Operations

The activities carried out in an organization.

Research

The process of observation and testing characterized by the scientific method.

Operations Research is a quantitative approach to decision making based on the scientific method of problem solving.

Definitions of Operations Research

- Operations research is the application of the methods of science to complex problems in the direction and management of large systems of men, machines, materials and money in industry, business, government and defence. The distinctive approach is to develop a scientific model of the system incorporating measurements of factors such as chance and risk, with which to predict and compare the outcomes of alternative decisions, strategies or controls. The purpose is to help management in determining its policy and actions scientifically.

  —*Operational Research Society, UK*

- Operations research is concerned with scientifically deciding how to best design and operate man–machine systems usually requiring the allocation of scarce resources.

  —*Operations Research Society, America*

- It is the use of scientific knowledge through interdisciplinary team effort for the purpose of determining the best utilization of limited resources.

  —*Hamdy A Taha*

- The application of scientific method to study of operations of large complex organizations or activities. It provides top level administrators with a quantitative basis for decisions that will increase the effectiveness of such organizations in carrying out their basic purpose.

  —*Committee on OR of National Research Council*

History of Operations Research

- **Strategic and tactical problems – World war-II**

The term 'operations research' was coined as a result of research on military operations during World War II. Since expecting adequate solutions from individuals or specialists in a single discipline was unrealistic, groups of individuals who collectively were considered specialists in mathematics, economics, statistics and probability theory, engineering, behavioural, and physical science were formed. In Britain, Col. P. M. S. Blackett was assigned the problem of analysing the coordination of radar equipment at gun sites

- **Adoption in other fields**

After the war, scientists who had been active in the military operations research (OR) groups made efforts to apply operations research approach to civilian problems. related to business. industry, research and development, etc.

- **Economic and industrial boom**

After World War II, economic and industrial boom resulted in continuous mechanization, automation, decentralization of operations and division of management functions. Therefore application of operations research to managerial decision-making became popular.

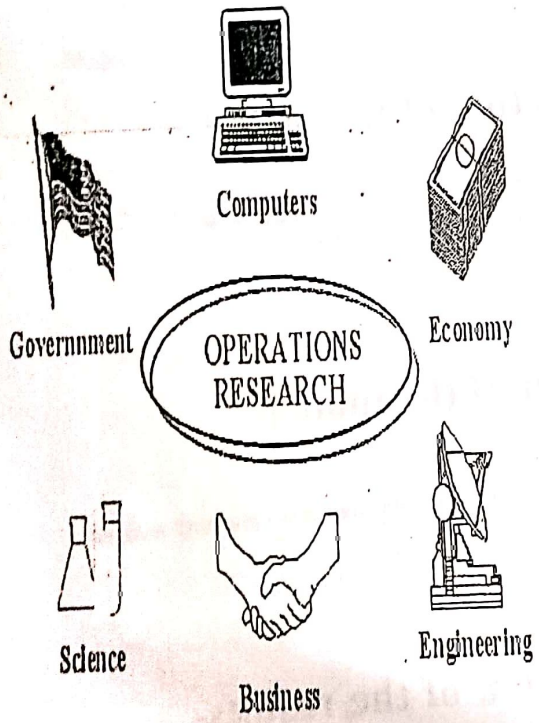
Characteristics of Operations Research

- Use of Scientific Methods
- Use of Models to represent the complex relationship
- Interdisciplinary approach
- Provision of a quantitative basis for decision making
- Methodological Approach
- Decision-making
- Operations Economy

Applications/Scope of Operations Research

- Areas
- Finance
- Production
- Marketing
- Personnel

- Techniques
  - Linear Programming
  - Decision Models
  - Network Theory
  - Inventory Control
  - Queuing Theory
  - Sequencing
  - Game Theory
  - Simulation



Process of Operations Research

Formulation & Definition of the problem-

Issues involved in the problem are discussed. Critical issues to be addressed are found out. Specifying the objective and its scope. Identifying the alternative courses of action. Specifying the constraints

Data Collection-

Involves translating the defined problem into a model. two sources- observation & Standard

Construction of the model-

process of capturing selected characteristics of a system or a process and then combining these into an abstract representation of the original. Mathematical model is used where characteristics of a system or process is established through a set of mathematical relationships

Solution of the model-

Validation of the model verifying the solution obtained. post optimality analysis

Implementation of the results-

Advantages of Operations Research

- Better Decision-making
- Better control
- Better Co-ordination
- Helpful in complex situations
- Resolve conflicts
- Improves Productivity

Limitations of Operations Research

- Magnitude of Computation
- Non-Quantifiable Factors
- Distance between User and Analyst
- Time and Money Costs
- Implementation

Unit 2- Linear Programming Problem

Definitions

Linear programming is a mathematical technique useful for allocation of 'scarce' or 'limited' resources to several competing activities on the basis of a given criterion of optimality.

The word 'linear' refers to linear relationship among variables in a model. That is, a change in one variable causes a resulting proportional change in other variable. The word 'programming' refers to the solution of a problem that involves the economic allocation of limited resources, by choosing a particular course of action or strategy among various alternative strategies in order to achieve a desired objective.

Components of an LPP

- Decision Variables.
- Objective Function
  - To maximize or minimize
- Constraints
  - Involving $\leq$, $=$, or $\geq$ sign
- Non-negativity Condition
  - Variables to be non-negative

Assumptions underlying Linear Programming

- Proportionality
- Additive
- Continuity
- Certainty
- Finite Choices

2

Graphic Solution to LPPs

- Plot constraints
- Mark feasible region which should be a convex set
- Evaluate corner points/use iso-profit or iso-cost lines to get optimal solution
  - Redundant constraints
  - Binding constraints
  - Non-binding constraints

Solutions to LPPs

- Unique Optimal Solution
- Multiple Optimal Solutions
- Infeasibility: No feasible solution
- Unbounded Solution

Advantages of Linear Programming

- Linear programming helps in attaining the optimum use of productive resources. It also indicates the use of productive factors effectively by selecting and distributing (allocating) these resources.
- Linear programming technique improves the quality of decisions. In other words, decision-making approach becomes more objective and less subjective.
- Linear programming technique provides possible and practical solutions by taking into account the constraints operating outside the problem.
- Linear programming also helps in re-evaluation of the outcome due to a course of action under changing conditions. This helps to adjust the remainder of the decision process for best results.

Limitations of Linear Programming

- Linear programming treats all relationships among decision variables as linear. However, in real life neither the objective function nor the constraints concerning business and industrial problems are linearly related to the variables.
- While solving an LP model, there is no guarantee to get an integer valued solution. Rounding off the solution to the nearest integer will not yield an optimal solution. In such cases, integer programming is used to ensure integer value to the decision variables.
- Linear programming model does not take into consideration the effect of time and uncertainty.
- Parameters of LP model are assumed to be constant but in real-life situations, they are neither known nor constant.
- A LP model has single objective function, whereas in real-life situations we may come across conflicting multi-objective problems. In such cases, a goal programming model is used to get satisfactory value of such objectives.

Q-5 To maintain good health, a person must fulfill certain minimum daily requirements of several kinds of nutrients. For the sake of simplicity let us assume that only three kinds of these needs to be considered calcium, protein vitamin A. also assume that the person's diet is to consist of only 2 food items, I & II whose prices & nutrient content's are given in the following table. Find out the optimal combination of the two food items that will satisfy the daily requirements & entail the least cost.

Foods	Calcium	Protein Qty	Vitamin A Qty	Cost per Unit
F1	10	5	2	6
F2	4	5	6	1
Daily Min. Requirements	20	20	12	

Q-6 A steel plant manufactures two grades of steel S_1 & S_2 Data given below shows the total resources consumed & profit per unit associated with S_1 & S_2 .iron and labor are the only resources which are consumed in the manufacturing process. The manager of the firm wishes to determine the different units of S_1 & S_2 which should be manufactured to maximize the total profit.

Resource utilized	Unit-requirement		Amount Available
	S_1	S_2	
Iron (kg)	30	20	300
labour (Hours)	5	10	110
Profit (Rs.)	6	8	

Q-7 A manufacturer produces two types of models M1 and M2.Each model of the type M1 requires 4 hours of grinding and 2 hours of polishing; whereas each model of M2 requires 2 hours of grinding and 5 hours of polishing. The manufacturer has 2 grinders and 3 polishers. Each grinder works for 40 hours a week and each polisher works 60 hours a week. Profit on M1 model is Rs.3.00 and on model M2 is Rs.4.00.Whatever produced in a week is sold in the market. How should the manufacturer allocate his production capacity to the two types of models, so that he makes maximum profit in a week?

Q-8 Alpha Limited produces & sells 2 different products under the brand name black & white. The profits per unit on these products are Rs. 50 & Rs. 40 respectively. Both black & white employ the same manufacturing process which has a fixed total capacity of 50,000 man-hours. As per the estimates of the marketing research department of Alpha Limited, there is a market demand for maximum 8,000 units of Black & 10,000 units of white. Subject to the overall demand, the products can be sold in any possible combination. If it takes 3 hours to produce one unit of black & 2 hours to produce one unit of white. formulate the about as a linear programming model.

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Q-9 A person wants to decide the constituents of a diet which will fulfill his daily requirements of proteins, Fats & carbohydrates at the minimum cost. The choice is to be made from four different types of foods. The yield per unit of these foods is:

Food Type	Yield Per Unit			Cost per Unit.
	Proteins	Fats	Carbohydrates	(Rs.)
1	3	2	6	45
2	4	2	4	40
3	8	7	7	85
4	6	5	4	65
Minimum Requirement	800	200	700	

Formulate the linear programming model for the problem.

Q-10 High Quality furniture Ltd. manufactures two products, tables & chairs. Both the products have to be processed through two machines M1 & M2 the total machine-hours available are: 200 hours ofM1 and 400 hours of M2 respectively. Time in hours required for producing a chair and a table on both the machines is as follows:

Time in Hours

Machine	Table	Chair
M1	7	4
M2	5	5

Profit from the Sale of table is Rs. 40 and that from a chair is Rs. 30, determine optimal mix of tables & chairs so as to maximize the total profit.

Q-11 Good Results Company manufactures & sells in the export market three different kinds of products P_1, P_2 & P_3. The anticipated sales for the three products are 100 units of P_1, 200 units of P_2 & 300 units of P_3. As per the terms of the contract Good results must produce at least 50 units of P_1 & 70 units of P_3. Following is the break – up of the various production lines:

Product	Production Hours per Unit				Unit Profit (Rs.)
	Department (A)	Department (B)	Department (C)	Department (D)	
P_1	0.05	0.06	0.07	0.08	15
P_2	0.10	0.12		0.30	20
P3	0.20	0.09	0.07	0.08	25
Available hours	40.00	45.00	50.00	55.00	

Management is free to establish the production schedule subject to the above constraints.

Formulate as a linear programming model assuming profit maximization criterion for Good Results Company.

Q-12 The management of Surya Chemicals is considering the optimal mix of two possible processes. The values of input & output for both these process are given as follows:

Process	Inputs Units		Outputs Units	
	I_1	I_2	O_1	O_2
X	2	6	3	7
Y	4	8	5	9

Maximum 500 units of Input I_1 and 300 units of I_2 are available to Surya Chemicals in the local market. The forecasted demand for outputs O_1 & O_2 are at least 5,000 units & 7,000 units respectively. The respective profits from process X & Y are Rs. 1,000 & Rs. 2,000 - per production run. You are required to formulate the above as a linear programming model.

Solve using Graphical Method

Q-13 A small scale industry manufactures electrical regulators, the assembly of which is being accomplished by a small group of skilled workers, both men and women. Due to the limitations of space and finance, the number of workers employed cannot exceed 11 and their salary bill not more than Rs. 60,000 per month. The male members of the skilled workers are paid Rs. 6000 per month, while the female worker doing the same work as the male member get Rs. 5000 member contributes Rs. 10,000 per month to total return of the industry, while the female worker contributes Rs. 8500 per month. Determine the number of male and female workers to be employed in order to maximize the monthly total return.**(maximization-bounded- unique)**

Q-14 Maximize: $z=2x_1 + x_2$ **(Bounded- Unique)**

Constraints:

$x_1 + 2x_2 \leq 360$

$x_1 + x_2 \leq 6$

$x_1 - x_2 \leq 2$

$x_1 - 2x_2 \leq 1$

$x_1, x_2 \geq 0$

Q-15 MD Ltd. has two bottling plants, one located at Solan and the other at Mohan Nagar. Each plant produces three drinks; whisky, beer and fruit juices name A, B, C respectively. The number of bottles produced per day is as follows:

	Plant at	
	Solan	Mohan Nagar
Whisky	1500	1500
Beer	3000	1000
Fruit Juices	2000	5000

A market survey indicates that during the month of April, there will be a demand of 20,000 bottles of whisky, 40,000 bottles of beer and 44,000 bottles of fruit juices. The operating costs per day for plants at Solan and Mohan Nagar are 600 & 400 monetary units. For how many days each plant is run in April so as to minimize the production cost while still meeting the market demand?**(Unbounded- Minimization-Unique)**

Q-16 Use the graphical method to solve the LP problem. (Minimize- Bounded with negative coordinates- Unique (negative) solution)

- Minimize $Z = -x_1 + 2x_2$

Subject to the constraints

- $-x_1 + 3x_2 \leq 10$
- $x_1 + x_2 \leq 6$
- $x_1 - x_2 \leq 2$

And $x_1, x_2 \geq 0$.

Q-17 Solve the following problem graphically (Maximize- Unbounded- Infinite)

Max $Z = 5X_1+4X_2$

Subject to $X_1-2X_2 \leq 1$

$X_1+2X_2 \geq 3$

$X_1, X_2 \geq 0$

Q-18 Solve the following problem graphically: (Maximize- Bounded- Unique)

Max $Z = -X_1+2X_2$

Subject to

$X_1-X_2 \leq -1$

$-0.5 +X_2 \leq 2$

$X_1, X_2 \geq 0$

Q-19 Maximize, $Z=4x_1 + 3x_2$ (Bounded- Multiple optimal)

Subject to the constraints

$3x_1 + 4x_2 \leq 24$

$8x_1 + 6x_2 \leq 48$

$x_1 \leq 5$

$x_2 \leq 6$

$x_1, x_2 \geq 0$

Q-20 Maximize: $z=4x_1 + 2x_2$ (Unbounded- Infeasible)

Constraints:

$2x_1 + 3x_2 \leq 18$

$x_1 + x_2 \geq 10$

$x_1, x_2 \geq 0$

Q-21 An advertising firm desires to reach two types of audiences- customers with annual income of more than Rs. 40,000 (target audience A) and customers with annual income of less than Rs. 40,000 (target audience B). The total advertising budget is Rs.2, 00,000. One programme of T.V advertising costs Rs.50, 000 and one programme of radio advertising costs Rs. 20,000. Contract conditions ordinarily require that there should be atleast3 programmes on T.V and the number of programmes on Radio must not exceed 5. Survey indicates that a single T.V programme reaches 7, 50,000 customers in target audience A and 1, 50,000 in target audience B. One Radio programme reaches 40,000 customers on target audience A and 2, 60,000 in target audience B. Formulate this as a LPP and determine the media mix to maximize the total reach using graphic method. (Maximize- Bounded- Unique)

Unit III: Transportation Problems

Inputs:

- Sources with availability
- Destinations with requirements
- Unit cost of transportation from various sources to destinations

Objective:

- To determine schedule of transportation to minimize total transportation cost

Transportation Method

Step 1

Balance the problem if Aggregate Demand and Aggregate Supply are unequal; place an M in the cost cell if some route is prohibited; and convert into equivalent minimization problem if it is a maximization problem.

Step 2

Obtain initial solution

- North-West Corner Rule: considers only demand and availability
- Least Cost Method: considers unit cost, demand and availability
- Vogel's Approximation Method: considers cost differences, demand and availability

Step 3

Test for optimality

For this, check if the number of occupied cells = $m + n - 1$

If yes, obtain u_i and v_j values and calculate Δ_{ij} values for unoccupied cells

- If all Δ_{ij} values are ≤ 0, it is optimal solution
- If all Δ_{ij} values are negative, it is unique optimal solution
- If some Δ_{ij} value/s = 0 while others are negative, there are multiple optimal solutions

In case of non-optimal solution go to step 4

Step 4

Improve non-optimal solution

(a) Begin with cell having largest Δ_{ij}, draw a closed path:

- ✓ Move alternately between rows and columns
- ✓ Stop only at occupied cells
- ✓ Start with a + sign in the cell of origin, place – and + signs alternately on cells on the path

(b) Consider cells with – sign, choose the least quantity in them, add it to each cell with + sign and subtract it from each cell with – sign

(c) Obtain revised solution and go back to step 3

Special Cases in Transportation Problem

Unbalanced Transportation Problem

A problem is known to be unbalanced problem if Aggregate Demand and Aggregate Supply are unequal; the first step is to make it balanced by adding a dummy row or dummy column as per the requirement & placing zero in all the cost cells of that dummy row or dummy column.

Transportation Problem of Maximization

If a problem is of maximization (instead of cost profit /saving is given) then convert it into equivalent minimization problem by subtracting all the values from the highest value.

Prohibition in Transportation Problem

If some routes are prohibited in the problem then place an M (which denotes vary high cost) in the cost cell which is prohibited and then solve in usual manner.

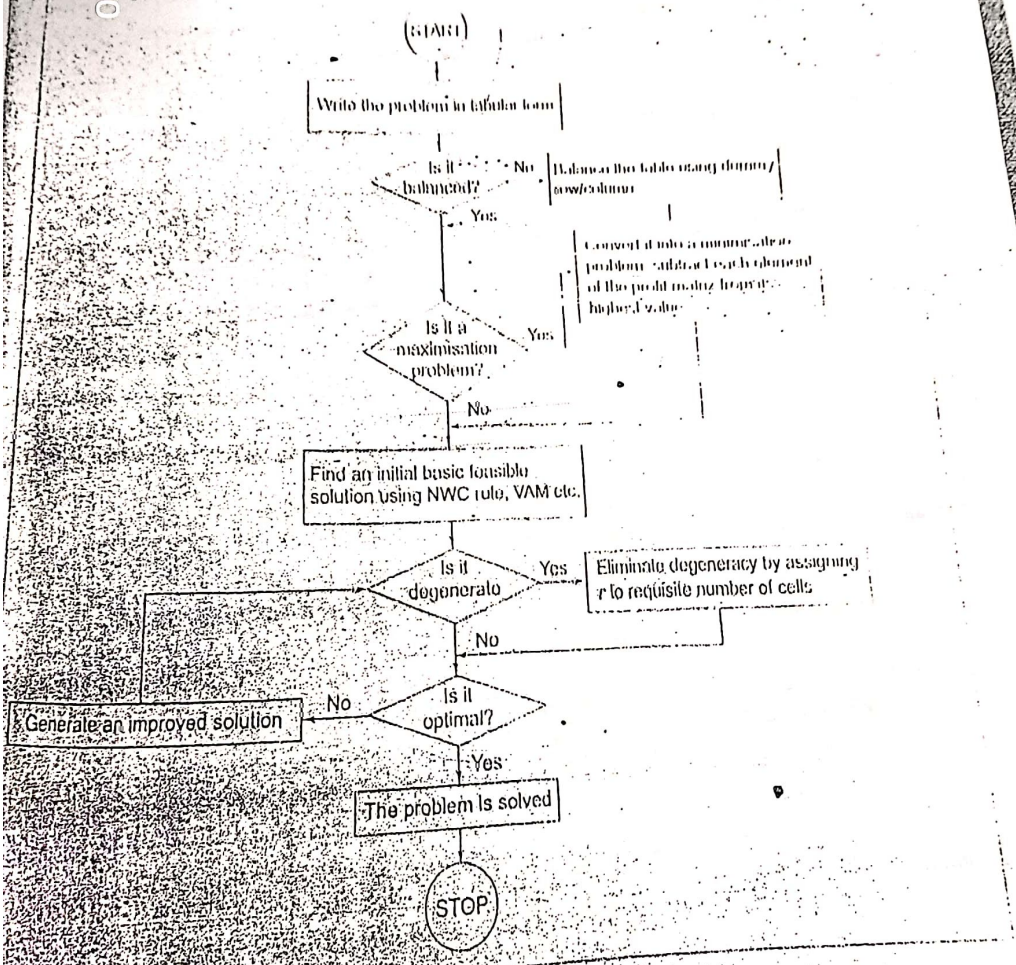
Degeneracy in Transportation Problem

If the basic feasible solution of a transportation problem with m origins and n destinations has fewer than $m+n-1$ positive c_{ij} (Occupied cells), the problem is said to be a degenerate transportation problem.

Degeneracy occurs at two stages:

1) Initial Stage
2) During testing the optimal solution. (Prevents testing optimality of the solution)

Degeneracy is removed by placing an infinitesimally small value ε (Epsilon) in each of the required number of independent cells



ure 5.4 : Schematic Presentation of Transportation Method

Q-1 Find out the initial feasible solution using:

i) North West Corner Method,
ii) Least Cost Method/ Matrix Minima Method
iii) Row Minima method
iv) Column Minima Method
v) Vogel's Approximation Method

		Distribution Centers				
		A	B	C	D	Supply
Plants	P	2	3	11	7	6
	Q	1	0	6	1	1
	R	5	8	15	9	10
Requirements		7	5	3	2	

Q-2 Find the optimal transportation cost using: MODI Method

	P	Q	R	S	Supply
A	12	10	12	13	500
B	7	11	8	14	300
C	6	16	11	7	200
Demand	180	150	350	320	1000

Q-3 Given the following transportation problem:

Warehouse	Market			Supply
	A	B	C	
1	10	12	7	180
2	14	11	6	100
3	9	5	13	160
4	11	7	9	120
Demand	240	200	220	

It is known that currently nothing can be sent from warehouse 1 to market A and from warehouse 3 to market C. Solve the problem and determine the least cost transportation schedule. Is the optimal solution obtained by you is unique? If not, what is/ are the other optimal solution/s?

Q-4 The following matrix gives the distance in kms. from source of supply to the destination. The shipping cost is Rs. 10per km. What shipping schedule should be used to minimize total cost?

	D1	D2	D3	D4	D5	Supply
S1	5	8	6	6	3	8
S2	4	7	7	6	5	5
S3	8	4	6	6	4	9
Demand	4	4	5	4	8	

Q-5 Determine a transportation schedule to minimize cost using NWC Rule.

	D1	D2	D3	Supply
S1	8	5	6	120
S2	15	10	12	80
S3	3	9	10	80
Demand	150	80	50	

Q-6 There are four warehouses located at A, B, C and D with capacity of 90, 50, 80 and 60 tons respectively. A supply has to be made in 3 cities P, Q and r with a demand of 120, 100 and 110 tons respectively. The shipping cost per ton is given below:-

	P	Q	R
A	7	10	5
B	12	9	4
C	7	3	11
D	9	5	7

- Shipments are prohibited from A to P. Find out the minimum cost.

Q-7 A company supplies the product from its warehouses to 4 cities. The profit per unit is given below.

Warehouse/City	1	2	3	4	Supply
A	6	6	11	15	80
B	4	6	10	12	120
C	6	4	7	6	150
D	4	10	14	14	70
E	8	8	7	9	90
Demand	100	200	120	80	

Q-8 The table below records transportation costs per unit of a product from origins O1, O2, O3 and O4 to destinations D1, D2, D3, D4 and D5. The capacities of the four origins are respectively 55, 45, 30 and 50 while the requirements of the five destinations are respectively 40, 20, 50, 30 and 40. Employing Vogel's Approximation Method, or otherwise, make the initial allocations to the origins to satisfy the requirements of the destinations and test the optimality of these allocations. Use Modified Distribution Method (MODI) for obtaining an optimal solution that would minimize the total cost of transportation.

Origin	Destination					Capacity
	D1	D2	D3	D4	D5	
O1	12	4	9	5	9	55
O2	8	1	6	6	7	45
O3	1	12	4	7	7	30
O4	10	15	6	9	1	50
Requirement	40	20	50	30	40	

Find the total cost implied by the solution. Is the solution unique? If not, find an alternative solution as well.

Q-9 A company has four warehouses and six stores. The warehouses altogether have a surplus of 22 units of a given commodity, divided among them as follows:

Warehouses	1	2	3	4
Surplus	5	6	2	9

The six stores altogether need 22 units of the commodity. Individual requirements at stores 1, 2,3,4,5 and 6 are 4, 4, 6, 2, 4 and 2 units respectively.

Cost of shipping one unit of commodity from warehouse to stores in rupees is given in the matrix below:

Warehouses	Stores					
	1	2	3	4	5	6
1	9	12	9	6	9	10
2	7	3	7	7	5	5
3	6	5	9	11	3	11
4	6	8	11	2	2	10

How the products should be shipped from the warehouses to the stores so that the transportation cost is minimum?

Q-10 A company has received a contract to supply gravel to three new construction projects located in towns A, B and C. The construction engineers have estimated that the required amounts of gravel which will be needed at these construction projects are:

Project Location	Weekly Requirements (Truckloads)
A	72
B	102
C	41

The company has 3 gravel pits located in towns X, Y and Z. The gravel required by the construction project can be supplied by three pits. The amount of gravel that can be supplied by each pit is as follows:

Plant:	X	Y	Z
Amount Available (Truckloads)	76	82	77

The company has computed the delivery cost from each pit to each project site. These costs (in Rs.) are shown in the following table:

Pit \ Project Location	A	B	C
	4	8	8
	16	24	16
	8	16	24

Schedule the shipment from each pit to each project in such a manner that it minimizes the total transportation cost within the constraints imposed by pit capacities and project requirements. Also find the minimum cost.

Q-11 Find the optimum solution to the following transportation problem in which the cells contain the transportation cost in rupees.

	W1	W2	W3	W4	W5	Available
F1	7	6	4	5	9	40
F2	8	5	6	7	8	30
F3	6	8	9	6	5	20
F4	5	7	7	8	6	10
Required	30	30	15	20	5	100

Warehouse Location(City)	A	B	C	D
Capacity (Tons)	90	50	80	60

The warehouse supply tobacco to cigarette companies in three cities that have the following demand:

Cigarette Company	Demand (Tons)
Bharat	120
Janta	100
Red Lamp	110

The following railroad shipping costs per tonne (in hundred rupees) have been determined:

Warehouse Location	Bharat	Janta	Red Lamp
A	7	10	5
B	12	9	4
C	7	3	11
D	9	5	7

Because of railroad construction, shipments are temporarily prohibited from warehouse at city A to Bharat Cigarette Company.

a) Find the optimum distribution for XYZ Tobacco Company.
b) Are there multiple optimum solutions? If yes, identify them.

Q-13 Find the initial solution for the transportation problem by VAM method. Also obtain the optimum solution by MODI Method.

Plant	Market				Supply
	M1	M2	M3	M4	
P1	6	4	9	1	40
P2	20	6	11	3	40
P3	7	1	0	14	50
P4	7	1	12	6	30
Demand	90	30	50	30	

Q-14 A company manufacturing air – coolers has two plants located at Mumbai and Kolkata with a capacity of 200 units and 100 units per week respectively. The company supplies the air – coolers to its four showrooms situated at Ranchi, Delhi, Lucknow and Kanpur which have a maximum demand of 75, 100, 100 and 30 units respectively. Due to the differences in raw material cost and transportation cost, the profit per unit in rupees differs which is shown in the table below:

	Ranchi	Delhi	Lucknow	Kanpur
Mumbai	90	90	100	110
Kolkata	50	70	130	85

Plan the production program so as to maximize the profit. The company may have its production capacity at both plans partly or wholly unused.

Q-15 A product is manufactured at four factories A, B, C and D. Their unit production costs are Rs. 2, Rs. 3, Re 1 and Rs. 5, respectively. Their production capacities are 50, 70, 30 and 50 units, respectively. These factories supply the product to four stores, demands of which are 25, 35, 105 and 20 units respectively. Unit transportation cost in rupees from each factory to each store is given in the table below:

	Stores			
Factories	I	II	III	IV
A	2	4	6	11
B	10	8	7	5
C	13	3	9	12
D	4	6	8	3

Determine the extent of deliveries from each of the factories to each of the stores, so that the total production and transportation cost is the minimum.

Q-16 A company has factories at F1, F2 and F3 that supply products to warehouses at W1, W2 and W3. The weekly capacities of the factories are 200, 160 and 90 units, respectively. The weekly warehouse requirements are 180, 120 and 150 units, respectively. The unit shipping costs (in rupees) are as follows:

Factory	Warehouse			Supply
	W1	W2	W3	
F1	16	20	12	200
F2	14	8	18	160
F3	26	24	16	90
Demand	180	120	150	450

Determine the optimal distribution for this company in order to minimize its total shipping cost.