

The cppad_mixed Capture Example and Speed Test

Bradley M. Bell

Applied Physics Laboratory,
Health Metrics and Evaluation,
University of Washington,
bradbell@uw.edu

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- ▶ make check, make speed, make install

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This is a `cppad_mixed` example; see [Roy04].

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- ▶ Compare Newton and quasi-Newton methods.

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- ▶ Interesting numerical AD method used to avoid overflow.

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- ▶ Compare Newton and quasi-Newton methods.
- ▶ Interesting numerical AD method used to avoid overflow.
- ▶ Optional constraints.

Data Model

Data Given Population Size and Capture Probability

- ▶ $y_{i,t}$ is the number of captures at location i and time t .

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Data Given Population Size and Capture Probability

- ▶ $y_{i,t}$ is the number of captures at location i and time t .
- ▶ N_i is the size of the population at location i .
- ▶ q_t is the probability of capture at time t .
- ▶ The conditional probability of $y_{i,t}$ given N_i and q_t is

$$\mathbf{p}(y_{i,t}|N_i, q_t) = \binom{N_i}{y_{i,t}} q_t^{y(i,t)} (1 - q_t)^{y(i,t)}$$

Population Size Given Fixed Effects

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- ▶ We use a Poisson distribution to model the probability of N_i given θ

$$\mathbf{p}(N_i|\theta) = \theta_0^{N(i)} \frac{\exp[-\theta_0]}{N_i!}$$

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$$\log[q_t/(1 - q_t)] = u_t + \theta_1$$

- ▶ The probability of capture at time t given θ and u is

$$q_t(\theta, u) = \mathbf{p}(q_t|\theta, u) = [1 + \exp(-u_t - \theta_1)]^{-1}$$

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$$\prod_{i=0}^{R-1} \left[\sum_{k=0}^{K-1} \theta_0^k \frac{\exp[-\theta_0]}{k!} \prod_{t=0}^{T-1} \binom{k}{y_{i,t}} q_t(\theta, u)^{y(i,t)} (1 - q_t(\theta, u))^{y(i,t)} \right]$$

Program Input

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- ▶ The user defined random likelihood for this example is

$$f(\theta, u) = -\log[\mathbf{p}(y|\theta, u)\mathbf{p}(u|\theta)]$$

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- ▶ There is no fixed constraint functions; i.e, $c(\theta)$.

▶

$$\hat{u}(\theta) = \operatorname{argmin} f(\theta, u) \text{ w.r.t. } u$$

- ▶ A is the random constraint matrix.
- ▶ The random constraint function is

$$0 = A\hat{u}(\theta) = \hat{u}_1(\theta) + \cdots + \hat{u}_{T-1}(\theta)$$

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- ▶ `number_times` positive integer , T 10

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- ▶ `max_population` positive integer , K 50

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- ▶ `mean_population` positive real , θ_0 5.0

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- ▶ `mean_logit_probability` real , θ_1 -0.5

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- ▶ `mean_population` positive real , θ_0 5.0
- ▶ `mean_logit_probability` real , θ_1 -0.5
- ▶ `std_logit_probability` positive real , θ_2 0.5

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- ▶ `number_locations` positive integer , R 50
- ▶ `number_times` positive integer , T 10
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- ▶ `mean_population` positive real , θ_0 5.0
- ▶ `mean_logit_probability` real , θ_1 -0.5
- ▶ `std_logit_probability` positive real , θ_2 0.5
- ▶ `quasi_fixed` true or false true

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- ▶ `number_locations` positive integer , R 50
- ▶ `number_times` positive integer , T 10
- ▶ `max_population` positive integer , K 50
- ▶ `mean_population` positive real , θ_0 5.0
- ▶ `mean_logit_probability` real , θ_1 -0.5
- ▶ `std_logit_probability` positive real , θ_2 0.5
- ▶ `quasi_fixed` true or false true
- ▶ `random_constraint` true or false false , true

Command Line Arguments

▶ random_seed	non-negative integer	0 , match previous
▶ number_fixed_samples	positive integer	1000
▶ number_locations	positive integer , R	50
▶ number_times	positive integer , T	10
▶ max_population	positive integer , K	50
▶ mean_population	positive real , θ_0	5.0
▶ mean_logit_probability	real , θ_1	-0.5
▶ std_logit_probability	positive real , θ_2	0.5
▶ quasi_fixed	true or false	true
▶ random_constraint	true or false	false , true
▶ trace_optimize_fixed	true or false	false

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▶ random_seed	non-negative integer	0 , match previous
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▶ number_times	positive integer , T	10
▶ max_population	positive integer , K	50
▶ mean_population	positive real , θ_0	5.0
▶ mean_logit_probability	real , θ_1	-0.5
▶ std_logit_probability	positive real , θ_2	0.5
▶ quasi_fixed	true or false	true
▶ random_constraint	true or false	false , true
▶ trace_optimize_fixed	true or false	false

Program Output

Seed, Memory, Timing Results

▶ actual_seed 1466520673 , 1466520673

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- ▶ actual_seed 1466520673 , 1466520673
- ▶ initialize_bytes 27,728,078 , 27,728,078

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▶ actual_seed	1466520673 , 1466520673
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▶ initialize_seconds	0.244 , 0.259

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- ▶ `std_logit_probability_std_avg` 0.12 , 0.12

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