mathcal{GGIW}\left(\xi_0\,;\,\zeta^{(u,j)}_0\right) \tag{30}$$

- Empty initial PMBM: $\mu^u_{0|0} = 0$ and $J_{0|0} = 0$.
- The state dependent probabilitites of detection and survival can be approximated as

$$p_{\mathrm{D}}(\xi)f(\xi) \approx p_{\mathrm{D}}(\hat{\xi})f(\xi) \qquad p_{\mathrm{S}}(\xi)f(\xi) \approx p_{\mathrm{S}}(\hat{\xi})f(\xi) \tag{31}$$

where $\hat{\xi} = \mathbb{E}[\xi] = \int \xi f(\xi)\mathrm{d}\xi$.
- The clutter Poisson rate λ is known and the spatial distribution is uniform, $c(\mathbf{z}) = A^{-1}$, where A is the volume of the surveillance region.

2) Targets detected for the first time: A target that is detected for the first time, resulting in a set of detections $\mathbf{C}$, has existence probablity

$$r_{\mathbf{C}} = \begin{cases} 1 & \text{if} \quad |\mathbf{C}| > 1 \\ \frac{\mathcal{L}_{\mathbf{C}}}{\kappa^{\mathbf{C}}+\mathcal{L}_{\mathbf{C}}} & \text{if} \quad |\mathbf{C}| = 1 \end{cases} \tag{32a}$$

The spatial distribiution,

$$f_{\mathbf{C}}(\xi) = \frac{\sum_{j=1}^{N^u_{k|k-1}} w^{u,j}_{k|k-1}\mathcal{L}^{(u,j,\mathbf{C})}_k \mathcal{GGIW}\left(\xi\,;\,\zeta^{u,j,\mathbf{C}}_{k|k}\right)}{\sum_{j=1}^{N^u_{k|k-1}} w^{u,j}_{k|k-1}\mathcal{L}^{(u,j,\mathbf{C})}_k} \tag{32b}$$

is multimodal, with one mode for each of the GGIW components in the predicted undetected spatial density $f^u_{k|k-1}$. The updated parameters $\zeta^{u,j,\mathbf{C}}_{k|k}$ and predicted likelihoods $\mathcal{L}^{(u,j,\mathbf{C})}_k$ are computed as outlined in Table II. Mixture reduction can be used to reduce this to a uni-modal GGIW density [7], [28]. The predicted likelihood is the weighted sum of the predicted likelihoods corresponding to each GGIW component in the predicted undetected spatial density,

$$\mathcal{L}_{\mathbf{C}} = \mu^u_{k|k-1} \sum_{j=1}^{N^u_{k|k-1}} w^{u,j}_{k|k-1}\mathcal{L}^{(u,j,\mathbf{C})}_k \tag{32c}$$

3) Existing MB *estimate:* The ith Bernoulli estimate, in the jth MB component, updated with a non-empty set $\mathbf{V}_{j,i} \neq \emptyset$ has probability of existence

$$r_{k|k}^{j,i,\mathbf{V}_{j,i}} = 1 \tag{33}$$

because, if the target was detected, then trivially it must exist. The updated spatial density and predicted likelihood,

$$f_{k|k}^{j,i,\mathbf{V}_{j,i}}(\xi) = \mathcal{GGIW}\left(\xi\,;\,\zeta_{k|k}^{j,i,\mathbf{V}_{j,i}}\right) \tag{34}$$

$$\mathcal{L}_{\mathbf{V}_{j,i}} = r_{k|k-1}^{j,i}\mathcal{L}_{k}^{j,i,\mathbf{V}_{j,i}} \tag{35}$$

are computed as outlined in Table II.

If instead the ith Bernoulli estimate, in the jth MB component, was updated with an empty set $\mathbf{V}_{j,i} = \emptyset$, the probability of existence is

$$r_{k|k}^{j,i,\mathbf{V}_{j,i}} = \frac{r_{k|k-1}^{j,i}q_{\mathrm{D}}^{j,i}}{1 - r_{k|k-1}^{j,i} + r_{k|k-1}^{j,i}q_{\mathrm{D}}^{j,i}} \tag{36}$$

where

$$q_{\mathrm{D}}^{j,i} = 1 - p_{\mathrm{D}}\left(\hat{\xi}_{k|k-1}^{(j,i)}\right) + p_{\mathrm{D}}\left(\hat{\xi}_{k|k-1}^{(j,i)}\right)\left(\frac{\beta_{k|k-1}^{(j,i)}}{\beta_{k|k-1}^{(j,i)}+1}\right)^{\alpha_{k|k-1}^{(j,i)}} \tag{37}$$

In this case the probability of existence is computed as the relative probability that the target either exists but was not detected, $r_{k|k-1}^{j,i}q_{\mathrm{D}}^{j,i}$, or does not exist, $1-r_{k|k-1}^{j,i}$. The updated state distribution is bi-modal,

$$\begin{aligned} f_{k|k}^{j,i,\mathbf{V}_{j,i}}(\xi) = & \frac{1 - p_{\mathrm{D}}\left(\hat{\xi}_{k|k-1}^{(j,i)}\right)}{q_{\mathrm{D}}^{j,i}}\mathcal{GGIW}\left(\xi_k\,;\,\zeta_{k|k-1}^{(j,i)}\right) \\ & + \frac{p_{\mathrm{D}}\left(\hat{\xi}_{k|k-1}^{(j,i)}\right)\left(\frac{\beta_{k|k-1}^{(j,i)}}{\beta_{k|k-1}^{(j,i)}+1}\right)^{\alpha_{k|k-1}^{(j,i)}}}{q_{\mathrm{D}}^{j,i}} \\ & \times \mathcal{G}\left(\gamma_k\,;\,\alpha_{k|k-1}^{(u,j)}, \beta_{k|k-1}^{(u,j)}+1\right) \\ & \times \mathcal{N}\left(\mathbf{x}_k\,;\,m_{k|k-1}^{(u,j)}, P_{k|k-1}^{(u,j)}\right) \\ & \times \mathcal{IW}_d\left(X_k\,;\,v_{k|k-1}^{(u,j)}, V_{k|k-1}^{(u,j)}\right) \end{aligned} \tag{38}$$

where the first mode corresponds to the case that the target was not detected, the second mode corresponds to the case that the target generated an empty set of measurements. Using gamma-mixture reduction, see [7], the bi-modal GGIW distribution can be reduced to a uni-modal GGIW distribution. The predicted likelihood is

$$\begin{aligned} \mathcal{L}_{\mathbf{V}_{j,i}} = & 1 - r_{k|k-1}^{j,i}p_{\mathrm{D}}\left(\hat{\xi}_{k|k-1}^{(j,i)}\right) \\ & + r_{k|k-1}^{j,i}p_{\mathrm{D}}\left(\hat{\xi}_{k|k-1}^{(j,i)}\right)\left(\frac{\beta_{k|k-1}^{(j,i)}}{\beta_{k|k-1}^{(j,i)}+1}\right)^{\alpha_{k|k-1}^{(j,i)}} \end{aligned} \tag{39}$$

4) Predicted likelihoods: The predicted partition likelihood and predicted MB likelihood are used to compute the weights of the MBM components in (25b), and are expressed as

$$\mathcal{L}_{\mathcal{P}} = \prod_{\substack{\mathbf{C}\in\mathcal{P} \\ |\mathbf{C}|>1}} \mathcal{L}_{\mathbf{C}} \times \prod_{\substack{\mathbf{C}\in\mathcal{P} \\ |\mathbf{C}|=1}} \left(\kappa^{\mathbf{C}} + \mathcal{L}_{\mathbf{C}}\right) \tag{40a}$$

$$\mathcal{L}_{\{\mathbf{V}_{j,i}\}_i} = \prod_i \mathcal{L}_{\mathbf{V}_{j,i}} \tag{40b}$$

B. Prediction

Let the posterior multi-object density at time k be a PMBM of the form (20). Then the predicted multi-object density at time $k+1$ is PMBM.

1) Undetected targets: The predicted PPP for the undetected targets has Poisson rate

$$\mu_{k+1|k}^{u} = \mu_{k+1}^{b} + \left\langle f_{k|k}^{u}; p_{\mathrm{S}}\right\rangle \mu_{k|k}^{u} = \mu_{k+1}^{b} + \mu_{k|k}^{u}\mathsf{P}_{\mathrm{S}}^{u} \tag{41}$$

where

$$\mathsf{P}_{\mathrm{S}}^{u} = \sum_{j=1}^{N_{k|k}^{u}} w_{k|k}^{(u,j)} p_{\mathrm{S}}\left(\hat{\xi}_{k|k}^{u,j}\right) \tag{42}$$

The predicted spatial distribution is

$$f_{k+1|k}^{u}(\xi_{k+1}) \tag{43}$$

$$\begin{aligned} = & \frac{\mu_{k+1}^{b}}{\mu_{k+1}^{b} + \mathsf{P}_{\mathrm{S}}^{u}\mu_{k|k}^{u}} \sum_{j=1}^{N_{k+1}^{b}} w_{k+1}^{(b,j)}\mathcal{GGIW}\left(\xi_{k+1}\,;\,\zeta_{k+1}^{(b,j)}\right) \\ & + \frac{\mu_{k|k}^{u}}{\mu_{k+1}^{b} + \mathsf{P}_{\mathrm{S}}^{u}\mu_{k|k}^{u}} \sum_{j=1}^{N_{k|k}^{u}} w_{k|k}^{(u,j)} p_{\mathrm{S}}\left(\hat{\xi}_{k|k}^{u,j}\right)\mathcal{GGIW}\left(\xi\,;\,\zeta_{k+1|k}^{(u,j)}\right) \end{aligned} \tag{44}$$

where the predicted parameters $\zeta_{k+1|k}^{(u,j)}$ are computed as outlined in Table III. The updated spatial distribution has $N_{k+1}^{b} + N_{k|k}^{u}$ GGIW components.

2) Detected targets: The predicted MBM for detected targets has weights and number of components $\mathcal{W}_{k+1|k}^{j} = \mathcal{W}_{k|k}^{j}$, $J_{k+1|k} = J_{k|k}$ and $I_{k+1|k}^{j} = I_{k|k}^{j}$. The probability of existence and spatial distribution are

$$r_{k+1|k}^{j,i} = p_{\mathrm{S}}\left(\hat{\xi}_{k|k}^{j,i}\right) r_{k|k}^{j,i} \tag{45}$$

$$f_{k+1|k}^{j,i}(\xi_{k+1}) = \mathcal{GGIW}\left(\xi\,;\,\zeta_{k+1|k}^{j,i}\right) \tag{46}$$

where the predicted parameters $\zeta_{k+1|k}^{(j,i)}$ are computed as outlined in Table III.

C. Complexity reduction

The number of components in the MBM increase rapidly [23], thus approximations are necessary. This section outlines some approximations that are based on clustering of the measurements, standard MTT ellipsoidal gating, and maximum likelihood association. Gating is commonly used in MTT to reduce the computational cost, see e.g. [26]. Clustering has previously been used sucessfully in extended target MTT, see e.g. [16], [18], [20], [21]. Note that other alternatives to reduce the complexity may exist, a topic for future work is to investigate the pros and cons of different methods.

For each MB component in the MB mixture the following is performed:

- Standard ellipsoidal gating is used to group measurements and targets. Given the gating decisions, the gating-groups are assumed to be statistically insulated such that they can be treated independently.
- Any gating-group with only measurements is only a candidate for new targets. Multiple different partitions are considered, computed using the Distant Partitioning method [19], [20]. Each partition gives a potential MB density with target estimates detected for the first time, and a corresponding likelihood can be computed. Of the multiple MB densities, only the maximum likelihood MB density is returned.
- Any gating-group that contains both measurements and targets is treated as follows:
  - Multiple partitions are considered, computed using the Distant Partitioning method [19], [20].
  - For each partition, all possible associations of the cells to previously undetected targets or previously detected targets are considered. If there are N cells, this gives 2^N alternatives.
  - Nearest neighbour association for the target estimates and the cells associated to detected targets.
- For each gating group this creates multiple updated MBs; these are truncated to only contain the alternatives that correspond to 99% of the likelihood.

An updated MB-mixture is obtained by considering all possible ways to combined the alternative updated MBs for the gating groups. This updated MB-mixture is pruned by discarding the MB-components with low weights. This procedure yields a low computational complexity, without sacrificing estimation performance.

VI. Simulation Study

In this section the results from a Monte Carlo simulation study are presented. The kinematic state is $\mathbf{x}_k = [\mathbf{p}_k,\ \mathbf{v}_k]^{\mathrm{T}} \in \mathbb{R}^4$ and describes the target's position $\mathbf{p}_k \in \mathbb{R}^2$ and velocity $\mathbf{v}_k \in \mathbb{R}^2$. The random matrix $X_k \in \mathbb{S}^2_{++}$ is two dimensional. The motion model $\mathbf{f}(\cdot)$ and process noise covariance $\mathbf{Q}$ are

$$\mathbf{f}(\mathbf{x}_k) = \begin{bmatrix} \mathbf{I}_2 & T_s\mathbf{I}_2 \\ \mathbf{0}_2 & \mathbf{I}_2 \end{bmatrix} \mathbf{x}_k,\ \mathbf{Q} = \mathbf{G}\sigma_a^2\mathbf{I}_2\mathbf{G}^{\mathrm{T}},\ \mathbf{G} = \begin{bmatrix} \frac{T_s^2}{2}\mathbf{I}_2 \\ T_s\mathbf{I}_2 \end{bmatrix} \tag{47}$$

where T_s is the sampling time and σ_a is the acceleration standard deviation. Because the kinematic state motion model is constant velocity, the extent transformation function M is an identity matrix, $M(\mathbf{x}_k) = \mathbf{I}_2$.

A scenario with 27 targets was randomly generated, see Figure 1. The scenario has 100 time steps, and the targets appear in, and disappear from, the surveillance area at different time steps. The probability of detection is set to $p_{\mathrm{D}} = 0.90$ and the probability of survival is set to $p_{\mathrm{S}} = 0.99$. The birth spatial density consists of four GGIW components, with positions in $[\pm 75\ ,\ \pm 75]^{\mathrm{T}}$. Extended target Poisson rates were randomly sampled in the interval $\gamma \in [7, 9]$ and the clutter Poisson rates was $\lambda = 20$. An estimate of the set of targets is obtained by taking the mean vector of all Bernoulli estimates with existence probability larger than 0.5 from the MB component with largest MB weight. This simple method for target extraction work reasonably well for the scenario considered here and, more importantly, the same extraction method can be used in all of the compared tracking filters. However, note that alternative target extraction methods are available that may improve performance in certain circumstances, see, e.g., [29].

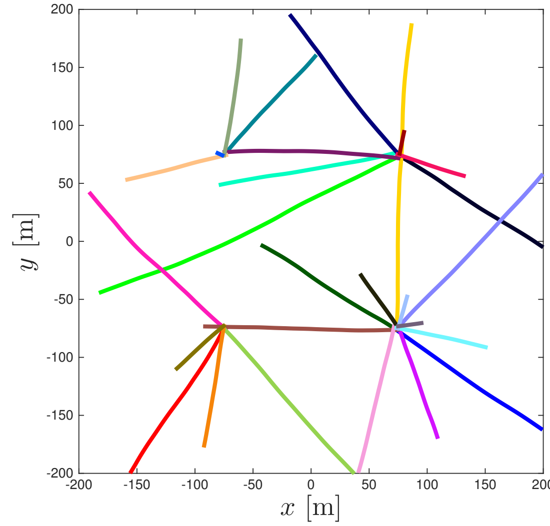

Fig. 1. Tracking scenario with 27 targets that each originate from one of four locations.

Average estimated cardinality distribution and OSPA are shown in Figure 2 and Figure 3, where the GGIW-PMBM filter is compared the GGIW-PHD, GGIW-CPHD and GGIW-LMB filters [7], [16], [18], [21]. The PMBM filter has lower variance than the three other filters, especially the PHD filter. Further, if a cardinality estimate is extracted by taking the most probable cardinality at each time step, then the PMBM filter has a lower estimation error than the other filters.

VII. Concluding Remarks

The paper presented a Gamma Gaussian inverse Wishart implementation of the Poisson Multi-Bernoulli Mixture conjugate prior for multiple extended target tracking. The implementation is derived for the standard extended target models, and both the update and the prediction equations are given. Further, approximations that simplify the complexity of the filter were suggested. A simulation study shows better performance compared to other extended target filters.

References

[1] K. Granström and M. Baum, "Extended Object Tracking: Introduction, Overview and Applications," *arXiv pre-print*, 2016. [Online]. Available: arxiv.org/abs/1604.00970

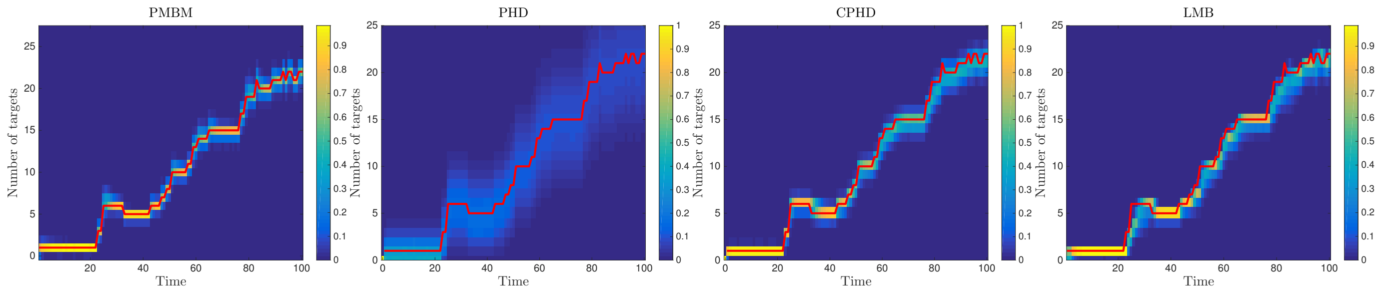

Fig. 2. Estimated cardinality distributions for the scenario in Figure 1, averaged over the Monte Carlo simulations. The cardinality distributions over time are shown as heat maps, the ground truth cardinality is shown as a red line.

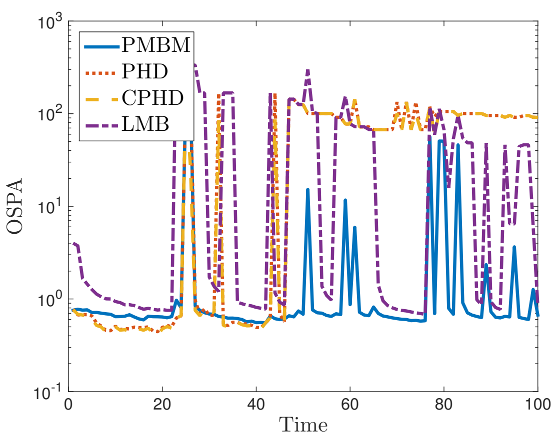

Fig. 3. Optimal Sub-Pattern Assigment metric for the scenario in Figure 1, averaged over the Monte Carlo simulations.

[2] K. Gilholm and D. Salmond, "Spatial distribution model for tracking extended objects," *IEE Proceedings of Radar, Sonar and Navigation*, vol. 152, no. 5, pp. 364–371, Oct. 2005.

[3] K. Gilholm, S. Godsill, S. Maskell, and D. Salmond, "Poisson models for extended target and group tracking," in *Proceedings of Signal and Data Processing of Small Targets*, vol. 5913. San Diego, CA, USA: SPIE, Aug. 2005, pp. 230–241.

[4] W. Koch, "Bayesian approach to extended object and cluster tracking using random matrices," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 44, no. 3, pp. 1042–1059, Jul. 2008.

[5] M. Feldmann, D. Fränken, and J. W. Koch, "Tracking of extended objects and group targets using random matrices," *IEEE Transactions on Signal Processing*, vol. 59, no. 4, pp. 1409–1420, Apr. 2011.

[6] M. Baum and U. Hanebeck, "Extended object tracking with random hypersurface models," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 50, no. 1, pp. 149–159, Jan. 2013.

[7] K. Granström and U. Orguner, "Estimation and Maintenance of Measurement Rates for Multiple Extended Target Tracking," in *Proceedings of the International Conference on Information Fusion*, Singapore, Jul. 2012, pp. 2170–2176.

[8] R. Mahler, *Statistical Multisource-Multitarget Information Fusion*. Norwood, MA, USA: Artech House, 2007.

[9] ——, *Advances in Multisource-Multitarget Information Fusion*. Norwood, MA, USA: Artech House, 2014.

[10] ——, "Multitarget Bayes filtering via first-order multi target moments," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 39, no. 4, pp. 1152–1178, Oct. 2003.

[11] ——, "PHD filters of higher order in target number," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 43, no. 4, pp. 1523–1543, Oct. 2007.

[12] B.-T. Vo, B.-N. Vo, and A. Cantoni, "The cardinality balanced multi-target multi-bernoulli filter and its implementations," *IEEE Transactions on Signal Processing*, vol. 57, no. 2, pp. 409–423, Feb. 2009.

[13] B.-T. Vo and B.-N. Vo, "Labeled random finite sets and multi-object conjugate priors," *IEEE Transactions on Signal Processing*, vol. 61, no. 13, pp. 3460–3475, Apr. 2013.

[14] R. Mahler, "PHD filters for nonstandard targets, I: Extended targets," in *Proceedings of the International Conference on Information Fusion*, Seattle, WA, USA, Jul. 2009, pp. 915–921.

[15] U. Orguner, C. Lundquist, and K. Granström, "Extended Target Tracking with a Cardinalized Probability Hypothesis Density Filter," in *Proceedings of the International Conference on Information Fusion*, Chicago, IL, USA, Jul. 2011, pp. 65–72.

[16] C. Lundquist, K. Granström, and U. Orguner, "An extended target CPHD filter and a gamma Gaussian inverse Wishart implementation," *IEEE Journal of Selected Topics in Signal Processing, Special Issue on Multi-target Tracking*, vol. 7, no. 3, pp. 472–483, Jun. 2013.

[17] M. Beard, S. Reuter, K. Granström, V. B.-T., B.-N. Vo, and A. Scheel, "A generalised labelled multi-bernoulli filter for extended multi-target tracking," in *Proceedings of the International Conference on Information Fusion*, Washington, DC, USA, Jul. 2015, pp. 991–998.

[18] ——, "Multiple extended target tracking with labelled random finite sets," *IEEE Transactions on Signal Processing*, vol. 64, no. 7, pp. 1638–1653, Apr. 2016.

[19] K. Granström, C. Lundquist, and U. Orguner, "A Gaussian mixture PHD filter for extended target tracking," in *Proceedings of the International Conference on Information Fusion*, Edinburgh, UK, Jul. 2010.

[20] ——, "Extended Target Tracking using a Gaussian Mixture PHD filter," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 48, no. 4, pp. 3268–3286, Oct. 2012.

[21] K. Granström and U. Orguner, "A PHD filter for tracking multiple extended targets using random matrices," *IEEE Transactions on Signal Processing*, vol. 60, no. 11, pp. 5657–5671, Nov. 2012.

[22] J. Williams, "Marginal multi-Bernoulli filters: RFS derivation of MHT, JIPDA, and association-based member," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 51, no. 3, pp. 1664–1687, Jul. 2015.

[23] K. Granström, M. Fatemi, and L. Svensson, "Poisson multi-Bernoulli conjugate prior for multiple extended object estimation," *arXiv pre-print*, 2016. [Online]. Available: arxiv.org

[24] U. Orguner, "A variational measurement update for extended target tracking with random matrices," *IEEE Transactions on Signal Processing*, vol. 60, no. 7, pp. 3827–3834, Jul. 2012.

[25] K. Granström and U. Orguner, "A New Prediction Update for Extended Target Tracking with Random Matrices," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 50, no. 2, Apr. 2014.

[26] Y. Bar-Shalom, P. K. Willett, and X. Tian, *Tracking and Data Fusion: A Handbook of Algorithms*. YBS Publishing, 2011.

[27] K. Granström and U. Orguner, "On Spawning and Combination of Extended/Group Targets Modeled with Random Matrices," *IEEE Transactions on Signal Processing*, vol. 61, no. 3, pp. 678–692, Feb. 2013.

[28] ——, "On the Reduction of Gaussian inverse Wishart mixtures," in *Proceedings of the International Conference on Information Fusion*, Singapore, Jul. 2012, pp. 2162–2169.

[29] S. Reuter, B.-T. Vo, B.-N. Vo, and K. Dietmayer, "The Labeled Multi-Bernoulli Filter," *IEEE Transactions on Signal Processing*, vol. 62, no. 12, pp. 3246–3260, Jul. 2014.