PHY407 Lab08

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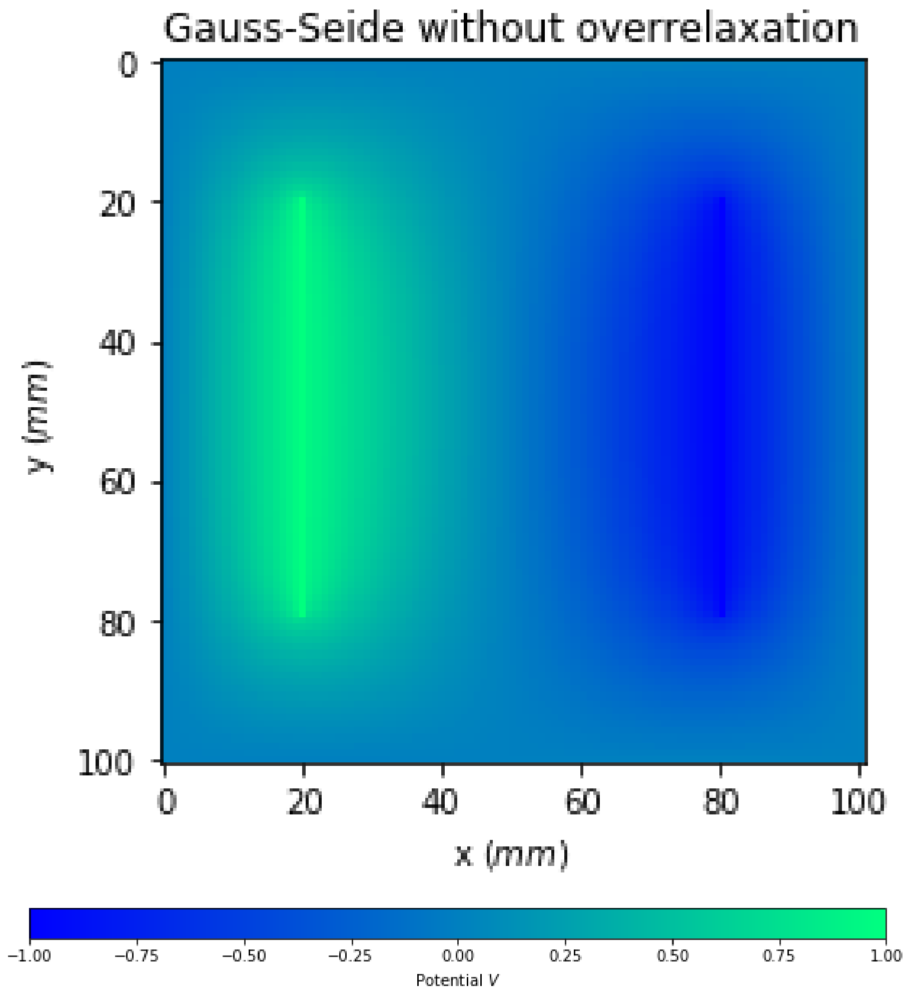
1 Electrostatics and Laplace's equation

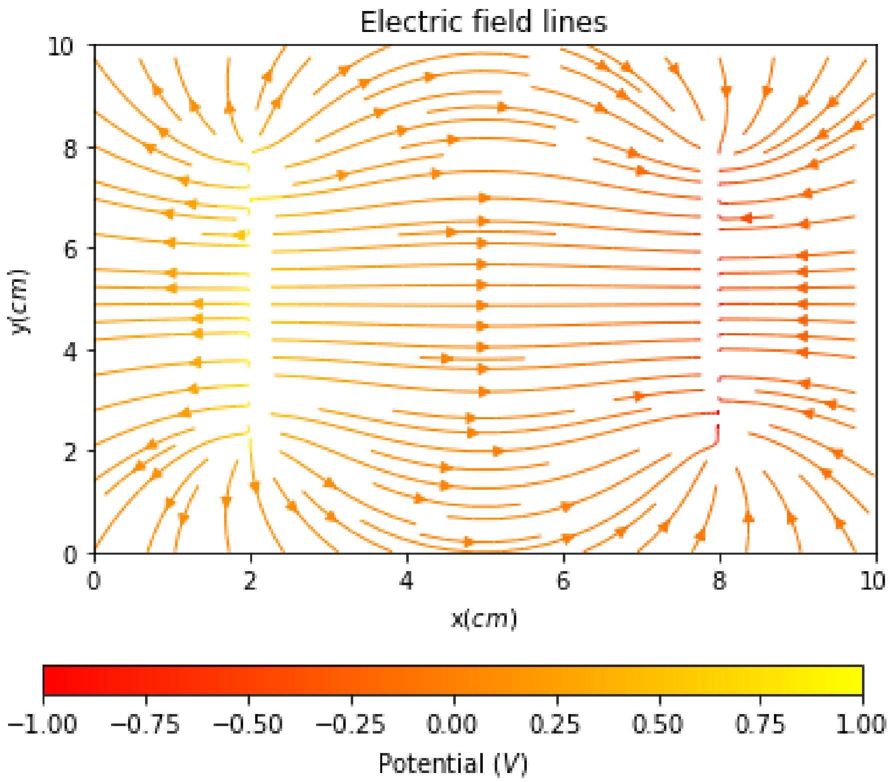
In this section we will use the Gauss-Seide method to find the potential at each point inside a box with two parallel capacitors of opposite potential contained inside and then show the spread of potential by plotting a contour plot. We will find the distribution of potential using both the none over-relaxation method, and the over-relaxation method with omega=0.1 and omega=0.5. We will also plot the electric field lines generated by the two capacitors.

1.a

First we will plot our results from the none over-relaxation Gauss-Seide method.



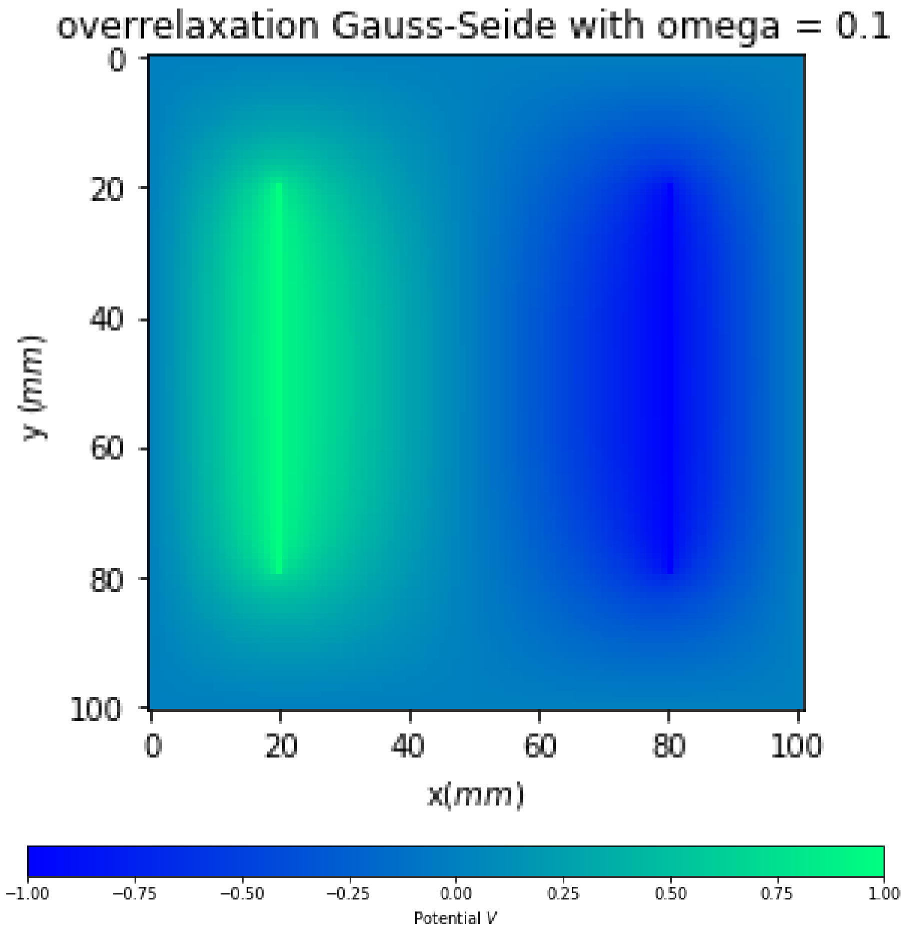
The graph suggests that the highest and lowest potentials occur on the capacitors which is expected. The potential gets closer to zero as you move further away from both capacitors and the potential is zero on the boundary of the box.



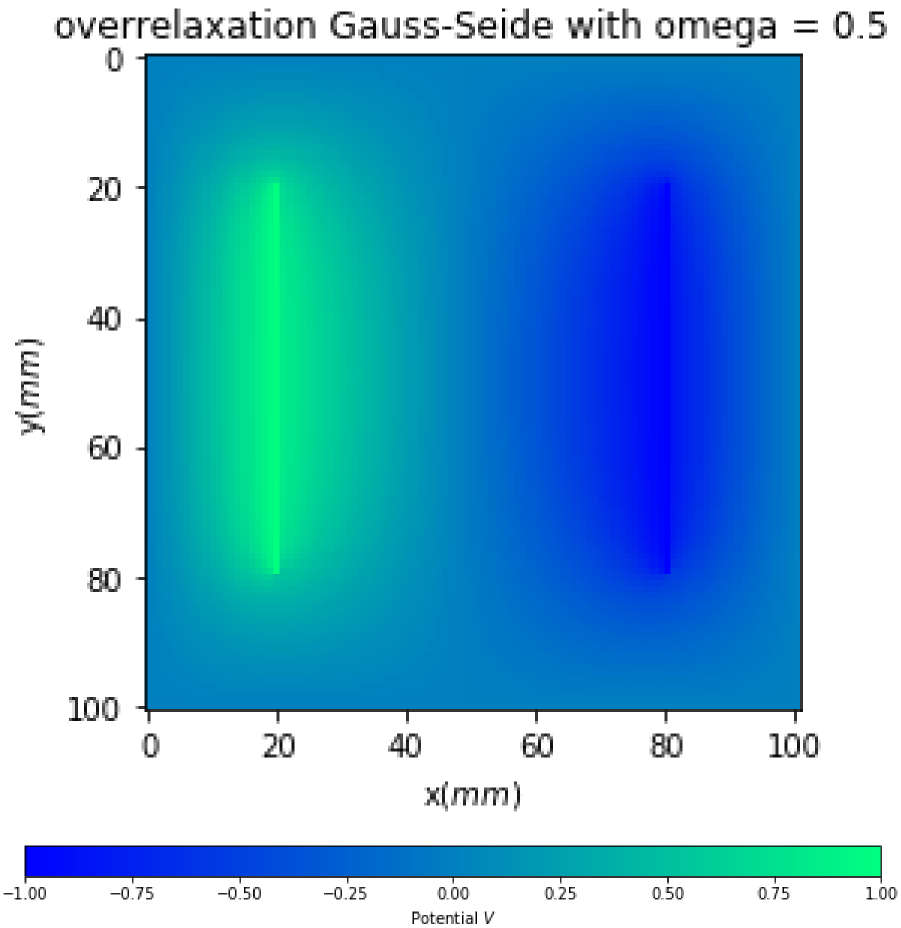
The electric field is pointing away from the capacitor with positive potential and towards the capacitor with negative potential. The electric field gets stronger (as in the absolute value of the electric field gets larger) when as you get closer to a capacitor. This is to be expected.

1.b

now we will plot our results from the over-relaxation version of the Gauss-Seide method. first we will plot the case where omega=0.1:



This looks very similar to the non-over-relaxation solution. The only noticable difference is that the time it took to run the code for this solution was 101.932 seconds while the non-over-relaxation solution took 112.652 second to run. So the over-relaxation method proves to be a bit faster when omega=0.1.



Similar to the previous solution, this graph looks like the non-over-relaxation solution as well (with very small differences in the distribution of potential) and the only noticeable difference is the code run time. This solution, with omega=0.5, took 68.556 seconds to run. The pattern seems to be that the run time of the code becomes lower as omega becomes higher as long as delta converges with the omega chosen.

2 Simulating the shallow water system, Part I

2.a

We know

$$\vec{F}(u,\eta) = [\frac{1}{2}u^2 + g\eta, (\eta - \eta_b)u] := (F_u, F_\eta)$$

In flux conservative form, $\vec{u} = (u, \eta)$:

$$\begin{aligned} \frac{\partial u}{\partial t} &= -g\frac{\partial \eta}{\partial x} - u\frac{\partial u}{\partial x} \\ &= \frac{d}{dx}(-g\eta - \frac{u^2}{2}) \\ &= -\frac{\partial}{\partial x}F_u \end{aligned} \tag{1}$$

and

$$\begin{aligned} \frac{\partial \eta}{\partial t} &= -\frac{\partial(u(\eta - \eta_b))}{\partial x} \\ &= -\frac{\partial}{\partial x}F_\eta \end{aligned} \tag{2}$$

So for the FTCS scheme following 1,

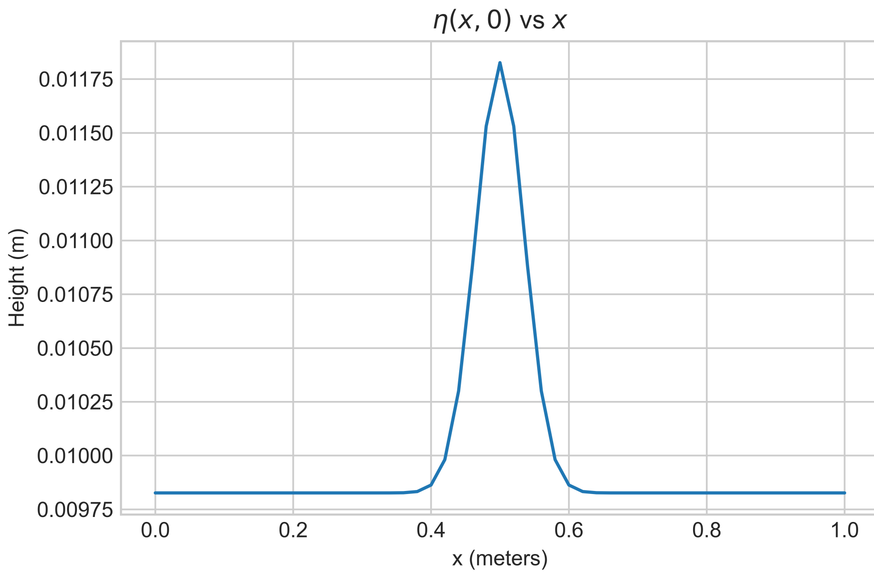
$$\begin{aligned} u_j^{n+1} &= u_j^n - \frac{\Delta t}{2\Delta x}(F_{u,j+1}^n - F_{u,j-1}^n) \\ &= u_j^n - \frac{\Delta t}{2\Delta x}[(\frac{1}{2}(u_{j+1}^n)^2 + g\eta_{j+1}^n) - (\frac{1}{2}(u_{j-1}^n)^2 + g\eta_{j-1}^n)] \end{aligned} \tag{3}$$

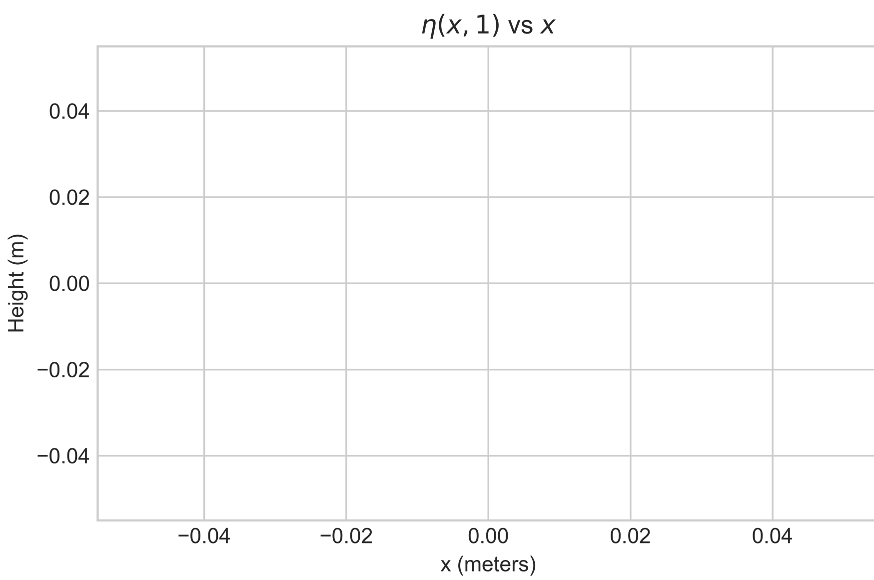
And for the FTCS scheme following 2,

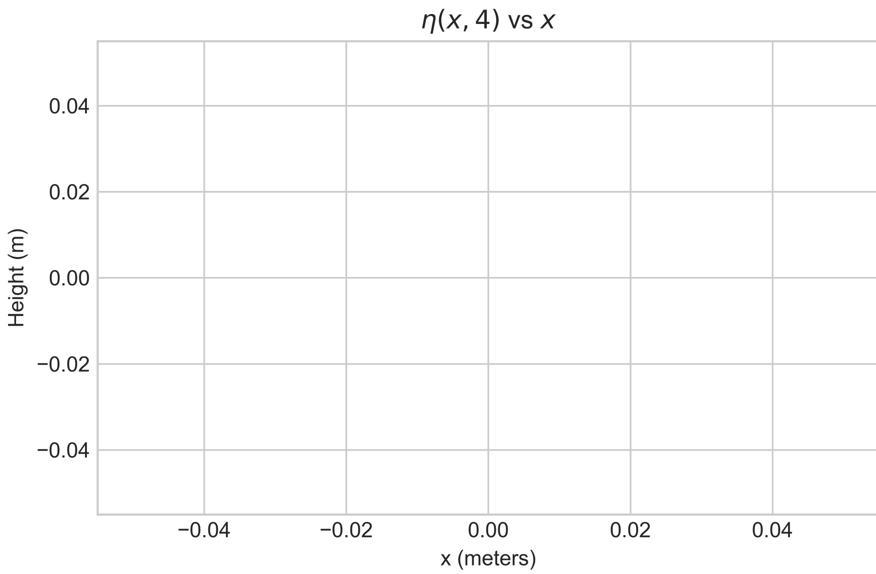
$$\begin{aligned} \eta_j^{n+1} &= \eta_j^n - \frac{\Delta t}{2\Delta x}(F_{\eta,j+1}^n - F_{\eta,j-1}^n) \\ &= \eta_j^n - \frac{\Delta t}{2\Delta x}[(u_{j+1}^n(\eta_{j+1}^n - \eta_{b,j+1})) - (u_{j-1}^n(\eta_{j-1}^n - \eta_{b,j-1}))] \end{aligned} \tag{4}$$

2.b

Here are the plots







As can be seen for $t = 0$ the algorithm worked. And it peaks at $x = 0.5m$ whcih was expected. But for RunTime errors, it wasn't possible to reach the plots with $t = 2, 4$.

2.c

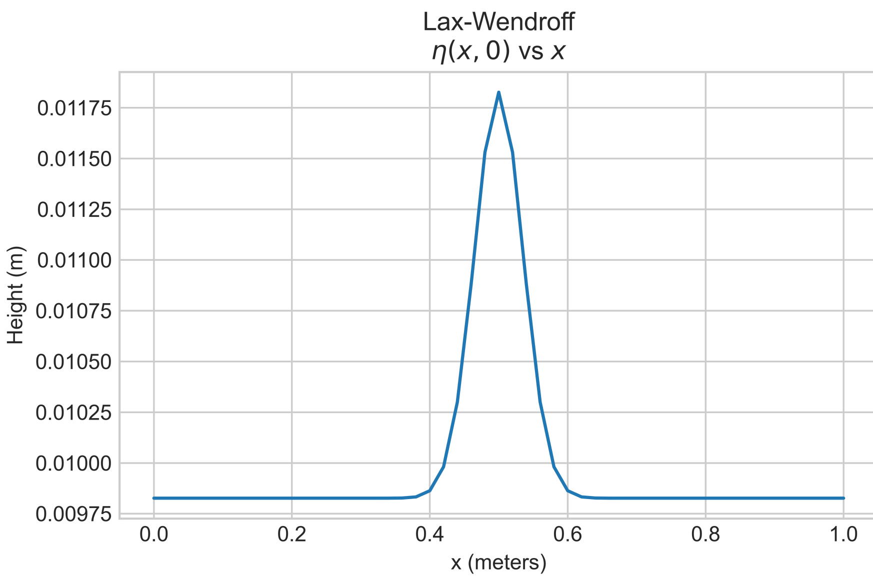
equation 6a in the handout is $\frac{du}{dt} + u\frac{du}{dx} = -g\frac{dn}{dx}$. if we let u,n=0, then this equation becomes $\frac{du}{dt} + 0\frac{du}{dx} = \frac{du}{dt} = -g\frac{dn}{dx}$
equation 6b in the handout is $\frac{dn}{dt} + \frac{d}{dx}(uh) = 0$.
h=n-n_b, n=0, and n_b is constant so h=n_b.
equation 6b becomes $\frac{dn}{dt} + \frac{d}{dx}(un_b) = \frac{dn}{dt} + n_b\frac{du}{dx} = 0 --> \frac{dn}{dt} = -n_b\frac{du}{dx}$

pages 429 and 430 in the Computational physics textbook derive the equation for $\| \lambda \|$

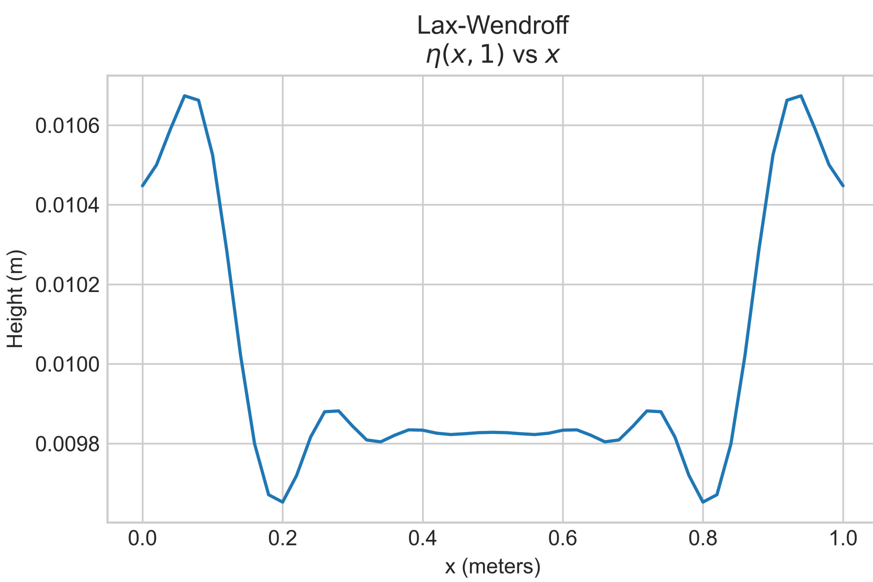
3 Simulating the shallow water system, Part II

In this section we will use the Two-Step Lax-Wendroff scheme to more accurately plot the shallow water system.

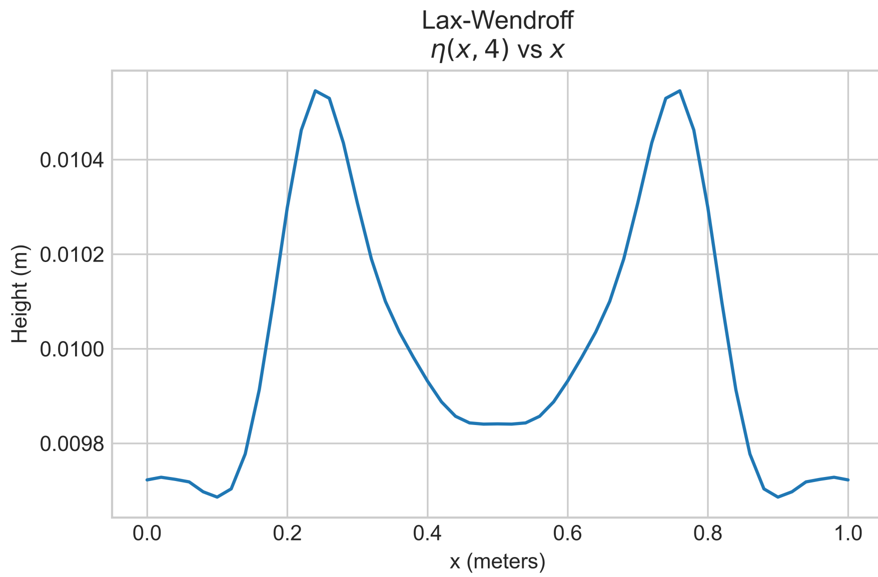
3.a



Lax-Wendroff
$\eta(x,0)$ vs x
0.01175
0.01150
0.01125
0.01100
0.01075
0.01050
0.01025
0.01000
0.00975
Height (m)
0.0
0.2
0.4
0.6
0.8
1.0
x (meters)

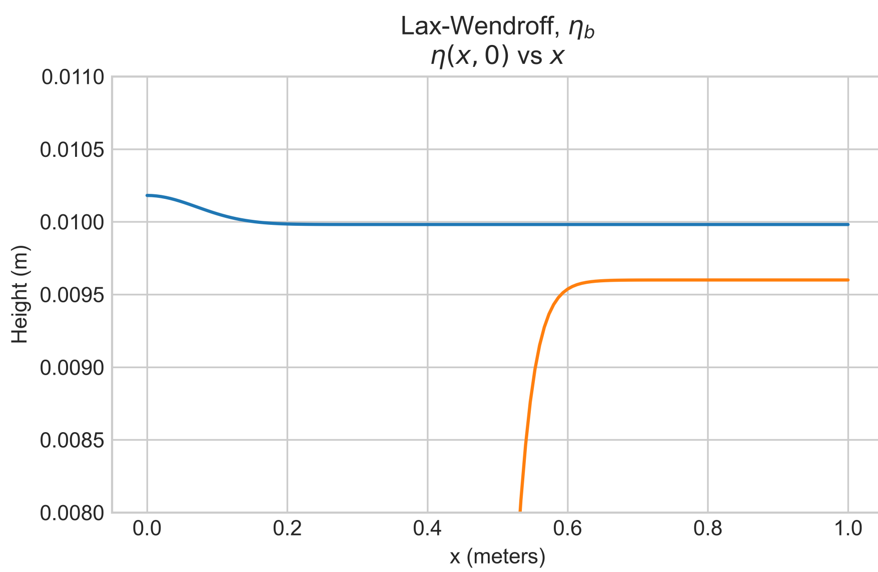


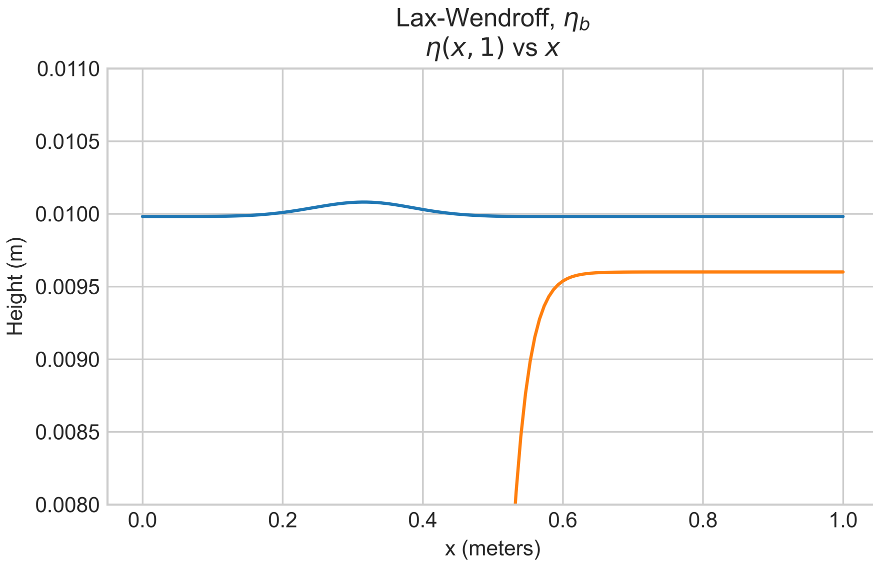
Lax-Wendroff
$\eta(x,1)$ vs x
0.0106
0.0104
0.0102
0.0100
0.0098
Height (m)
0.0
0.2
0.4
0.6
0.8
1.0
x (meters)

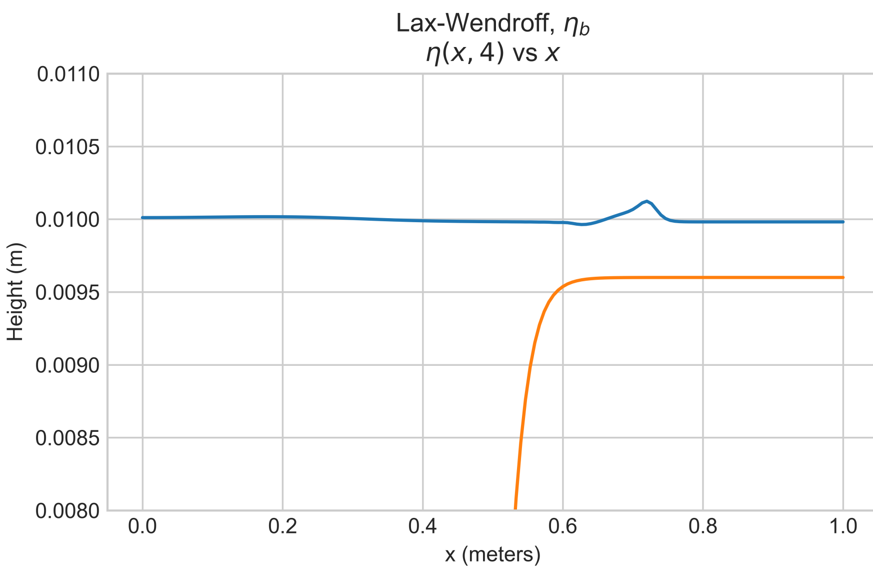


3.b

In this section, the bottom topography is changed with function. So the wave form and speed changes with propagation will be observed.







Orange lines are representing the bottom topography. Although the bump at the height of the wave is not very visible from the graphs, the speed values will give an understanding of the changes:

```
speed of wave at t= 0  0.0 m/s
speed of wave at t= 1s  0.0178 m/s
speed of wave at t= 2s  0.0337 m/s
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```
speed of wave at t= 3s  0.045 m/s
speed of wave at t= 4s  0.0493 m/s
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So as can be seen, as wave propagates, it gets faster.