

## Improvement of RANS models by machine learning for a bump configuration

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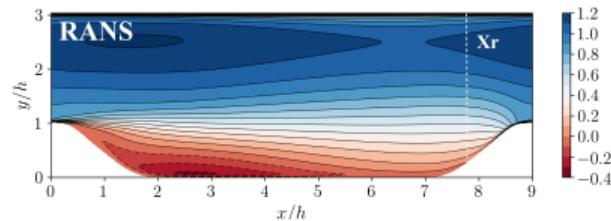
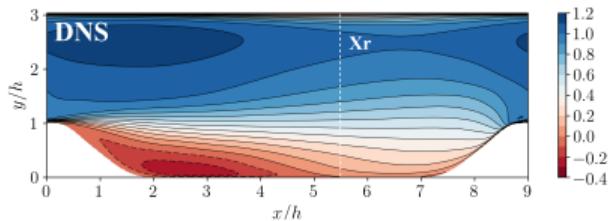
2022 Symposium on Turbulence Modeling: Roadblocks, and the Potential for Machine Learning  
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## Motivation

RANS simulations are widely used in engineering but they lack accuracy.

- flows with adverse pressure gradient, large separations, curvature, swirl, ...



Periodic hill configuration at  $Re=2800$  : mean streamwise velocity field.

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## Requirements

- High fidelity test cases, reflecting critical physics

## Recap of 2021 work

Solve the exact equations :  $\mathcal{R}_e(u_e(\mathbf{x}, t)) = 0$

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Build a NN to construct the correction term as a function  
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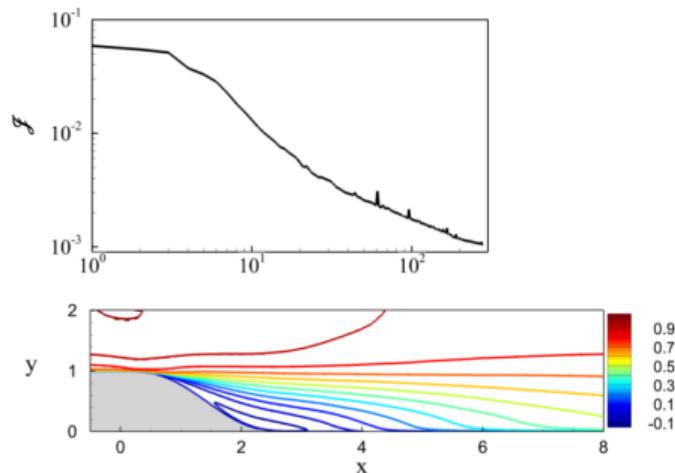
Computations are performed using the NN augmented  
turbulence model

# Data assimilation following Franceschini et al. (2020, PRF)

- Boussinesq correction :

$$\frac{\partial u_i u_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial(2\nu S_{ij})}{\partial x_j} + \frac{\partial \tau_{ij}^{SA}}{\partial x_j} + \mathbf{f}_{u_i}$$

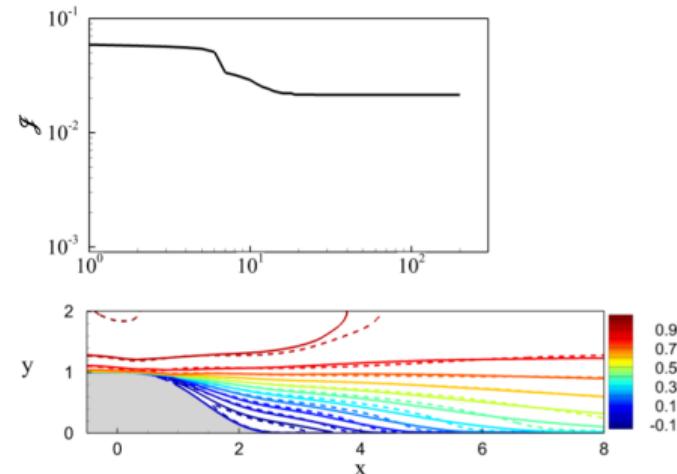
$\tilde{f}_u$ -corr. DENSE REF



- Eddy-viscosity correction :

$$u_j \frac{\partial \tilde{\nu}}{\partial x_j} = P - D + T + \mathbf{f}_{\tilde{\nu}}$$

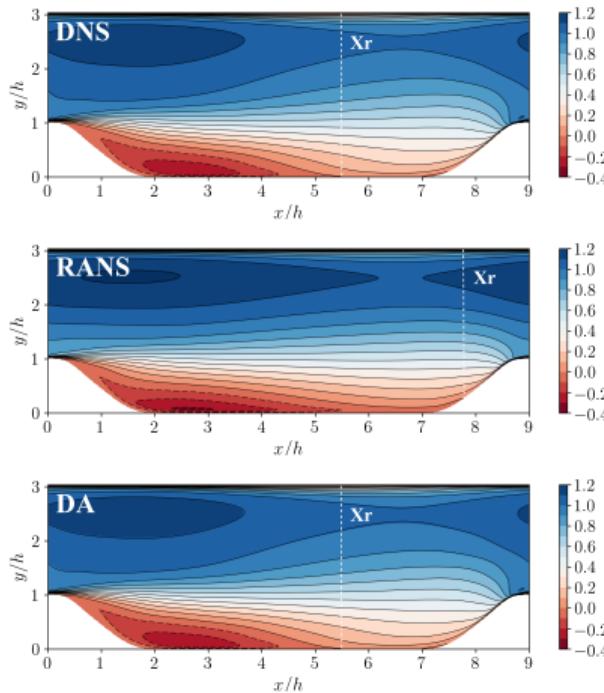
$\tilde{f}_{\tilde{\nu}}$ -corr. DENSE REF



# Data assimilation - Volpiani et al. (2021, PRF)

Periodic hill configuration ( $Re_b = 2800$ )

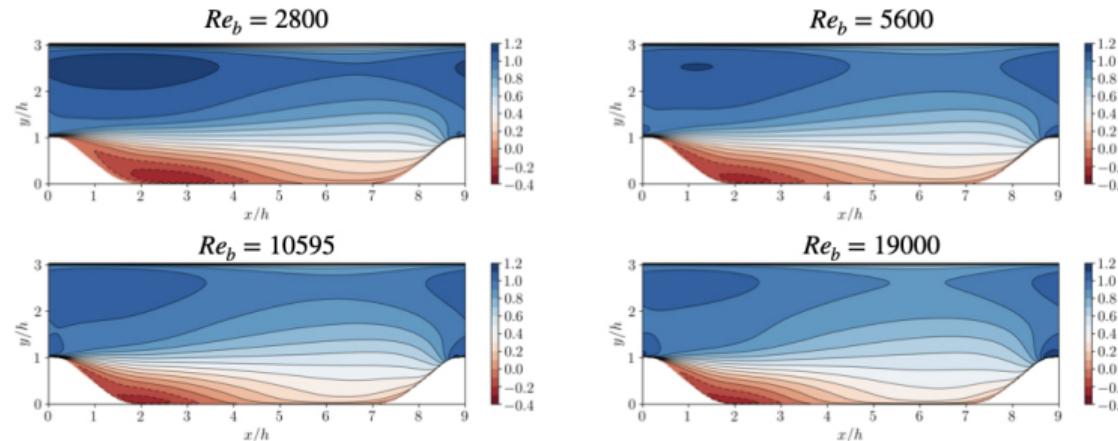
Mean streamwise velocity field



# Machine learning strategy - Volpiani et al. (2021, PRF)

*Database of training flows to predict flow past periodic hills*

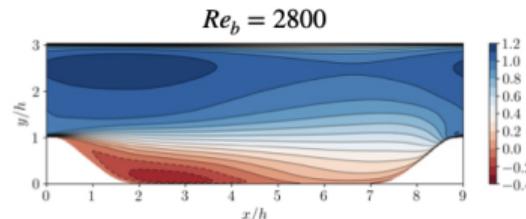
Training flow scenario	Selected cases	Fit on training data only	Fit on entire data without rotation
Scenario I	PH-2800, PH-10595, PH-19000	96.5 %	95.6 %
Scenario II	PH-2800, PH-5600, PH-10595	96.2 %	93.5 %



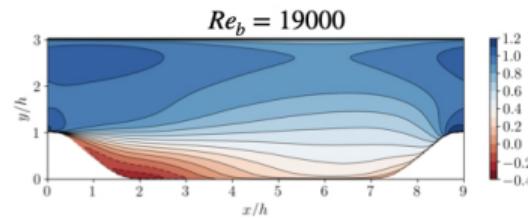
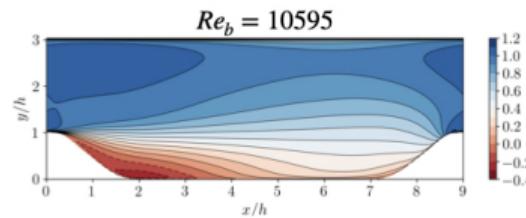
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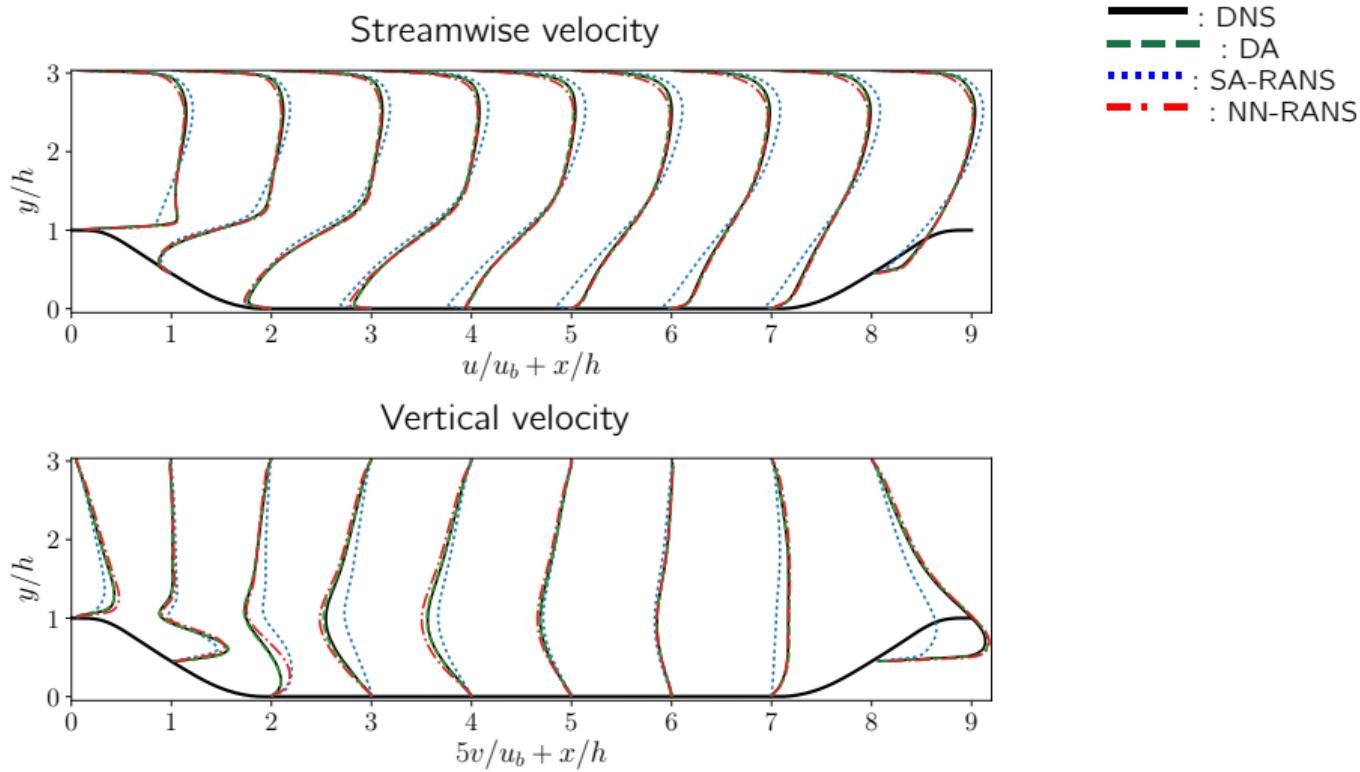


$Re_b = 5600$

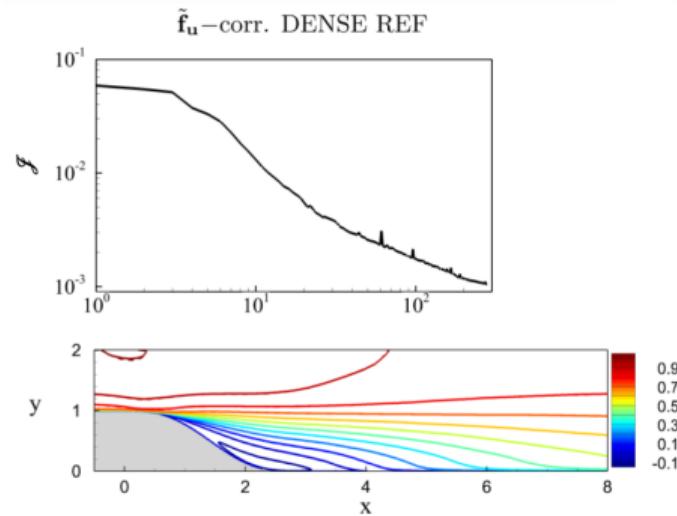


# RANS using a neural network-based model

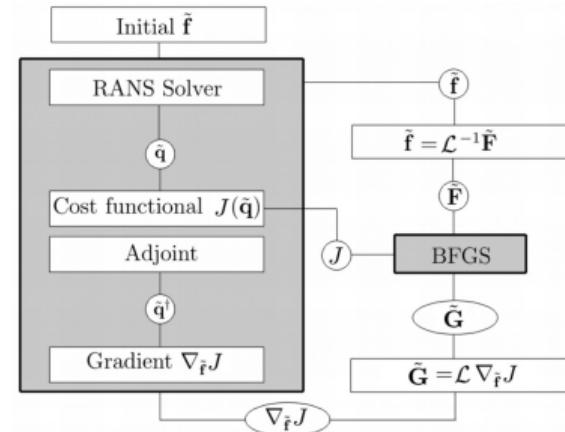
Interpolation in Reynolds number : scenario I is used to predict flow at  $Re_b = 5600$



# Data assimilation following Franceschini et al. (2020, PRF)



Assimilation of velocity measurements based on the volume force  $f_{u_i}$



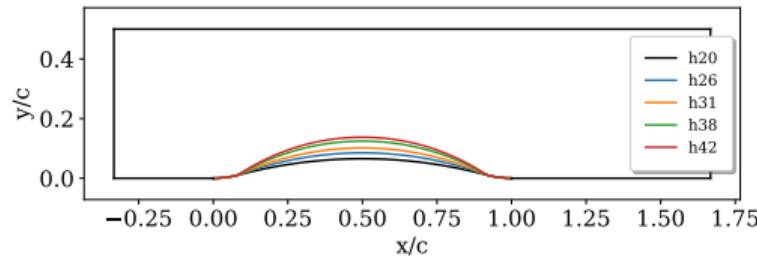
Coupling between the optimization algorithm (BFGS) and the fluid solver.

# Eddy-viscosity correction

RANS equations :

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0, \quad \frac{\partial \bar{u}_i \bar{u}_j}{\partial x_j} = -\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j} [2(\nu + \nu_t^{SA} + \Delta\nu_t) S_{ij}]$$

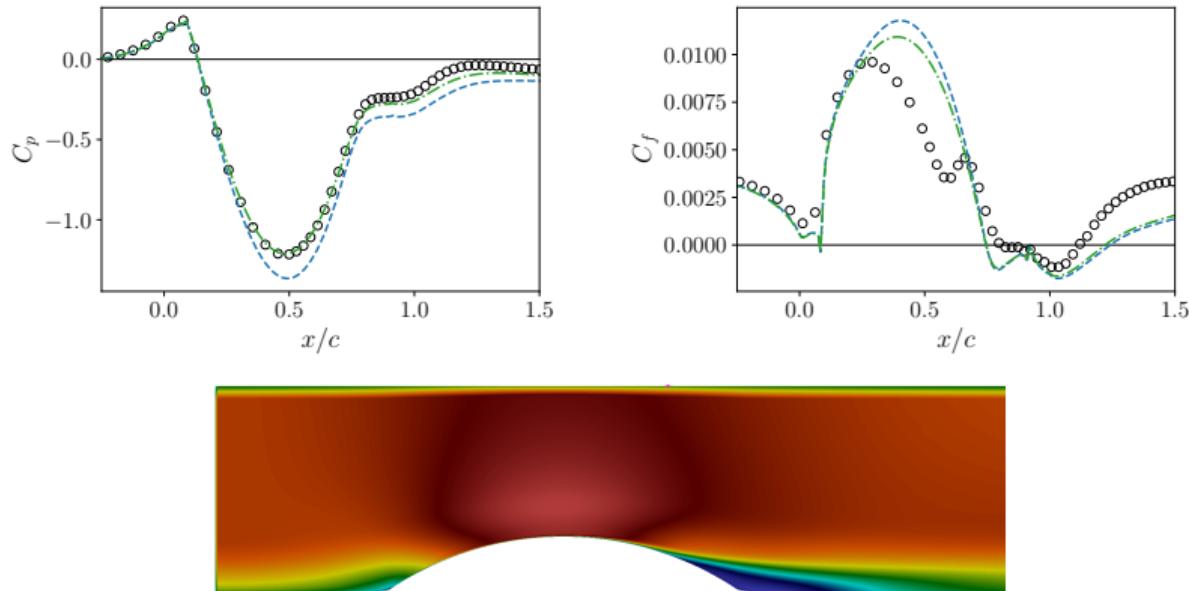
- $\nu_t^{SA}$  is determined by solving the one equation Spalart-Allmaras (SA) turbulence model
- $\Delta\nu_t$  is determined using the LES statistical data :  $\nu_t^{LES} = \frac{-\overline{u'_i u'_j} \partial_j \bar{u}_i}{2S_{ij} S_{ij}} \approx \frac{\max(0, -\overline{u'_i u'_j} \partial_j \bar{u}_i)}{\max(0, 2S_{ij} S_{ij}) + \epsilon}$



Family of bumps from Matai & Durbin (JFM, 2019).

# Baseline RANS-SA simulation for the flows over a family of bumps

Discussion on boundary conditions : [slip wall](#), as [LES \(shorter domain\)](#), LES bump h42.



Streamwise velocity for bump h42 from Matai & Durbin (JFM, 2019).

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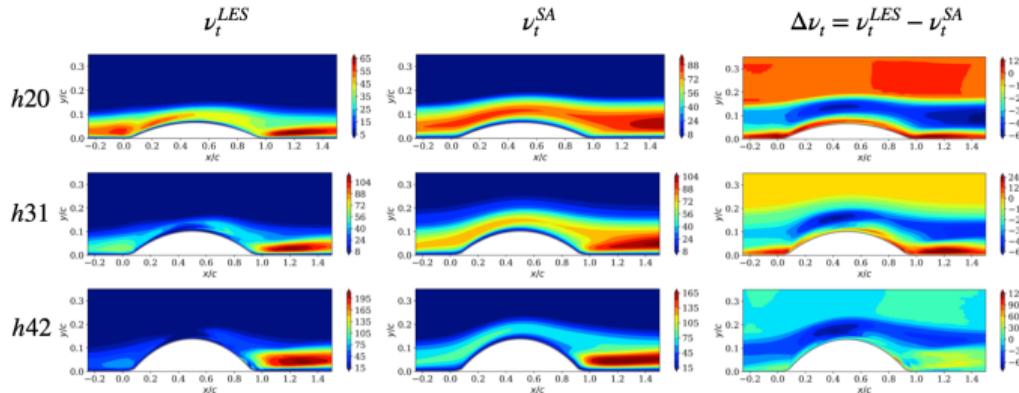
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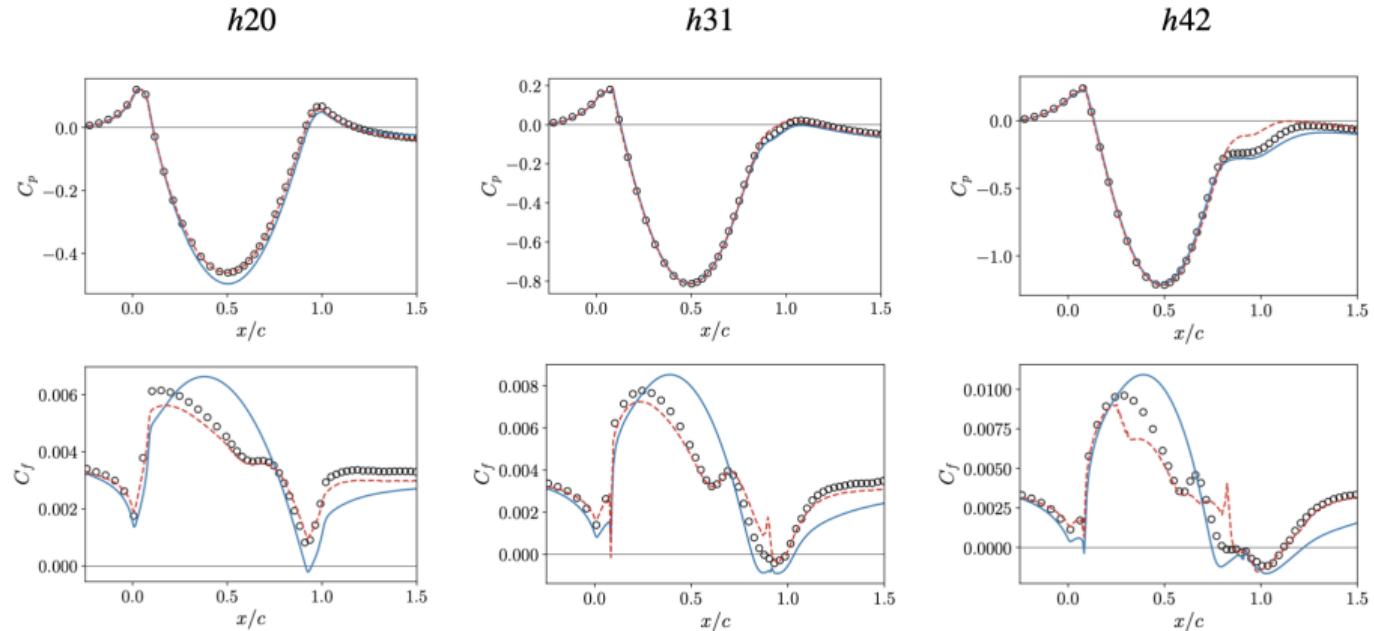
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# Eddy-viscosity correction



Corrected  $C_p$  and  $C_f$  profiles : RANS-SA, RANS- $\nu_t^{LES}$ .

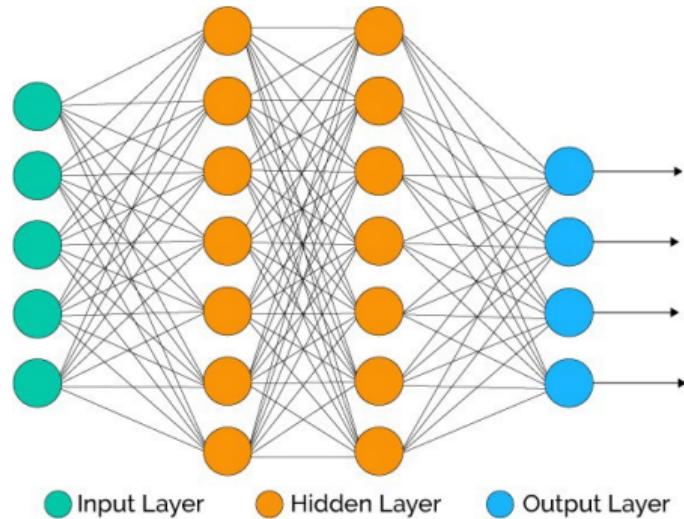
# Machine learning strategy

## Input layer :

Feature	Description	Formula
$q_1$	Q-criterion	$\frac{\ \Omega\ ^2 - \ S\ ^2}{\ \Omega\ ^2 + \ S\ ^2}$
$q_2$	Ratio of pressure normal stresses to shear stresses	$\sqrt{\frac{\partial p}{\partial x_k} \frac{\partial p}{\partial x_k}} + \frac{1}{2} \frac{\partial u^2}{\partial x_k}$
$q_3$	Gorle et al. marker	$\left\  \bar{U}_i \bar{U}_j \frac{\partial \bar{u}_i}{\partial x_j} \right\  + \sqrt{\frac{\bar{U}_i \bar{U}_j \bar{U}_l \bar{U}_k \frac{\partial \bar{u}_i}{\partial x_j} \frac{\partial \bar{u}_k}{\partial x_l}}{\bar{U}_k \frac{\partial \bar{p}}{\partial x_k}}}}$
$q_4$	Streamline pressure gradient	$\left\  \bar{U}_k \frac{\partial \bar{p}}{\partial x_k} \right\  + \sqrt{\frac{\nu_t}{\nu_t + 100\nu} \frac{\bar{U}_i \bar{U}_j \bar{U}_l \bar{U}_k \frac{\partial \bar{u}_i}{\partial x_j} \frac{\partial \bar{u}_k}{\partial x_l}}{c_{b1} \tilde{S} \tilde{\nu}}}}$
$q_5$	Viscosity ratio	$\left  c_{b1} \tilde{S} \tilde{\nu} \right  + c_{w1} f_w \left( \frac{\tilde{\nu}}{d} \right)^2$
$q_6$	SA ratio of production to destruction	$\left  c_{b1} \tilde{S} \tilde{\nu} \right  + c_{w1} f_w \left( \frac{\tilde{\nu}}{d} \right)^2$
$q_7$	SA ratio of production to diffusion	$\left  c_{b1} \tilde{S} \tilde{\nu} \right  + \frac{c_{w2}}{\sigma} \frac{\partial \bar{v}}{\partial x_k} \frac{\partial \bar{u}^2}{\partial x_k}$
$q_8$	Turbulence intensity	$k_{qcr} + \frac{1}{2} \bar{U}^2$

Hidden layer :  $4 \times 80$  neurons

Output layer :  $\Delta \nu_t$  or  $\nu_t^{LES}$



● Input Layer

● Hidden Layer

● Output Layer

# Predicting the viscosity discrepancy $\Delta\nu_t$ using NN1

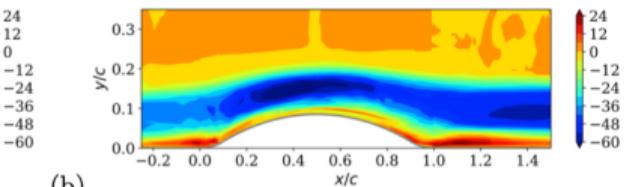
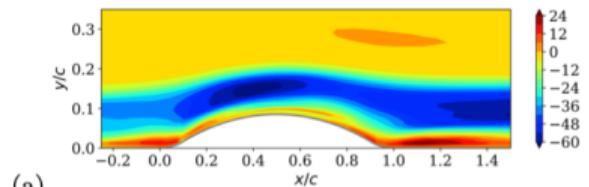
Training cases : h20, h31, h42

Validation cases : h26, h38

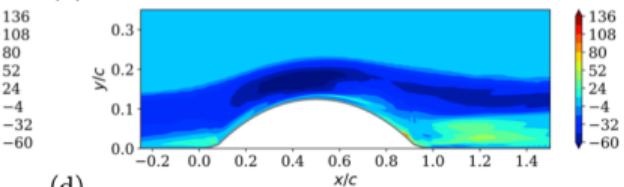
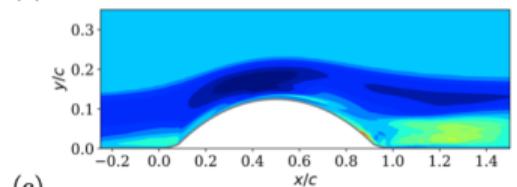
$$\Delta\nu_t$$

$$\Delta\nu_t^{NN}$$

*h26*



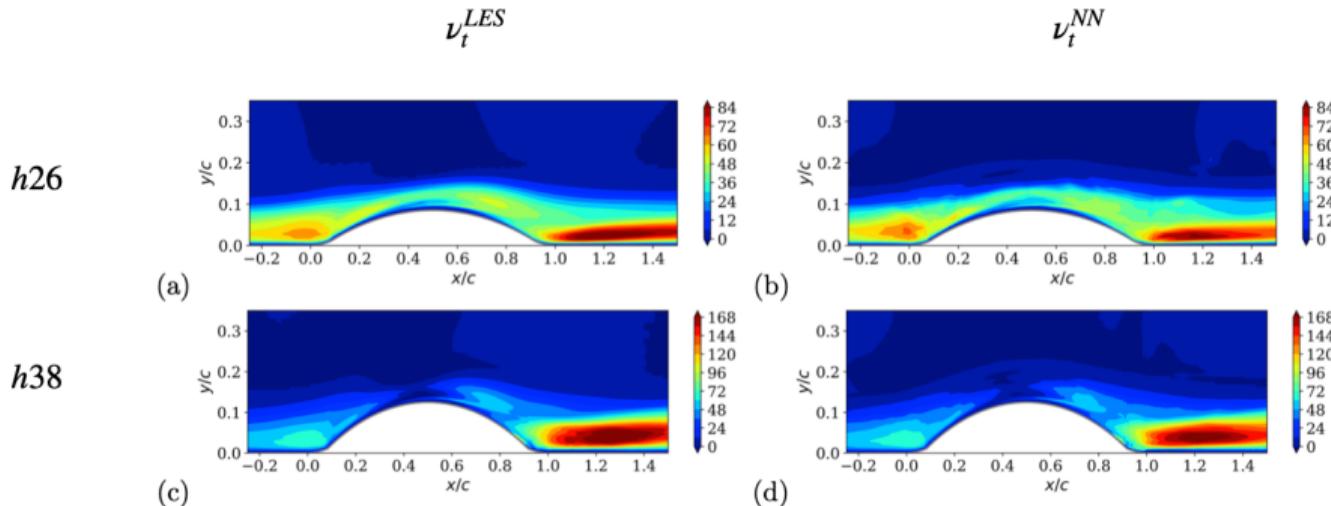
*h38*



# Predicting the viscosity discrepancy $\nu_t^{LES}$ using NN2

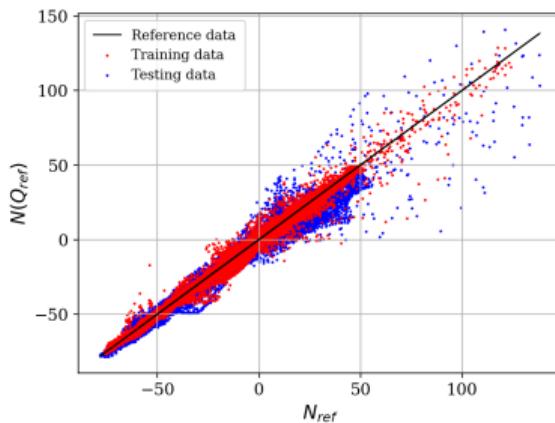
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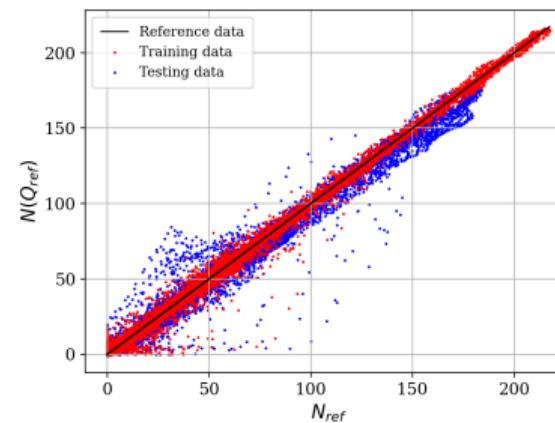


# Comparison between NN1 vs NN2

NN1 -  $\Delta\nu_t^{NN}$

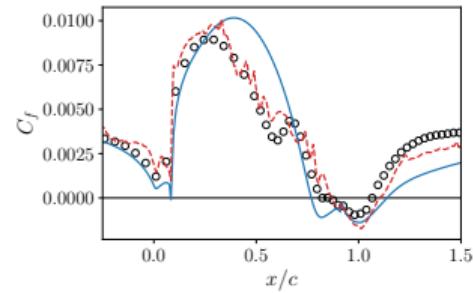
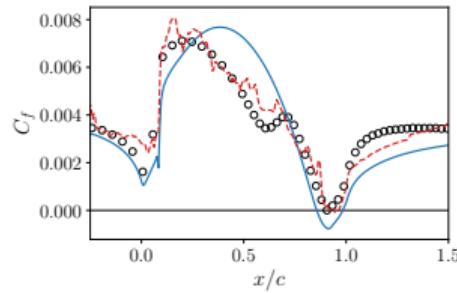


NN2 -  $\nu_t^{NN}$



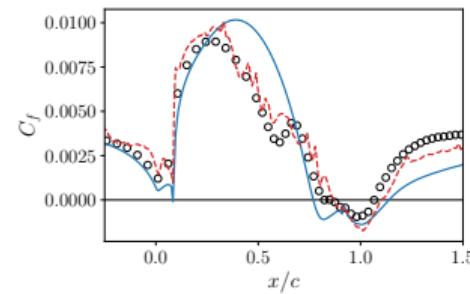
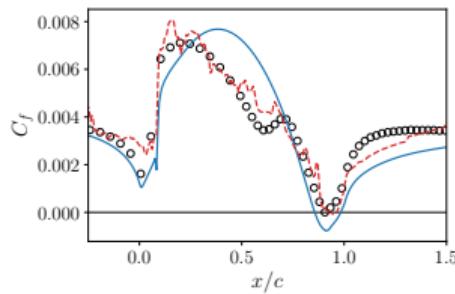
# A posteriori results using NN1 and NN2

NN1 -  $\Delta\nu_t^{NN}$

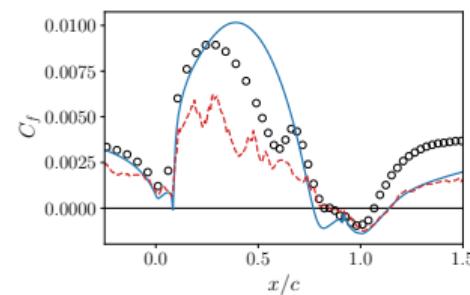
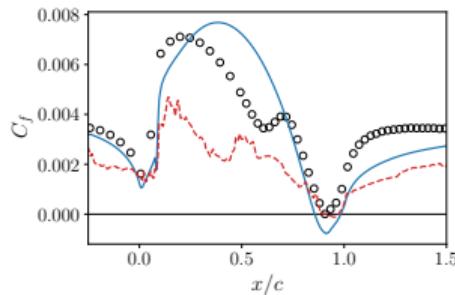


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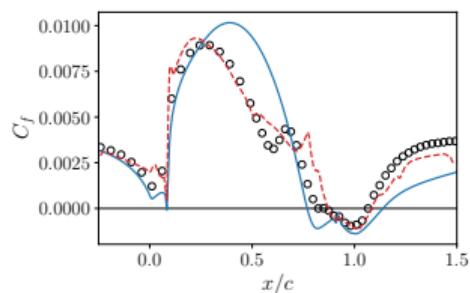
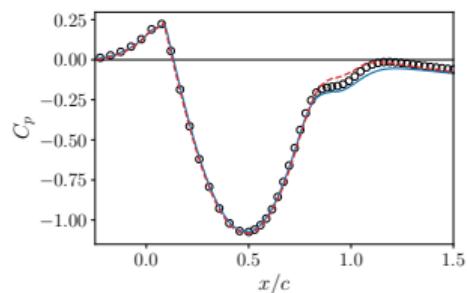
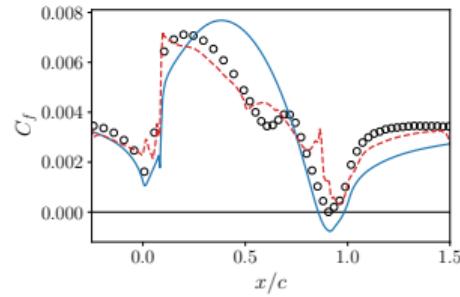
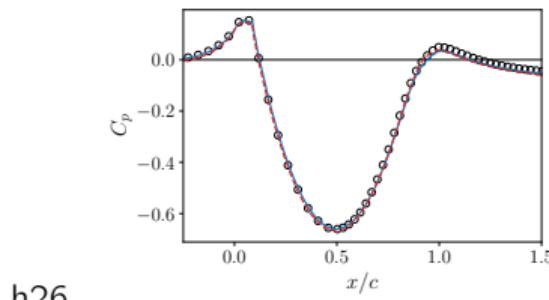


NN1 -  $\Delta\nu_t^{NN}$  no treatment



# A posteriori results using NN1 and NN2

NN2 -  $\nu_t^{NN}$



## Conclusions

- In the past, we employed DA+ML to improve RANS models
- This strategy gives accurate results but it is quite complex and time consuming
- More recently, we employed ML to correct directly the unknown eddy-viscosity term in the RANS equations
- Advantages : simple, fast, no need to transport any turbulent variables
- Although not perfect, the NN-based model improved  $C_p$  and  $C_f$  predictions
- **Machine learning-based turbulence models can be used to improve CFD capabilities**

## Publication

- Volpiani et al. (2021). Machine learning-augmented turbulence modelling for RANS simulations of massively separated flows. *Physical Review Fluids*.
- Volpiani, Bernardini & Francesquini (2022). Neural network-based eddy-viscosity correction for RANS simulations of flows over bi-dimensional bumps. *International Journal of Heat and Fluid Flow*.