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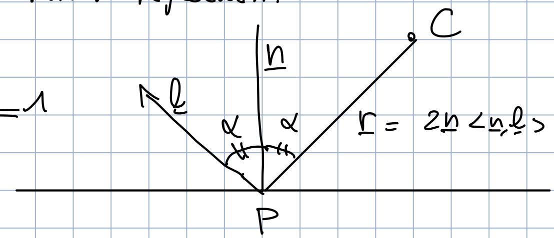
Exercise 2

Ground $y = \emptyset$

Light to source direction $\underline{l} = [1\ 2\ 2]^T$ $\quad r(t) = t\underline{d} + \underline{o}$ ray light

Camera $C = (4, 6, 7)$

? Position P on ground y where camera observes perfect mirror reflection?



Since $y = \emptyset$ and $P \perp y$, then P has normal $\underline{n} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $\hat{n} = \|n\| = 1$

$$\|\underline{l}\| = \sqrt{\underline{l}^T \underline{l}} = \sqrt{[1\ 2\ 2] \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}} = \sqrt{1+4+4} = \sqrt{9} = 3$$

$$\hat{\underline{l}} = \frac{\underline{l}}{\|\underline{l}\|} = \frac{\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}}{3} = \frac{1}{3} \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

then $\underline{r} = 2\hat{n}\langle \hat{n}, \hat{\underline{l}} \rangle - \hat{\underline{l}} \implies 2 \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \cdot [0\ 1\ 0] \cdot \frac{1}{3} \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} - \frac{1}{3} \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} =$

$$= \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} [0\ \tfrac{1}{3}\ 0] \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} - \begin{bmatrix} 1/3 \\ 2/3 \\ 2/3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2/3 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} - \begin{bmatrix} 1/3 \\ 2/3 \\ 2/3 \end{bmatrix} = \begin{bmatrix} 0 \\ 4/3 \\ 0 \end{bmatrix} - \begin{bmatrix} 1/3 \\ 2/3 \\ 2/3 \end{bmatrix} = \begin{bmatrix} -1/3 \\ 2/3 \\ -2/3 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} -1 \\ 2 \\ -2 \end{bmatrix}$$

Since $\|\underline{r}\| = 1$ then $\underline{r} = \hat{\underline{r}}$,

Moreover, in $\hat{\underline{r}} = t\underline{d} + \underline{o}$, $\underline{o}$ of $\hat{\underline{r}}$, AKA P, has $y = \emptyset$

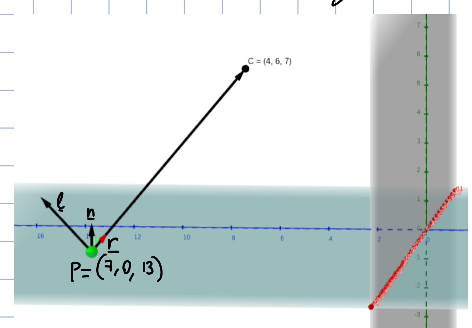
Find P, t for which camera C intersects $\hat{\underline{r}}$

$$P = C - t\hat{\underline{r}}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ 7 \end{bmatrix} - t \cdot \frac{1}{3} \cdot \begin{bmatrix} -1 \\ 2 \\ -2 \end{bmatrix} \overset{\substack{P \in y=\emptyset \\ P(y=0)}}{\implies} \begin{bmatrix} x \\ \emptyset \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ 7 \end{bmatrix} + \begin{bmatrix} t/3 \\ -2t/3 \\ 2t/3 \end{bmatrix} \implies -\frac{2t}{3} + 6 = \emptyset \implies t = -6 \cdot \frac{-3}{2} \implies t = 9$$

$$\implies \begin{bmatrix} x \\ \emptyset \\ z \end{bmatrix} = \begin{bmatrix} 4+3 \\ 6-6 \\ 7+6 \end{bmatrix} = \begin{bmatrix} 7 \\ \emptyset \\ 13 \end{bmatrix} = P$$

$\boxed{\text{Task 1 ✓}}$



Plane reflecting $\frac{1}{2}$ light for diffuse and specular reflection.

Shininess coefficient of plane $K = 2$.

Plane not emit light, no ambient illumination, directed light intensity $I = 1$.

No color in scene, everything grey.

? Compute intensity of light I at P

Assume any distance you want.

Intensity I at point P $\Rightarrow$

$I_e = 0$ not emitting light

$p_a \cdot I_a = 0$ no ambient illumination

$K = 2$ shininess

$p_d = p_s = \frac{1}{2}$

$n = 1$ only 1 single light

$I_{j=1} = 1$ intensity of the single light

everything grey $\rightarrow$ $(\underset{R}{128}, \underset{G}{128}, \underset{B}{128})$

Phong Lighting Model

$$I = I_e + p_a \cdot I_a + \sum_{j=1}^{n} \left(p_d \cdot \cos\phi_j + p_s \cdot \cos^K \alpha_j\right) \cdot I_j$$

$$I = 0 + 0 + \sum_{j=1}^{1} \left(p_d \cdot \cos\phi_j + p_s \cdot \cos^K \alpha_j\right) \cdot I_j$$

$$\cos(\phi) = \langle \underline{\hat{n}}, \underline{\hat{\ell}} \rangle = [0\ 1\ 0]^T \cdot \frac{1}{3} \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

$$I = \left(\frac{1}{2} \cdot [0\ 1\ 0]^T \cdot \frac{1}{3} \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} + \frac{1}{2} \cdot \langle \underline{v}, \underline{\hat{r}} \rangle^2\right) \cdot 1$$

$\underline{v} = \underline{\hat{r}}$ since maximal intensity on perfect reflection direction

so $\langle \underline{v}, \underline{\hat{r}} \rangle = \langle \underline{\hat{r}}, \underline{\hat{r}} \rangle = \|\underline{\hat{r}}\|^2 = 1$

$$I = \left(\frac{1}{6} \cdot 2 + \frac{1}{2} \cdot 1^2\right) \cdot 1 = \frac{1}{3} + \frac{1}{2} = \frac{5}{6}$$

With the extended Phong Model we need to take into account the distance for every diffusion and specular light we get from light sources:

$$I = I_e + p_a \cdot I_a + \sum_{j=1}^{n} \left(p_d \cdot \cos(\phi_j) + p_s \cdot \cos^K(\alpha_j)\right) I_j \cdot att(d)$$

In this case we assume directional light distance such that there's no attenuation

Hence $att(d) = 1$ and $I = \frac{5}{6} \cdot att(d) = \frac{5}{6} \cdot 1 = \frac{5}{6}$

Giving us back the previous Phong Model and keeping the former result $I = \frac{5}{6}$ valid

$\boxed{\text{Task 2 } \checkmark}$

Exercise 3

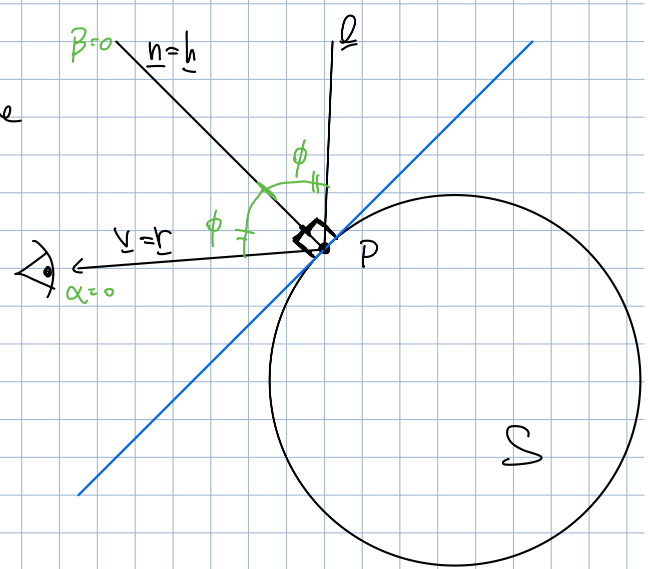
ray $\underline{y}$ intersecting sphere S at point P with normal $\underline{n}$ at P, $\underline{v}$ direction towards camera, $\underline{\ell}$ direction to light source, $\underline{r}$ reflected direction.

Prove: if all vectors coplanar, angle β between half vector $\underline{h} = \frac{\underline{\ell} + \underline{v}}{2}$ and $\underline{n}$ equals half angle α between $\underline{v}$ and $\underline{r}$

Since all vectors are coplanar, $\exists$ geometric plane containing all of them, so a coordinate is the same for all of them and 2Dimensional representation is accurate.

Assuming all vectors are normalized.

The viewpoint sees point P along sphere's tangent on point P.

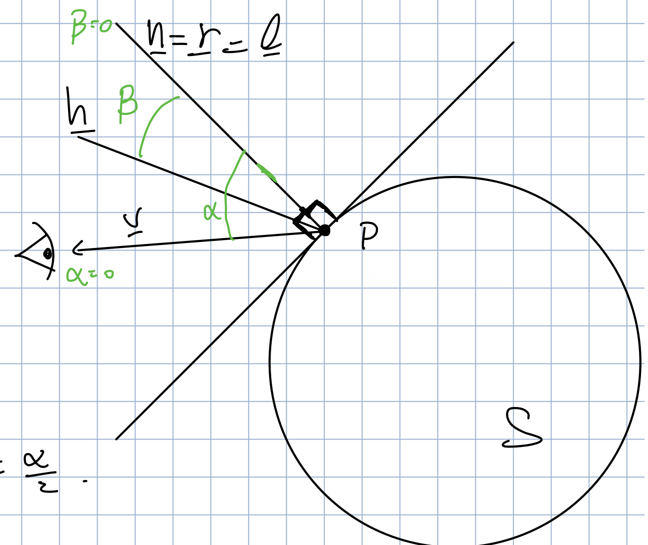


<u>For trivial case</u>:

if $\underline{v}=\underline{r}$, then $\angle(\underline{v},\underline{r})=0^\circ=\alpha$

and $\underline{h}=\frac{1}{2}(\underline{\ell}+\underline{v})=\underline{n}$, so $\angle(\underline{n},\underline{h})=0^\circ=\beta$

Hence $\beta=\frac{\alpha}{2}$ holds



<u>Clean case</u>:

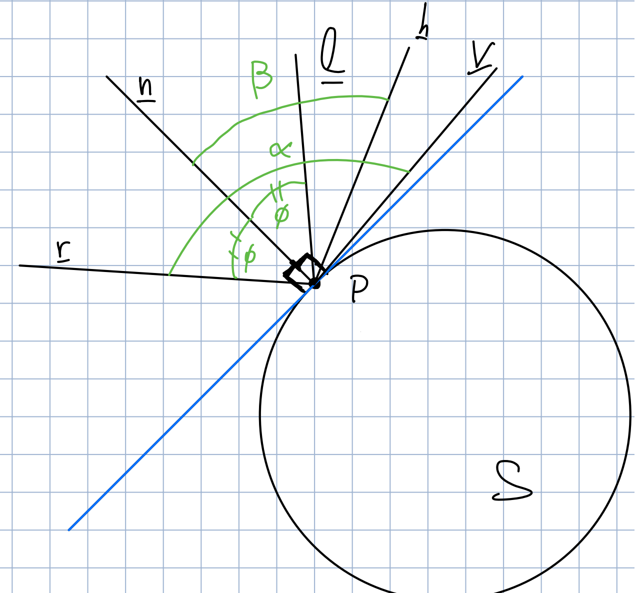
if light $\underline{\ell}$ comes from normal $\underline{n}$, then $\underline{\ell}=\underline{n}=\underline{r}$ since reflected directly. This shows clearly hold out of halfway vector $\beta=\frac{\alpha}{2}$.

<u>General case</u>:

Since halfway vector is the unit vector exactly halfway between view direction $\underline{v}$ and light direction $\underline{\ell}$:

$$h=\frac{1}{2}(\underline{\ell}+\underline{v}) \quad \text{and} \quad \angle(\underline{n},\underline{h})=\beta$$

Moreover, normal $\underline{n}$ alway between $\underline{\ell}$ and $\underline{r}$ by definition of specular reflection.



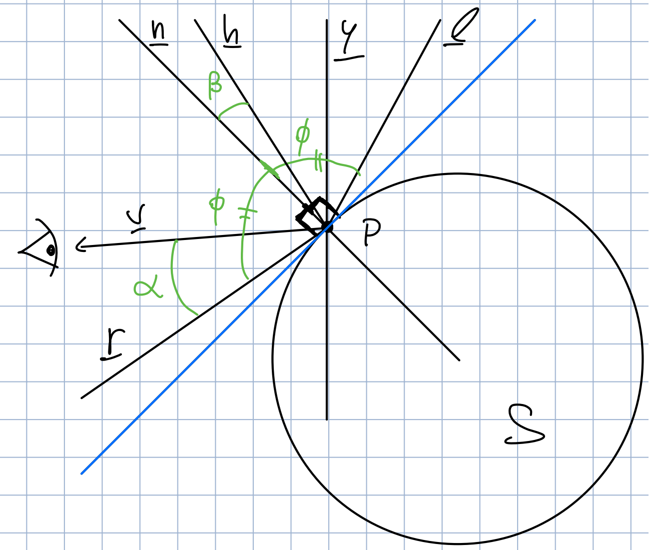
Note $\angle(\underline{v},\underline{r})=\alpha$

Angles α,β are directly correlated through $\underline{v}$ since h is given from $\frac{1}{2}(\underline{\ell}+\underline{v})$ with $\frac{1}{2}$ proportion in angle degrees.

Example: if $\angle(\underline{r},\underline{n})$ increase by $+1^\circ$,

$\angle(\underline{r},\underline{n})$ and $\angle(\underline{n},\underline{\ell})$ increase by $+\frac{1}{2}^\circ$

So increase get distributed evenly



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