Individual Risk, Inequality, and Monetary Policy

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Inequalities and macroeconomics

- **Macro =** « Growth » **+** « Business Cycle »
- Today inequalities permeate both subfields
- **Growth**
  => contributions of **automation, innovations, rents** (…) to inequalities
- **Business Cycles?**

Towards a « new synthesis » ?

- **50s & 60s: Neoclassical** synthesis (Hicks, Samuelson, Solow...)

  = neoclassical model + price inertia

- **80s & 90s: New Keynesian** synthesis (Mankiw, Gali, Woodford...)

  = RBC + nominal rigidities

- **2010s: Heterogenous-Agent New Keynesian** (« HANK ») synthesis

  = New Keynesian model + inequalities

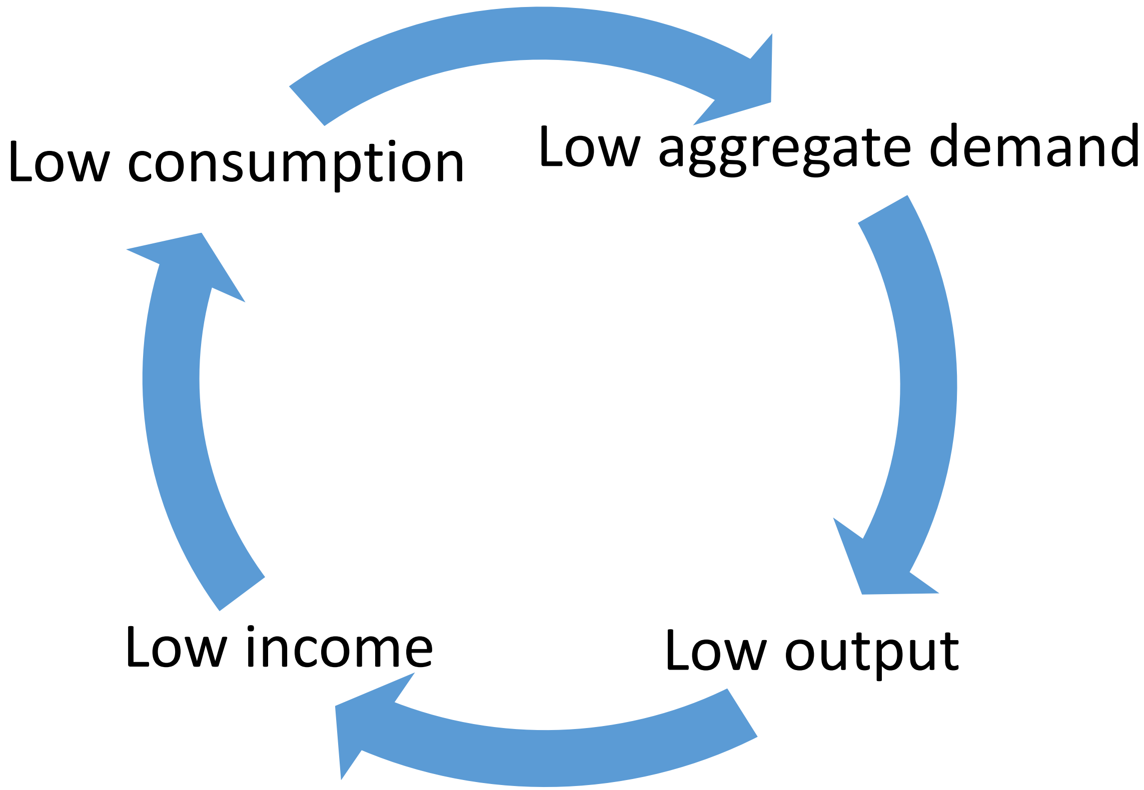
Towards a « new synthesis » ?

- **2010s: Heterogenous-Agent New Keynesian** (« HANK ») synthesis
- New consensus:
  - AD fluctuations not only matter but are **dominant**
  - Heterogeneity **does** matter in the propagation of business cycles
  - Monetary policy matters for stabilization **and** inequalities
- Helps structure current policy debates

Towards a « new synthesis » ?

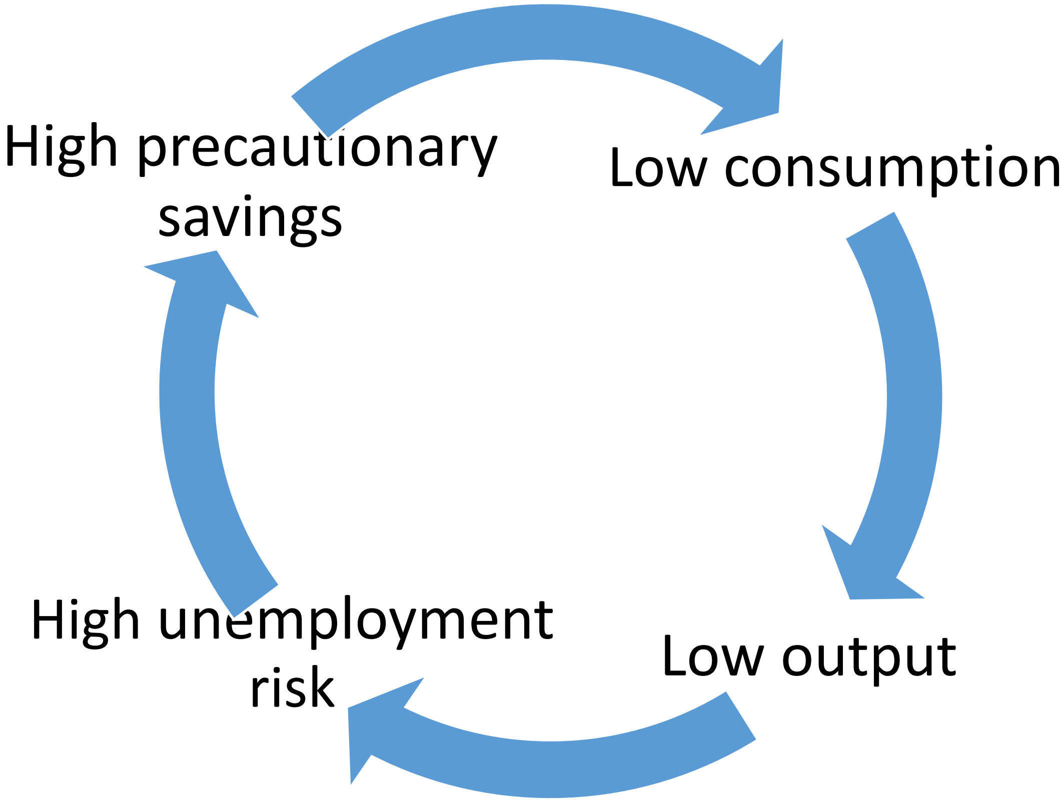
- **2010s: Heterogenous-Agent New Keynesian** (« HANK ») synthesis
- Stresses 3 potential amplification mechanisms (there may be more):
  - (New) Keynesian Cross
  - Precautionary-saving spiral
  - Flight to liquidity

(New) Keynesian Cross



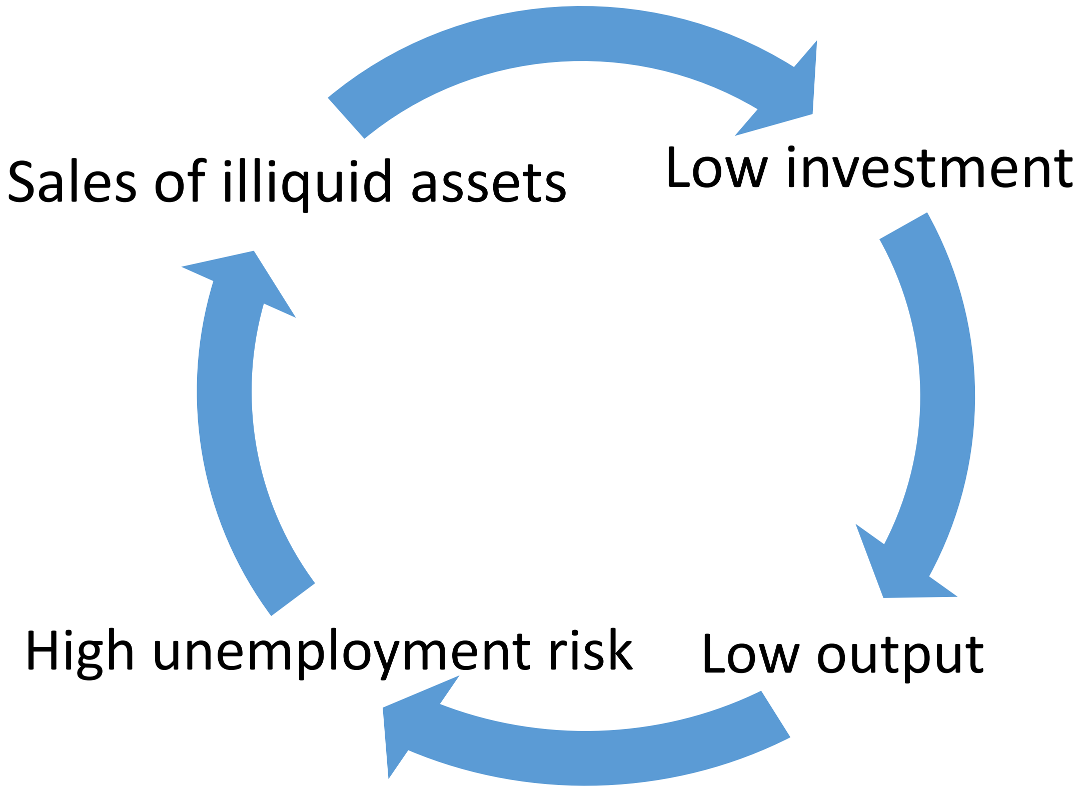
Key : distribution of MPCs

Precautionary Saving Spiral

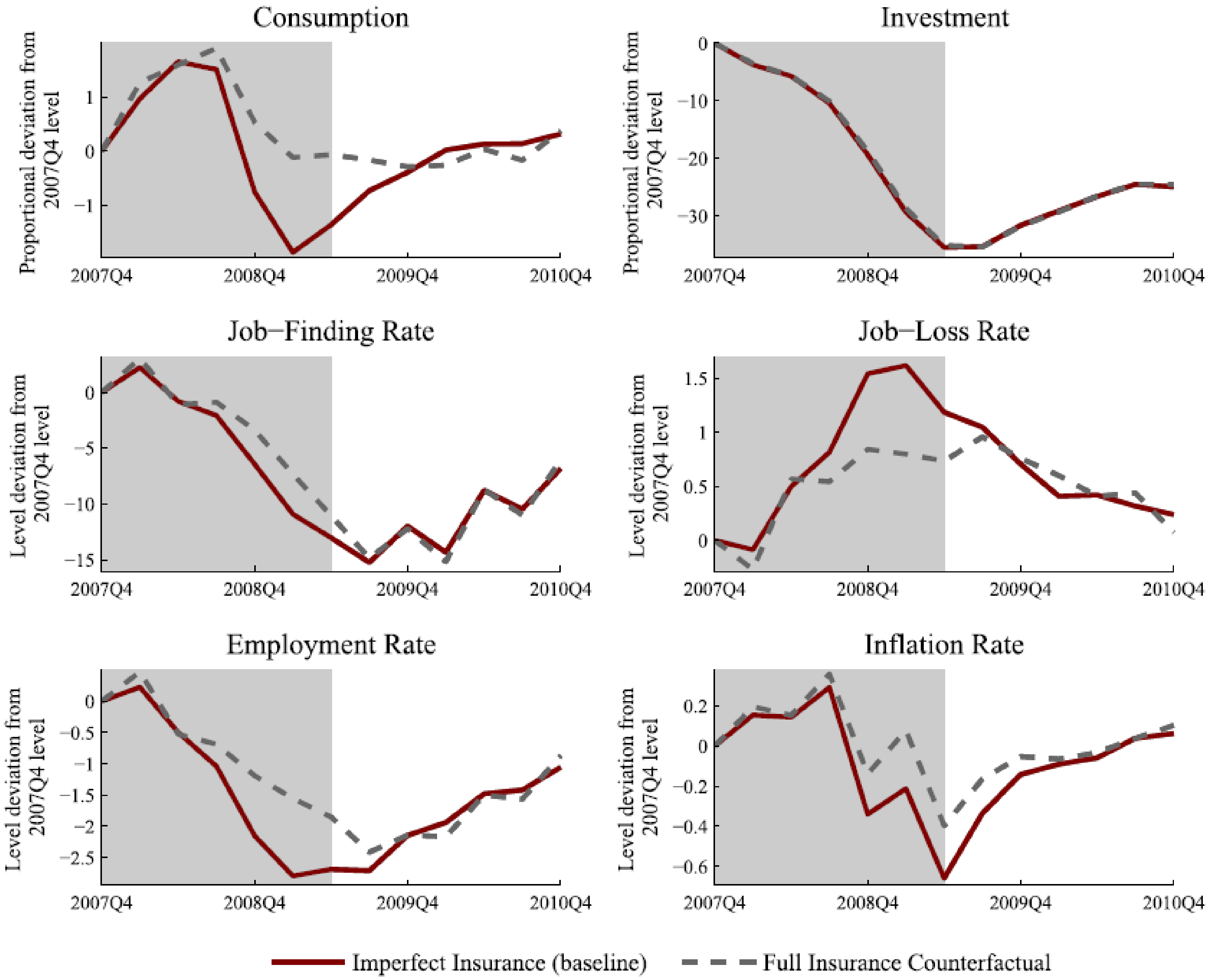


Key : Cyclicality of unemployment risk

Flight to Liquidity



Key : transaction costs in financial markets


Consumption
Proportional deviation from 2007Q4 level
1
0
−1
2007Q4
2008Q4
2009Q4
2010Q4
Investment
Proportional deviation from 2007Q4 level
0
−10
−20
−30
Job−Finding Rate
Level deviation from 2007Q4 level
0
−5
−10
−15
Job−Loss Rate
Level deviation from 2007Q4 level
1.5
1
0.5
0
Employment Rate
Level deviation from 2007Q4 level
0
−0.5
−1
−1.5
−2
−2.5
Inflation Rate
Level deviation from 2007Q4 level
0.2
0
−0.2
−0.4
−0.6
Imperfect Insurance (baseline)
Full Insurance Counterfactual

A Simple Example

- NK Cross logic at work in 2-agent NK models
- Precautionary-saving spiral at work in 3-agent “HANK & SaM”
  (e.g. Ravn & Sterk 2017, 2021; Challe 2020)
- Useful, but none of these have a full cross-sectional distribution
- **Next slides**: tractable CARA-Normal framework
  (Acharya & Dogra 2020, Acharya, Challe & Dogra 2022)

Firms

Firm choices lead to

- **NKPC**

$$\pi_t = \widetilde{\beta}\pi_{t+1} + \kappa \hat{y}_t$$

- **wage**

$$\omega_t = \boldsymbol{\omega}(y_t), \quad \boldsymbol{\omega}'(y_t) > 0$$

Policy

Monetary policy:

$$1 + i_t = (1 + r)\Pi_t^{\phi_\pi}$$

Fiscal policy:

$$g_t + \frac{T_t}{P_t} = \tau_t \omega_t$$

where

- g_t is real gov't spending
- $\tau_t = \boldsymbol{\tau}(y_t)$ is a (potentially cyclical) proportional labor-income tax
- T_t is a lump-sum transfer (balances the budget)

Households

Mass 1 of HH solve

$$\max_{\{c_t^i, A_{t+1}^i\}_{t=0}^{\infty}} \quad -\frac{1}{\gamma}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t e^{-\gamma c_t^i}$$

subject to

$$P_t c_t^i + \frac{1}{1+i_t} A_{t+1}^i = A_t^i + P_t \underbrace{\left[(1-\tau_t)\,\omega_t \ell_t^i + d_t + \frac{T_t}{P_t}\right]}_{y_t^i}$$

$$\ell_t^i \sim \mathcal{N}(1, \boldsymbol{\sigma}_\ell^2(y_t))$$

Implies that $y_t^i \sim \mathcal{N}(y_t - g_t, \boldsymbol{\sigma}^2(y_t))$, where $\boldsymbol{\sigma}^2(y_t) = (1 - \boldsymbol{\tau}(y_t))^2 \boldsymbol{\omega}(y_t)^2 \boldsymbol{\sigma}_\ell^2(y_t)$

Note that HH can **borrow or lend** (to each other)

Households

CARA-Normal framework produces **linear** decisions rules

Conjecture & verify that

$$c_t^i = \mathcal{C}_t + \mu_t \left(a_t^i + y_t^i \right)$$

where

- $a_t^i \equiv \frac{A_t^i}{P_t}$ is real value of nominal bond holdings
- $a_t^i + y_t^i$ is "cash on hand" (the relevant individual state variable)
- μ_t is the **MPC** (assume no HtM HHs for now; relaxed later)

$$\frac{1}{\mu_t} = 1 + \frac{1}{\mu_{t+1}(1 + r_t)}$$

Aggregate Demand

The MPC μ_t affects the **pass-through** from income shocks to consumption

Since HH care about consumption risk, μ_t partly determine AD

Iterating forward

$$\mu_t^{-1} = \sum_{s=0}^{\infty} \frac{1}{\prod_{k=0}^{s-1}(1 + r_{t+k})}$$

Higher interest rates make self-insurance through the bond market **harder**

Aggregate Demand

Individual Euler:

$$c_t^i = -\frac{\ln \beta(1+r_t)}{\gamma} + \mathbb{E}_t\, c_{t+1}^i - \frac{\gamma \mu_{t+1}^2 \boldsymbol{\sigma}^2(y_{t+1})}{2}$$

Linear aggregation:

$$c_t = -\frac{\ln \beta(1+r_t)}{\gamma} + c_{t+1} - \frac{\gamma \mu_{t+1}^2 \boldsymbol{\sigma}^2(y_{t+1})}{2}$$

Mket clearing $y_t = c_t + g_t$

$$y_t = -\frac{\ln \beta(1+r_t)}{\gamma} + y_{t+1} - \frac{\gamma \mu_{t+1}^2 \boldsymbol{\sigma}^2(y_{t+1})}{2} + g_t - g_{t+1}$$

- intertemporal substitution channel (as in RANK)
- income-risk channel
- **self-insurance channel**

The cyclicality of income risk

In equilibrium, y_t^i is i.i.d. with mean $\overline{y}_t$ and variance

$$\boldsymbol{\sigma}^{\mathbf{2}}(y_t) = \left[\Big(1 - \boldsymbol{\tau}(y_t)\Big)\boldsymbol{\omega}(y_t)\right]^2 \boldsymbol{\sigma}_{\boldsymbol{\ell}}^{\mathbf{2}}(y_t)$$

so **cyclicality of income risk** $\dfrac{d\boldsymbol{\sigma}^{\mathbf{2}}(y)}{dy}$ equals

$$2\boldsymbol{\sigma}(y)\boldsymbol{\sigma}_{\boldsymbol{\ell}}(y)\left\{\underbrace{\left(1 - \boldsymbol{\tau}\left(y\right)\right)\boldsymbol{\omega}'\left(y\right)}_{\substack{\text{cyclicality of}\\ \text{real wages}}} - \underbrace{\boldsymbol{\tau}'\left(y\right)\boldsymbol{\omega}\left(y\right)}_{\substack{\text{cylicality of}\\ \text{taxes}}}\right\} + \underbrace{\frac{\boldsymbol{\sigma}^{\mathbf{2}}\left(y\right)}{\boldsymbol{\sigma}_{\boldsymbol{\ell}}^{\mathbf{2}}\left(y\right)}\frac{d\boldsymbol{\sigma}_{\boldsymbol{\ell}}^{\mathbf{2}}(y)}{dy}}_{\substack{\text{cyclicality of}\\ \text{employment risk}}}$$

endogenous - and depends (inter alia) on tax-transfer system

Linearized demand block

$$\begin{aligned} \widehat{y}_t &= \Theta \widehat{y}_{t+1} - \frac{1}{\gamma}(i_t - \pi_{t+1}) - \Lambda \widehat{\mu}_{t+1} + \widehat{g}_t - \widehat{g}_{t+1} \\ \widehat{\mu}_t &= \widetilde{\beta} \widehat{\mu}_{t+1} + \widetilde{\beta}(i_t - \pi_{t+1}) \end{aligned}$$

where

$$\Theta = 1 - \frac{\gamma \mu^2}{2} \frac{d\boldsymbol{\sigma}^{\mathbf{2}}(\boldsymbol{y}^*)}{dy} \qquad \text{and} \qquad \Lambda = \gamma \mu^2 \boldsymbol{\sigma}^2(y^*)$$

Linearized demand block

$$\widehat{y}_t = \Theta\widehat{y}_{t+1} - \frac{1}{\gamma}(i_t - \pi_{t+1}) - \Lambda\widehat{\mu}_{t+1} + \widehat{g}_t - \widehat{g}_{t+1}$$

$$\widehat{\mu}_t = \widetilde{\beta}\widehat{\mu}_{t+1} + \widetilde{\beta}(i_t - \pi_{t+1})$$

where

$$\Theta = 1 - \frac{\gamma\mu^2}{2}\frac{d\boldsymbol{\sigma^2(y^*)}}{dy} \qquad \text{and} \qquad \Lambda = \gamma\mu^2\boldsymbol{\sigma}^2(y^*)$$

- RANK $(\sigma = 0)$: $\Theta = 1,\ \Lambda = 0$

Linearized demand block

$$
\begin{aligned}
\widehat{y}_t &= \Theta\widehat{y}_{t+1} - \frac{1}{\gamma}(i_t - \pi_{t+1}) - \Lambda\widehat{\mu}_{t+1} + \widehat{g}_t - \widehat{g}_{t+1} \\
\widehat{\mu}_t &= \widetilde{\beta}\widehat{\mu}_{t+1} + \widetilde{\beta}(i_t - \pi_{t+1})
\end{aligned}
$$

where

$$\Theta = 1 - \frac{\gamma\mu^2}{2}\frac{d\boldsymbol{\sigma}^{\mathbf{2}}(\boldsymbol{y}^*)}{dy} \qquad \text{and} \qquad \Lambda = \gamma\mu^2\boldsymbol{\sigma}^2(y^*)$$

- RANK $(\sigma = 0)$: $\Theta = 1$, $\Lambda = 0$
- Procyclical risk $\left(\frac{d\boldsymbol{\sigma}^2}{dy} > 0\right)$: $\Theta < 1$, discounted Euler eq
- Acyclical risk $\left(\frac{d\boldsymbol{\sigma}^2}{dy} = 0\right)$: $\Theta = 1$, but still $\Lambda > 0$: precautionary savings channel
- Countercyclical risk $\left(\frac{d\boldsymbol{\sigma}^2}{dy} < 0\right)$: $\Theta > 1$, explosive Euler eq

Full Linearized Model

Demand block

$$\begin{aligned}
\widehat{y}_t &= \Theta \widehat{y}_{t+1} - \frac{1}{\gamma}(i_t - \pi_{t+1}) - \Lambda \widehat{\mu}_{t+1} + \widehat{g}_t - \widehat{g}_{t+1} \\
\widehat{\mu}_t &= \widetilde{\beta} \widehat{\mu}_{t+1} + \widetilde{\beta}(i_t - \pi_{t+1})
\end{aligned}$$

Standard Phillips curve, Taylor rule:

$$\begin{aligned}
\pi_t &= \widetilde{\beta} \pi_{t+1} + \kappa \widehat{y}_t \\
i_t &= \Phi_\pi \pi_t
\end{aligned}$$

where $\tilde{\beta} = \frac{1}{1+r}$

Determinacy when $\Phi_\pi = 0$, $\pi_t = 0$ and $\widehat{g}_t = 0$

$$\widehat{y}_t = \Theta \widehat{y}_{t+1}$$

Determinacy when $\Phi_\pi = 0$, $\pi_t = 0$ and $\widehat{g}_t = 0$

$$\widehat{y}_{t+1} = \Theta^{-1}\widehat{y}_t$$

Does a unique bounded $\{\hat{y}_t\}$ solve this? YES (determinacy), NO (indeterminacy)

Determinacy when $\Phi_\pi = 0$, $\pi_t = 0$ and $\widehat{g}_t = 0$

$$\widehat{y}_{t+1} = \Theta^{-1}\widehat{y}_t$$

Does a unique bounded $\{\hat{y}_t\}$ solve this? YES (determinacy), NO (indeterminacy)

- HANK - acyclical risk $(\Theta = 1)$/RANK: Indeterminacy
- HANK - procyclical risk $(\Theta < 1)$: Determinacy
- HANK - countercyclical risk $(\Theta > 1)$: Indeterminacy

An income risk-adjusted Taylor principle

Now return to full log-linear model, still assuming $\widehat{g}_t = 0$ for simplicity

After some substitutions, the dynamics can be written in matrix form as follows:

$$\begin{bmatrix} \widehat{y}_{t+1} \\ \widehat{\pi}_{t+1} \\ \widehat{\mu}_{t+1} \end{bmatrix} = \begin{bmatrix} \frac{1}{\Theta} + \frac{\kappa\left(\gamma^{-1}-\Lambda\right)}{\widetilde{\beta}\Theta} & \frac{\left(\gamma^{-1}-\Lambda\right)\left(\Phi_\pi-\widetilde{\beta}^{-1}\right)}{\Theta} & -\frac{\Lambda}{\widetilde{\beta}\Theta} \\ -\frac{\kappa}{\widetilde{\beta}} & \frac{1}{\widetilde{\beta}} & 0 \\ -\frac{\kappa}{\widetilde{\beta}} & \frac{1}{\widetilde{\beta}} - \Phi_\pi & \frac{1}{\widetilde{\beta}} \end{bmatrix} \begin{bmatrix} \widehat{y}_t \\ \widehat{\pi}_t \\ \widehat{\mu}_t \end{bmatrix}$$

No predetermined variables $\Rightarrow$ **3 roots outside unit circle** needed for eq'm uniqueness

An income risk-adjusted Taylor principle

Acharya & Dogra (2020) show that a NSC for this is

$$\Phi_\pi > 1 + \frac{\gamma}{\kappa}\left[\frac{\left(1-\tilde{\beta}\right)^2}{\left(1-\tilde{\beta}\right)+\gamma\tilde{\beta}\Lambda}\right](\Theta - 1)$$

where $\widetilde{\beta} = \frac{1}{1+r}$

- procyclical risk ($\Theta < 1$): determinacy more likely
- acyclical risk ($\Theta = 1$): determinacy requires $\Phi_\pi > 1$ (as in RANK)
- countercyclical risk ($\Theta > 1$): determinacy less likely (as HANK & SaM model)

Forward guidance

- Suppose Fed announces at t a rate cut at date $t+k$
- In RANK

$$\widehat{y}_t = -\frac{1}{\gamma}\sum_{k=0}^{\infty}(i_{t+k} - \pi_{t+k+1})$$

- With fixed prices, date $t+k$ rate cut equally as effective as date t cut
- With sticky prices, date $t+k$ rate cut more effective than date t cut

Constant prices

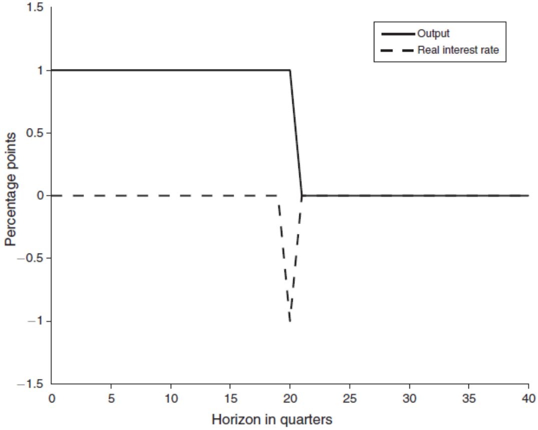


FIGURE 1. RESPONSE OF OUTPUT TO A 1-QUARTER DROP IN THE REAL INTEREST RATE 20 QUARTERS IN THE FUTURE

Sticky prices

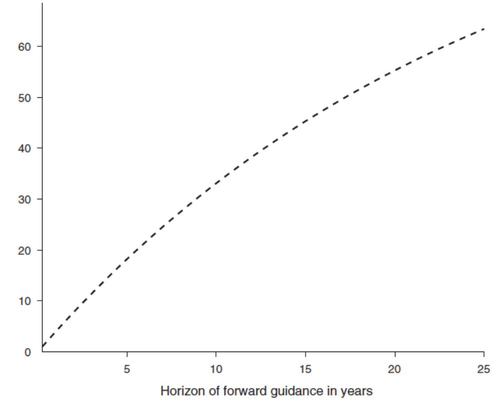


FIGURE 2. RESPONSE OF CURRENT INFLATION TO FORWARD GUIDANCE ABOUT INTEREST RATES AT DIFFERENT HORIZONS RELATIVE TO RESPONSE TO EQUALLY LARGE CHANGE IN CURRENT REAL INTEREST RATE

Source: McKay et al. (2016)

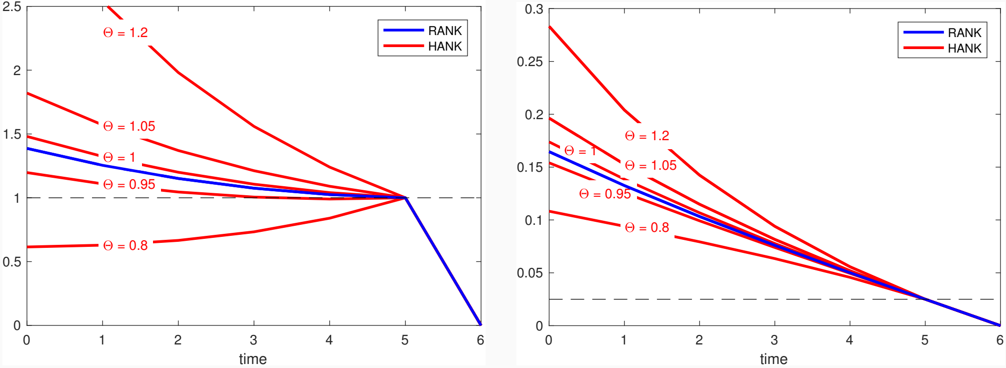
Forward guidance

- In HANK

$$\widehat{y}_t = -\frac{1}{\gamma}\sum_{k=0}^{\infty}\Theta^k(i_{t+k} - \pi_{t+k+1}) - \Lambda\sum_{k=0}^{\infty}\Theta^k\sum_{s=1}^{\infty}\tilde{\beta}(i_{t+k+s} - \pi_{t+k+s+1})$$

- With fixed prices:
  - sufficiently procyclical risk ($\Theta << 1$), date $t+k$ rate cut less effective than date t rate cut
  - acyclical risk ($\Theta = 1$), date $t+k$ rate cut more effective (precautionary savings)
    - Lower future $r_t \Rightarrow \mu_t \downarrow$. Lower pass through of income risk into consumption risk, weakens precautionary savings motive.
  - with countercyclical risk ($\Theta > 1$), date $t+k$ rate cut more effective

Response to cut in i_t 5 periods in the future



Output

Inflation

Fiscal multipliers in liquidity trap

- fiscal multipliers can be very (implausibly) large in RANK at the ZLB
- Consider liquidity trap lasting T periods, $\widehat{g}_t = g > 0$ during trap, zero thereafter
- In RANK:
  - with **fixed** prices

$$\frac{\partial \widehat{y}_t}{\partial g} = 1, \quad 0 \leq t \leq T$$

    and independent of duration of trap. Indeed:

$$\widehat{y}_T = \underbrace{\widehat{y}_{T+1}}_{=0} + \underbrace{\widehat{g}_T}_{=g} - \underbrace{\widehat{g}_{T+1}}_{=0}$$

$$\widehat{y}_{T-1} = \underbrace{\widehat{y}_T}_{=g} + \underbrace{\widehat{g}_{T-1}}_{=g} - \underbrace{\widehat{g}_T}_{=g}$$

$$\ldots$$

$$\widehat{y}_0 = \widehat{y}_1 = g$$

  - With **sticky** prices, multiplier increasing in duration of trap (due to inflation feedback loop)

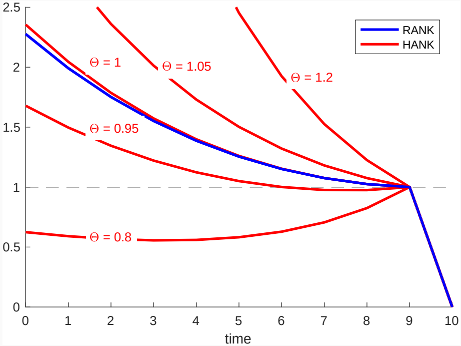
Fiscal multipliers in liquidity trap

- In HANK with **fixed** prices:

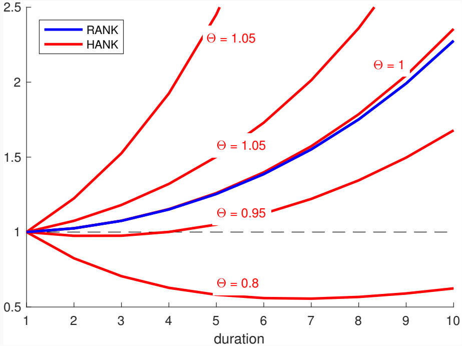
$$\frac{\partial \widehat{y}_t}{\partial g} = \Theta^{T-t-1}, 0 \leq t \leq T$$

  - with procyclical risk ($\Theta < 1$), decreasing in duration of trap
  - with acyclical risk ($\Theta = 1$), independent of duration of trap
  - with countercyclical risk ($\Theta > 1$), increasing in duration of trap
- With sticky prices...

Fiscal multipliers



$\frac{d\widehat{y}_t}{dg}$ in a 10 period liquidity trap



$\frac{d\widehat{y}_0}{dg}$ as a function of liquidity trap duration

MPC heterogeneity

- Many HANK models have heterogenous MPC with (near) HtM households
- Suppose $\eta \in (0, 1)$ measure of households hand to mouth with income $y_t^i = \chi y_t$
  - $\frac{dy_t^i}{dy_t} = \chi =$ **cyclical sensitivity of income of constrained households**
  - Average MPC is $(1-\eta) \times \mu_t + \eta \times 1 > \mu_t$
- Aggregate Euler eq becomes (abstracting from gov't spending)

$$y_t = y_{t+1} - \frac{\Xi}{\gamma} \ln[\beta(1+r_t)] - \Xi \frac{\gamma \mu_{t+1}^2 \boldsymbol{\sigma}^2(y_{t+1})}{2}, \qquad \Xi = \frac{1-\eta}{1-\eta\chi}$$

- Resource constraint:

$$y_t = c_t = \eta \chi y_t + (1-\eta) c_t^u \qquad \Rightarrow \qquad y_t = \Xi c_t^u$$

Ξ is 'static' response of GDP to consumption of unconstrained

Effect on monetary policy transmission

Kaplan et al. (2018) argue monetary transmission can be decomposed into **two** effects

- **direct** effect via unconstrained households (intertemporal substitution in consumption)
- **indirect** effect via constrained households (New Keynesian cross)

Consider Euler equation again

$$y_t = y_{t+1} - \frac{\Xi}{\gamma} \ln[\beta(1+r_t)] - \Xi \frac{\gamma \mu_{t+1}^2 \boldsymbol{\sigma}^2(y_{t+1})}{2}, \qquad \Xi = \frac{1-\eta}{1-\eta\chi}$$

- HTM income equally cyclically sensitive ($\Xi = 1$): exact offsetting of direct & indirect effects
- HTM income less cyclically sensitive ($\Xi < 1$): dampens response to interest rates
- HTM income more cyclically sensitive ($\Xi > 1$): stronger response to interest rates

Optimal monetary policy

Acharya, Challe, Dogra (2019) study **optimal** monetary policy

- utilitarian Social Welfare function can be written as:

$$\sum_{t=0}^{\infty} \beta^t \underbrace{u(c_t, \ell_t)}_{\substack{\text{welfare of} \\ \text{rep. agent}}} \times \underbrace{\Sigma_t}_{\substack{\text{welfare cost} \\ \text{of inequality}}}$$

- In RANK $\Sigma = 1$; In HANK $\Sigma > 1$
- Evolution of Σ depends on consumption risk:

$$\ln \Sigma_t = \frac{\gamma}{2} \mu_t^2 \sigma_{y,t}^2 + \ln[1 - \vartheta + \vartheta \Sigma_{t-1}]$$

- new tradeoff - monetary policy affects both σ_y^2 and μ_t^2
- optimal mp: more accomodative than RANK in recessions

LQ problem in RANK

$$\min \frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left\{(\widehat{y}_t - \widehat{y}_t^e)^2 + \frac{\varepsilon}{\kappa}\pi_t^2\right\} \qquad s.t. \qquad \pi_t = \beta\pi_{t+1} + \kappa(\widehat{y}_t - \widehat{y}_t^e) + \frac{\varepsilon}{\Psi}\widehat{\varepsilon}_t$$

yielding the "*targeting rule*"

$$\boxed{\underbrace{(\widehat{y}_t - \widehat{y}_t^e)}_{\text{output gap}} + \underbrace{\varepsilon\widehat{p}_t}_{\text{price level}} = 0}$$

LQ problem in HANK

$$\min \frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left\{\left[(1-\varpi)\underbrace{\left(\widehat{y}_t-\varkappa(\Omega)\widehat{y}^e_t\right)}_{\text{consumption risk}}+\varpi\left(\widehat{y}_t-\widehat{y}^e_t\right)\right]^2+\frac{\varepsilon}{\kappa\Upsilon(\Omega)}\pi_t^2\right\}$$

s.t.

$$\pi_t=\beta\pi_{t+1}+\kappa\left(\widehat{y}_t-\widehat{y}^e_t\right)+\frac{\varepsilon}{\Psi}\widehat{\varepsilon}_t$$

LQ problem in HANK

$$\min \frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left\{\left[(1-\varpi)\underbrace{\left(\widehat{y}_t-\varkappa(\Omega)\widehat{y}_t^e\right)}_{\text{consumption risk}}+\varpi\left(\widehat{y}_t-\widehat{y}_t^e\right)\right]^2+\frac{\varepsilon}{\kappa\Upsilon(\Omega)}\pi_t^2\right\}$$

s.t.

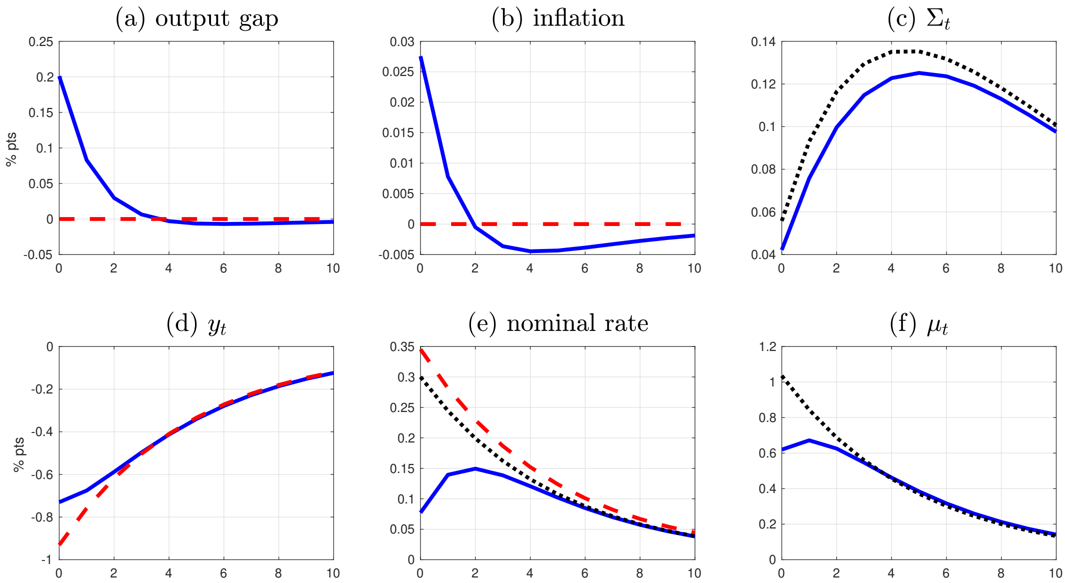
$$\pi_t=\beta\pi_{t+1}+\kappa\left(\widehat{y}_t-\widehat{y}_t^e\right)+\frac{\varepsilon}{\Psi}\widehat{\varepsilon}_t$$

yielding the "*targeting rule*"

$$\boxed{\underbrace{[1-\delta(\Omega)]\,\widehat{y}_t}_{\text{output stabilization}}+\delta(\Omega)\left(\widehat{y}_t-\widehat{y}_t^e\right)+\frac{\varepsilon}{\Upsilon(\Omega)}\widehat{p}_t=0}$$

where $\delta=(1-\varpi)\varkappa+\varpi$

Optimal dynamics following a negative productivity shock



Optimal dynamics following a positive markup shock