High-Fidelity CFD Workshop 2021

Mesh Motion Test Suite

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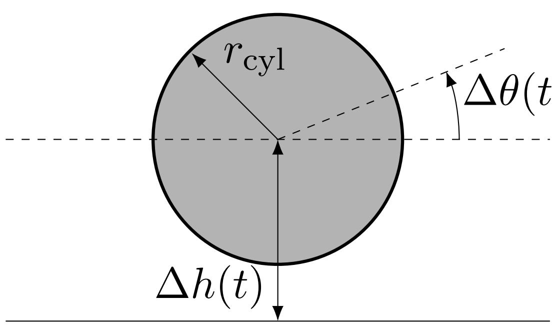
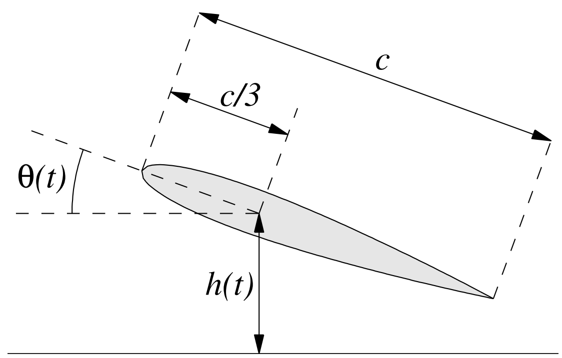


Flow in a moving cylinder



Heaving-Pitching NACA 0012

1 Summary

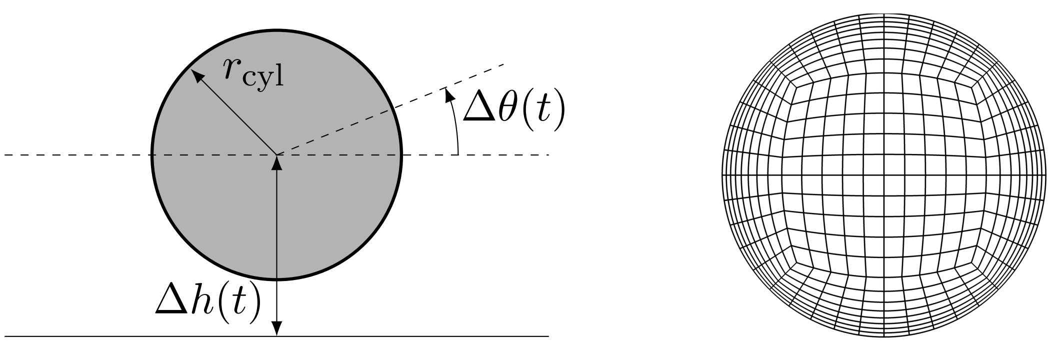
This problem is aimed at testing the accuracy and performance of high-order flow solvers for problems with deforming domains. Two geometries are considered: a cylinder and an airfoil. The cylinder cases involve a smaller domain and are intended to serve as verification simulations. The NACA 0012 problem is larger and has exhibited spread in the results in previous workshops. For both geometries, multiple motions are defined, and for the cylinder case, simulations at multiple Reynolds numbers are requested. The sections below describe the setup of each case. The outputs are defined similarly for both geometries, and a uniform data submission format is outlined in the Requirements section.

2 Cylinder Cases

These cases involve computing flow inside a cylinder undergoing three different motions, including translation, rotation, and deformation. In addition, three different Reynolds numbers are considered for each motion.

2.1 Geometry

The reference geometry for this problem is a perfect cylinder for which several types of motion are prescribed. The center of motion coincides with the geometric center of the cylinder, and the fluid domain of interest is the cylinder interior volume. Figure 2 shows a diagram of the problem geometry and the fluid domain.



(a) Geometry diagram. (b) Fluid domain.

Figure 2: Cylinder problem description.

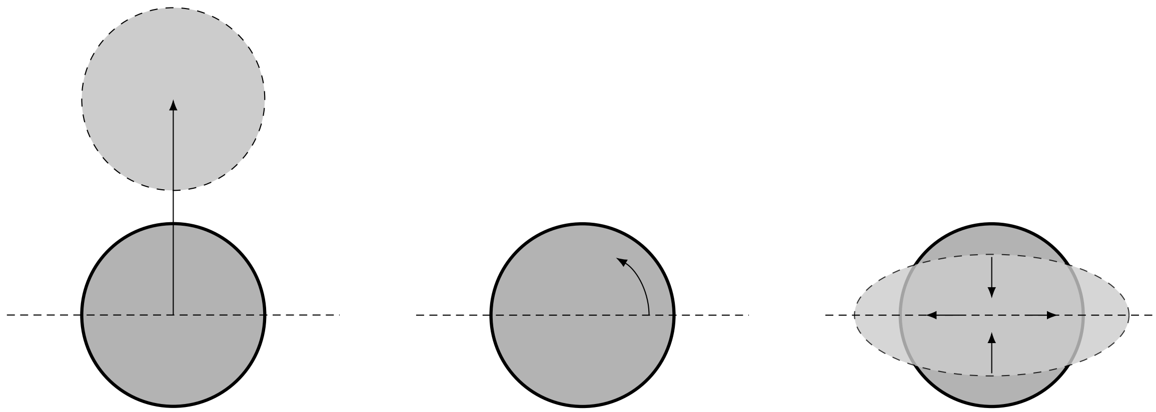
2.2 Motion

Three prescribed motions are defined for this problem. They are designed to target the properties of translation (Motion 1), rotation (Motion 2), and deformation (Motion 3) as shown in Figure 3.

The motion descriptions for translation ($\Delta h(t)$), rotation ($\Delta\theta(t)$), and deformation ($r(r_0, \theta, t)$) are given in Table 1. Relevant constants for all motions are listed in Table 2. All cases shall be run from $t = 0$ until $t = 2$ for a total duration of 2 time units. r_{cyl} is the initial radius of the cylinder wall for all motions. r_0 is any radius at the initial time $r|_{t=0}$, A_θ is a rotation amplitude (Motion 2), and A_a is an amplification factor for the deformation of a circle into an ellipse (Motion 3).

The prescribed-motion function for Motions 1 & 2 is defined as

$$\alpha(t) = t^2\,(3 - t)\,/4$$



(a) Motion 1: Translation (b) Motion 2: Rotation (c) Motion 3: Deformation

Figure 3: Cylinder motion descriptions.

	Motion 1	Motion 2	Motion 3
$\Delta h(t)$	$\alpha(t)$	0	0
$\Delta\theta(t)$	0	$A_\theta \cdot \alpha(t)$	0
$r(r_0, \theta, t)$	r_0	r_0	$\beta(r_0, \theta, t)$

Table 1: Cylinder prescribed-motion test cases, $t \in [0, 2]$

which goes from 0 to 1 on the interval $t = [0, 2]$. The prescribed-motion for Motion 3 is of a cylinder deforming into an ellipse such that the interior area remains constant during deformation. The deformation is defined as a continuous mapping that occurs along radial axes as

$$\beta(r_0, \theta, t) = \frac{b(r_0, t)}{\sqrt{1 - [e(r_0, t)\cos\theta]^2}}$$

where the semi-major axis, semi-minor axis, and eccentricity are defined to be

$$a(r_0, t) = \psi(t) r_0 \qquad \psi(t) = 1 + (A_a - 1)\alpha(t) \qquad b(r_0, t) = \frac{r_0}{\psi(t)} \qquad e(t) = \sqrt{1 - \psi(t)^{-4}}$$

Note, $\alpha(t)$ is the same function defined previously for the translation and rotation motions.

The deformation mapping prescribed in Motion 3 is a continuous mapping along radial axes as a function of any initial radius (r_0) and the corresponding angular location (θ) at a given time (t), as illustrated in Figure 4. Note, this mapping does not preserve the *distribution* of nodes in a discretization, which is demonstrated in Figure 5.

2.3 Governing Equations and Flow Conditions

The governing equations for this problem are the 2D compressible Euler and Navier-Stokes equations with a constant ratio of specific heats equal to 1.4 and a Prandtl number of 0.72. For the Euler calculations, the cylinder interior is prescribed to have no normal velocity on the wall. For viscous calculations, the cylinder interior is prescribed with a no-slip, adiabatic wall boundary condition.

The initial condition at time $t = 0$ is given by the conserved-variable state vector

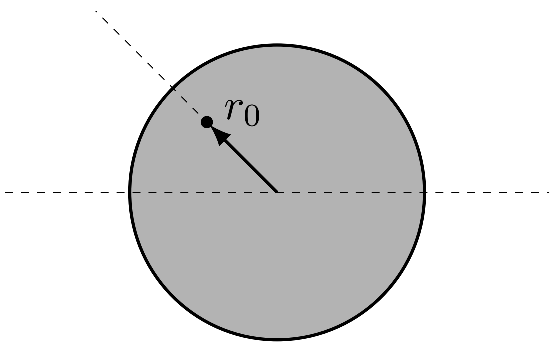
$$\boldsymbol{u}|_{t_0} = [\rho, \rho v_1, \rho v_2, \rho E]|_{t_0} = [1, 0, 0, 50.]$$

For each test case, a range of Reynolds numbers should be simulated. The reference velocity is chosen to be 1.0 and the reference length scale is the cylinder diameter, $d = 2r_{\text{cyl}} = 1.0$.

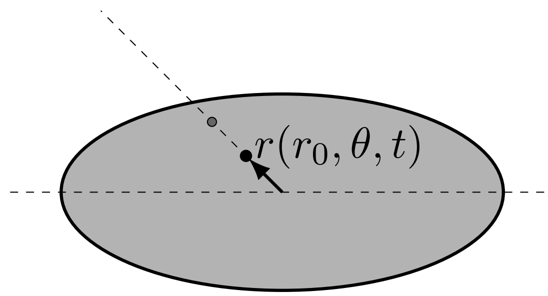
Reynolds numbers: (Euler $Re = \infty$), ($Re = 1000$), ($Re = 10$)

r_{cyl}	0.5
A_θ	π
A_a	1.5

Table 2: Cylinder motion constants.

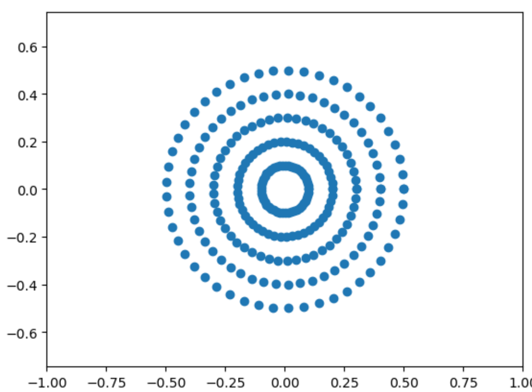


(a) Initial state.

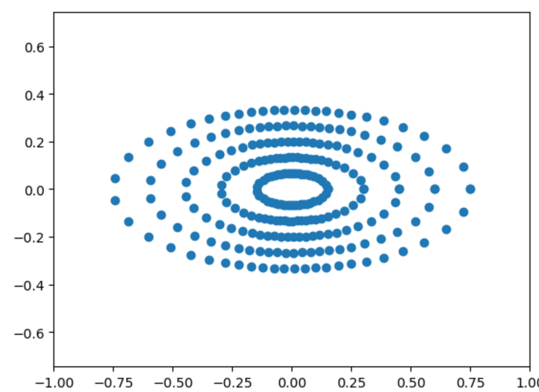


(b) Mapping at time t.

Figure 4: Cylinder problem: diagram of radial deformation mapping.



(a) $t = 0$



(b) $t = 2$

Figure 5: Discrete representation of Motion 3 deformation mapping.

3 Airfoil Cases

These cases involve a NACA 0012 airfoil undergoing a smooth flapping-type motion, starting from rest at zero angle of attack and ending at a one chord length higher position at the end of the motion at time T. Three motions are considered at one Reynolds number, Re = 1000, based on the chord length.

3.1 Geometry

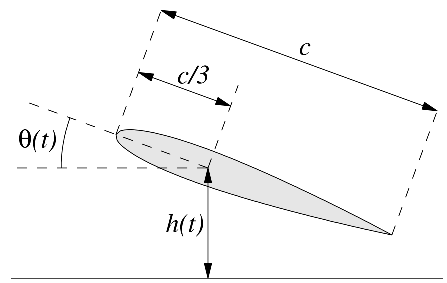
The geometry consists of a NACA 0012 airfoil with chord length $c = 1$, with geometry modified to give zero trailing edge thickness:

$$y(x) = \pm 0.6(0.2969\sqrt{x} - 0.1260x - 0.3516x^2 + 0.2843x^3 - 0.1036x^4), \quad x \in [0, 1].$$

The far-field boundary should be located at least 100 chord-lengths away from the airfoil.

3.2 Motion

The airfoil undergoes a smooth upward motion of one chord length for the duration of $T = 2$ time units, by heaving and pitching about a point located at the airfoil 1/3 chord location (see figure). We consider three different motions, with different properties and difficulties. We first define the following polynomials:



$$b_1(t) = t^2(t^2 - 4t + 4)$$
$$b_2(t) = t^2(3 - t)/4$$
$$b_3(t) = t^3(-8t^3 + 51t^2 - 111t + 84)/16$$

In terms of these, we define the vertical displacement $h(t)$ and the pitching angle $\theta(t)$ for the three motions according to below:

Motion 1 (Pure heaving)

$$\begin{cases} h(t) = b_2(t) \\ \theta(t) = 0 \end{cases}$$

Motion 2 (Flow aligning)

$$\begin{cases} h(t) = b_2(t) \\ \theta(t) = A_2 \cdot b_1(t) \end{cases}$$

Motion 3 (Energy extracting)

$$\begin{cases} h(t) = b_3(t) \\ \theta(t) = A_3 \cdot b_1(t) \end{cases}$$

where the constants $A_2 = 60\pi/180$ and $A_3 = 80\pi/180$.

3.3 Governing Equations and Flow Conditions

The governing equations for this problem are the 2D compressible Navier-Stokes equations with a constant ratio of specific heats equal to 1.4 and a Prandtl number of 0.72. Two boundary conditions are imposed: far-field characteristic conditions at the outer domain and no-slip adiabatic wall condition on the moving airfoil.

The free-stream has a Mach number $M_\infty = 0.2$ and is horizontal. The Reynolds number based on the chord of the airfoil is Re = 1000. The initial condition at time $t = 0$ is the steady-state solution for the initial position $h = 0$, $\theta = 0$. To simplify post-processing, we assume convenient units in which the airfoil chord is $c = 1$ and the free-stream density and speed are unity, so that the free-stream conservative state vector is

$$[\rho, \rho u, \rho v, \rho E] = \left[1,\ 1,\ 0,\ 0.5{+}1/[M^2\gamma(\gamma - 1)]\right].$$

4 Outputs

The requested output quantities are defined similarly for both the cylinder and the airfoil cases. The first output is the work (energy) that the fluid exerts on the surface of the cylinder/airfoil during the motion, which can be written as

$$W = \int_0^T \boldsymbol{F}(t) \cdot \boldsymbol{v}_0 dt + \int_0^T \boldsymbol{\tau}(t) \cdot \boldsymbol{\omega} dt = \int_0^T F_y(t)\dot{h}(t)dt + \int_0^T \tau_z(t)\dot{\theta}(t)dt \tag{1}$$

Here, $\boldsymbol{F}(t) = [F_x(t), F_y(t)]$ is the force imparted by the fluid on the surface, $\tau(t) = [0, 0, \tau_z(t)]$ is the torque imparted by the fluid on the surface about the reference pivot point (cylinder center, airfoil

1/3 chord), $\boldsymbol{v}_0 = \dot{h}(t)$ is the velocity of the pivot point, and $\omega_0 = [0, 0, \dot{\theta}]$ is the angular velocity of the cylinder/airfoil about the pivot point. Note, that this output can be equivalently computed as

$$W = \int_0^T \int_{\text{surface}} \vec{v}_G(t) \cdot \vec{f}_{\text{surf}}(t) ds dt \tag{2}$$

where $\vec{v}_G(t)$ is the velocity of the surface and $\vec{f}_{\text{surf}}(t)$ is the surface stress vector.

The second output is the vertical impulse from the fluid onto the surface during the motion,

$$I = \int_0^T F_y(t) dt \tag{3}$$

5 Requirements

1. Perform the indicated simulation for the test cases. Calculate the quantities W and I for each case, and perform a grid/timestep convergence study to get the values as accurate as possible. Record the work units.

2. Provide the work units, the converged output values, nDOFs in the discretization (spatial and temporal), and the distance to the far-field boundary (aifoil case) for each simulation. Submit this data to the case organizers, using the template shown below.

Geometry	Motion	Re	W	I	space nDOF	time nDOF	WU
Cylinder	1	∞	-	-	-	-	-
⋮							
Airfoil	1	1000	-	-	-	-	-
⋮							

Table 3: Template for team contributions. The spatial nDOF does not include the equation state rank. The temporal nDOF is the number of time steps times the number of stages per time step (in a multistage time integration).