Università
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**Faculty
of Informatics**

Computer Graphics (Fall 2023)

Assignment 4: Transformations and Barycentric Coordinates

October 18, 2023

Exercise 1 [5 points]

Let us consider a 2D coordinate system with points $p_1 = (1,1)^T$, and $p = (1.5, 2.5)^T$. Additionally, let's define a vector $\mathbf{u} = p - p_1$. Perform the following tasks:

Task 1 [1 points] Construct two matrices, $R_{90}, T \in \mathrm{R}^{3\times3}$, such that the first one performs counter-clockwise rotation around the center of the coordinates system by angle $90°$, and the second one performs translation by vector $\mathbf{t} = (1, -2)^T$, in homogeneous coordinates.

Task 2 [1 points] Represent the points p_1 and p as well as the vector $\mathbf{u}$ using homogeneous coordinates and transform them first using the R_{90}, and then T matrices. Convert the obtained points and vector back into Cartesian coordinates and denote them by p_1', p', and $\mathbf{u}'$. Draw all the points and vectors before and after the transformation, and verify that $\mathbf{u}' = p' - p_1'$. What influence did the matrix T have on the $\mathbf{u}'$?

Task 3 [1 points] Construct a scaling matrix, $S \in \mathrm{R}^{3\times3}$, which scales everything in x and y direction by a factor of 2. Compute new points p_1'', p'' and vector $\mathbf{u}''$ by transforming p_1', p', and $\mathbf{u}'$ using matrix S. Again perform the operation in homogeneous coordinates, and then, convert the results to Cartesian coordinates and visualize them.

Task 4 [2 points] Construct matrices $R_{90}^{-1}, T^{-1}, S^{-1} \in \mathrm{R}^{3\times3}$, which perform inverse transformations to R_{90}, T, S, i.e., clockwise rotation by $90°$, translation by vector $\mathbf{t}' = (-1, 2)^T$, and scaling by factor 0.5. Compute matrix $M = R_{90}^{-1} T^{-1} S^{-1}$, and verify that transforming points p_1'', p'', and $\mathbf{u}''$ with this matrix results in the initial values of p_1, p, and $\mathbf{u}$.
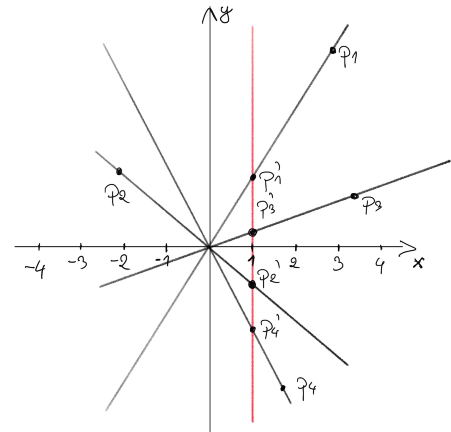
Exercise 2 [4 points]

Consider a triangle made of three points $p_1 = (6, 0, 4)$, $p_2 = (2, 0, 0)$, and $p_3 = (2, 4, 4)$. Verify whether point $p = (4, 1, 3)$ belongs to the interior of the triangle.

Exercise 3 [3 points]

Using barycentric coordinates, prove that the centroid of a triangle divides its medians in ratio 2:1.

Exercise 4 [3 points]

Consider a transformation that projects all the points $p_i \in \mathrm{R}^2$ onto the line $x = 1$. For each point, the projection is performed along the line that passes through the center of the coordinate system and the point (see the image on the right). Each point p'_i is a result of applying this transformation to point p_i. Construct a matrix $M \in \mathrm{R}^{3 \times 3}$ which realizes this transformation using homogeneous coordinates. More specifically, after converting a point p_i into the homogeneous coordinates, multiplying it with the matrix M, and transforming it back to the Cartesian coordinates, you should obtain point p'_i. Note that this is not an affine transformation. Comment on how the matrix M transforms the points lying on the y-axis. Interpret the results both in the homogeneous and Cartesian coordinates.

Bonus exercise 5 [2 points]

Consider the same task as in Exercise 4, but the line onto which the points are projected can be now arbitrary, and it is defined by a line equation $y = ax + b$, where $a, b \in \mathrm{R}$ are constants. Derive the matrix M for this more general case.

Submission

Submit a single PDF including all the calculations you did to solve the assignments to iCorsi. The file should contain information about all authors and it should be named with their surnames. You should not use any external tools, other than a simple calculator, which can provide intermediate or final results.

Solutions must be returned on October 25 2023 via iCorsi3