CS 101 Fall 2024 - Quiz 5
November, 4, 2024 - 20 Minutes **Name:** **Student ID:**

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1. (2 points) Honor Code

I promise that I will complete this quiz independently and will not use any electronic products or paper-based materials during the quiz, nor will I communicate with other students during this quiz.

I will not violate the Honor Code during this quiz.

√ True ◯ False

2. (8 points) True or False

Determine whether the following statements are true or false.

(a) (1') A Huffman Coding Tree is a full binary tree.

√ True ◯ False

(b) (1') When using Huffman coding to encode characters, the length of the code assigned to each character may be the same.

√ True ◯ False

(c) (1') Heap sort is a stable sorting algorithm. ◯ True √ False

(d) (1') The time complexity of heap sort is $O(n\log(n))$ in both the best-case and the worst-case.

√ True ◯ False

(e) (1') If a binary min-heap is implemented using an array, then the time complexity for finding the maximum value in the heap is $O(\log(n))$. ◯ True √ False

(f) (1') In a heap, element A is an ancestor of element B ($A \neq B$). If the heap continually pops the top element out, element B must still be in the heap when element A is popped out.

√ True ◯ False

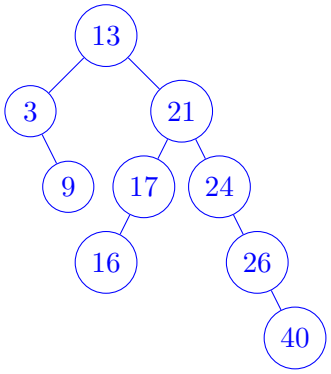
(g) (1') In a BST, there may be a subtree that is not a BST. ◯ True √ False

(h) (1') The worst-case of finding the maximum element in a BST is $O(n)$. √ True ◯ False

3. (5 points) BST

(a) (3') Elements $[13, 21, 3, 17, 24, 26, 40, 9, 16]$ are inserted sequentially into an empty BST. Please draw the BST after the insertions.

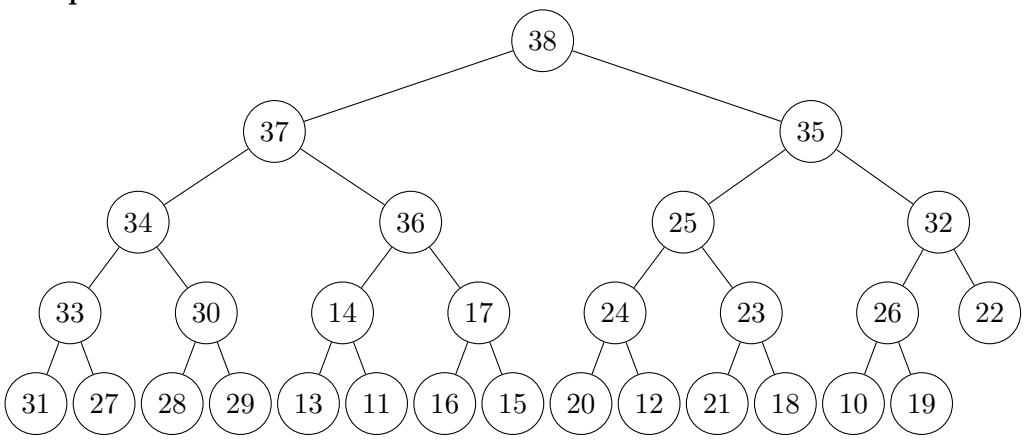
Solution:



(b) (2') If element x is inserted into the BST above as the right child of element 9, please provide all possible integer values for x (all elements are distinct).

Solution: $10, 11, 12$

4. (7 points) Heap



(a) (1') Is this heap a max-heap or a min-heap?

Solution: max-heap

(b) (3') Suppose that you pop the key from the heap above. Write down all the elements that are involved in one (or more) comparison.

Solution: 14,17,19,34,35,36,37. The compares are 35-37, 19-37, 34-36, 19-36, 14-17, 19-17

(c) (3') Suppose that inserting the key x was the last operation performed in the binary heap in the figure. That is, after inserting x, the heap is shown in the figure above. Write down all possible values of x.

Solution: 19,26,32,35. To insert a node in a binary heap, we place it in the next available lead node and swim it up. Thus, 19,26,32,35 and 38 are the only keys that we might move. But, the last inserted key could not have been 38, because, then, 35 would have been the old root (which would violate heap order because the left child of the root is 37).

5. (8 points) Fill in the blanks

(a) (4') Using Huffman Coding to encode the characters $[a, b, c, d, e, f]$ with frequencies $[2, 3, 4, 8, 7, 6]$. The length of the code of character f is _____2_____. Suppose the Huffman Code of all characters is required to have as many zeros as possible (each character is counted once). For instance, $[a : 1; b : 00; c : 01]$ (3 zeros) has more zeros than $[a : 0; b : 10; c : 11]$ (2 zeros). Then the Huffman Code for the character c is _____001_____.

(b) (4') In a binary max-heap with n elements and duplicated elements are not allowed, the 6^{th} largest element can be found in time $O($_____$\log n$_____$)$ if we can only access the top of the heap. And the 6^{th} largest element can be found in time $O($_____1_____$)$ if we can access the array storing the heap.