Lecture 12

	Discrete离散	Continuous连续
PMF / PDF	$P(X=x)$	$f_x(x)=F'_X(x)$
CDF	$F_x(x)=P(X\leq x)$	$F_X(x)=P(X\leq x)$
E(X)	$E(x)=\sum xP(X=x)$	$E(X)=\int xf(x)dx$
LOTUS	$E(g(x))=\sum g(x)P(X=x)$	$E(g(X))=\int_{-\infty}^{\infty}g(x)f(x)dx$

PDF - Probability Density Function

Definiton:

R.v X has PDF f(x) if $P(a\leq X\leq b)=\int_a^b f(x)dx$ for all a and b

To be valid:

1. $f(x)\geq 0$
2. $\int_{-\infty}^{\infty}f(x)dx=1$

If X has PDF f, the CDF is $F(x)=P(X\leq x)=\int_{-\infty}^{x}f(t)dt$

If X has CDF F (and X is continous), then $f(x)=F'(x)$ by FTC

$P(a\leq X\leq b)=\int_a^b f(x)dx=F(b)-F(a)$

Var & Std

Variance - 方差

$Var(X)=E(X-EX)^2$

Another Way to Express Var:

$Var(X)=E(X^2-2X(EX)+(EX))=E(X^2)-2E(X)E(X)+(EX)^2$

$=E(X^2)-(EX)^2$

Notation $EX^2=E(X^2)$

Standard deviation - 标准差

$SD(X)=\sqrt{Var(X)}$

Uniform - 均匀分布

$Unif(a,b)$

probability length

PDF

f(x) = c, if a <= x <= b; 0, otherwise

CDF

$1 = \int_a^b cdx = c = \frac{1}{b-a}$

$$F(X) = \int_{-\infty}^{x} f(t)dt = 0, if x < a; 1, if x > b; x - a/x - b, if a <= x <= b;$$

E(X)

$E(X) = \int_a^b \frac{x}{b-a} dx = \frac{a+b}{2}$

$Y = X^2, E(X^2) = E(Y) = \int_{-\infty}^{\infty} x^2 f(x)dx$

law of the unconscious statistician (LOTUS)

$E(g(X)) = \int_{-\infty}^{\infty} g(x)f(x)dx$

Var(U)

Let $U \sim Unif(0,1),\ E(U) = 1/2$,$E(U^2) = \int_0^1 u^2 f(u)du = 1/3$

$Var(U) = 1/3 - 1/4 = 1/12$

Universality

> Uniform is Universal

Let $U \sim Unif(0,1)$ F be CDF (assume F is strictly increasing and continuous)

Then Let $X = F^{-1}(U)$, Then $X \sim F$

Proof:

$P(X \leq x) = P(F^{-1}(U) \leq x) = P(U \leq F(x)) = F(x)$