

## Encoding and Classifier

- The classical data was embedded in two ways in the form of amplitude embedding and angle embedding.
- In the amplitude-embedding technique, data is encoded into the amplitudes of a quantum state. A normalized classical ***N-dimensional*** data point ***x*** is represented by the amplitudes of a ***n-qubit quantum state***.

$$|\Psi\rangle = \sum x_i |i\rangle$$

Where *i* is the quantum basis.

- Angle embedding, Encodes ***N*** features into the rotation angles of ***n*** qubits, where ***N ≤ n***. The rotations can be chosen as either ***RX, RY or RZ*** gates, as defined by the rotation.
- The Embeddings were done using the PennyLane library in python.

### Amplitude Embedding

- Amplitude Embedding needs only two qubits to encode the classical data.
- For the Variation Quantum Classifier, first we implemented one layer of ***RX, RZ and CNOT*** gates with ***3*** layers which gave a ***test accuracy of 0.2*** and ***highest train accuracy of 0.4***.
- We considered batch size of 5 and ***NesterovMomentumOptimizer*** as our optimizer
- Then I changed the ansatz into implementing ***RX, RY and RZ and a CNOT*** gate which with ***3*** layers gave better training results (Some iterations image):

Iter:	1		Cost:	3.4104577		Accuracy:	0.4366667
Iter:	2		Cost:	2.5536059		Accuracy:	0.5233333
Iter:	3		Cost:	1.5775721		Accuracy:	0.4233333
Iter:	4		Cost:	2.3110371		Accuracy:	0.5233333
Iter:	5		Cost:	1.8661167		Accuracy:	0.4233333
Iter:	6		Cost:	1.2871659		Accuracy:	0.4733333
Iter:	7		Cost:	2.6443048		Accuracy:	0.4800000
Iter:	8		Cost:	1.1493262		Accuracy:	0.5233333
Iter:	9		Cost:	1.3527157		Accuracy:	0.6733333
Iter:	10		Cost:	1.7157764		Accuracy:	0.3800000

- Next Considering the number of layers as ***4*** gave:

Iter:	1	Cost: 1.3446573	Accuracy: 0.5700000
Iter:	2	Cost: 1.8204442	Accuracy: 0.5233333
Iter:	3	Cost: 1.4120008	Accuracy: 0.4233333
Iter:	4	Cost: 1.4381492	Accuracy: 0.4333333
Iter:	5	Cost: 2.7283498	Accuracy: 0.1733333
Iter:	6	Cost: 1.1202866	Accuracy: 0.6600000
Iter:	7	Cost: 0.9492884	Accuracy: 0.5733333
Iter:	8	Cost: 1.3184546	Accuracy: 0.3733333
Iter:	9	Cost: 1.9484108	Accuracy: 0.4266667
Iter:	10	Cost: 0.8060176	Accuracy: 0.6566667

- We also got a testing accuracy of **0.74**

```
# Compute accuracy
predictions = [np.sign(variational_classifier(weights, bias, x)) for x in X_test]
acc = accuracy(Y_test, predictions)

print(
    " Cost: {:.7f} | Accuracy: {:.7f} ".format( cost(weights, bias, X, Y), acc)
)
```

Cost: 0.8060176 | Accuracy: 0.7416667

- Next **5** layers was considered:

Iter:	1	Cost: 1.2976096	Accuracy: 0.5633333
Iter:	2	Cost: 1.7779056	Accuracy: 0.3333333
Iter:	3	Cost: 0.6407656	Accuracy: 0.7933333
Iter:	4	Cost: 1.7630178	Accuracy: 0.2300000
Iter:	5	Cost: 1.7932884	Accuracy: 0.5633333
Iter:	6	Cost: 1.2295164	Accuracy: 0.3466667
Iter:	7	Cost: 1.7967084	Accuracy: 0.2033333
Iter:	8	Cost: 1.3215029	Accuracy: 0.4233333
Iter:	9	Cost: 1.4793313	Accuracy: 0.3133333
Iter:	10	Cost: 0.9544546	Accuracy: 0.7400000

- It gave a test accuracy of **0.833** the highest test accuracy.
- Considering number of layers as **six** gave us :

Iter:	1	Cost: 1.5990938	Accuracy: 0.4266667
Iter:	2	Cost: 1.9411440	Accuracy: 0.3300000
Iter:	3	Cost: 1.5537574	Accuracy: 0.4500000
Iter:	4	Cost: 2.0847227	Accuracy: 0.5233333
Iter:	5	Cost: 1.7075924	Accuracy: 0.5233333
Iter:	6	Cost: 1.2553577	Accuracy: 0.3633333
Iter:	7	Cost: 1.0514950	Accuracy: 0.5766667
Iter:	8	Cost: 1.4040993	Accuracy: 0.1900000
Iter:	9	Cost: 1.6749637	Accuracy: 0.3066667
Iter:	10	Cost: 2.3157339	Accuracy: 0.4233333

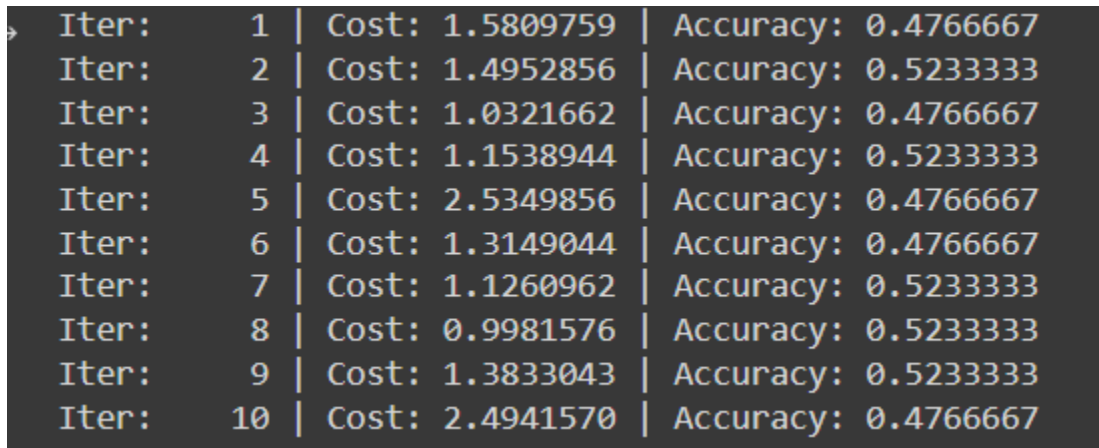
- It gave a test accuracy of **0.4** which is significantly lower than that of five layers.
- Also considered **seven** layers which gave:

Iter:	1	Cost: 0.8171923	Accuracy: 0.6666667
Iter:	2	Cost: 3.2291236	Accuracy: 0.5233333
Iter:	3	Cost: 0.9597792	Accuracy: 0.7466667
Iter:	4	Cost: 1.5642177	Accuracy: 0.4766667
Iter:	5	Cost: 1.7429029	Accuracy: 0.5766667
Iter:	6	Cost: 2.2471905	Accuracy: 0.4366667
Iter:	7	Cost: 1.2662306	Accuracy: 0.3866667
Iter:	8	Cost: 1.9547518	Accuracy: 0.1733333
Iter:	9	Cost: 1.5926295	Accuracy: 0.1966667
Iter:	10	Cost: 1.3634367	Accuracy: 0.4766667

- The test accuracy was **0.5177** which is better than six layers but worse than five layers

## Angle Embedding

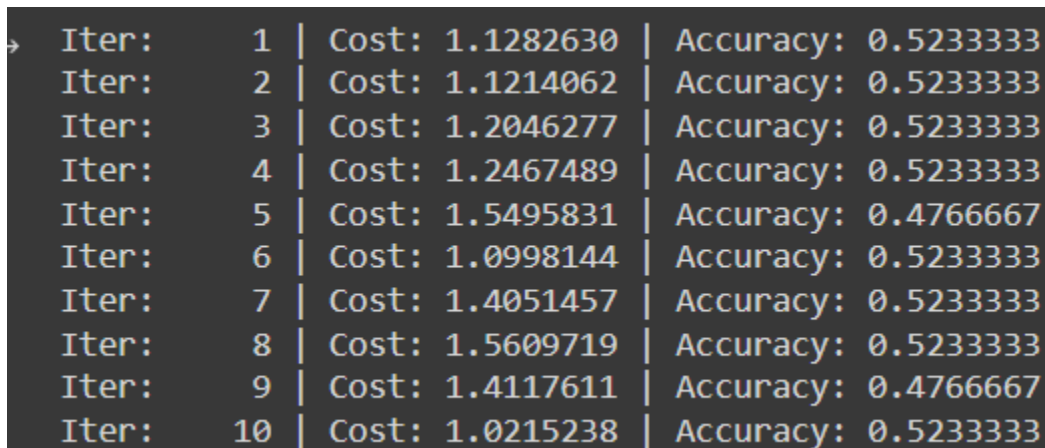
- For angle embedding , we need four qubits to encode the classical data
- We considered batch size of 5 and ***NesterovMomentumOptimizer*** as our optimizer
- The ansatz considered was **RX,RY and RZ** and a **CNOT** gate which with **3** layers gave training results as (Some iterations image):



A screenshot of a terminal window showing training results for a quantum circuit with 3 layers. The output is a table with 10 rows, each representing an iteration. Each row contains the iteration number, the cost value, and the accuracy value, separated by vertical bars. The accuracy values alternate between 0.4766667 and 0.5233333.

Iter:	1	Cost: 1.5809759	Accuracy: 0.4766667
Iter:	2	Cost: 1.4952856	Accuracy: 0.5233333
Iter:	3	Cost: 1.0321662	Accuracy: 0.4766667
Iter:	4	Cost: 1.1538944	Accuracy: 0.5233333
Iter:	5	Cost: 2.5349856	Accuracy: 0.4766667
Iter:	6	Cost: 1.3149044	Accuracy: 0.4766667
Iter:	7	Cost: 1.1260962	Accuracy: 0.5233333
Iter:	8	Cost: 0.9981576	Accuracy: 0.5233333
Iter:	9	Cost: 1.3833043	Accuracy: 0.5233333
Iter:	10	Cost: 2.4941570	Accuracy: 0.4766667

- Next Considering the number of layers as **4** gave:



A screenshot of a terminal window showing training results for a quantum circuit with 4 layers. The output is a table with 10 rows, each representing an iteration. Each row contains the iteration number, the cost value, and the accuracy value, separated by vertical bars. The accuracy values are mostly 0.5233333, with a few instances of 0.4766667 at iterations 5 and 9.

Iter:	1	Cost: 1.1282630	Accuracy: 0.5233333
Iter:	2	Cost: 1.1214062	Accuracy: 0.5233333
Iter:	3	Cost: 1.2046277	Accuracy: 0.5233333
Iter:	4	Cost: 1.2467489	Accuracy: 0.5233333
Iter:	5	Cost: 1.5495831	Accuracy: 0.4766667
Iter:	6	Cost: 1.0998144	Accuracy: 0.5233333
Iter:	7	Cost: 1.4051457	Accuracy: 0.5233333
Iter:	8	Cost: 1.5609719	Accuracy: 0.5233333
Iter:	9	Cost: 1.4117611	Accuracy: 0.4766667
Iter:	10	Cost: 1.0215238	Accuracy: 0.5233333

- It gave a test accuracy of **0.4833**.

- Next Considering the number of layers as **5** gave:

```
➞ Iter:    1 | Cost: 1.0045051 | Accuracy: 0.5233333
   Iter:    2 | Cost: 1.2125953 | Accuracy: 0.4766667
   Iter:    3 | Cost: 2.6445697 | Accuracy: 0.5233333
   Iter:    4 | Cost: 1.8856087 | Accuracy: 0.5233333
   Iter:    5 | Cost: 1.8997313 | Accuracy: 0.4766667
   Iter:    6 | Cost: 1.0875426 | Accuracy: 0.5233333
   Iter:    7 | Cost: 1.0829376 | Accuracy: 0.4766667
   Iter:    8 | Cost: 1.6559267 | Accuracy: 0.4766667
   Iter:    9 | Cost: 1.1080120 | Accuracy: 0.4766667
   Iter:   10 | Cost: 1.4331608 | Accuracy: 0.5233333
```

- It gave a test accuracy of **0.4677**.
- Next Considering the number of layers as **6** gave:

```
➞ Iter:    1 | Cost: 1.7537068 | Accuracy: 0.4766667
   Iter:    2 | Cost: 1.1955090 | Accuracy: 0.5233333
   Iter:    3 | Cost: 2.3393624 | Accuracy: 0.4766667
   Iter:    4 | Cost: 1.0095663 | Accuracy: 0.4766667
   Iter:    5 | Cost: 1.3053505 | Accuracy: 0.4766667
   Iter:    6 | Cost: 1.1883827 | Accuracy: 0.4766667
   Iter:    7 | Cost: 1.8977823 | Accuracy: 0.5233333
   Iter:    8 | Cost: 1.3684752 | Accuracy: 0.4766667
   Iter:    9 | Cost: 1.6650727 | Accuracy: 0.4766667
   Iter:   10 | Cost: 1.0005680 | Accuracy: 0.5233333
```

- It gave a test accuracy of **0.4833**.

## ANALYSIS:

- Amplitude embedding needs only two qubits to encode the data as compared to four for angle embedding. Since near-term quantum computers only have a limited number of qubits, amplitude embedding is a better option.
- Other encoding such as kernel, basis were considered and researched on. Since basis encoding needs a lot of qubits (classical data has values in 10000's which would need 14 qubits to embed) while the kernel method was a bit too complex and hence I went with angle and amplitude embedding which were relatively better and simpler.
- Amplitude embedding for the chosen ansatz gave better results and higher accuracy. The ansatz chosen for angle embedding saturated in terms of accuracy and couldn't improve significantly even when layers were increased which shows that the chosen ansatz is not a good one
- For Amplitude embedding, for the chosen ansatz **five layers gave the best results** with a **test accuracy of 0.833** and also gave the best training accuracy too.
- For Angle embedding, almost all layers gave similar results with training accuracy saturating at **0.5233**. There were slight variations for the given ansatz with different number of layers and the best result came for four layers which gave a test accuracy of **0.4833**
- The ansatz chosen was simple and hence the training and test results were not very impressive. A better ansatz involving further more complex gates and entanglements would hopefully give better results.