Multiplication

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} * \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} x_1y_1x_4y_4 + 2x_3y_3x_2y_2 \\ x_2y_2x_4y_4 \\ x_1y_3y_2x_4 + y_1x_3x_2y_4 \\ x_2y_2x_4y_4 \end{bmatrix}$$

Scalar Multiplication

$$k * x = \begin{bmatrix} kx_1 \\ x_2 \\ kx_3 \\ x_4 \end{bmatrix} = x * k$$

Scalar Addition
Scalar Subtraction

$$x \pm k = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \pm \begin{bmatrix} k \\ 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} x_1 \pm kx_2 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \text{ and } k \pm x = \pm(x \pm k)$$

Inversion

$$x^{-1} = \begin{pmatrix} x_1x_2x_4^2 \\ \alpha \\ -x_2^2x_3x_4 \\ \alpha \end{pmatrix} \quad \text{provided that} \quad \alpha = x_1^2x_4^2 - 2x_2^2x_3^2 \neq 0$$

Division

$$x/y = x * y^{-1}$$

Scalar Division

$$x/k = x * \left(\frac{1}{k}\right) = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} * \begin{bmatrix} 1 \\ k \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} x_1 \\ kx_2 \\ x_3 \\ kx_4 \end{bmatrix} \quad k \neq 0, x_2 \neq 0, x_4 \neq 0$$

Scalar Division

$$k/x = k * x^{-1}$$

Equal?

$x = y$ if $x_1 = y_1$, $x_2 = y_2$, $x_3 = y_3$, and $x_4 = y_4$

Similar?

x is similar to y if $x_1y_2 = y_1x_2$ and $x_3y_4 = y_3x_4$

Norm of x

$\left(\frac{x_1}{x_2}\right)^2 + \left(\frac{x_3}{x_4}\right)^2$, a floating-point number measuring x

Absolute value of x

$\sqrt{\text{Norm of } x}$, a floating-point number

Less than?

$x < y$ is true if Norm of $x <$ Norm of y; false, otherwise.