Income-Based vs. Rent-Based Housing Subsidies in the Presence of Search Frictions

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Abstract

Many developed countries provide housing subsidies to low-income renters. Recipients in the U.S. pay a fixed percentage of their income towards housing, and the subsidy covers the remaining cost. In other countries, the subsidy is instead calculated as a percentage of the contract rent. This paper documents evidence of search and matching frictions in the rental housing market and examines how these frictions impact the costs and benefits of income-based and rent-based housing subsidies. I develop a directed search model of the rental housing market and analyze the equilibrium effects of subsidies. Under rent-based subsidies, higher matching frictions increase costs for both the households and the subsidy provider. On the other hand, when subsidies are based on income, households bear most of the additional costs of increased matching frictions. In general, income-based subsidies distort households' search behavior more than when subsidies are rent-based.

1 Introduction

Since 2001, between 40 and 50% of U.S. rental households have been cost-burdened[1]. In particular, low-income households are the most likely to face these burdens: 83% of renter households earning less than $15,000 were cost-burdened in 2018 (Joint Center for Housing Studies of Harvard University, 2020). This disparity is not unique to the U.S. For example, the median share of gross household income spent on housing among all renter households in the United Kingdom in 2013 was 30%. The same figure among the lowest quintile of renter households was about 45% (Carliner and Marya, 2016). Many developed countries have recently sought to ease the affordability challenges for their low-income renters using housing subsidies. Subsidy recipients in the U.S. must pay a fixed percentage of their income towards housing, and the subsidy covers the remaining rent. In contrast, the housing policies in other countries (such as the U.K. and France) calculate a household's subsidy as a percentage of the contract rent set by the landlord. This difference in the structure of housing subsidies could have important consequences for the decisions of low-income households.

This paper makes two main contributions. First, this paper documents evidence of search frictions in the housing rental market by showing that households who change locations (and are thus likely to have fewer outside options) pay more for their current housing, *ceteris paribus*. This corresponds to the search behavior of households in the theoretical model. Unmatched households search for housing with high rent in order to move out of the less attractive unmatched state quickly, while matched households have their current housing as a better outside option and thus can search for the lower-rent housing that takes longer to find. Second, this paper examines how these search and matching frictions impact the costs

[1] As is standard in the literature, cost-burdened households are defined as those that pay more than 30% of their income for housing.

and benefits of income-based and rent-based housing subsidy programs.

It is important to consider search and matching frictions when analyzing the cost-efficiency of housing subsidies as these subsidies can distort residents' incentives to search for less expensive housing. This will raise the cost of providing housing subsidies as households will live in more expensive units even though lower rent units would otherwise be available. Whether subsidies are calculated as a percentage of the household's income or as a percentage of the contract rent distorts residents' incentives differently and thus cause different equilibrium outcomes in the housing market. The results show that income-based subsidies distort the residents' search decisions more than when subsidies are rent-based. In addition, matching frictions increase the cost of providing subsidies and decrease their ability to make housing more affordable for low-income households.

This paper models the rental housing market using directed search as in Menzio and Shi (2010b). One of the key mechanisms in the model is that households and landlords face a trade-off between the contract rent and the time it takes to find a match. In the model, housing subsidies are a lump sum transfer calculated as a percentage of either the household's income or current contract rent. With income-based housing assistance, the household's contribution to the rent is a constant based on their income and the subsidy pays for the remaining portion of the rent, up to a subsidy ceiling. Above this ceiling, the subsidy amount does not change and the household must pay for the difference. When housing subsidies are rent-based, households pay a set percentage of the rent up to the ceiling rent, and the full amount in excess of the ceiling.

The results show that with housing amenities fixed and under income-based housing assistance, the subsidy ceiling is a corner solution. Low-income households with current rents above the subsidy ceiling search for lower rent units, but they stop searching once they obtain a unit at the subsidy ceiling if they would not get the benefit from any further rent decreases. This behavior is consistent with McMillen and Singh's (2018) empirical finding that rents in Los Angeles tend to cluster around the subsidy ceiling (called the "Fair Market Rent"), after controlling for location, quality, and other characteristics of the property.

In contrast, when housing subsidies are based on rent, households search for lower rent units regardless of whether their current rent is above or below the subsidy ceiling. This is because the household's own contribution toward rent continues to fall even when the total contract rent is below the subsidy ceiling (although the household's savings are less than a dollar for each dollar by which the rent falls). Thus, the distortionary effects of rent-based subsidies are less than those caused by income-based subsidies.

While this result is intuitive and may be common to other models without search, the matching frictions have other implications for the housing outcomes in equilibrium. In particular, when matching frictions are stronger (meaning it is harder to find a match for a given market tightness) and all other parameters are held equal, households pay more in rent. This occurs because the lower match-finding probability makes households search in higher rent submarkets given their current state. As a result, they start out paying more for rent when they initially move from the unmatched to the matched state, and they move more slowly down the rent distribution. This result holds regardless of the housing policy, although who incurs the extra costs depends on the subsidy structure.

When subsidies are based on income, higher matching frictions cause unmatched households to search in submarkets with rents even higher above the subsidy ceiling. It also takes longer for matched households to move down the rent distribution to the subsidy ceiling.

Consequently, households bear the cost of the higher matching frictions while the subsidized amount remains fixed at its ceiling. Under rent-based subsidies, unmatched households also search in higher rent submarkets above the subsidy ceiling due to higher matching frictions, and matched households take longer to move to lower rent housing. Thus, as with income-based subsidies, households initially bear the cost of the higher matching frictions. However, since households paying relatively low rent search in submarkets below the subsidy ceiling, the slower pace of movement down the rent distribution means that both the household and the subsidized components of rent will be higher due to higher matching frictions.

The rest of the paper is organized as follows. Section 2 discusses previous literature related to this paper. Section 3 provides the details of the data used in this paper, and Section 4 presents empirical evidence of search frictions in the rental housing market. Section 5 describes the model, while Section 6 defines an equilibrium in the model. Section 7 provides the details of the calibration and estimation of the model. Section 8 uses the model to evaluate the housing policies, and Section 9 analyzes the interactions between the housing subsidies and matching frictions. Lastly, Section 10 concludes.

2 Literature Review

Many previous studies focus on the effects of housing subsidies in the short run (with inelastic supply) by examining the changes in rents after a major housing subsidy policy change. Previous literature examining the income-based voucher program in the U.S. includes Susin (2002) and Eriksen and Ross (2015). Studies analyzing rent-based housing assistance in Europe and New Zealand include Fack (2006), Gibbons and Manning (2006), Laferrère and Le Blanc (2004), and Hyslop and Rea (2018). These studies generally find that rents paid by assisted households increase after housing subsidy policies become more generous, after controlling for tenant, landlord, and housing unit characteristics (including unit quality). Some of the studies (Susin, 2002; Eriksen and Ross, 2015; Laferrère and Le Blanc, 2004) find that housing subsidies also increase rents paid by unassisted low-income households who compete with assisted households in the rental market. Thus, most prior studies conclude that landlords reap the benefits of housing subsidies in the short run.

Most prior studies that focus on the long-term effects of housing subsidies compare demand-side assistance to tenants with supply-side (project-based) policies that build public housing or incentivize private investors to construct new units for low-income renters (Sinai and Waldfogel, 2002; Olsen, 2001; Merril and Joseph, 1980). They find that demand-side housing subsidies have been much more cost effective and have the best outcomes for the target households. While most of the demand-side subsidies in these studies are income-based, Merril and Joseph (1980) also compare the effects of rent-based housing subsidies and lump sum transfers (fixed amounts that are not based on income or rent). In particular, they present some evidence that households receiving rent-based assistance live in more expensive housing units than those who receive a lump sum subsidy, which they interpret as evidence of shopping costs.

None of the aforementioned papers take into account the search behavior of households, which is important when analyzing housing subsidies to the potential for these policies to distort the households' incentives. This paper relates to the literature using directed search models of the labor market to study economies with heterogeneity (Menzio and Shi, 2010b; Schaal, 2011; Kaas and Kircher, 2015). While several papers have used directed search to model the housing market for homeowners (Karahan and Rhee, 2013; Hedlund, 2016a and

2016b; Zhu, Wright, and Gaumont, 2017; Albrecht, Gautier, and Vroman, 2016) or random search to model the rental housing market (Stull, 1978; Guasch and Marshall, 1985; Kim, 2018; Niedermayer and Wang; 2018), to my knowledge, this is the first paper to model the rental housing market using directed search.

3 Data

The primary data source used in this paper is the American Housing Survey (AHS). The AHS is a national longitudinal survey of housing units which is administered by the U.S. Census Bureau and the U.S. Department of Housing and Urban Development (HUD). The survey was conducted annually between 1973 and 1981, and has been conducted biannually in odd-numbered years since then. I will be using the 1997-2011 AHS as a new sample of housing units was drawn in 1997 and again in 2015. The AHS provides detailed information about characteristics of the housing unit, as well as characteristics of the current residents when occupied. The survey also asks recent movers the reason that they moved out of their previous residence.

Table 1 shows the averages for the main AHS variables used in this paper. The first column includes all privately-owned housing units in the AHS, except for units subject to rent control and vacant units that are held for occasional or seasonal use. The second column also excludes owner-occupied and vacant for sale units, and the third column further drops all vacant units.

Comparing columns (1) and (3), renter-households tend to have lower housing costs and lower income, live in slightly older and smaller units with 1 to 3 bedrooms, move more

Table 1: Descriptive Statistics (Source: American Housing Survey)

	Total	Vacant for Rent or Renter-Occupied	Renter-Occupied
Vacancy Rate	0.06	0.11	-
Fraction of Units Used as Rentals	0.32	-	-
Average Real Housing Costs (2006 Dollars)	1011	-	787
Avg. Real HH Income (2006 Dollars/Month)	5291	-	3194
Avg. Tenure Length (Years)	11.11	-	4.21
Frac. of HH's with Housing Gov't Assistance	-	-	0.07
Avg. Size of Housing Unit (SF)	1847	1172	1181
Avg. Age of Housing Unit (Years)	37	41	41
Frac. in Urban Areas	0.71	0.84	0.84
Frac. of Units that are Studios	0.01	0.02	0.02
Frac. of Units with 1 Bedroom	0.11	0.28	0.28
Frac. of Units with 2 Bedrooms	0.27	0.43	0.43
Frac. of Units with 3 Bedrooms	0.42	0.22	0.22
Frac. of Units with 4 Bedrooms	0.16	0.05	0.05
Frac. of Units with 5 or More Beds	0.04	0.01	0.01
Avg. Age of Head of HH (Years)	49	-	41
Frac. of HH's with Female Head	0.42	-	0.50
Frac. of HH's with Married Head	0.54	-	0.31
Frac. of Recent Movers Exogenously Separated from Previous Residence	0.12	-	0.14
N	603,925	179,013	155,217

often, are more prone to move for exogenous reasons[2], and live in urban areas. The head of renter-households is also younger on average and more likely to be female and unmarried. In addition, about 7% of renter households in the AHS receive government assistance in the form of housing subsidies or vouchers. A comparison of columns (2) and (3) shows that the characteristics of vacant units for rent are fairly similar to those of occupied rental units.

One portion of the analysis also uses the American Community Survey (ACS), which is a national cross-sectional survey conducted by the U.S. Census Bureau annually since 2001. The ACS includes person- and household-level characteristics. It also has data on some

[2] A household is classified as having moved for an exogenous reason if a private company or person wanted to use their previous residence, they were forced to leave by the government, their previous residence was lost to a disaster (fire, flood, etc.), or they moved due to a new job or job transfer.

housing characteristics including the geographic location of the household's current and previous housing unit.

4 Evidence of Search Frictions

When analyzing the costs and benefits of housing policies, search frictions are important to consider as housing subsidies could distort households' decisions. This would arise if the household does not fully internalize the benefit (in regard to potential cost savings) from continuing to search for housing. Renters report taking an average of 2.7 months to find housing, according to ZillowGroup's *Consumer Housing Trends Report* (2018) and Apartments.com and Google's *Online Search Behavior and Trends of Apartment Renters* (2015). In addition, landlords in the AHS with vacant rentals report that their units have remained unoccupied for about 3.1 months, on average. Thus, there seems to be some friction that prevents searching households and landlords from instantly matching.

In models with search frictions that allow matched agents to search, we would expect agents in the unmatched state to look for more expensive goods as they would likely have less desirable outside options. Table 2 shows the percentage of recently moved households[3] in the AHS whose current housing costs are higher/lower and quality is better/worse, sorted by the primary reason the household moved[4]. Logically, it would make sense for housing costs to increase (decrease) if a household moves to a better (worse) housing unit, which

[3]Recently moved households are those who have moved into their current housing unit within one year of the interview date.

[4]The rows of Table 2 would sum to one except that the columns where costs or quality have stayed the same are not shown. Households who did not report, did not know, or refused to answer the questions about the change to their housing costs or quality were dropped from the sample prior to these calculations. The sample also excludes households who are currently living in public housing or receive housing subsidies or vouchers from the government.

would be columns (1) and (4). Households would be even better off if their housing quality increased and costs decreased, which is true for households in column (3). However, rational households would not voluntarily move to lower quality housing that costs more, which is what occurs for households in column (2).

Table 2: Percentage of Recent Movers whose Housing Costs/Quality Increased (Decreased), by Reason for Moving (Source: American Housing Survey)

Main Reason For Moving	(1) $\uparrow$ *cost*, $\uparrow$ *quality*	(2) $\uparrow$ *cost*, $\downarrow$ *quality*	(3) $\downarrow$ *cost*, $\uparrow$ *quality*	(4) $\downarrow$ *cost*, $\downarrow$ *quality*
Forced to leave by gov't	29.6	8.6	9.6	5.2
Private company/person wanted to use it	21.3	20.4	4.7	7.4
Disaster loss	20.8	15.6	10.8	6.0
New job or job transfer	24.2	14.1	7.2	8.2
Married	21.2	17.5	7.2	7.3
Widowed	10.6	12.3	12.4	15.3
Divorced or separated	8.6	12.2	7.2	26.5
To be closer to work/school/other	23.0	14.3	8.8	6.3
To establish own household	24.8	19.3	4.6	4.4
Needed a larger house or apartment	51.5	3.9	8.0	1.3
Wanted better quality housing	48.2	1.4	15.9	0.8
Change from owner to renter	16.2	5.5	11.9	20.9
Wanted less expensive housing	5.6	3.6	29.7	19.5
Other financial/employment reason	12.9	6.2	19.1	18.1
Other family/personal reason	21.2	13.0	10.9	8.6
Other housing related reason	29.5	8.4	12.8	7.6
Other	26.8	9.1	12.3	8.7
All reasons of equal importance	31.7	6.8	13.8	7.8

The top rows of Table 2 are households whose primary reason for moving was an exogenous separation. The second set of rows includes moving due to a change in relationship status, the third set of rows lists reasons that are primarily endogenous choices to move, and the bottom rows are other categories and "All reasons of equal importance." Comparing the percentage of households in columns (2) and (3) in the three groups, we see that in general, households that were exogenously separated from their previous housing move into more expensive and lower quality housing units more often than households that chose to move (endogenous group), and the opposite is true for less expensive and higher quality housing units. Since the households in the exogenous group would most likely have to move quickly as they have a less desirable outside option (relative to those in the endogenous group), this provides further evidence of search frictions as their time constraints prevent them for waiting to find their ideal housing unit.

As the AHS does not include data about the previous residence's actual rental amount or characteristics, it is also useful to consider how a household's current rent depends on the reason they moved, conditional on housing unit and household characteristics. Table 3 shows the regression results of the following equation:

$$\begin{aligned} GrossRent_{it} &= \beta_1 ExogenouslySeparated_{it} + \beta_2 HHIncome_{it} \\ &+ \beta_3 RentSubsidyRecipient_{it} + \beta_4 X_{it} + \beta_5 Z_{it} + \gamma_t + \epsilon_{it} \end{aligned} \tag{1}$$

where i denotes households; t denotes the interview year; $GrossRent_{it}$ is monthly housing costs including the contract rent, utilities (water, electricity, and gas), and fuel costs[5] (in 2005 dollars); $ExogenouslySeparated_{it}$ is an indicator for whether the household moved from the previous residence for an exogenous reason[6]; $HHIncome_{it}$ is the household's monthly income (in 2005 dollars); $RentSubsidyRecipient_{it}$ is an indicator for whether the household receives housing subsidies from the government; X_{it} is a vector of houshold characteristics; and Z_{it} is a vector of characteristics of the housing unit where household i is currently living[7]. The results show that households pay about \$83 (or 7%) more for housing if they were exogenously

[5] As some tenants' contract rent includes utilities and/or fuel, gross monthly rent is more comparable across households than using the contract rent alone.

[6] Exogenous reasons for moving are defined as: forced to leave by the government, private company or person wanted to use the housing unit, disaster loss, and moving for a new job or job transfer. Including marital status changes as exogenous reasons for moving does not significantly affect the results.

[7] The household characteristics in X_{it} include the sex, race, highest educational attainment, marital status, and a quadratic in age of the head of household. The housing unit characteristics include the square footage, number of bedrooms, number of units in the building, metro status, a quadratic in age, and the region where the housing unit is located.

separated from their previous residence when defining recent movers as those who moved in the last 2 years. The difference is \$57 (or 6%) among households who moved in the last year. This provides further evidence that households previously in the unmatched state must pay more for housing, which could be attributed to search frictions.

Table 3: Rent Paid by Recent Movers Who Moved for Exogenous vs. Endogenous Reasons (Source: American Housing Survey)

	Moved in Last 2 Years		Moved in Last Year	
	$GrossRent$	log($GrossRent$)	$GrossRent$	log($GrossRent$)
	(1)	(2)	(3)	(4)
ExogenouslySeparated	83.355***	0.072***	56.572***	0.058***
	(10.446)	(0.007)	(7.901)	(0.008)
HHIncome	0.024***		0.039***	
	(0.009)		(0.002)	
log(HHIncome)		0.122***		0.111***
		(0.003)		(0.004)
RentSubsidyRecipient	-327.561***	-0.560***	-286.963***	-0.525***
	(15.614)	(0.012)	(11.371)	(0.022)
Household Demographics	Yes	Yes	Yes	Yes
Housing Unit Characteristics	Yes	Yes	Yes	Yes
Year Fixed Effects	Yes	Yes	Yes	Yes
N	93,278	90,747	32,700	31,638
R^2	0.306	0.371	0.356	0.372

Another instance when moving households could face time constraints or have fewer outside options during their housing search is if they relocate to a different geographical area. Households new to an area would likely face higher search costs if they know less about the area (for example, which neighborhoods are safer or have more amenities), and so it would be more difficult to decide on housing before arriving to the new location. Furthermore, if they do not know many people in their new location, the household would have fewer options for temporary housing while they search for a more permanent place. Consequently, we would expect households who are moving to a new location to pay more than recent movers who have stayed in the same geographic area, and these frictions would likely be positively correlated with the distance between the old and new locations.

Tables 4 and 5 provide evidence that households new to an area pay more for rent. Table 4 shows the regression results using the following equation:

$$\log(GrossRent_{it}) = \beta_1 ChangeLocation_{it} + \beta_2 \log(HHIncome_{it}) + \beta_3 ChangeLocation_{it} * \log(HHIncome_{it}) + \beta_4 X_{it} + \beta_5 Z_{it} + \gamma_t + \epsilon_{it} \tag{2}$$

where i denotes households; t denotes the interview year; $GrossRent_{it}$ is monthly housing costs including the contract rent, utilities (water, electricity, and gas), and fuel costs (in 2005 dollars); $ChangeLocation_{it}$ is an indicator for whether the household's previous residence was in a different state or PUMA[8]; $HHIncome_{it}$ is the monthly household income (in 2005 dollars); X_{it} is a vector of houshold characteristics; and Z_{it} is a vector of characteristics of the housing unit where household i is currently living[9]. Table 5 uses a similar regression equation except with gross monthly rent and household income in levels. In Tables 4 and 5, column (1) uses data from the 2001-2017 ACS, while the remaining columns use data for 2005-2017 as the PUMA was not reported before 2005. Columns (1) and (2) define relocating households as those who moved to a new state. Column (3) characterizes relocating as moving to a new state or to a new PUMA within the same state. The sample for column (4) excludes households that moved to a new state in order to examine the effect on rent of staying within the state but moving to a new PUMA.

The results show that households who recently moved (within one year of the interview date)

[8] Public Use Microdata Areas (PUMA) are the lowest level of geography in the public release of the ACS. PUMAS include at least 100,000 people, are contained within states, and are redrawn every 10 years based on population information from the decennial census. Appendix Figure 1 shows a map of the 2010 PUMA designations for the Minneapolis-St. Paul metropolitan area.

[9] The household characteristics used in the regression described by equation (2) include the sex, race, highest educational attainment, employment status, and a quadratic in age of the head of household. The housing unit characteristics include the number of bedrooms, number of units in the building, a quadratic in age, metro status and the state where the housing unit is located.

Table 4: Rent Paid by Recent Movers Who Relocated vs. Stayed in Same Area, in logs (Source: American Community Survey)

	State Change (1)	State Change (2)	State or PUMA Change (3)	PUMA Change (4)
ChangeLocation	0.142***	0.137***	0.138***	0.128***
	(0.013)	(0.014)	(0.012)	(0.015)
log(HHIncome)	0.061***	0.060***	0.063***	0.062***
	(0.002)	(0.003)	(0.003)	(0.003)
ChangeLocation*log(HHIncome)	-0.010***	-0.009***	-0.012***	-0.014***
	(0.001)	(0.002)	(0.001)	(0.002)
Household Demographics	Yes	Yes	Yes	Yes
Housing Characteristics	Yes	Yes	Yes	Yes
Year Fixed Effects	Yes	Yes	Yes	Yes
Years	2001-2017	2005-2017	2005-2017	2005-2017
N	1,255,440	1,120,975	1,120,975	931,998
R^2	0.392	0.400	0.400	0.389

to a new state or PUMA pay about 13% more for housing than households who have recently moved to a residence in the same area. Table 5 indicates that recently moved households who change states pay about $75 more per month relative to recently moved households who relocate within the state. The effect on rents is about a third of that size (but still statistically significant) when considering households who move to a different PUMA within the same state. This is not surprising (and in fact reassuring) as the identification comes from the premise that households are more constrained when moving far away, and a household could be categorized as changing PUMAs if they move to a different part of the same city.

Looking at the positive coefficient on $HHIncome_{it}$, households with higher income pay more for rent, as to be expected. However, the negative coefficient on the interaction term shows that the relocation costs are slightly lower for higher income households. This could result if higher income households have more outside options (for example, staying in hotels that cost more per night than full-time housing units while they search) even when they are changing locations. Nevertheless, the net effect after taking the interaction term into account indicates that search frictions affect relocating households regardless of income.

Table 5: Rent Paid by Recent Movers Who Relocated vs. Stayed in Same Area, in Levels (Source: American Community Survey)

	State Change (1)	State Change (2)	State or PUMA Change (3)	PUMA Change (4)
ChangeLocation	72.139***	75.991***	45.971***	20.599***
	(8.952)	(9.601)	(6.036)	(4.454)
HHIncome	0.003***	0.003***	0.003***	0.003***
	(1.580e-04)	(1.435e-04)	(1.434e-04)	(1.385e-04)
ChangeLocation*HHIncome	-2.058e-04**	-2.611e-04***	-2.301e-04***	-2.169e-04***
	(8.626e-05)	(8.933e-05)	(6.510e-05)	(4.712e-05)
Household Demographics	Yes	Yes	Yes	Yes
Housing Characteristics	Yes	Yes	Yes	Yes
Year Fixed Effects	Yes	Yes	Yes	Yes
Years	2001-2017	2005-2017	2005-2017	2005-2017
N	1,255,440	1,120,975	1,120,975	931,998
R^2	0.469	0.478	0.477	0.469

5 Model

This section presents a model of the rental housing market. One of the key features of the model is that both households and landlords choose in which submarket they search, but they face a trade-off between the contract rent and the time it takes to find a match. As a result of this trade-off, unmatched households will search in high-rent markets so that they can move out of their (inferior) outside option housing relatively quickly. Once matched, they will search in lower-rent markets and move down the rent distribution until an exogenous shock once again forces them into the unmatched state. While this result would also transpire if search was random, using a directed search model is useful because it admits a block-recursive equilibrium, in which the equilibrium outcomes do not depend on the endogenous distributions generated within the model.

5.1 Environment

The economy is populated by a continuum of infinitely-lived households with measure one and a continuum of potential landlords with positive measure. Each household is endowed with income $y \in \mathbb{R}_{++}$ and has a periodical utility function $v(c, a)$ defined over non-housing consumption c and housing amenities a, where $v : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is twice-continuously differentiable, strictly increasing, and weakly concave with respect to both arguments. Households maximize their expected sum of periodical utilities discounted at the factor $\beta \in (0, 1)$. Landlords are risk neutral and maximize their expected sum of periodical profits discounted at the factor $\beta \in (0, 1)$.

Time is discrete and continues forever. The state of the economy at the beginning of a period is denoted by $\psi = (g_u, g_m)$ where $g_u : \mathbb{R}_{++} \to [0, 1]$ is the measure of households who are unmatched at each income level, and $g_m : \mathbb{R}_{++} \times \mathbb{R} \to [0, 1]$ is the measure of matched households with contract rent r and income y. Each period is divided into four stages: separation, depreciation, and construction; search; matching; and paying rent to live in housing. In the first stage, housing units (both unmatched and matched) fully depreciate with probability $\delta_h \in (0, 1)$, and potential landlords can construct a vacant housing unit at sunk cost S_c. Also, matched households in remaining units are forced to move from their current housing unit with probability $\delta \in (0, 1)$. Households cannot voluntarily move into the unmatched state.

During the search stage, a household gets the opportunity to search for a housing unit with probability $\lambda_u \in (0, 1]$ if they began the period unmatched or with probability $\lambda_m \in (0, 1]$ if they began the period matched. However, if the household began the period matched but was forced to separate from their housing unit (either because of the δ or the δ_h event), they cannot search immediately. Landlords with vacant housing units can pay a posting cost of $k > 0$ to search.

Housing search is directed to specific submarkets, which are indexed by the rent $r \in \mathbb{R}$ and the income of searching households y. Landlords with a vacant unit can choose in which submarket (y, r) they will search. A household with income y who received the search opportunity chooses the rent r they are willing to pay, and enters submarket (y, r). The ratio of landlords to households in each market is denoted by $\theta(y, r, \psi)$; both landlords and households take the market tightness of each submarket as given.

During the matching stage, households and landlords in each submarket meet through a frictional process. A searching household finds a landlord with probability $p(\theta(y, r, \psi))$, where $p : \mathbb{R}_+ \to [0, 1]$ is twice-continuously differentiable, strictly increasing, and strictly concave, with $p(0) = 0$ and $p'(0) < \infty$. Likewise, a landlord with a vacant unit finds a searching household with probability $q(\theta(y, r, \psi)) = \frac{p(\theta(y,r,\psi))}{\theta(y,r,\psi)}$. The function $q : \mathbb{R}_+ \to [0, 1]$ is twice-continuously differentiable, strictly decreasing, and convex, with $q(0) = 1$ and $q'(0) < 0$, and assumed such that $p(q^{-1}(\cdot))$ is concave.

When a landlord and household meet in submarket (y, r), the landlord offers the household a contract with fixed rent r. If the household accepts the contract, the two agents form a new match; otherwise, the housing unit remains vacant and the household returns to their previous housing state for the remainder of the period. If a previously matched household meets a new landlord, the previous landlord cannot make a counteroffer to retain the household.

In the last stage, matched households pay their landlord their contract rent r. There is no borrowing or saving, so they consume the remainder of their income, $c = y - r$, and enjoy housing amenities a_m from the housing unit. Unmatched households live in their outside option[10], pay b to receive housing amenities $a_u < a_m$, and consume the remainder of their income, $c = y - b$. Matched landlords receive rent r from their tenant and pay a maintenance cost m_c, which is a common parameter across housing units.

5.2 Household's Problem

Consider a currently matched household with income y who has the opportunity to search for a new match. They must decide the rent to which they will direct their search, taking as given the tightness $\theta(y, r)$ and consequently the probability of finding a match $p(\theta(y, r))$ in each submarket. If we let $R\big(y, W(y, r)\big)$ be the search value function of a household with income y and rent r in their current match, then the household will solve

$$R\big(y, W(y,r), \psi\big) = \max_{r'} \Big\{ p(\theta(y, r', \psi)) \left[W(y, r', \psi) - W(y, r, \psi) \right] \Big\} \tag{3}$$

where $W(y, r, \psi)$ is the household's lifetime value while in a match at the beginning of the fourth stage (paying rent to live in the housing unit), defined as

$$\begin{aligned} W(y, r, \psi) =& v(y - r, a_m) + \beta \mathbb{E}_{\hat{\psi}} \Bigg[(1 - \delta_h)(1 - \delta) \Bigg(W(y, r, \hat{\psi}) \\ &+ \lambda_m \max \Big\{ R\big(y, W(y, r, \hat{\psi}), \hat{\psi}\big), 0 \Big\} \Bigg) + \left[(1 - \delta_h)\delta + \delta_h \right] U(y, \hat{\psi}) \Bigg] \end{aligned} \tag{4}$$

$U(y)$ denotes the value of an unmatched household at the beginning of the fourth stage, defined by

$$U(y, \psi) = v(y - b, a_u) + \beta \mathbb{E}_{\hat{\psi}} \Big[U(y, \hat{\psi}) + \lambda_u \max \Big\{ R\big(y, U(y, \hat{\psi}), \hat{\psi}\big), 0 \Big\} \Big] \tag{5}$$

[10]The outside option can be thought of as staying in a hotel or at a friend or family member's house.

5.3 Landlord's Problem

Next, consider a landlord currently in a match with a household paying rent r out of their income y. The landlord will take as given the household's search policy $(y, \tilde{r})$ (the submarket where they search for a new match), which solves (3). Then, if we let V denote a landlord's value of holding a vacant unit and $J(y, r)$ be the landlord's lifetime value of a match at the beginning of the fourth stage, we have

$$J(y,r,\psi) = r - m_c + \beta(1-\delta_h)\mathbb{E}_{\hat{\psi}}\Big[(1-\delta)(1-\lambda_m p(\theta(y,\tilde{r},\hat{\psi})))J(y,r,\hat{\psi}) \\ + \Big(\delta + (1-\delta)\lambda_m p(\theta(y,\tilde{r},\hat{\psi}))\Big)V(\hat{\psi})\Big] \tag{6}$$

During the search stage, a landlord with a vacant unit who chooses to pay the posting cost k directs their search to submarket (y, r) by solving

$$V(\psi) = \max_{y,r}\Big\{ -k + q(\theta(y,r,\psi))J(y,r,\psi) + (1-q(\theta(y,r,\psi)))\beta(1-\delta_h)\mathbb{E}_{\hat{\psi}}\Big[V(\hat{\psi})\Big]\Big\} \tag{7}$$

Since each landlord is free to choose their submarket, the expected value of visiting any submarket must be the same across active submarkets. As a result, the tightness of any active submarket satisfies

$$\frac{V(\psi) + k - \beta(1-\delta_h))\mathbb{E}_{\hat{\psi}}\Big[V(\hat{\psi})\Big]}{J(y,r,\psi) - \beta(1-\delta_h)\mathbb{E}_{\hat{\psi}}\Big[V(\hat{\psi})\Big]} \geq q(\theta(y,r,\psi)) \tag{8}$$

and $\theta(y, r, \psi) \geq 0$ with complementary slackness[11].

Finally, potential landlords considering entering the market will solve

$$\max\Big\{V(\psi) - S_c, 0\Big\} \tag{9}$$

No new housing units will be created if the cost S_c exceeds the benefit $V(\psi)$. However, if the

[11] The minimum rent at which submarkets are active can be calculated by setting $J, p(\theta) = 0$ in equation (6) and solving for r.

benefit of constructing a new unit is larger than the cost, potential landlords will construct new units, which will put downward pressure on $V(\psi)$ from the increased competition in the search market. As a result, if $\delta_h > 0$, then in any steady state new units must be created to replace depreciating units, and so the value of the a vacancy will equal the construction cost, $V(\psi) = S_c$.

6 Equilibrium

Definition 6.1. A *recursive equilibrium* in this economy is a set of value functions for households $\{W, U, R\}$, a search policy function for households $\tilde{r}$, a landlord's value function while in a match J, a landlord's value of holding a vacant housing unit V, a market tightness function θ, and a transition probability function for the aggregate state of the economy Φ, such that

1. Given the market tightness function, the households' value functions solve (3), (4), and (5), and $\tilde{r}$ is the policy function associated with (3).
2. Given the market tightness function and the household's policy function $\tilde{r}$, the landlord's value function in a match J solves (6).
3. Given the market tightness function and the landlord's matched value, a landlord's value of vacant unit V solves (9).
4. Given the value function of matched and unmatched landlords (J and V), the market tightness function θ satisfies (8).
5. Φ is derived from the policy functions of households and exogenous separation, depreciation, and searching probabilities.

6.1 Block Recursive Equilibrium

The solution in a recursive equilibrium is a system of functional equations that depend on the distribution of agents across states, where the distribution itself is endogenously generated within the model. Solving for the equilibrium is generally a difficult task given the high dimensionality of this distribution. However, given the structure of the market, a block recursive equilibrium exists and reduces the dimensionality problem.

Definition 6.2. A BRE (*block recursive equilibrium*) is a recursive equilibrium such that the value functions, policy function, and market tightness function do not depend on the endogenous aggregate distributions of workers across matched states, ψ.

Proposition 6.1. *A block recursive equilibrium exists and is unique.*

Menzio and Shi (2010a) prove the existence and uniqueness of a BRE in a similar economy without heterogeneity, and Menzio and Shi (2010b) prove existence for the economy with ex-ante heterogeneity as in this paper. The model in this paper is a particular case of the class of models for which their proofs apply. Thus, I appeal to the proofs in Menzio and Shi (2010a, 2010b), and now discuss the useful qualities of the BRE and the critical modeling assumptions that allow for its existence.

BRE are useful because the equilibrium market tightness function does not depend on the solution to the household's search problem. As a result, the market tightness function can be obtained using the free entry condition, and the solution can then be used to solve the household's problem.

The existence of a BRE relies crucially on the assumption that the search technology in the housing market is directed, and the free entry of landlords. First, if markets were characterized by random search, then landlords posting rent offers for their vacant units would need to know the distribution of households across employment states in order to determine the probability of acceptance at each rent offer. In addition, households would need to know the distribution of rent offers when setting their acceptance rule. Second, free entry is also crucial because it pins down the tightness in each submarket.

7 Calibration and Estimation

This section presents the calibration and estimation of the model parameters and functional forms. Some of the choices reflect calibrations in prior literature, while other parameters are chosen to match data moments. The calibration is summarized in Table 6, with further descriptions provided in the following subsections. The model is then used to analyze housing subsidies.

Table 6: Model Parameters

Parameter	Description	Value	Source
Precalibrated			
ω	CES share param. (utility)	0.5	Bajari et al (2012)
ν	CES elasticity (utility)	0.3	Kahn (2009)
σ	CRRA param. (utility)	2.0	Hedlund (2016), Karahan & Rhee (2013)
y	Income	1.0,1.9,3.2,4.5	AHS (average of quartile groups)
b	Cost to live in outside option	0.4	Equal to maintenance costs
a_u	Unmatched housing amenity	0.3	Normalization
γ	Matching function param.	0.2	Search duration distribution
β	Discount factor	0.996	Match 5% annual interest rate
δ	Separation prob.	0.0168	AHS (reason for moving)
δ_h	Housing depreciation	0.003	Malpezzi et al (1987); Shilling et al (1991)
λ_m, λ_u	Search opporunity prob.	0.1,0.5	Genesove (2003)
c_m	Maintenance cost	0.4	NAA Survey of Operating Inc & Exp
Within Model			
a_m^y	Matched housing amenity	0.33,0.47,7.47,15.59	Targets search pol of unmatched HHs
κ	Vacancy posting cost	10^{-4}	Targets average search
S_C	Construction cost	0.10	duration of renters

7.1 Preferences and Income

The model period is one month. The functional form for the utility function is CES nested inside CRRA, as in Hedlund (2016a, 2016b) and Bajari, Chan, Krueger, and Miller (2012):

$$v(c,a) = \frac{\left[\left(\omega c^{\frac{\nu-1}{\nu}} + (1-\omega)a^{\frac{\nu-1}{\nu}}\right)^{\frac{\nu}{\nu-1}}\right]^{1-\sigma}}{1-\sigma} \tag{10}$$

I set the CES share parameter to be $\omega = 0.5$, which is similar to the estimate in Bajari, Chan, Krueger, and Miller (2012). They estimate the elasticity of substitution ν between housing and nonhousing consumption to be about 3.3. However, Kahn (2009) finds evidence that the elasticity is well below one, which is consistent with several other housing studies (Hanushek and Quigley, 1980; Polinsky and Ellwood, 1979; Flavin and Nakagawa, 2008), and so I will follow Kahn (2009) by setting $\nu = 0.3$. I will use $\sigma = 2$ for the CRRA parameter, following Hedlund (2016a, 2016b) and Karahan and Rhee (2013).

In the AHS, the average monthly income for households in each quartile of the income distribution (after winsorizing) is \$1,030, \$1,921, \$3,224, and \$4,537 (in 2005 dollars), respectively, when limiting the sample to households who rent a privately-owned 2 bedroom unit in urban areas, without rent control or rent subsidies, and not doubled up[12]. Therefore, the set of fixed monthly household incomes I consider is $\{1.0, 1.9, 3.2, 4.5\}$.

The optimal rental search policy for unmatched households is closely related to the relationship between a household's utility while unmatched versus matched. I set $b = 0.4$ so

[12] Doubled up households are those that include at least one adult in addition to the householder, the householder's spouse or partner, and children below the age of 21, which is consistent with the definition by the U.S. Department of Housing and Urban Development (Eggers and Moumen, 2013). I drop these households from the sample so that all of the households have a maximum of two income earners. Defining adult children as those 18 years or older has negligible effects on the estimates.

the cost for unmatched households to live in the outside option equals housing maintenance costs (see Section 7.4), and $a_u = 0.3$ as the amenity value. Then, I estimate the amenity value of a match a_m separately for each income group. For this estimation, I matched the rental search policy for households living in the outside option to the the average rent paid by households in the AHS who have been living in their current unit less than a year and were exogenously separated from their previous unit (among the same subsample used to calibrate the income parameters). In the data, low-income households pay \$661 on average, while medium-low-income households pay \$735, medium-high-income households pay \$875, and high-income households pay \$1,021. The a_m estimation yields the values 0.33, 0.47, 7.47, and 15.59 for the households ranging from low to high income, respectively.

7.2 Matching Technology

Following Menzio and Shi (2010a), Hedlund (2016a, 2016b), and Karahan and Rhee (2013), I assume that the probability a searching household finds a match is

$$p(\theta) = \theta(1 + \theta^{\gamma})^{\frac{-1}{\gamma}} \tag{11}$$

The average time spent searching for a new housing unit is 2.7 months, and about 40% of searching renters find housing in a month or less, according to ZillowGroup's *Consumer Housing Trends Report* (2018) and Apartments.com and Google's *Online Search Behavior and Trends of Apartment Renters* (2015). Data from the American Housing Survey shows that most vacant rental units have remained unoccupied for 3.1 months, on average. This motivates a value of 0.2 for γ. I then estimate that the vacancy posting cost κ is 10^{-4} and the landlord's fixed cost to enter the market S_c to 0.1, in order to match the renter's search duration.

7.3 Separation, Depreciation, and Search Opportunities

In order to match an annual interest rate of 5%, the monthly discount factor of households and landlords is 0.996. For the exogenous match separation parameter, approximately 20% of recent movers in the AHS were separated from their previous rental unit for exogenous reasons. Thus, I set $\delta = 0.0168$. Harding, Rosenthal, and Sirmans (2007) estimate that owner-occupied housing depreciates approximately 2.5% per year. I calibrate the depreciation parameter δ_h to the slightly higher annual rate of 3.5% because Malpezzi, Ozanne, and Thibodeau (1987) and Shilling, Sirmans, and Dombrow (1991) find that renter-occupied housing usually depreciates faster. According to Genesove (2003), the Property Owners and Managers Survey found that 15% of rental units are rented with no lease, 40% have a lease of less than a year, 43% have a lease of exactly one year, and 2.3% of units have a lease of more than a year. Consequently, I set $\lambda_m = 0.1$ so that matched households receive a search opportunity slightly more than once a year on average. I set the λ_u parameter to be 0.5 for unmatched households.

7.4 Maintenance Costs

The remaining parameter in the model is the housing maintenance cost for landlords in a match, m_c. Each year, the National Apartment Association conducts a Survey of Operating Income and Expenses in Rental Apartment Communities. The surveys for 2016 through 2019 show that landlords' monthly operating expenses range between \$355 and \$470 (in 2005 dollars) with an average of \$405, and so I set $c_m = 0.4$ to match this statistic.

8 Results and Policy Analysis

8.1 Results with No Rent Subsidies

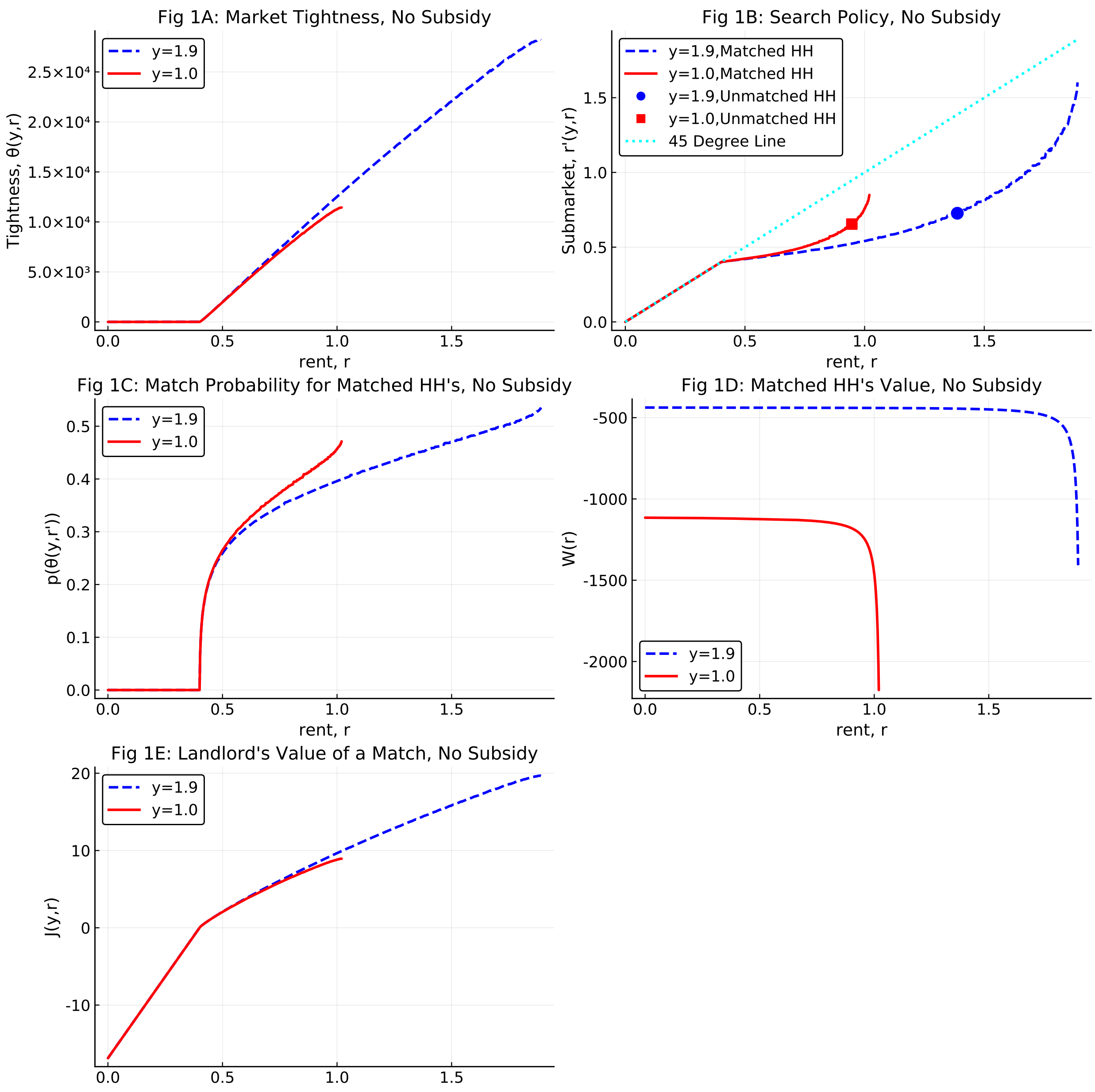
Figure 1 shows the equilibrium market tightness, search policies, probability of finding a new match[13], value function for matched households, and landlord's value from a match, for households with income in $\{1.03, 1.9\}$ without any rent subsidies. (Appendix Figure 1 shows the corresponding results for households with income in $\{3.2, 4.5\}$. The levels of the results are higher, but the same relationships hold as with the lower income households.)

We see that matched households have a higher value when they have a higher income, given any rent level, which makes sense since their higher income allows them to consume more of the non-housing good. In addition, Figures 1B and 1C show that, given a rent level in the current match, households will search in higher rent submarkets if they have lower income, and consequently their probability of finding a new match is higher. This increased probability of separating with the current match decreases a landlord's value of matching with low-income households (although the difference is minimal given the parameters), which also decreases the market tightness for low-income submarkets. Thus, the red solid line for low-income households in Figures 1A and 1E is at or below the blue dashed line for medium-low-income households.

Looking at Figure 1B, the search policy for matched households is strictly increasing in the current rent. This is because the utility function is strictly increasing in consumption (and thus strictly decreasing in rent), and households must balance the probability and gain from finding a match when choosing the submarket for search. Low-income households search in

[13] Figure 1C shows the probability the household finds a new match in their optimal search market. The x-axis is the household's rent r in their current match, and the y-axis is the match-finding probability $p(\theta(y, r'))$, where r' is itself a function of y, r as shown in Figure 1B.

Figure 1: Equilibrium with No Subsidies, Lower Income Households



In Panel B, the y-coordinates of the markers indicate the submarkets in which unmatched households search.

the submarket with 0.66 as the rent, while households with medium-low income search in the 0.73 submarket. Once they find a match, they move down the rent distribution until they are exogenously forced into the unmatched state by the separation or the housing depreciation shock. For both income levels, submarkets with rents below about 0.4 are inactive since landlords would earn negative profits from offering lower rents.

8.2 Results with Income-Based Rent Subsidies

Currently in the U.S., voucher recipients are required to contribute 30% of their adjusted income for housing and the remainder is subsidized, up to the fair market rent (FMR). Households may choose to rent a unit exceeding the FMR limit, but the subsidy remains fixed as if their rent was the FMR. When a household first moves into a unit, they cannot pay more than 40% of their adjusted income for rent, although I am ignoring this latter constraint for simplicity. In addition, I do not cap the number of available vouchers[14]. Therefore, I am modeling income-based rent subsidies as

$$S_{IncBased} = \min(R, \bar{R}) - 0.3Income \tag{12}$$

Simulating the model without rent subsidies[15] yields a median rent of 0.58. Thus, $\bar{R} = 0.58$ represents the FMR subsidy ceiling[16]. Eligibility for housing subsidies depends on a household's income level and household composition, as well as the income distribution of the geographic region. I have set the policy in my analysis as if households at the two

[14]U.S. housing policy limits the number of housing vouchers by location. Most areas have long waiting lists to receive vouchers. For simplicity, I am treating housing subsidies as an entitlement program, as it is in most countries other than the U.S.

[15]I simulated 25,000 households at each income level for 12,000 periods, and I burned the first 2,000 periods of data.

[16]HUD calculates the Fair Market Rent as either the 40^{th} or 50^{th} percentile of estimated local housing rents for a unit with a similar number of bedrooms in the Metropolitan Statistical Area from the prior year. FMR calculations do not depend on a specific household's income.

lower income levels can receive housing subsidies, in order to see if the effects of the policy depend on income[17]. Under this subsidy structure, low-income households pay at least 0.3 (equivalent to \$300) towards the rent and the maximum subsidy they can receive is 0.28 (equivalent to \$280), while medium-low-income households pay at least 0.57 (equivalent to \$570) towards the rent and can receive a maximum of 0.01 from the subsidy (equivalent to \$10).

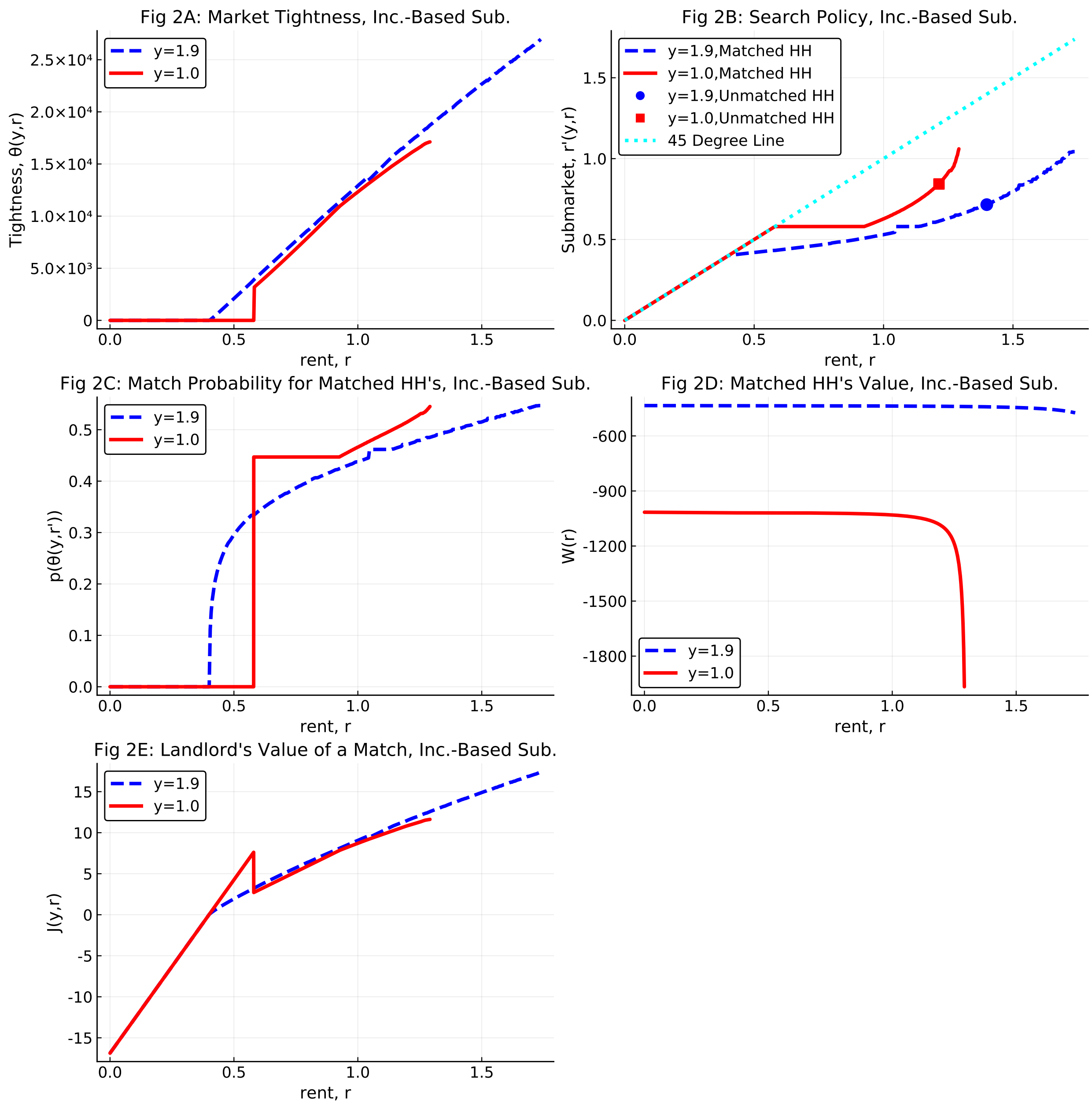
Figure 2 displays the results when households receive rent subsidies based on their income level. Note that the x-axis shows the contract rent that the landlord receives, which includes the rent subsidy in addition to what the household pays. Figure 2B shows that low-income households do not search for a new match if the contract rent in their current match is below 0.58. This occurs because households would pay the same amount of their income towards housing even if they were to find a match with lower rent. When their current rent is between 0.58 and 0.91, they search in the $(y_l, 0.58)$ submarket so they can minimize their own contribution to the rent but with a high match-finding probability. Low-income households with contract rent higher than 0.91 search in submarkets above the subsidized rent ceiling but below their current rent in order to move down the rent distribution faster.

When receiving income-based subsidies, medium-low-income households must contribute at least 0.57 (30% of 1.9) towards rent. This means they would prefer to reject the rent subsidy when their contract rent is below 0.57.[18] Consequently, the search behavior does not change from Figure 1B for households (at contract rents below about 1.1) that would search in

[17] Allowing the two lower income groups to receive housing subsidies does not alter the effects of housing subsidies on each income group individually as there is no interaction in the submarkets between the two income groups

[18] In contrast, low-income households must contribute only 0.3 (30% of 1.0) under the income-based subsidy policy. Since 0.3 is lower than the rent in any active submarket without subsidies (firms would earn negative profits with a contract rent of 0.3), low-income households never choose to reject the subsidy.

Figure 2: Equilibrium with Income-Based Subsidies



In Panel B, the y-coordinates of the markers indicate the submarkets in which unmatched households search.

submarkets below 0.57 under the"no subsidies" housing policy.

The flat portion of the blue dashed line in Figure 2B shows that medium-low-income households that would normally have search policies at or slightly above 0.57 instead search in the $(y_{ml}, 0.58)$ submarket. This is because for all contract rents between 0.57 and 0.58, the household's contribution to the rent would be 0.57. However, the highest match-finding probability over this range of contract rents is in the $(y_{ml}, 0.58)$ submarket, so it is never optimal to search for contract rents $r' \in [0.57, 0.58)$. Households with contract rents above 1.17 search in submarkets slightly lower than what they would have without subsidies, since the small extra income from the subsidy (0.01) allows them to search in markets with lower match-finding probabilities.

Households in the unmatched state search in the 0.86 submarket if they have low income and 0.74 if they have medium-low income. Recall that the search policies for the unmatched low-income and medium-low-income households without rent subsidies were 0.66 and 0.73, respectively. They search in higher rent submarkets under income-based subsidies for two reasons. First, the extra income from the subsidy loosens their budget constraint and so allows them to have higher contract rents for a given amount of the non-housing good. Second, households only receive the subsidy once they find a match, which increases the relative value of being in the matched state compared to the unmatched state. Thus, they search in a higher submarket in order to increase the probability of finding a match.

This second effect is especially strong for low-income households. The maximum subsidy amount for low-income households (0.28) is much higher than for medium-low-income households (0.01). Consequently, although low-income households still have lower total income, their search policy while unmatched (0.84) is still higher than that of unmatched medium-low-income households (0.76).

Note that there is a discontinuity in the landlord's value function when matched with low-income households at rent 0.58 in Figure 2E. If a landlord could match with a household for rent slightly below 0.58, they would have higher expected profits because the household would not search for a different match. This means that there is a lower probability of separating, and so the landlord would have a higher lifetime value. However, no landlord would be in a match with these low contract rents in equilibrium since submarkets below 0.58 are inactive for low-income households.

8.3 Results with Rent-Based Subsidies

France and other European countries provide rent subsidies that are based on a household's rent level. The rent subsidy is calculated as a percentage of the contract rent, up to a ceiling rent which is calculated at the Metropolitan Statistical Area (similar to the FMR in the U.S. subsidy structure), minus a minimum participation rent, according to the following formula:

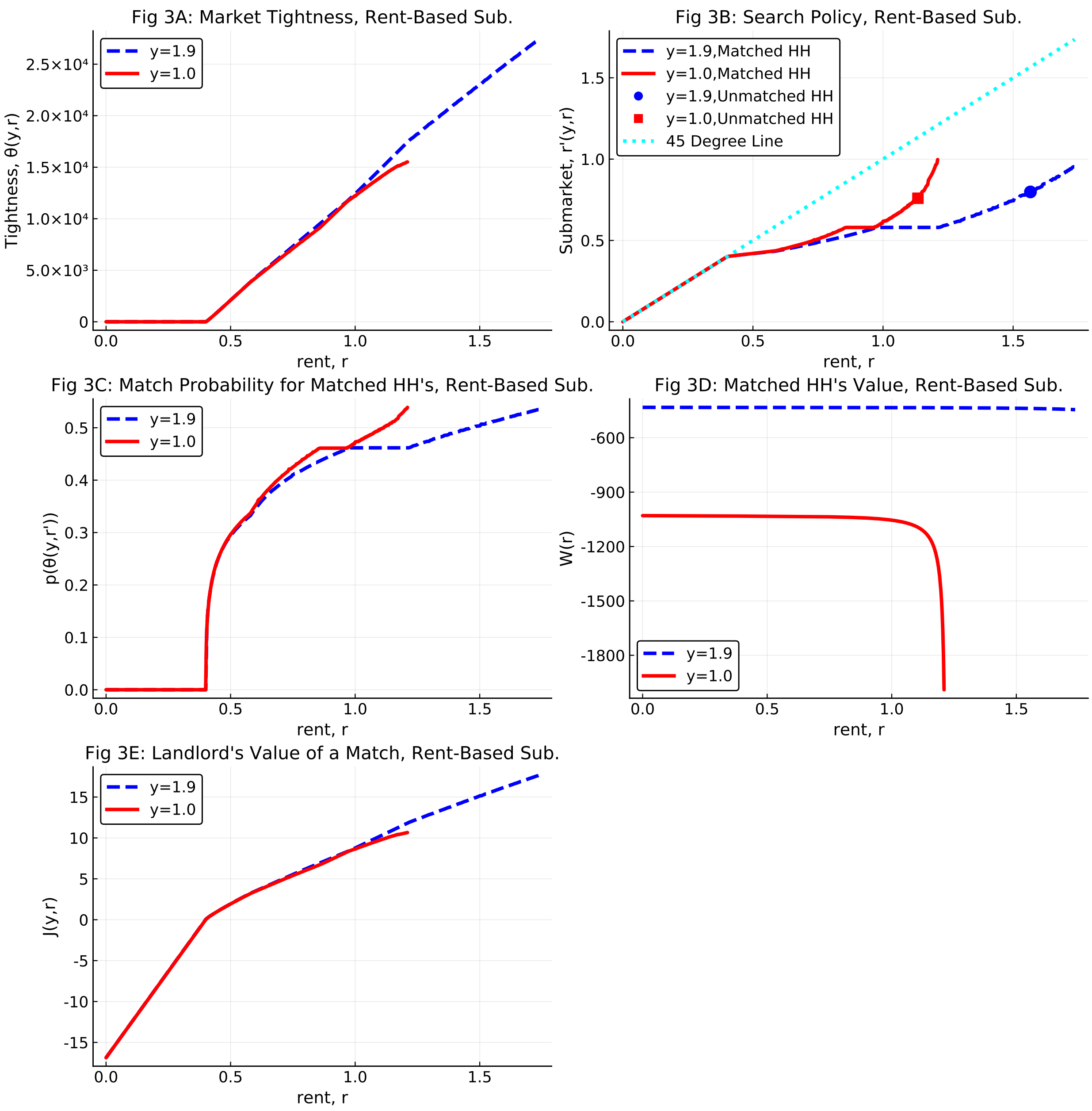
$$S_{RentBased} = s(Inc)[\min(R, \bar{R}) - R_0(Inc)] \tag{13}$$

where the percentage $s(Inc)$ is a decreasing function of income and the minimum participation rent $R_0(Inc)$ is increasing in income. For simplicity of the analysis, I have made $s(Inc) = 0.5$ and $R_0(Inc) = 0.2$ constant in income[19]. The rent ceiling is $\bar{R} = 0.58$ to match the parameter in the Income-Based Subsidies section.

Figure 3 shows the results with rent-based housing subsidies. As with the income-based

[19] In countries with rent-based subsidies, $s(Inc)$ and $R_0(Inc)$ are complicated functions based on the household income and family composition. This paper sets them as constants for simplicity. See Fack (2006) for a more complete description of how these functions are determined for France's housing policy.

Figure 3: Equilibrium with Rent-Based Subsidies



In Panel B, the y-coordinates of the markers indicate the submarkets in which unmatched households search.

subsidies, Figure 3B shows there is a positive measure of rents at which matched households search in the submarket with the subsidy ceiling as the rent. This occurs for low-income households with current rent between 0.85 and 1.0, and between about 1.0 and 1.3 for medium-low-income households. However, unlike with income-based subsidies, matched households below these levels search in submarkets with rents lower than the subsidy ceiling as these matches would lower their required contribution to the rent payment. Furthermore, even matched households below the subsidy ceiling search for lower rent housing, although the slope of the optimal submarket line over this interval (between 0.4 and 0.58 in Figure 3B) is lower than at higher contract rents. This is because the household's required contribution decreases by half as much as the decrease to the total contract rent.

9 Effects of Matching Frictions

Matching frictions is one of the key features of the model. The primary parameter governing the level of these frictions in the results is γ in the household's match-finding probability function, $p(\theta) = \theta(1+\theta^{\gamma})^{\frac{-1}{\gamma}}$. With this functional form, the probability a household or a landlord finds a match is increasing in γ (while holding market tightness θ constant), which means there is a negative relationship between matching frictions and γ.

9.1 Effects of Matching Frictions With No Subsidies

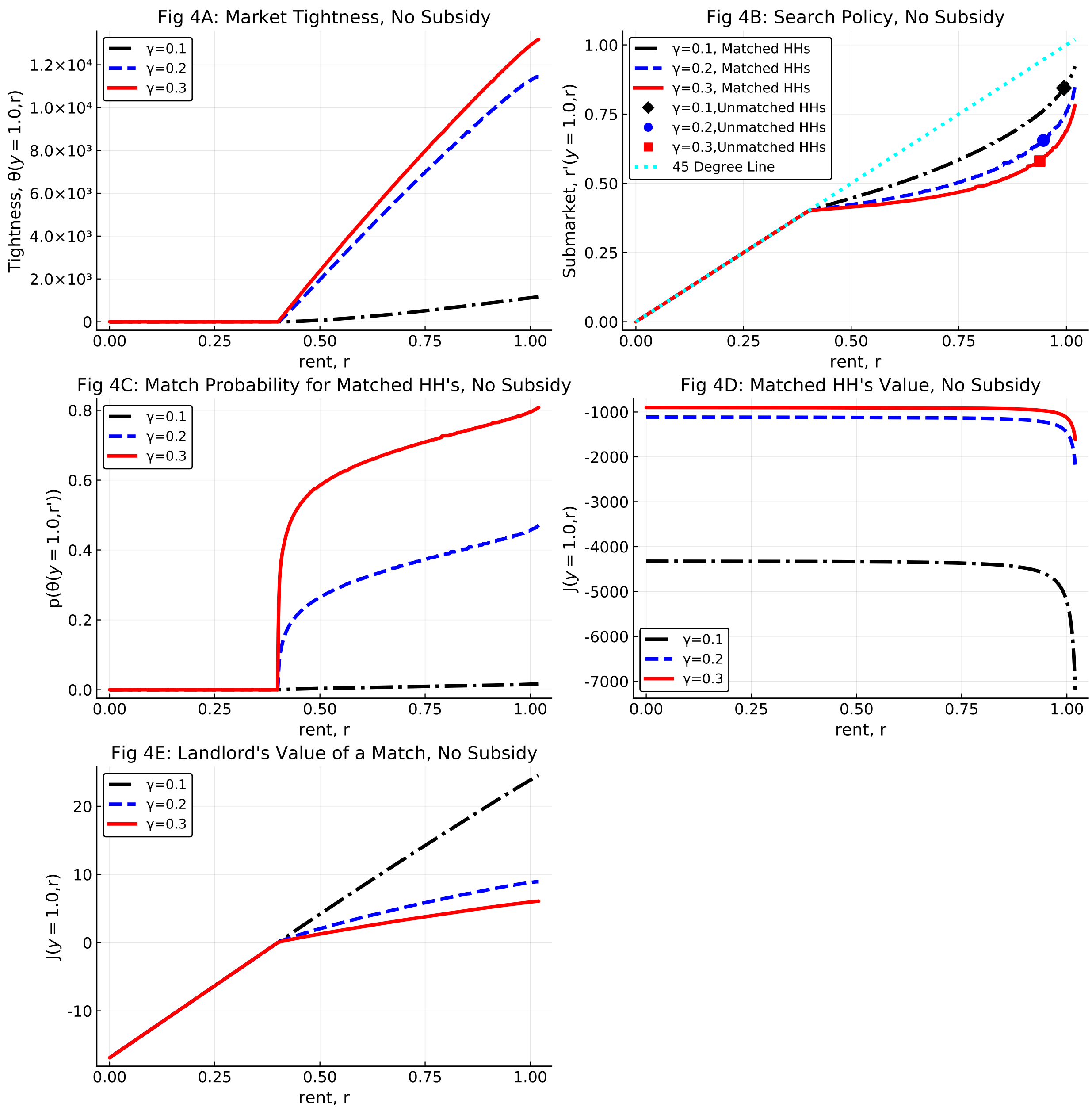
Figure 4 displays the market tightness, search policy, match probability for matched households, value function for matched households, and the landlord's value of a match when there are no subsidies and households income is 1.0, for $\gamma \in \{0.1, 0.2, 0.3\}$. When γ is lower, the market tightness would have to be much higher in order for a household's match-finding probability to be the same. However, Figure 4A shows that the market tightness in a submarket with a given rent is, in fact, much lower in equilibrium with a lower γ.

This occurs because the match-finding probability of a landlord with a vacant unit is also decreasing in γ. Fewer potential landlords choose to construct a housing unit, even though a landlord's value of a match is much higher in Figure 4E (since it would also be higher for a tenant to find a new match). As a result, the equilibrium market tightness and the household's match-finding probability is much lower in each submarket. This also means that households choose a higher rent submarket (relative to when γ is high) in order to increase their chances of finding a match (although Figure 4C shows the match-finding probability in the household's optimal submarket for search is still lower). Overall, this means that households start out paying higher rent and move down the rent distribution at a slower pace. Thus, households must pay more rent for otherwise identical housing units when matching frictions are higher.

Table 7 shows simulation results for the average rent for low- and medium-low-income households under the various subsidy structures and matching friction parameters.[20] The third and sixth columns show the average contract rent while in a match for each of the income groups. The fourth and seventh columns show the average subsidy amount received, and the fifth and eighth columns indicate the household's portion of the rental payment. Looking at the top panel (with no rent subsidies), low-income households pay 0.45 (equivalent to \$450) for rent on average when γ is 0.3, and almost twice that amount (0.83) when γ is 0.1. Similarly, medium-low-income households pay about 2.5 times more rent due to the higher matching frictions. In fact, both with and without housing subsidies, higher matching

[20] I simulated 25,000 households at each income level for 12,000 periods, and I burned the first 2,000 periods of data.

Figure 4: Effects of Changing Matching Frictions, No Subsidies



In Panel B, the y-coordinates of the markers indicate the submarkets in which unmatched households search.

frictions cause households to pay more for rent.

Table 7: Average Rents (from Simulation), Subsidy Rent Ceiling $\bar{R} = 0.58$

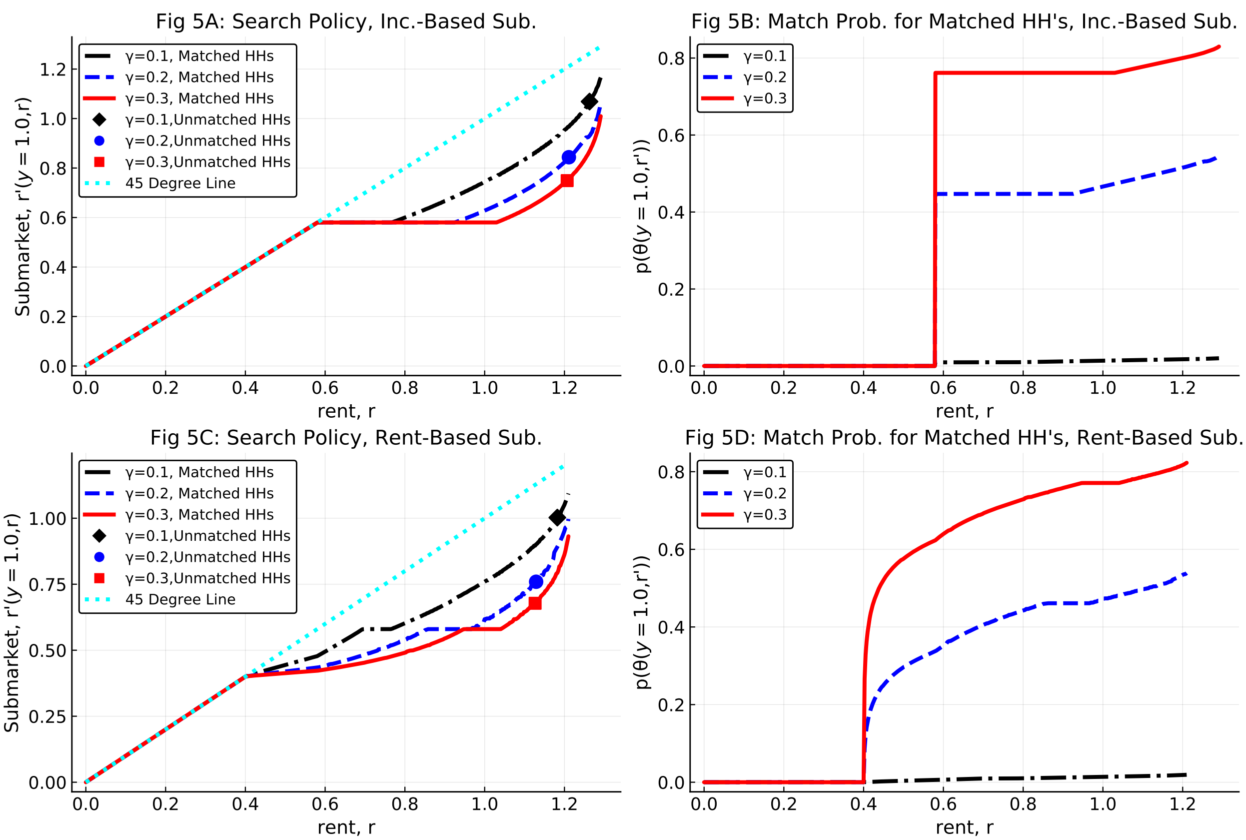
		Low-Income			Medium-Low-Income		
Subsidy	γ	Rent	Subsidy	HH Portion	Rent	Subsidy	HH Portion
None	0.1	0.83	-	0.83	1.13	-	1.13
	0.2	0.52	-	0.52	0.54	-	0.54
	0.3	0.45	-	0.45	0.45	-	0.45
Inc-Based	0.1	1.05	0.28	0.77	1.13	0.01	1.12
	0.2	0.66	0.28	0.38	0.53	0.00	0.53
	0.3	0.62	0.28	0.34	0.45	0.00	0.45
Rent-Based	0.1	0.99	0.19	0.80	1.25	0.19	1.06
	0.2	0.55	0.15	0.40	0.56	0.15	0.41
	0.3	0.47	0.12	0.35	0.47	0.12	0.35

9.2 Effects of Matching Frictions with Subsidies

Figure 5 shows the search policy and match probability for matched households with income-based and rent-based subsidies, using values of $\gamma \in \{0.1, 0.2, 0.3\}$, 1.0 for household income, and $\bar{R} = 0.58$ for the subsidy rent ceiling. As previously discussed, when subsidies are income-based, households never search for units with rent below the subsidy ceiling. However, Figures 5A and 5B shows that when γ is lower, households search in a higher rent submarket given their current state, and their probability of finding a match is lower. As a result, matched households live in units strictly above the subsidy ceiling for a longer duration. The subsidy amount per matched household is the same regardless of the value of γ, but households have to pay more for the rent above the subsidy ceiling due to the higher matching frictions.

In contrast, the structure of rent-based subsidies incentivizes households to search in lower-rent submarkets even when their current rent is at or below the subsidy ceiling in order to decrease their portion of the rent. With a higher value of γ (lower matching frictions), unmatched households search in lower rent submarkets but still above the 0.58 subsidy ceil-

Figure 5: Effects of Matching Frictions with Subsidies



In Panels A and C, the y-coordinates of the markers indicate the submarkets in which unmatched households search.

ing. This means that the subsidized amount is the same for recently unmatched households, but the household's own contribution is lower. After finding a match, the household moves down the rent distribution (and below the subsidy ceiling) at a faster rate due to the higher match-finding probability. As a result, both the subsidy and the household's portion of the rent are lower when there are lower matching frictions and rent-based subsidies due to the household's incentives to search for less expensive housing.

Looking at Table 7 when subsidies are based on income, low-income households pay 0.77 (equivalent to \$770, or 77% of their income) towards the rent when γ is 0.1, but they pay the much smaller amount of 0.34 (equivalent to \$340, or 34% of their income) when γ is 0.3. While the average amount the low-income households pay is somewhat smaller under income-based subsidies compared to no subsidies, the total contract rent is much higher since the subsidies distort the household's search behavior. The average rent and average household contribution to the rent is fairly similar for medium-low-income households, however, because the subsidized portion of rent is very small.

Under rent-based subsidies, both the households' rental payment and the subsidy amount decrease when matching frictions decrease. Changing γ from 0.1 to 0.3 (decreasing matching frictions) decreases the low-income household's contribution to the rent from 0.80 to 0.35. The medium-low-income households' contribution falls from 1.06 to 0.35, while the average subsidized amount decreases from 0.19 to 0.12 (equivalent to a change from \$190 to \$120) for both income groups[21].

Comparing the results with and without subsidies shows that income-based subsidies de-

[21]The subsidy amount tends to be similar for both income groups with rent-based subsidies due to the simplifying assumption that $s(Inc)$ and $R_0(Inc)$ in (13) are constants. The housing policy could make the subsidies more generous for low-income households than medium-low-income households by setting $s(Inc), R_0(Inc)$ as decreasing functions of income.

crease low-income households' portion of the rent by about 0.10 (equivalent to $100) on average and costs the subsidy-provider 0.28 per matched household (equivalent to $280). Rent-based subsidies save low-income households about 0.08 (equivalent to $80) on average and medium-low-income households about 0.10 (equivalent to $100), while costing the subsidy-provider about 0.15 (equivalent to $150) per matched household. However, both forms of rental subsidies cost the most and are least effective at decreasing the household's portion of the rent when matching frictions are high. Furthermore, improving the match technology (decreasing match frictions) is even more effective than offering subsidies at lowering households' rent payments.

The analysis up to this point has used $\bar{R} = 0.58$ as the subsidy rent ceiling in order to isolate the effects from changing matching frictions. However, changing the matching friction parameter results in a different equilibrium rent distribution, which means the subsidy rent ceiling would also be different if it was set to be the median rent in the equilibrium without subsidies. The median rent in equilibrium is 1.57, 0.58, and 0.42 when γ is 0.2, 0.1, and 0.3, respectively. Table 8 shows the simulation results for the average rent for low- and medium-low-income households when using these median rents for $\bar{R}$. For high matching frictions ($\gamma = 0.1$), both income- and rent-based subsidies decrease the households' contribution to the rent by a greater amount when using 1.57 as $\bar{R}$ instead of 0.58. However, providing the subsidies also costs about 1.00 more (equivalent to $1,000) per matched household. Overall, the prior result still holds: both forms of rental subsidies are least cost effective when matching frictions are high, even when using the median rent (in the equilibrium without subsidies) as the subsidy rent ceiling.

Table 8: Average Rents (from Simulation), Median Rent as Subsidy Rent Ceiling $\bar{R}$

			Low-Income			Medium-Low-Income		
Subsidy	γ	$\bar{R}$	Rent	Subsidy	HH Portion	Rent	Subsidy	HH Portion
None	0.1	-	0.83	-	0.83	1.13	-	1.13
	0.2	-	0.52	-	0.52	0.54	-	0.54
	0.3	-	0.45	-	0.45	0.45	-	0.45
Inc-Based	0.1	1.57	1.97	1.27	0.70	1.94	1.00	0.94
	0.2	0.58	0.66	0.28	0.38	0.53	0.00	0.53
	0.3	0.42	0.47	0.12	0.35	0.45	0.00	0.45
Rent-Based	0.1	1.57	1.39	0.59	0.80	1.61	0.66	0.95
	0.2	0.58	0.55	0.15	0.40	0.56	0.15	0.41
	0.3	0.42	0.46	0.11	0.35	0.46	0.11	0.35

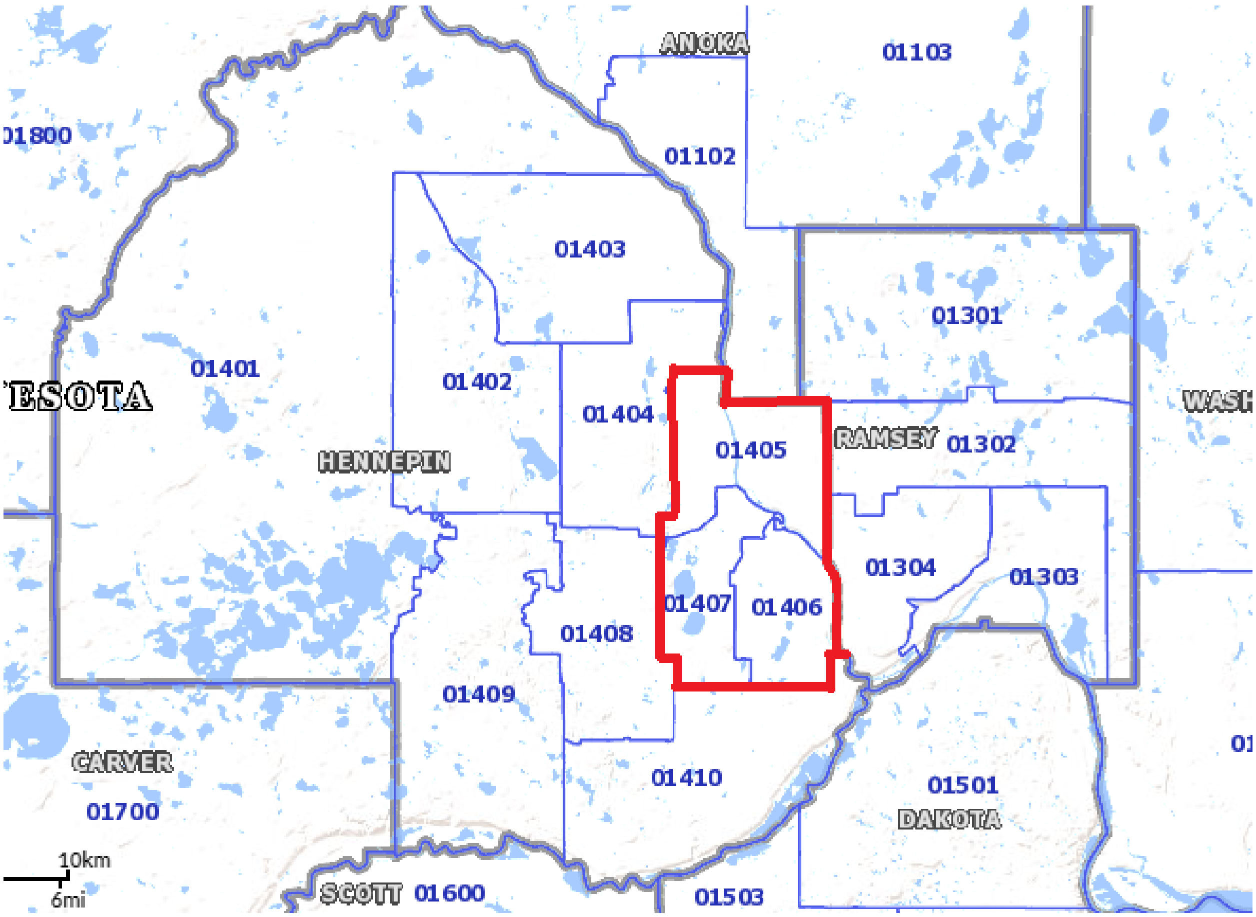
10 Conclusion

This paper documents evidence of search and matching frictions in the rental housing market and examines how these frictions impact the costs and benefits of income-based and rent-based housing subsidies. I develop a directed search model of the rental housing market, and compare the equilibrium outcomes under the different housing policies. The results show that income-based subsidies distort household search decisions more than when subsidies are rent-based. When matching frictions increase, households pay more in rent regardless of the housing policy. Under income-based housing subsidies, households bear the extra costs of the higher matching frictions. When subsidies are instead calculated as a fraction of the rent and matching frictions increase, costs increase for both the household and the subsidy provider. In addition, both forms of rental subsidies cost the most and are least effective at lowering households' rental payments when matching frictions are high.

While the model in this paper includes ex-ante heterogeneity in the household incomes, future research should allow for heterogeneity in housing unit quality (the amenity value of match). In addition to being more realistic, this extension would provide another margin of adjustment for households, which could have important implications in particular for households at the subsidy ceiling with income-based subsidies.

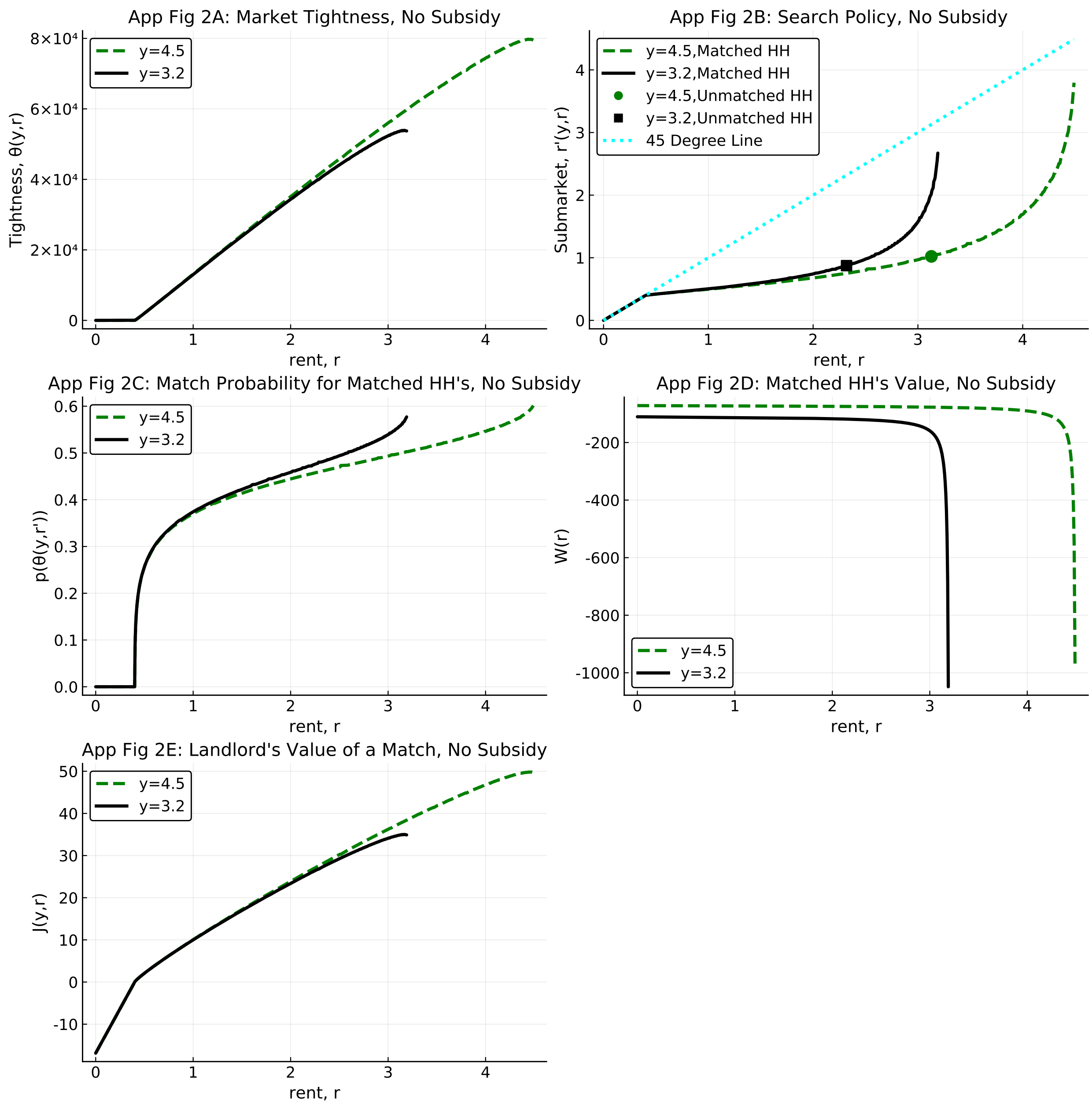
Appendix

Appendix Figure 1: 2010 PUMA Designations for the Minneapolis-St. Paul Metropolitan Area



Source: U.S. Census Bureau.
The red line denotes the border of Minneapolis, MN.

Appendix Figure 2: Equilibrium with No Subsidies, Higher Income Households



In Panel B, the y-coordinates of the markers indicate the submarkets in which unmatched households search.

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