Potential Functions

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Potential Function

- A *potential function* is a differentiable real-valued function,
  $U : \Re^n \rightarrow \Re$ (*n*-dimension real-number → real number)
- The value of a potential function can be viewed as *energy*, and the gradient of the potential is *force*.
- The *gradient* is a *vector field* which points in the direction that locally maximally increases *U* at point *q*:

$$\nabla U(q) = DU(q)^T = \left[\frac{\partial U}{\partial q_1}(q), \ldots, \frac{\partial U}{\partial q_n}(q) \right]^T$$

- The *gradient* ∇ of *U* is also called the *Jacobian D* of *U* that maximally increases the function *U* at point *q*.

Potential Field

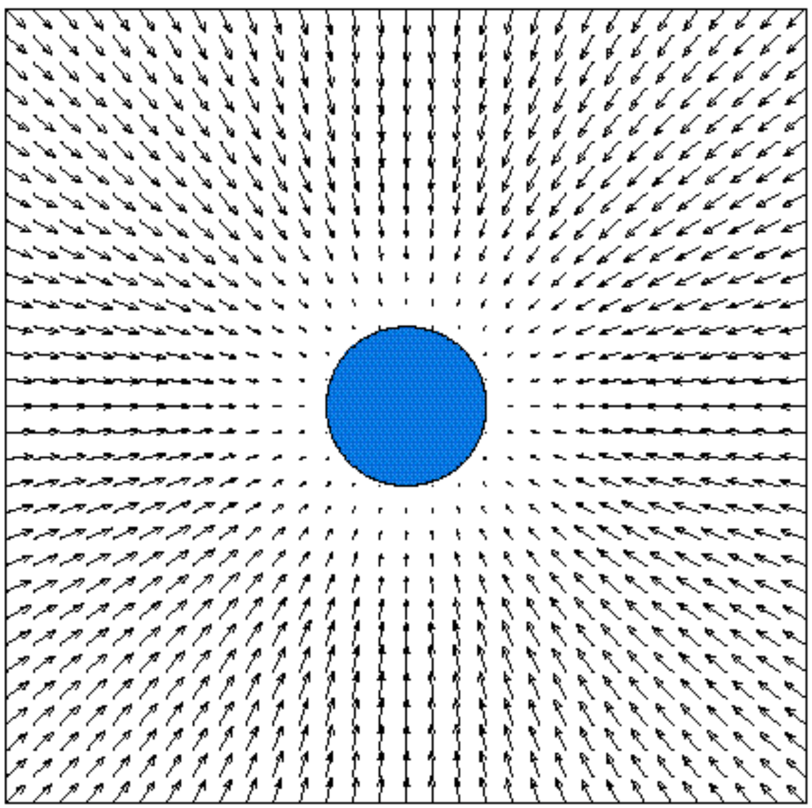
- A potential function $U(q)$ defines attractive $U_{\text{att}}(q)$ and repulsive $U_{\text{rep}}(q)$ potential fields in free space.
- The robot moves towards the goal by following a gradient field, avoiding collisions with obstacles by adding the attractive and repulsive potential fields:

$$U(q) = U_{\text{att}}(q) + U_{\text{rep}}(q)$$

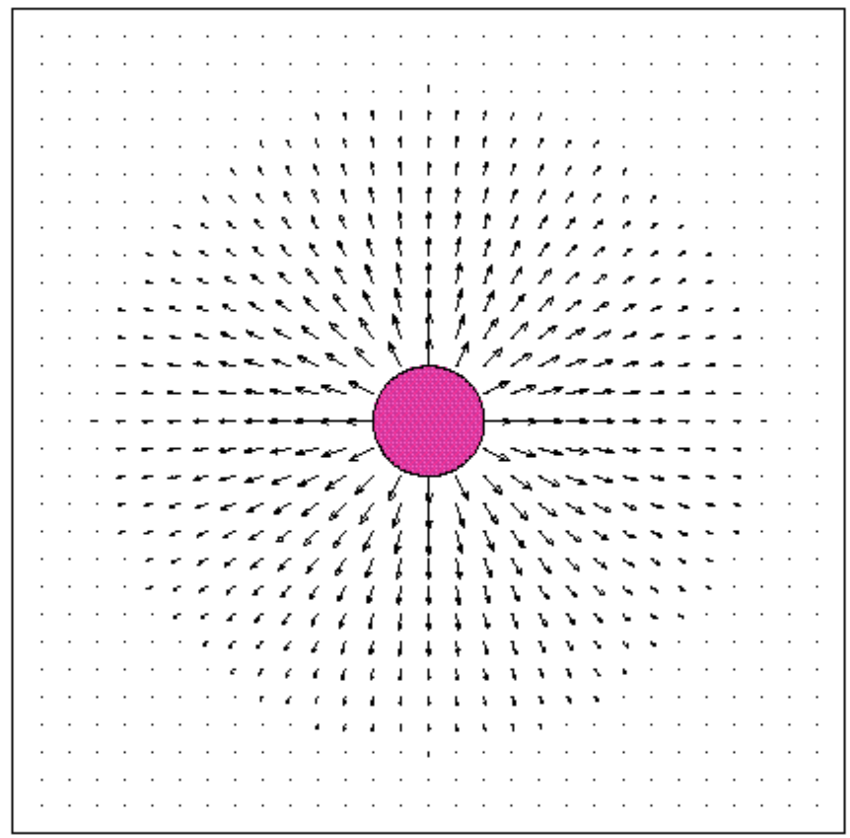
- The potential fields may include multiple obstacles including dynamic obstacles.
- The robot is initially treated as a point (robot orientation θ may be neglected).

Potential Field

- The goal attracts the robot while obstacles repel it
- Move-to-Goal: Attractive potential field
- Avoid-Obstacles: Repulsive potential field



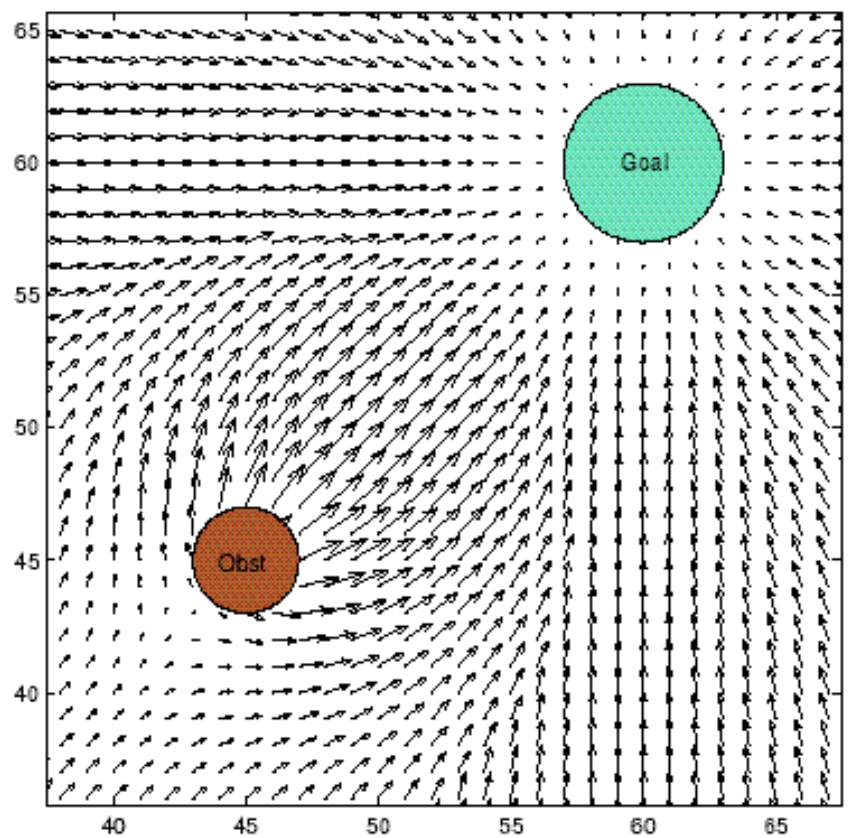
Attractive

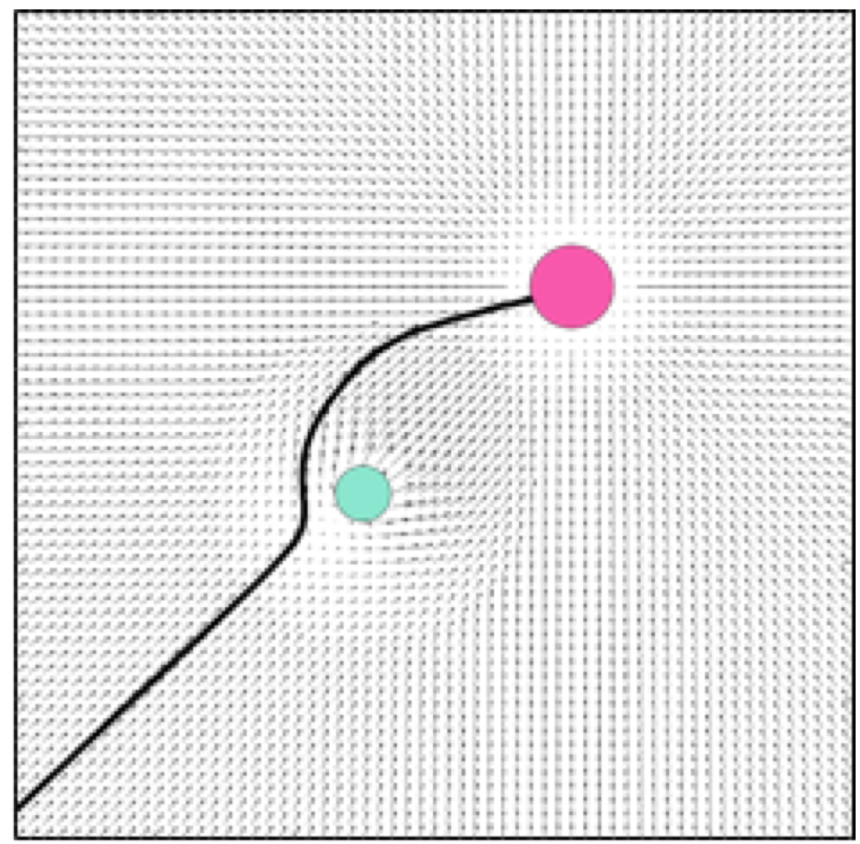


Repulsive

Potential Field

- The sum of the attractive and repulsive fields draws the robot to the goal while deflecting it from obstacles
- Robot follows the potential field to reach the goal as a ball would roll down a mountain.





Potential Field

- Assume a differentiable artificial potential field function $U(q)$ where the corresponding artificial force $F(q)$ at $q = (q_1, q_2, \ldots, q_n)$ is given by

$$F(q) = -\nabla U(q)$$

where the gradient vector of U (at location q), for a 2d space, is given by

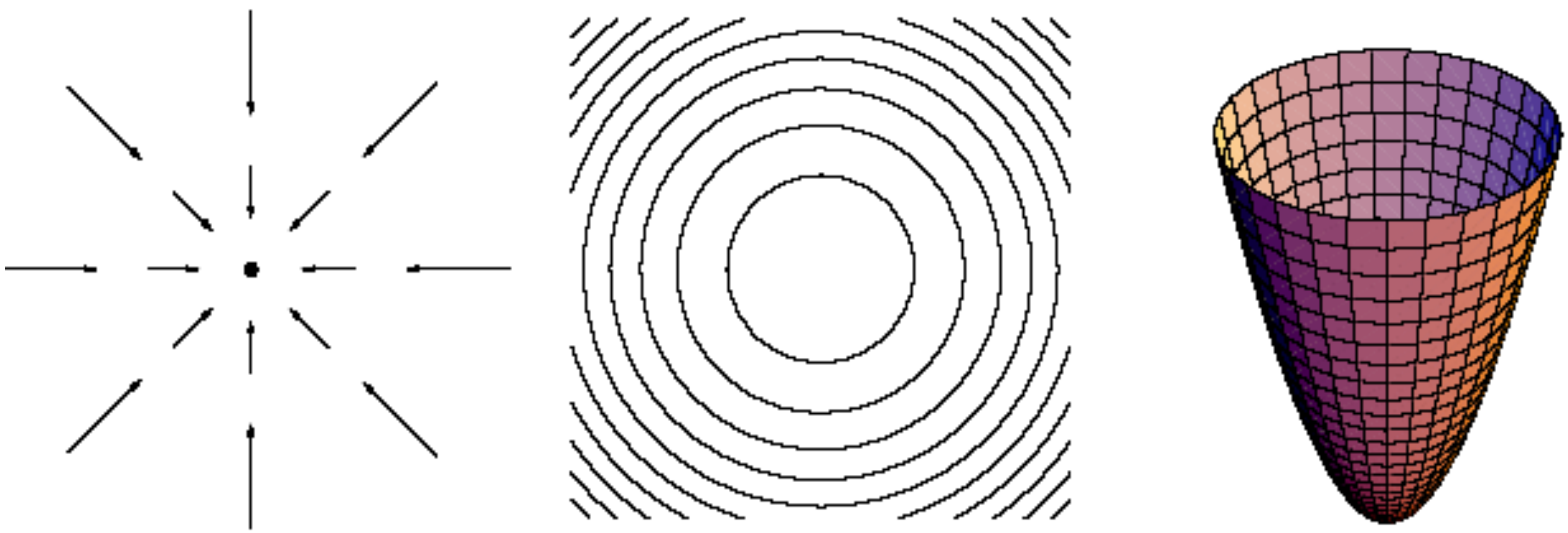
$$\nabla U(q) = \left(\frac{\partial U(q)}{\partial q_1}, \frac{\partial U(q)}{\partial q_2} \right)$$

- The potential field forces acting on the robot are

$$F(q) = F_{att}(q) + F_{rep}(q) = -\nabla U_{att}(q) - \nabla U_{rep}(q)$$

Attractive Potential Field

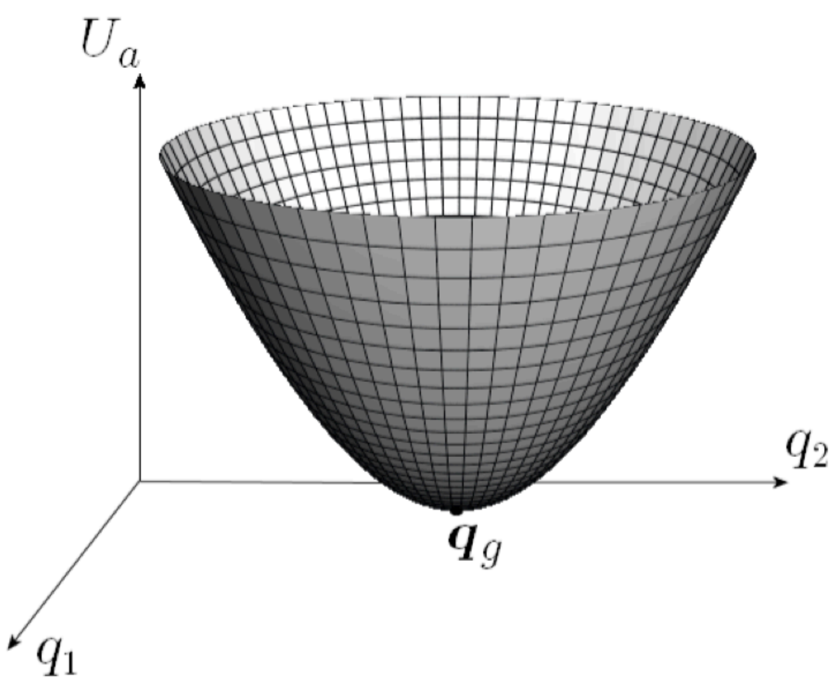
- The further away q is from q_{goal} the larger the magnitude of the vector and the larger the attraction towards the goal.
- The closer to the goal, the slower the speed of the robot.



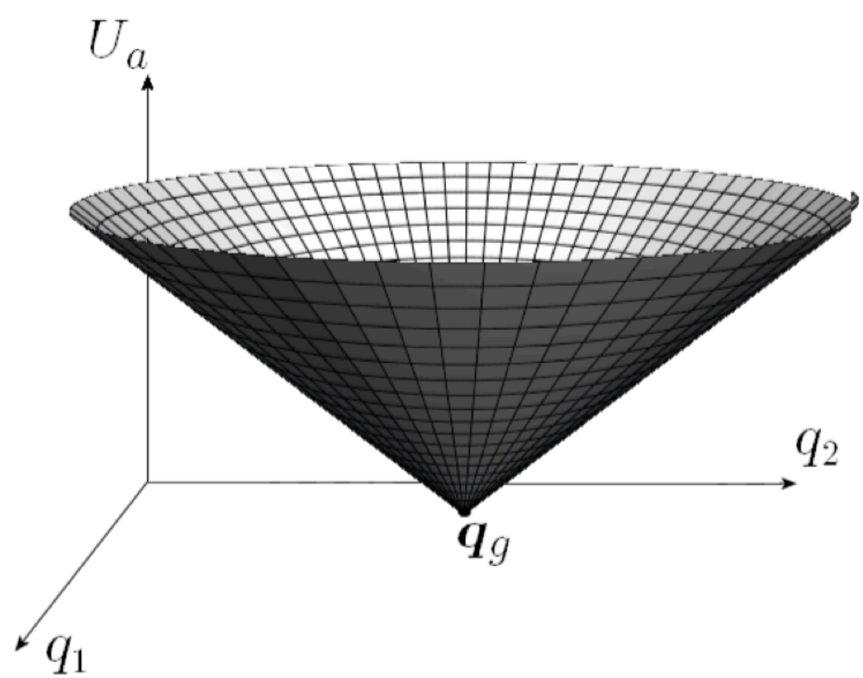
(a) Attractive gradient vector field with goal at center
(b) Contour plot where circles correspond to constant $U_{att}(q)$ values
(c) Graph of the attractive potential field

Attractive Potential Field

- Attractive potential field defines gradients with magnitudes proportional to the Euclidean distance from q to q_{goal}, $d(q,q_{\text{goal}}) = ||q - q_{\text{goal}}||$.
- Paraboloidal vs Conical potential function example.



paraboloidal



conical

Attractive Potential Field

- The simplest potential function is a **conic** function with scaled distance *d* and parameter k_{att} to the goal:

$$U(q) = k_{att} d(q, q_{goal})$$

- The resulting attractive gradient is constant:

$$\nabla U(q) = \frac{k_{att}}{d(q, q_{goal})}(q - q_{goal})$$

- Note that $(q\text{-}q_{\text{goal}})/d(q,q_{\text{goal}})$ defines a unit vector.
- The negative gradient will trace a path towards the goal with magnitude k_{att}.

Attractive Potential Field

- Another continuously differentiable potential function is a **parabolic** function that grows quadratically with distance *d(q,q*$_{\text{goal}}$) to the q_{goal} such that the magnitude of the attractive gradient decreases as the robot approaches it:

$$U_{att}(q) = \frac{1}{2} k_{att} d^2(q, q_{goal})$$

- The resulting attractive gradient is linear:

$$\nabla U_{\text{att}}(q) = \nabla\left(\frac{1}{2} k_{att} d^2\left(q, q_{goal}\right)\right) = \frac{1}{2} k_{att} \nabla d^2\left(q, q_{goal}\right)$$

$$= \frac{1}{2} k_{att} 2d\left(q, q_{goal}\right) \nabla d\left(q, q_{goal}\right)$$

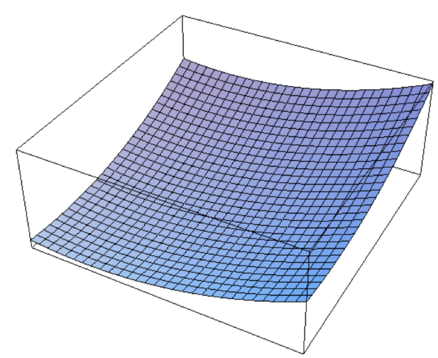
$$= k_{att} d\left(q, q_{goal}\right) \frac{\left(q - q_{goal}\right)}{d\left(q, q_{goal}\right)} = k_{att}\left(q - q_{goal}\right)$$

Attractive Potential Field

- While the gradient $\nabla U_{\text{att}}(q)$ converges linearly to 0 as q approaches q_{goal}, it grows without bounds as q moves away from q_{goal}, producing a velocity that may be too large at q_{start}.
- Quadratic and conic potentials can be combined:
  - quadratic potential attracts the robot linearly at distance d^*_{goal} closer to q_{goal}
  - conic potential attracts the robot constantly at distance d^*_{goal} away from q_{goal}

$$U_{att}(q) = \begin{cases} \frac{1}{2} k_{att} d^2(q, q_{goal}), & d(q, q_{goal}) \le d^*_{goal} \\ d^*_{goal} k_{att} d(q, q_{goal}) - \frac{1}{2} k_{att} (d^*_{goal})^2 & d(q, q_{goal}) > d^*_{goal} \end{cases}$$

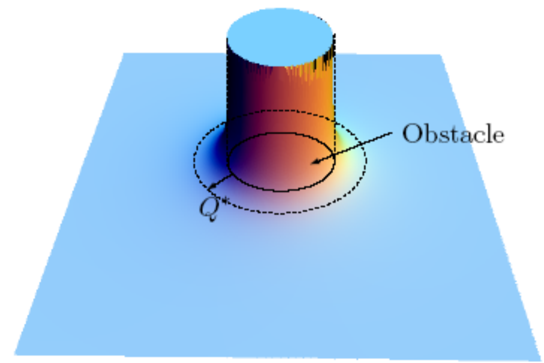
$$\nabla U_{att}(q) = \begin{cases} k_{att}(q - q_{goal}), & d(q, q_{goal}) \le d^*_{goal} \\ \dfrac{d^*_{goal} k_{att}(q - q_{goal})}{d(q, q_{goal})} & d(q, q_{goal}) > d^*_{goal} \end{cases}$$



Repulsive Potential Field

- Strength of repulsive field depends on proximity to obstacle, the closer the robot is, the stronger the repulsive force, defined by inverse distance $d(q,q_{obs})$.
- Potential fields and gradients are defined with factor $d(q,q_{\mathrm{obs}}) = d^*_{\mathrm{obs}}$ (note: in the fig $Q^* = q_{\mathrm{obs}}$) allowing the robot to ignore far away obstacles, and k_{rep} is a gain on the repulsive field, both set heuristically.

$$U_{rep}(q) = \begin{cases} \frac{1}{2} k_{rep} \left(\frac{1}{d(q,q_{obs})} - \frac{1}{d^*_{obs}} \right)^2, & d(q,q_{obs}) \le d^*_{obs} \\ 0, & d(q,q_{obs}) > d^*_{obs} \end{cases}$$



$$\nabla U_{rep}(q) = \begin{cases} \frac{1}{2} k_{rep} 2 \left(\frac{1}{d(q,q_{obs})} - \frac{1}{d^*_{obs}} \right) \nabla \left(\frac{1}{d(q,q_{obs})} - \frac{1}{d^*_{obs}} \right), & d(q,q_{obs}) \le d^*_{obs} \\ 0, & d(q,q_{obs}) > d^*_{obs} \end{cases}$$

$$= \begin{cases} k_{rep} \left(\frac{1}{d^*_{obs}} - \frac{1}{d(q,q_{obs})} \right) \frac{1}{d^2(q,q_{obs})} \nabla d(q,q_{obs}), & d(q,q_{obs}) \le d^*_{obs} \\ 0, & d(q,q_{obs}) > d^*_{obs} \end{cases}$$

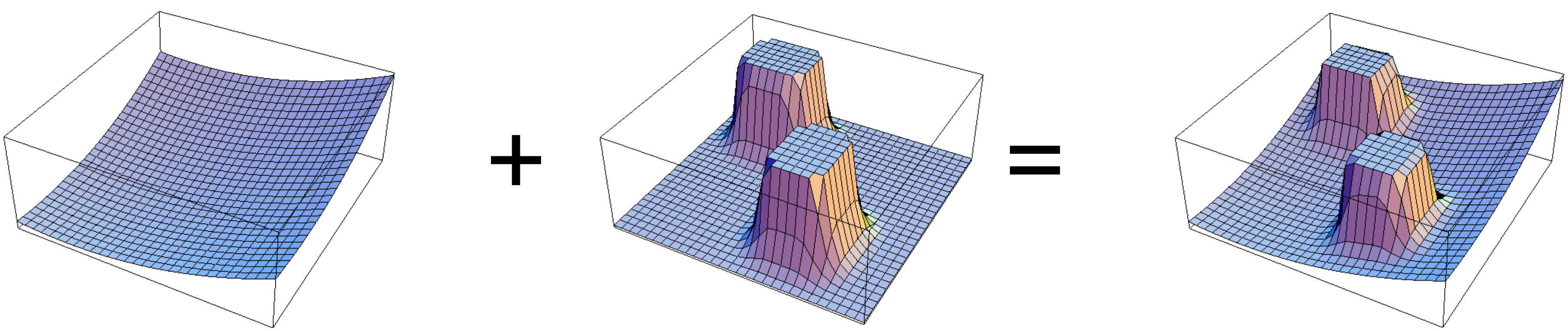
Potential Fields

- An example of a combined attractive and repulsive potential field and corresponding gradients is as follows:

$$U(q) = U_{att}(q) + U_{rep}(q) \qquad U_{rep}(q) = \sum_{i=1}^{n} U_{rep_i}(q)$$

$$U_{att}(q) = \begin{cases} \frac{1}{2} k_{att} d^2(q, q_{goal}), & d(q, q_{goal}) \le d^*_{goal} \\ d^*_{goal} k_{att} d(q, q_{goal}) - \frac{1}{2} k_{att} (d^*_{goal})^2 & d(q, q_{goal}) > d^*_{goal} \end{cases}$$

$$U_{rep_i}(q) = \begin{cases} \frac{1}{2} k_{rep_i} \left(\frac{1}{d(q, q_{obs})} - \frac{1}{d^*_{obs_i}} \right)^2, & d(q, q_{obs}) \le d^*_{obs} \\ 0, & d(q, q_{obs}) > d^*_{obs} \end{cases}$$



Gradient Descent

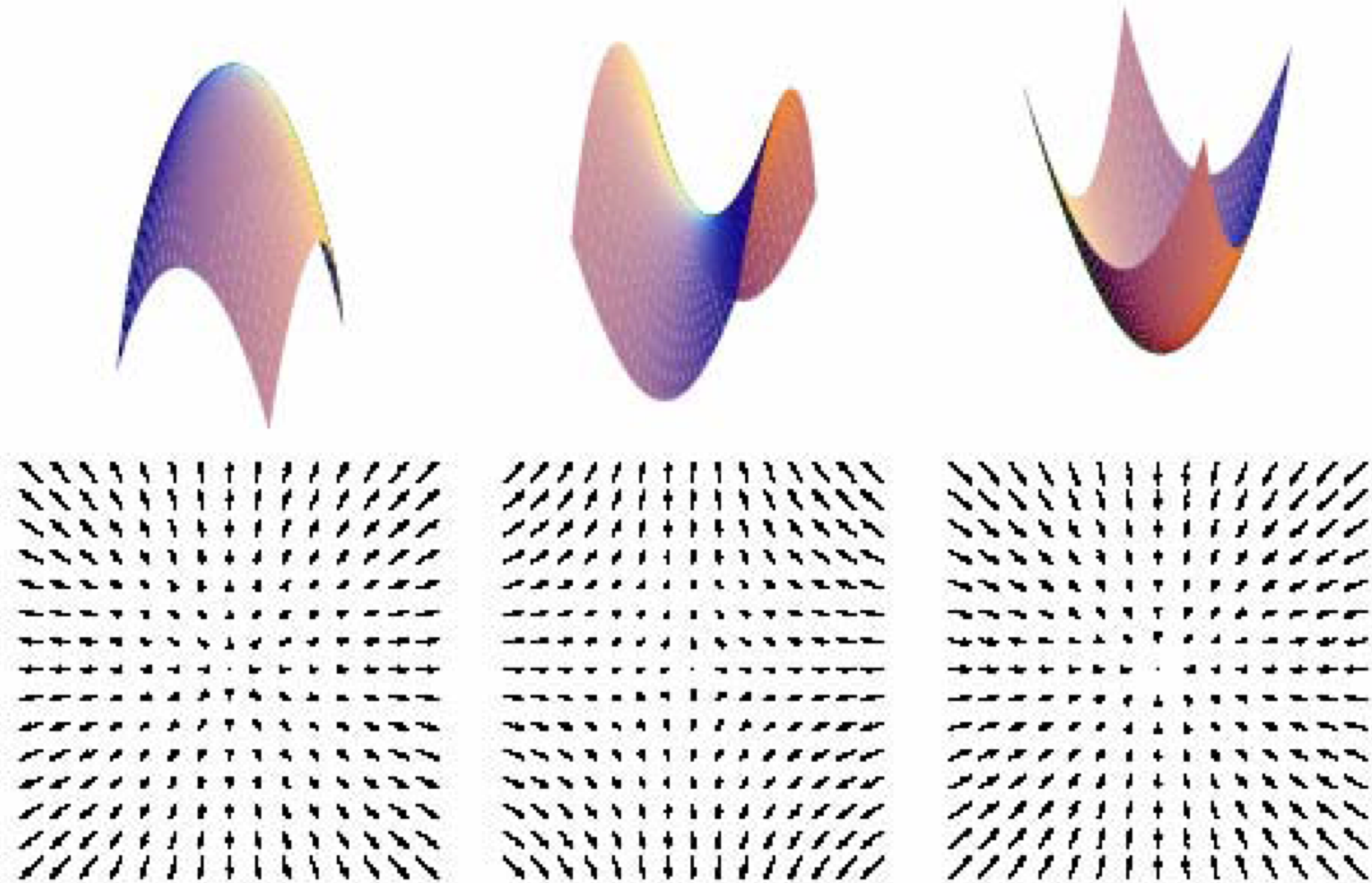
- The potential function approach directs a robot as if it were a particle moving in a gradient vector field.
- Potential functions can be viewed as a landscape where robots move from a “high-value” to a “low-value” energy state.
- The robot follows a path “downhill” by following the negative gradient of the potential function, also know as gradient descent:

$$\dot{c}(t) = -\nabla U(c(t))$$

- Where $\dot{c}(t)$ can be considered the velocity of a point along curve $c(t)$.
- The robot terminates motion at a *critical point q** where the gradient vanishes, either at a *maximum*, *minimum* or *saddle*:

$$\nabla U(q^*) = 0$$

Critical Points



(Maximum) (Saddle) (Minimum)

The Hessian

- The type of critical point where $\nabla U(q^*) = 0$ can be determined by its second derivative, the Hessian matrix *H*.
- If the Hessian is nonsingular (Det(*H*) ≠ 0), the critical point is a unique point:
  - if *H* is positive definite, the critical point is a *minimum*
  - if *H* is negative definite, the critical point is a *maximum*
  - if *H* is indefinite, the critical point is a *saddle* point
- The robot usually terminates its motion at a local minimum, and a local maximum is usually a starting point since gradient descent decreases *U*.
- Local minima are stable as any motion will return the robot to the minimum.

$$H = \begin{bmatrix} \frac{\partial^2 U}{\partial q_1^2} & \cdots & \frac{\partial^2 U}{\partial q_1 \partial q_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 U}{\partial q_n \partial q_1} & \cdots & \frac{\partial^2 U}{\partial q_n^2} \end{bmatrix}$$

Single Object Distance

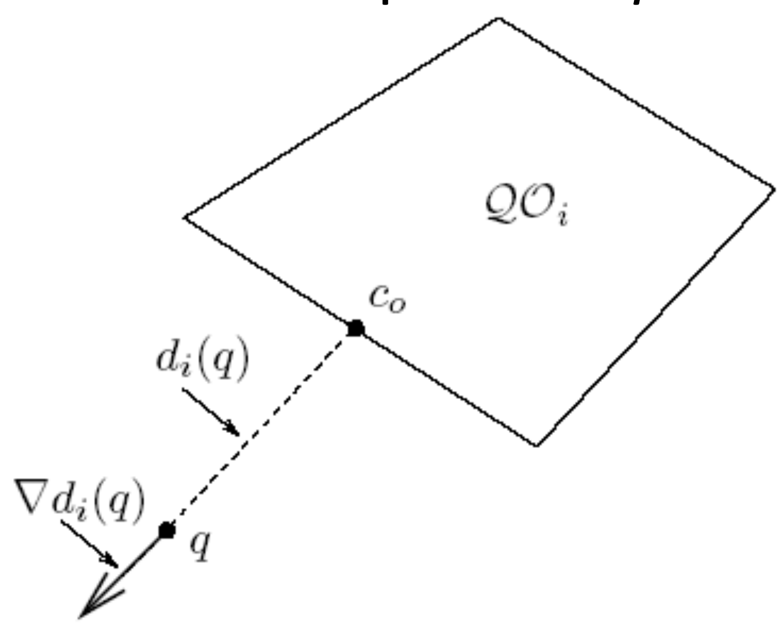
- To avoid oscillations at points two-way equidistant from obstacles, the repulsive potential function is defined in terms of distances to *individual* obstacles (instead of a general repulsive potential function), where $d_i(q)$ is the distance to obstacle QO_i, and the min operators returns the smallest distance $d(q,c)$ for all points c in QO_i:

$$d_i(q) = \min_{c \in QO_i} d(q,c)$$

- For convex obstacles QO_i where c is the closest point to q then the gradient of $d_i(q)$ is

$$\nabla d_i(q) = \frac{q-c}{d(q,c)}$$

- The gradient of $\nabla d_i(q)$ describes the direction that maximally increases the repulsive force to QO_i from q.



Gradient Descent

Algorithm: Gradient Descent

Input: A means to compute the gradient $\nabla U(q)$ at a point q

Output: A sequence of points { $q(0)$, $q(1)$, …. $q(i)$ }

$q(0)=q_{start}$
$i = 0$
while $\nabla U(q(i)) \neq 0$ do
    $q(i+1) = q(i) - \alpha(i)\nabla U(q(i))$
    $i=i+1$
end while

Comments: $\alpha(i)$ is a scalar determining the step of the i-th iteration. It should be small enough not to jump into obstacles while large enough not to require extensive computations

Gradient Descent

Algorithm: Gradient Descent

Input: A means to compute the gradient $\nabla U(q)$ at a point q

Output: A sequence of points { $q(0)$, $q(1)$, ... $q(i)$ }

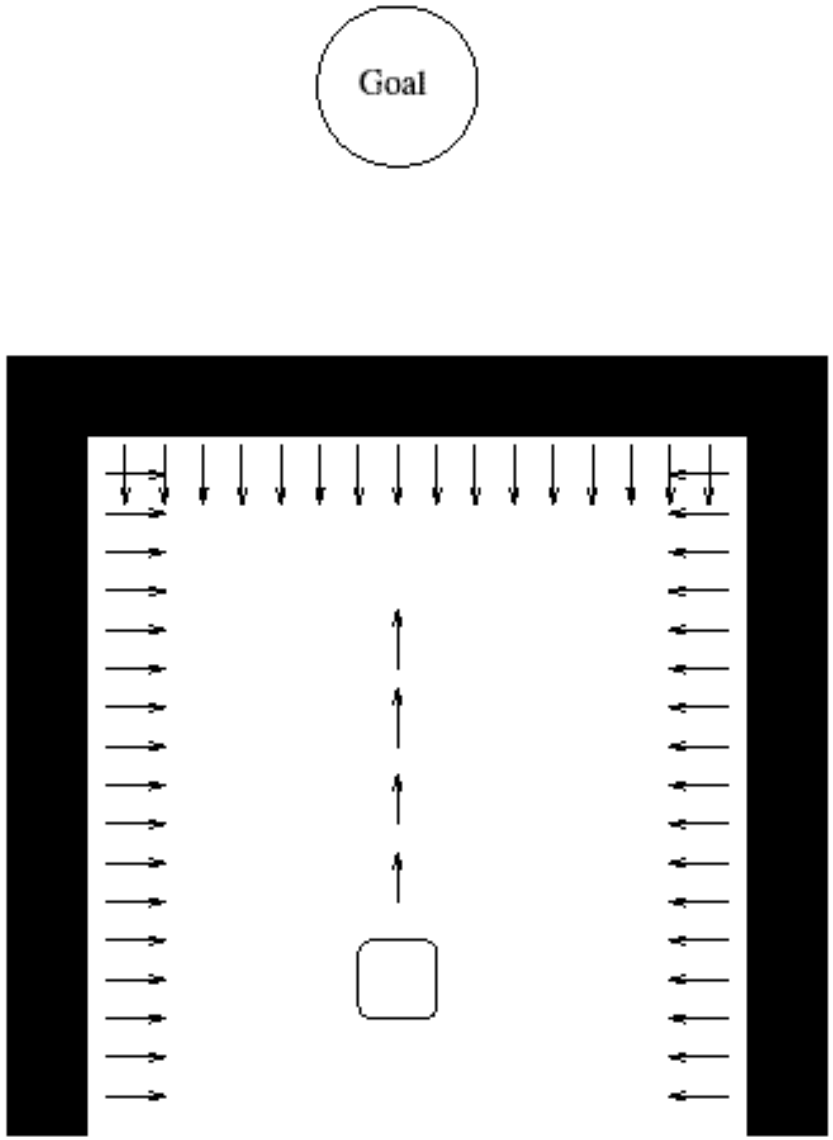
```
q(0)=q_start
 i = 0
while ||∇U(q(i)) || > ε  do
      q(i+1) = q(i) - α(i)∇U(q(i))
      i=i+1
end while
```

Comments: Since it is unlikely to be able to exactly satisfy the condition $\nabla U(q(i)) = 0$, this condition is replaced by an ε sufficiently small.

Problems with Potential Fields

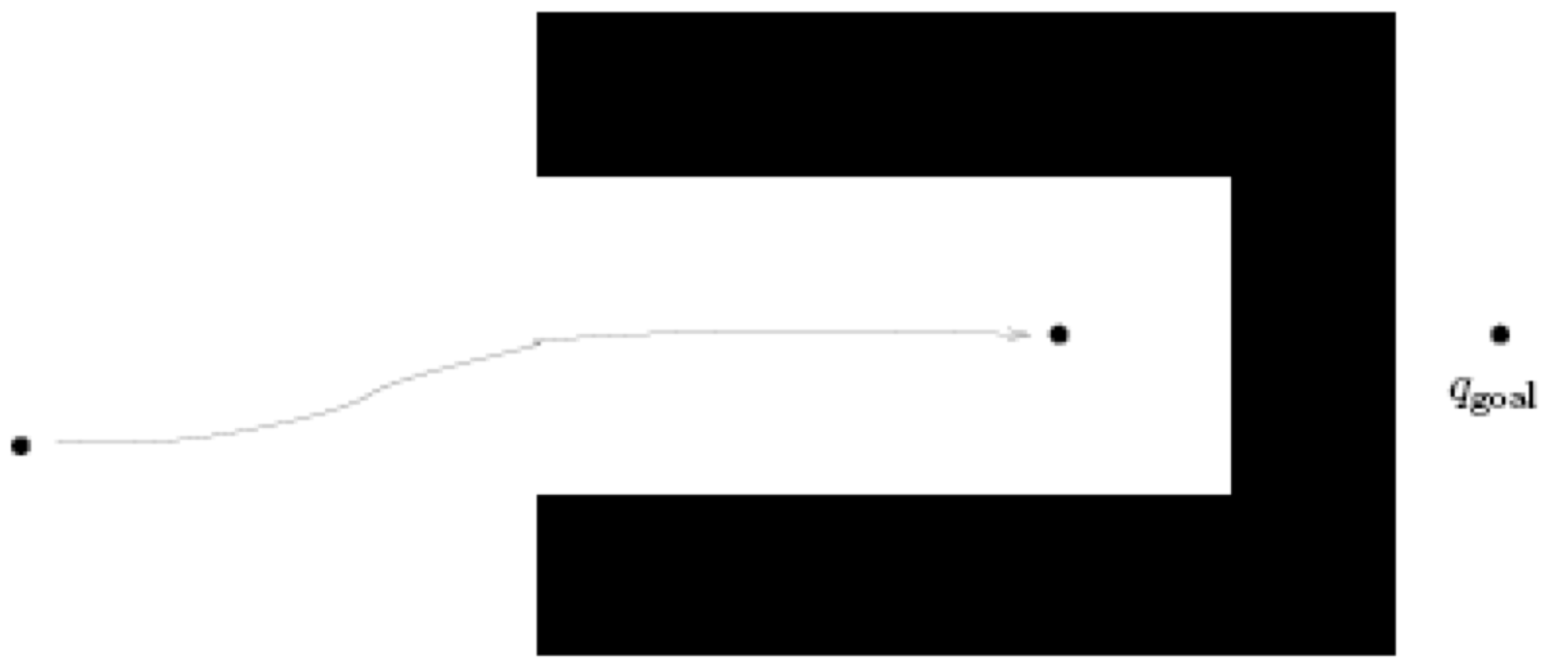
- *Local minima*
  - Attractive and repulsive forces can balance, so robot makes no progress.
  - Closely spaced obstacles, or dead end.
- *Unstable oscillation*
  - The dynamics of the robot/environment system can become unstable.
  - High speeds, narrow corridors, sudden changes.

Local Minimum



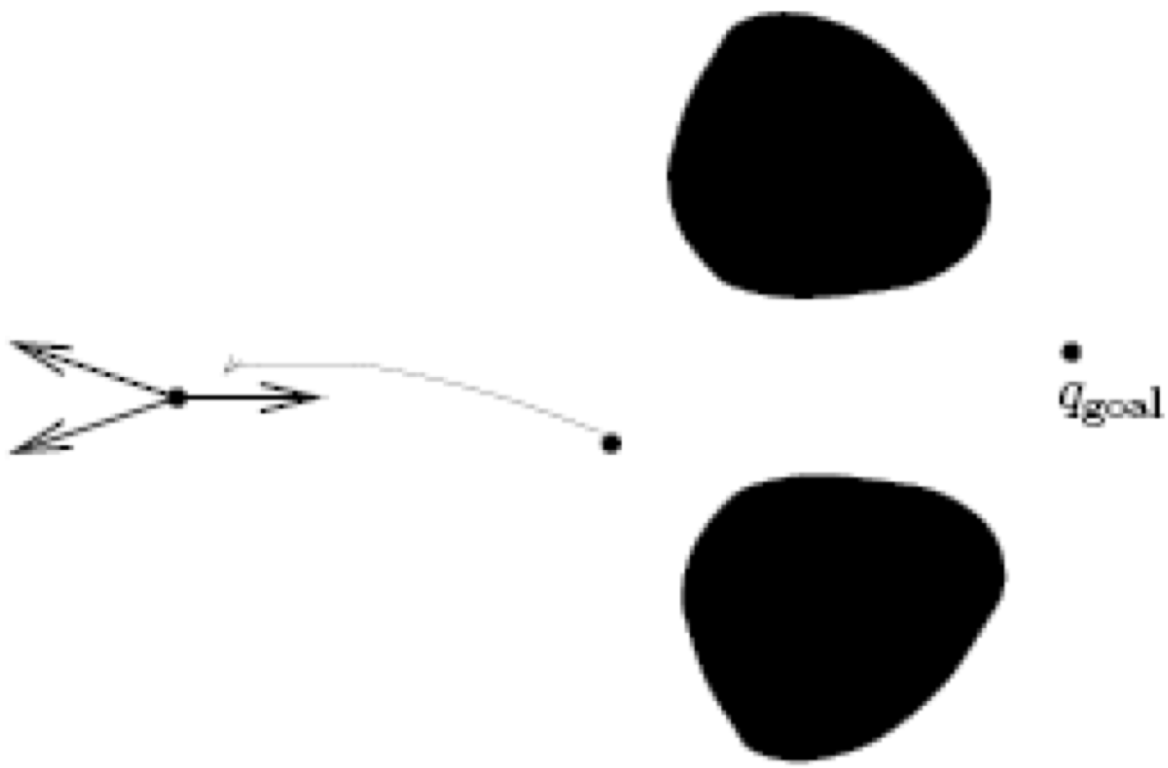
Local Minima

- How to know that there is only a single (global) minimum to guarantee that gradient descent will find a path to $q_{\text{goal.}}$



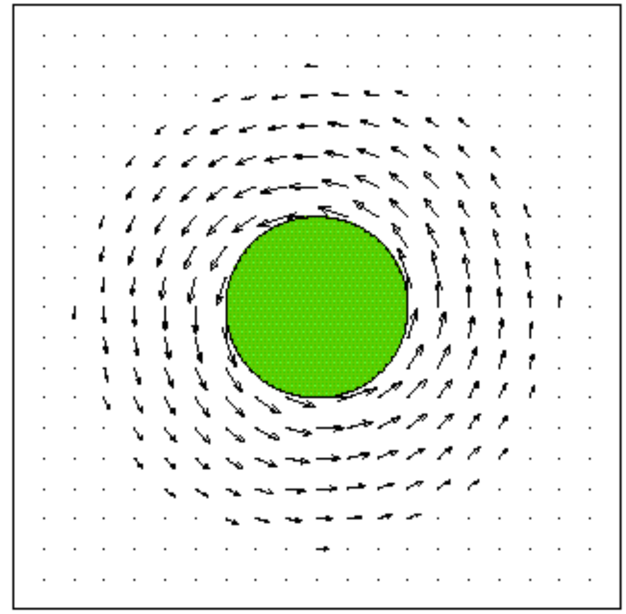
Local Minima

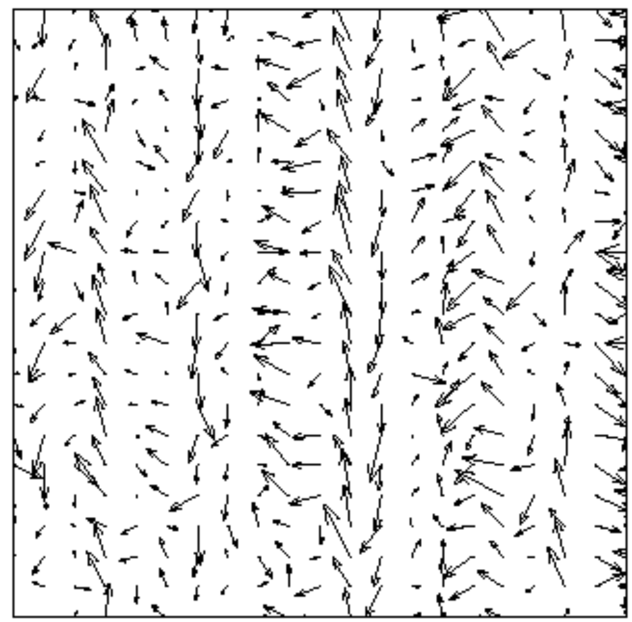
- Local minima may occur in multiple obstacle configurations.
- In order to avoid local minima random walks may be added when stuck in a specific location.



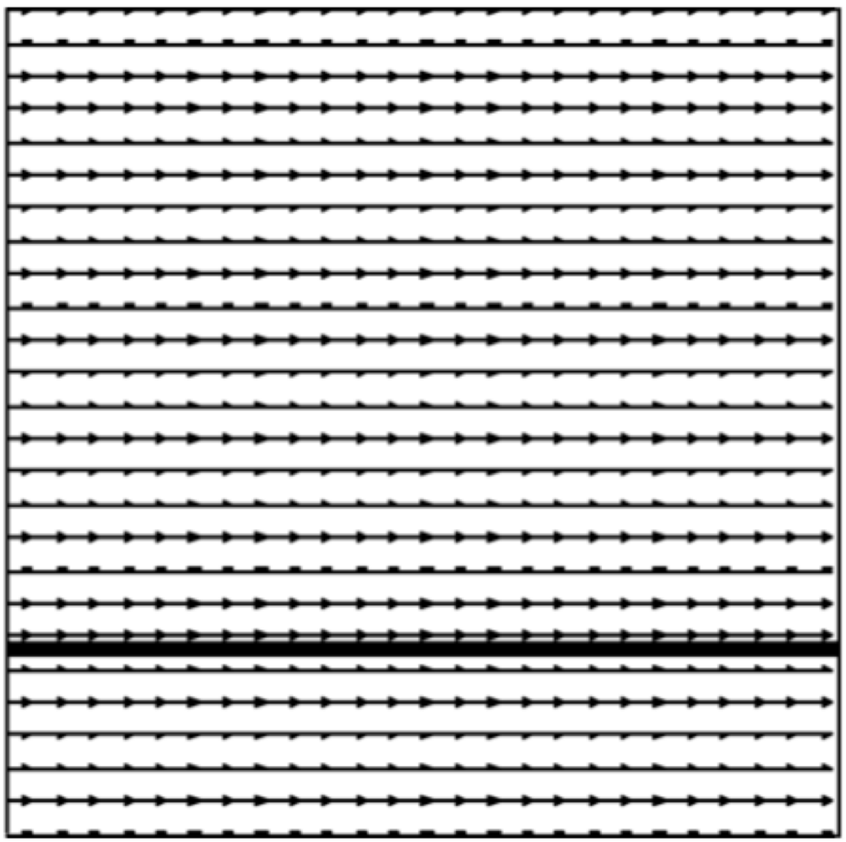
Rotational and Random Fields

- Adding a *rotational field* around obstacles
  - Breaks symmetry
  - Avoids some local minima
  - Guides robot around groups of obstacles
- A *random field* gets the robot unstuck.
  - Avoids some local minima.

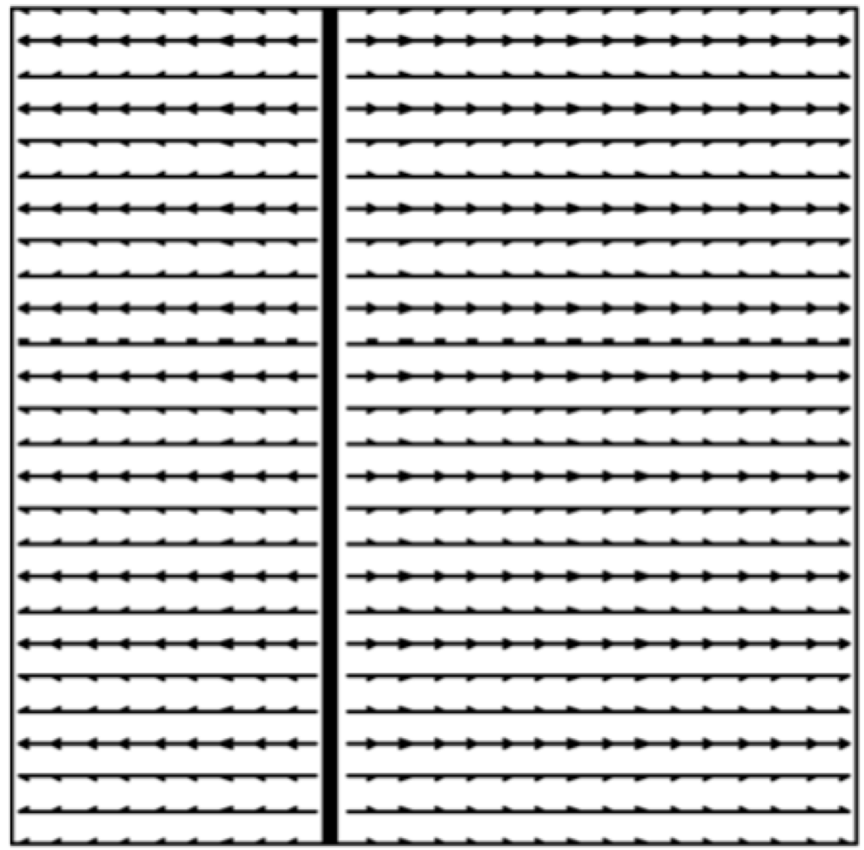




Potential Field



Uniform

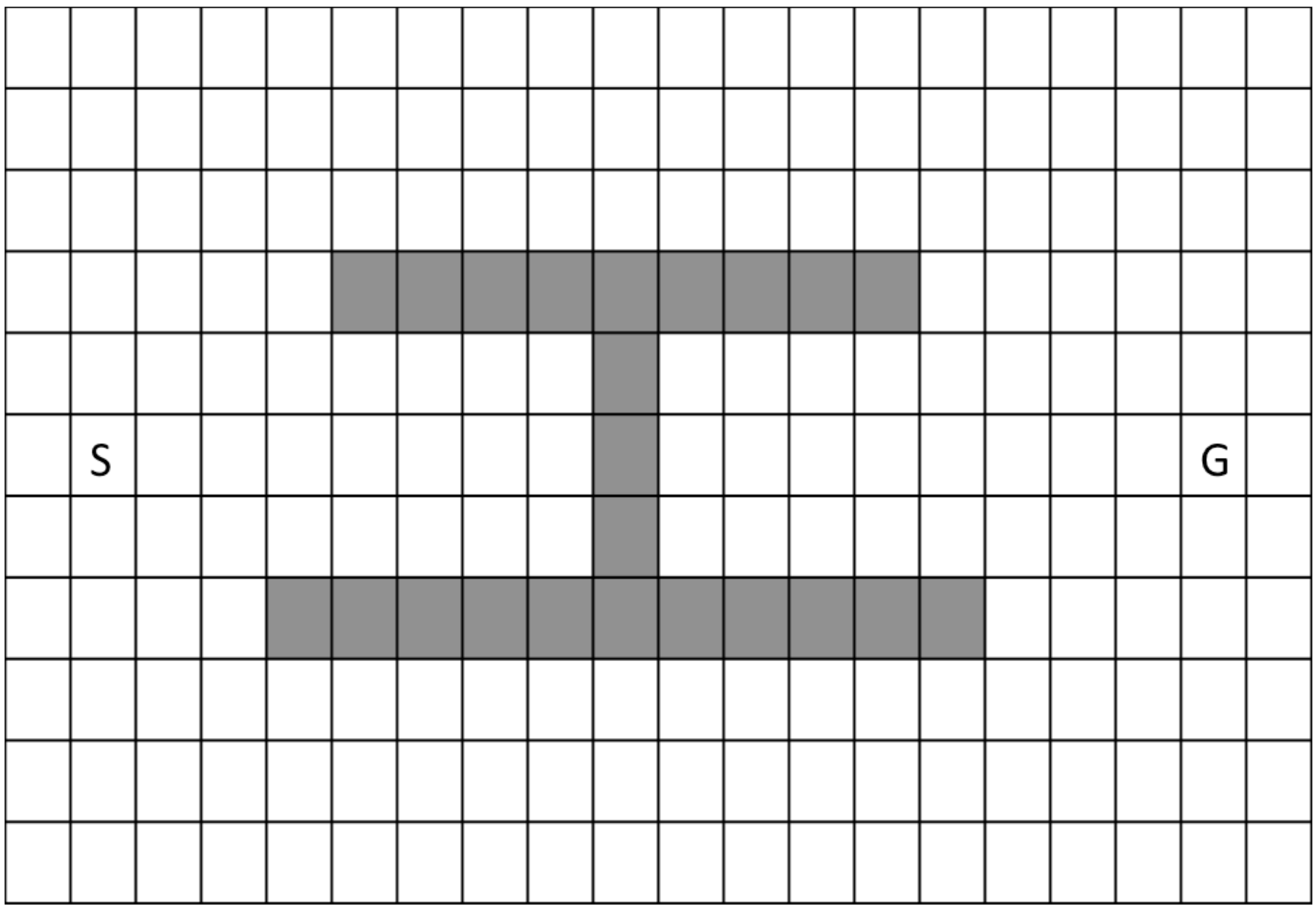


Perpendicular

Potential Functions

Spring 2017

Apply the Potential Function algorithm to the scene below to show the path from start “S” to goal “G” where darker elements correspond to obstacles. Use 8-point connectivity. Show repulsive fields that apply 1 grid cell away from obstacles, and attractive fields for the rest of the grid. Show final motion path from start to goal



Potential Functions

Fall 2017

Robot moves from start “S” to goal “G” using <u>8-point</u> connectivity where darker elements correspond to obstacles.

Attractive potential field is calculated as the number of cells *N* between the current robot location and “G”. Note that the force may intersect obstacles.

21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	1	1
21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	G	1
21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	1	1

Repulsive potential field is calculated as *N*=10 (orientation away from obstacles) using the 8-point connectivity and only applies 1 grid cell away from obstacles.

			10	10	10																	
			10	10	10																	
			10	10	10																	

Combined attractive and repulsive potential fields

21	20	19	28	27	26	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	1	1
21	20	19	28	27	26	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	G	1
21	20	19	28	27	26	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	1	1

Potential Functions

Fall 2017

Grid cell numerical values according to the attractive potential fields.

21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	6	6	6	6	6	6	6
21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	5	5	5	5	5	5
21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	4	4	4	4	4
21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	3	3	3	3
21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	2	2	2
21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	1	1
21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	G	1
21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	1	1
21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	2	2	2
21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	3	3	3	3
21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	4	4	4	4	4
21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	5	5	5	5	5	5

Potential Functions

Fall 2017

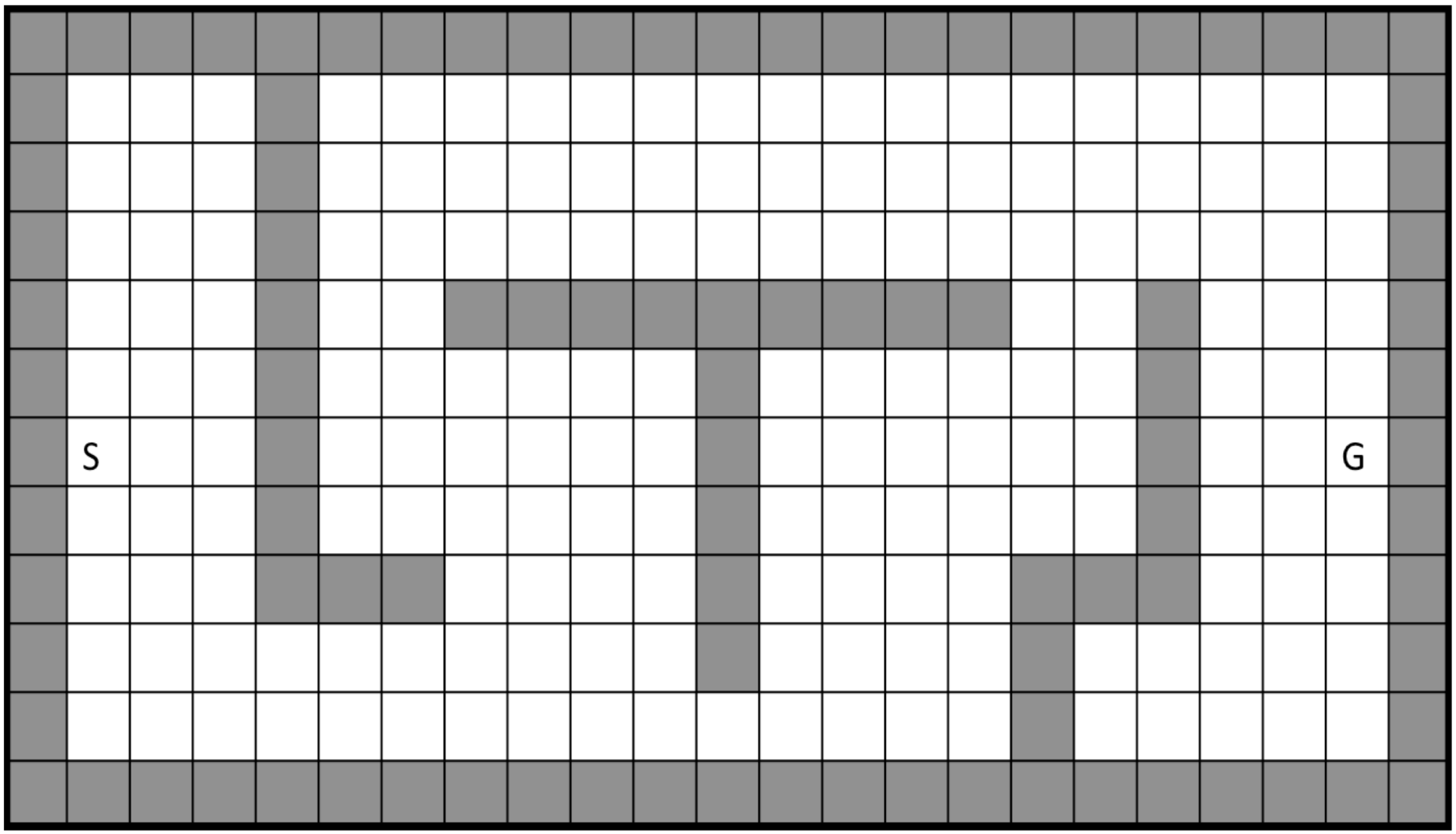
Grid cell according to the repulsive potential fields.

10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
10	10		10	10	10																10	10
10	10		10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10		10	10
10	10		10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10		10	10
10	10		10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10		10	10
10	10		10	10	10					10	10	10					10	10	10		10	10
10	10		10	10	10	10	10			10	10	10			10	10	10	10	10		10	10
10	10		10	10	10	10	10			10	10	10			10	10	10	10	10		10	10
10	10		10	10	10	10	10			10	10	10			10	10	10	10	10		10	10
10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10

Potential Functions

Fall 2017

1. Show the grid cell numerical values from “S” to “G” according to the combined attractive and repulsive potential fields.
2. Highlight the shortest path from “S” to “G” (if there is one).



Potential Functions

Fall 2017

1. Show the grid cell numerical values from “S” to “G” according to the combined attractive and repulsive potential fields.
2. Highlight the shortest path from “S” to “G” (if there is one).

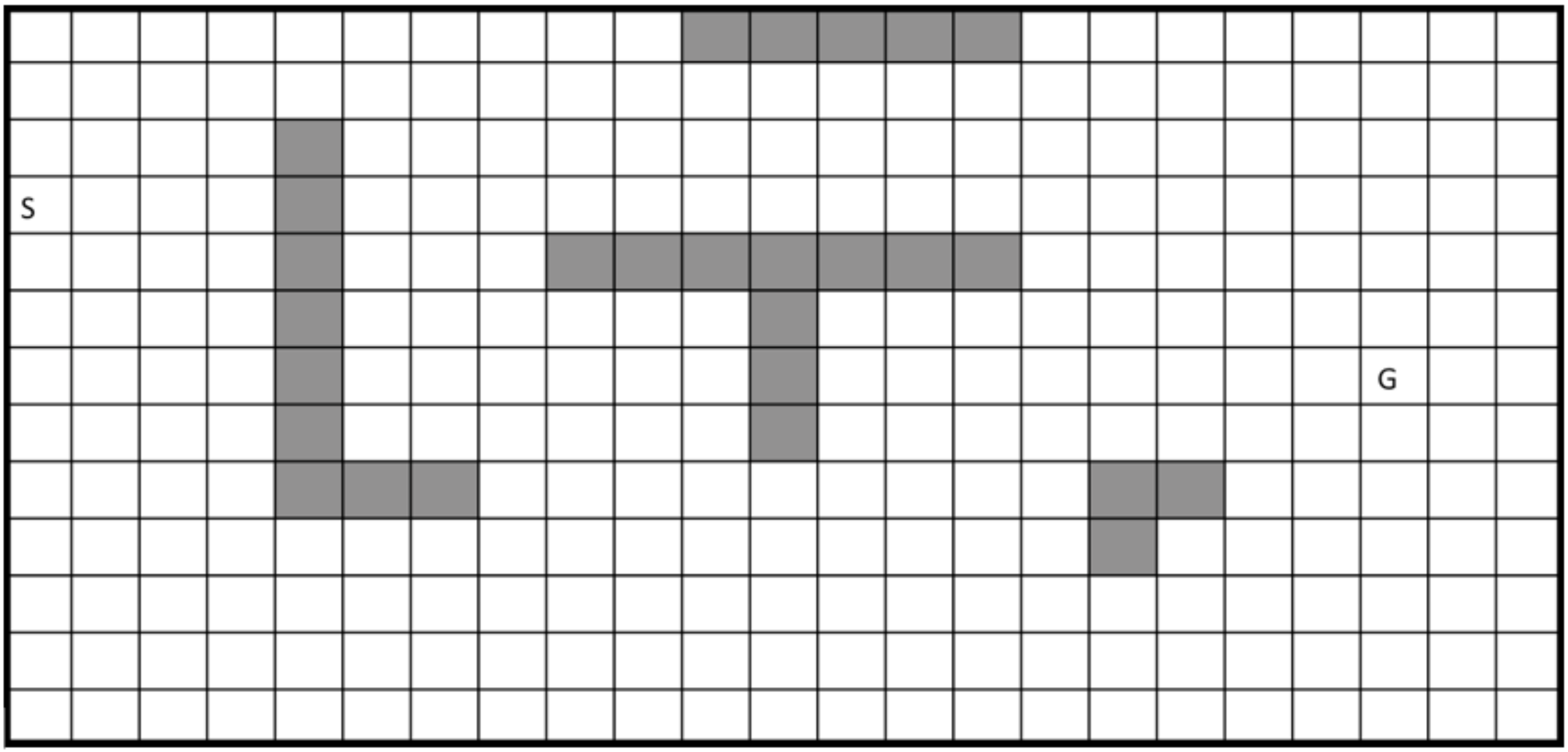
31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	16	16	16	16	16	16	16
31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	15	15	15	15	15	15
31	30	19	28	27	26	15	14	13	12	11	10	9	8	7	6	5	4	4	4	4	14	14
31	30	19	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	13	3	13	13
31	30	19	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	2	12	12
31	30	19	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	11	1	11	11
31	S	19	28	27	26	15	14	13	12	21	20	19	8	7	6	5	14	13	12	1	G	11
31	30	19	28	27	26	25	24	13	12	21	20	19	8	7	16	15	14	13	12	1	11	11
31	30	19	28	27	26	25	24	13	12	21	20	19	8	7	16	15	14	13	12	2	12	12
31	30	19	28	27	26	25	24	13	12	21	20	19	8	7	16	15	14	13	13	3	13	13
31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	14	14	14	14	14
31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	15	15	15	15	15	15

ROBOT GETS STUCK

Potential Functions

Spring 2018

1. Show the grid cell numerical values from “S” to “G” according to the combined attractive and repulsive potential fields.
2. Highlight the shortest path from “S” to “G”.
3. Can the robot get stuck?



Potential Functions

Spring 2018

1. Show the grid cell numerical values from “S” to “G” according to the combined attractive and repulsive potential fields (see example above).
2. Highlight the shortest path from “S” to “G”.
3. Can the robot get stuck?

20	19	18	17	16	15	14	13	12	11	20	19	18	17	16	16	6	6	6	6	6	6	6
20	19	18	27	26	25	14	13	12	11	20	19	18	17	16	15	5	5	5	5	5	5	5
20	19	18	27	26	25	14	13	12	11	10	9	8	7	6	5	4	4	4	4	4	4	4
S	19	18	27	26	25	14	23	22	21	20	19	18	17	16	15	4	3	3	3	3	3	3
20	19	18	27	26	25	14	23	22	21	20	19	18	17	16	15	4	3	2	2	2	2	2
20	19	18	27	26	25	14	23	22	21	20	19	18	17	16	15	4	3	2	1	1	1	2
20	19	18	27	26	25	14	13	12	11	20	19	18	7	6	5	4	3	2	1	G	1	2
20	19	18	27	26	25	24	23	12	11	20	19	18	7	6	15	14	13	12	1	1	1	2
20	19	18	27	26	25	24	23	12	11	20	19	18	7	6	15	14	13	12	2	2	2	2
20	19	18	27	26	25	24	23	12	11	10	9	8	7	6	15	14	13	13	3	3	3	3
20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	15	14	14	4	4	4	4	4
20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	5	5	5	5	5	5	5
20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	6	6	6	6	6	6	6	6

Potential Functions

Spring 2018

1. Show the grid cell numerical values from “S” to “G” according to the combined attractive and repulsive potential fields.
2. Highlight the shortest path from “S” to “G”.
3. Can the robot get stuck?

20	19	18	17	16	15	14	13	12	11	20	19	18	17	16	16	6	6	6	6	6	6	6
20	19	18	27	26	25	14	13	12	11	20	19	18	17	16	15	5	5	5	5	5	5	5
20	19	18	27	26	25	14	13	12	11	10	9	8	7	6	5	4	4	4	4	4	4	4
S	19	18	27	26	25	14	23	22	21	20	19	18	17	16	15	4	3	3	3	3	3	3
20	19	18	27	26	25	14	23	22	21	20	19	18	17	16	15	4	3	2	2	2	2	2
20	19	18	27	26	25	14	23	22	21	20	19	18	17	16	15	4	3	2	1	1	1	2
20	19	18	27	26	25	14	13	12	11	20	19	18	7	6	5	4	3	2	1	G	1	2
20	19	18	27	26	25	24	23	12	11	20	19	18	7	6	15	14	13	12	1	1	1	2
20	19	18	27	26	25	24	23	12	11	20	19	18	7	6	15	14	13	12	2	2	2	2
20	19	18	27	26	25	24	23	12	11	10	9	8	7	6	15	14	13	13	3	3	3	3
20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	15	14	14	4	4	4	4	4
20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	5	5	5	5	5	5	5
20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	6	6	6	6	6	6	6	6

- It can get stuck if following a different path.

Potential Functions

Robot moves from start "S" to goal "G" using 8-point connectivity where darker elements correspond to obstacles.

21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	2	2	2
21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	1	1
21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	G	1
21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	1	1
21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	2	2	2

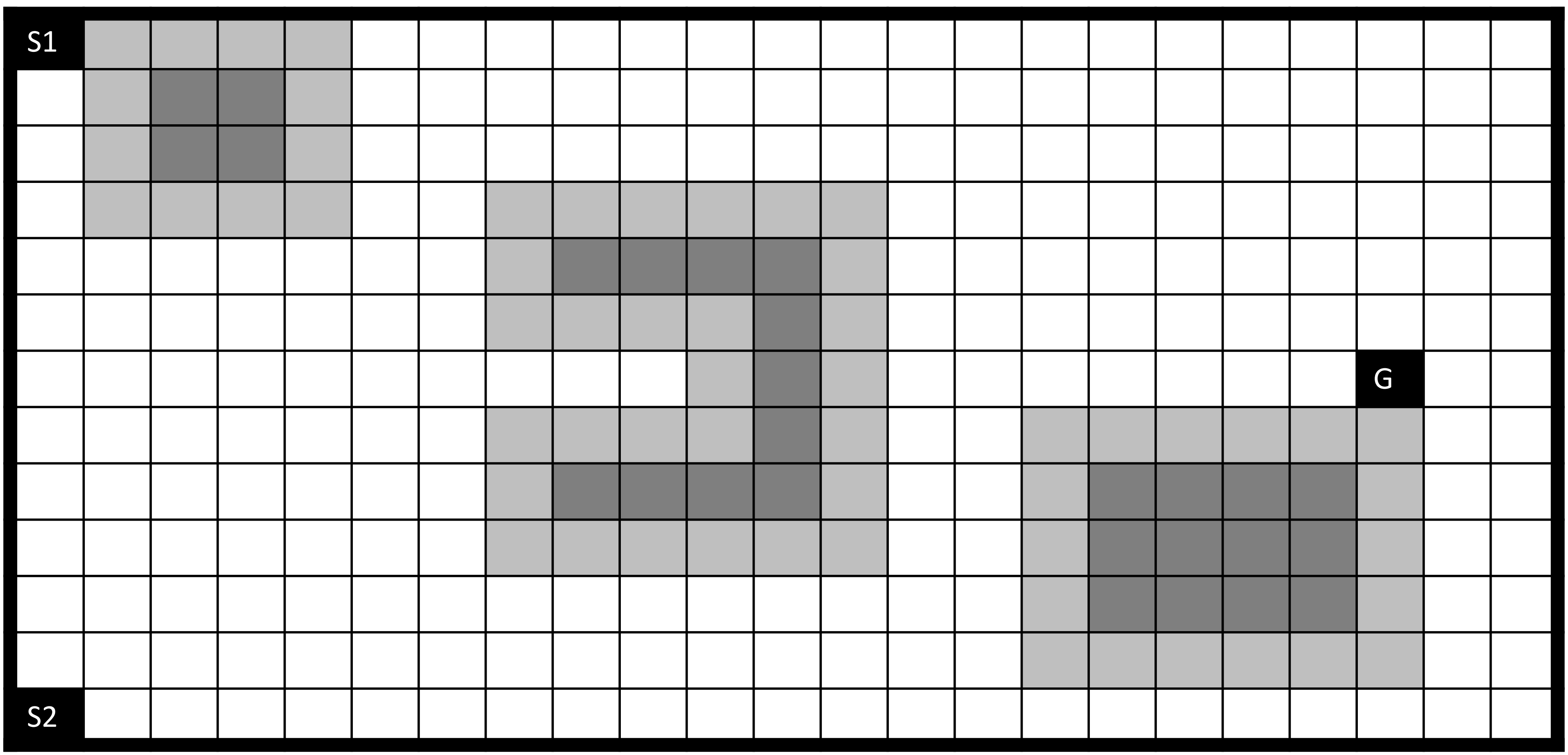
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	40	40	40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	40	80	40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	40	40	40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	2	2	2
21	20	19	58	57	56	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	1	1
21	20	19	58	97	56	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	G	1
21	20	19	58	57	56	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	1	1
21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	2	2	2

Potential Functions

Fall 2018

Consider the map below where darkest elements correspond to obstacles and lighter elements correspond to "1-cell" obstacle repulsive field.
Using the Potential Function algorithm, show in the diagram, if it exists, a path from "S1" to "G". If there is no such path, then show at least one path where the robot gets stuck and is not able to reach "G".



Potential Functions

Fall 2018

Using the Potential Function algorithm, show in the diagram, if it exists, a path from "S1" to "G". <u>If there is no such path</u>, then show at least one path where the robot gets stuck and is not able to reach "G".

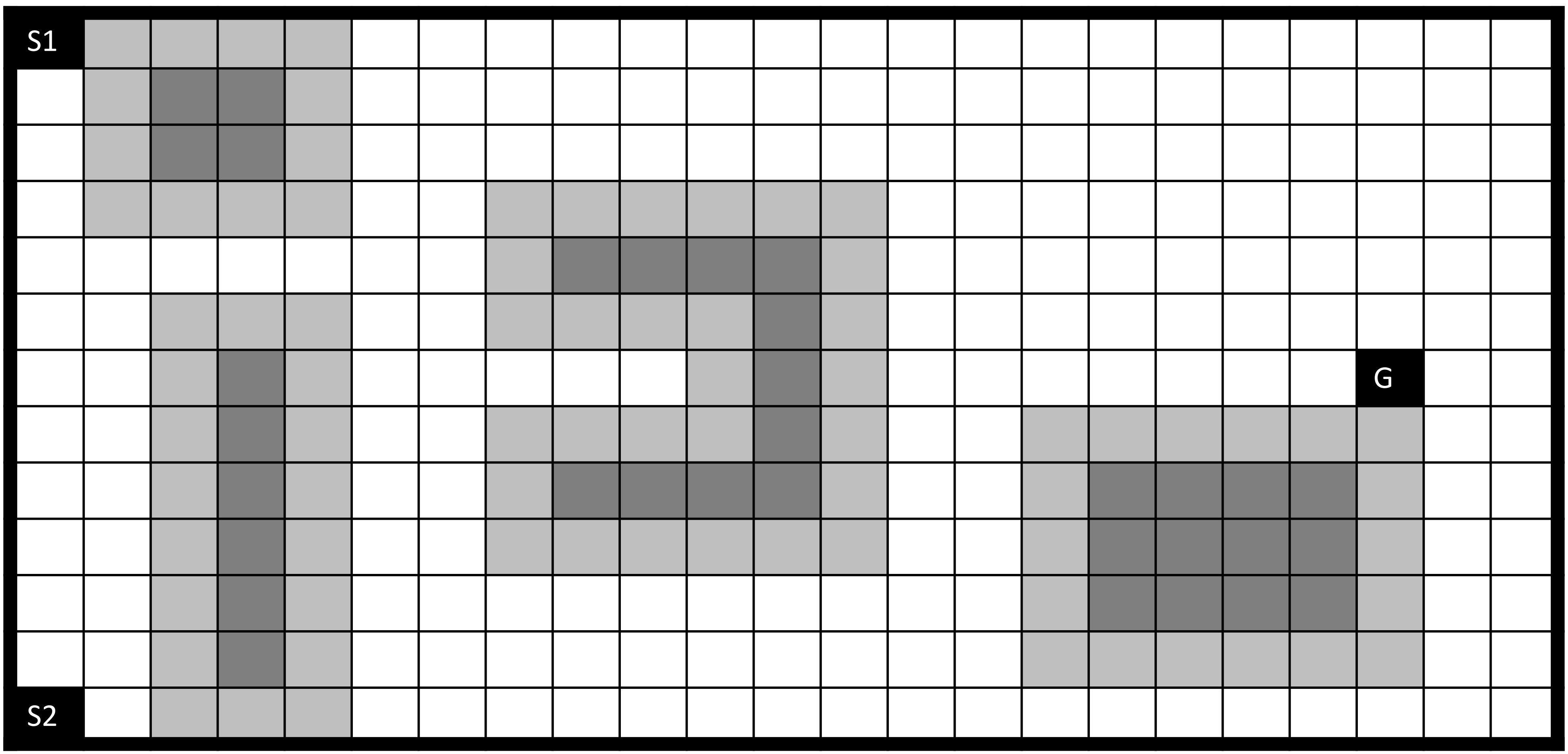
S1	59	58	57	56	15	14	13	12	11	10	9	8	7	6	6	6	6	6	6	6	6	6
20	59	98	97	56	15	14	13	12	11	10	9	8	7	6	5	5	5	5	5	5	5	5
20	59	98	97	56	15	14	13	12	11	10	9	8	7	6	5	4	4	4	4	4	4	4
20	59	58	57	56	15	14	53	52	51	50	49	48	7	6	5	4	3	3	3	3	3	3
20	19	18	17	16	15	14	53	92	91	90	89	48	7	6	5	4	3	2	2	2	2	2
20	19	18	17	16	15	14	53	52	51	50	89	48	7	6	5	4	3	2	1	1	1	2
20	19	18	17	16	15	14	13	12	11	50	89	48	7	6	5	4	3	2	1	G	1	2
20	19	18	17	16	15	14	53	52	51	50	89	48	7	6	45	44	43	42	41	41	1	2
20	19	18	17	16	15	14	53	92	91	90	89	48	7	6	45	84	83	82	82	42	2	2
20	19	18	17	16	15	14	53	52	51	50	49	48	7	6	45	84	83	83	83	43	3	3
20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	45	84	84	84	84	44	4	4
20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	45	45	45	45	85	45	5	5
S2	19	18	17	16	15	14	13	12	11	10	9	8	7	6	6	6	6	6	6	6	6	6

It's not possible to reach "G" from either "S1" or "S2"

Potential Functions

Spring 2019

Using the Potential Function algorithm, show in the diagram, if it exists, a path from "S1" to "G". <u>If there is no such path</u>, then show at least one path where the robot gets stuck and is not able to reach "G".



Potential Functions

Spring 2019

Using the Potential Function algorithm, show in the diagram, if it exists, a path from "S1" to "G". <u>If there is no such path</u>, then show at least one path where the robot gets stuck and is not able to reach "G".

S1	59	58	57	56	15	14	13	12	11	10	9	8	7	6	6	6	6	6	6	6	6	6
20	59	98	97	56	15	14	13	12	11	10	9	8	7	6	5	5	5	5	5	5	5	5
20	59	98	97	56	15	14	13	12	11	10	9	8	7	6	5	4	4	4	4	4	4	4
20	59	58	57	56	15	14	53	52	51	50	49	48	7	6	5	4	3	3	3	3	3	3
20	19	18	17	16	15	14	53	92	91	90	89	48	7	6	5	4	3	2	2	2	2	2
20	19	18	17	16	15	14	53	52	51	50	89	48	7	6	5	4	3	2	1	1	1	2
20	19	58	57	56	15	14	13	12	11	50	89	48	7	6	5	4	3	2	1	G	1	2
20	19	58	97	56	15	14	53	52	51	50	89	48	7	6	45	44	43	42	41	41	1	2
20	19	58	97	56	15	14	53	92	91	90	89	48	7	6	45	84	83	82	82	42	2	2
20	19	58	97	56	15	14	53	52	51	50	49	48	7	6	45	84	83	83	83	43	3	3
20	19	58	97	56	15	14	13	12	11	10	9	8	7	6	45	84	84	84	84	44	4	4
20	19	58	97	56	15	14	13	12	11	10	9	8	7	6	45	45	45	45	85	45	5	5
S2	19	58	57	56	15	14	13	12	11	10	9	8	7	6	6	6	6	6	6	6	6	6

It's not possible to reach "G" from either "S1" or "S2"

Potential Functions

Spring 2019

Consider the map below where darkest elements correspond to obstacles and lighter elements correspond to “1-cell” obstacle repulsive field.
<u>NOTE: In this example, potential field attraction values have been arbitrarily entered.</u>
Using the Potential Function algorithm, show in the diagram, if it exists, a path from “S2” to “G”. <u>If there is no such path</u>, then show at least one path where the robot gets stuck and is not able to reach “G”.

S1	40	40	40	40	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30
28	40	80	80	40	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20
26	40	80	80	40	15	14	13	12	11	10	9	8	10	6	10	10	10	10	10	10	10	10
24	40	40	40	40	15	14	40	40	40	40	40	40	10	6	8	8	8	3	3	3	3	3
22	20	18	17	16	15	14	40	80	80	80	80	40	10	6	5	8	8	2	2	2	2	2
20	19	18	17	16	15	14	40	40	40	40	80	40	10	6	5	4	8	2	1	1	1	2
20	19	40	40	40	15	14	13	12	11	40	80	40	10	6	5	4	3	2	1	G	1	2
20	20	40	80	40	15	14	40	40	40	40	80	40	10	6	40	40	40	40	40	40	1	2
22	22	40	80	40	15	14	40	80	80	80	80	40	10	6	40	80	80	80	80	40	2	2
24	24	40	80	40	15	14	40	40	40	40	40	40	10	6	40	80	80	80	80	40	3	3
26	26	40	80	40	15	14	13	12	11	10	9	8	7	6	40	80	80	80	80	40	4	4
28	28	40	80	40	15	14	13	12	11	10	9	8	7	6	40	40	40	40	40	40	5	5
S2	30	40	40	40	15	14	13	12	11	10	9	8	7	6	6	6	6	6	6	6	6	6

Potential Functions

Spring 2019

Consider the map below where darkest elements correspond to obstacles and lighter elements correspond to "1-cell" obstacle repulsive field.
Using the Potential Function algorithm, show in the diagram, if it exists, a path from "S2" to "G". <u>If there is no such path</u>, then show at least one path where the robot gets stuck and is not able to reach "G".

S1	40	40	40	40	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30
28	40	80	80	40	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20
26	40	80	80	40	15	14	13	12	11	10	9	8	10	6	10	10	10	10	10	10	10	10
24	40	40	40	40	15	14	40	40	40	40	40	40	10	6	8	8	8	3	3	3	3	3
22	20	18	17	16	15	14	40	80	80	80	80	40	10	6	5	8	8	2	2	2	2	2
20	19	18	17	16	15	14	40	40	40	40	80	40	10	6	5	4	8	2	1	1	1	2
20	19	40	40	40	15	14	13	12	11	40	80	40	10	6	5	4	3	2	1	G	1	2
20	20	40	80	40	15	14	40	40	40	40	80	40	10	6	40	40	40	40	40	40	1	2
22	22	40	80	40	15	14	40	80	80	80	80	40	10	6	40	80	80	80	80	40	2	2
24	24	40	80	40	15	14	40	40	40	40	40	40	10	6	40	80	80	80	80	40	3	3
26	26	40	80	40	15	14	13	12	11	10	9	8	7	6	40	80	80	80	80	40	4	4
28	28	40	80	40	15	14	13	12	11	10	9	8	7	6	40	40	40	40	40	40	5	5
S2	30	40	40	40	15	14	13	12	11	10	9	8	7	6	6	6	6	6	6	6	6	6

It's not possible to reach "G" from "S2"