

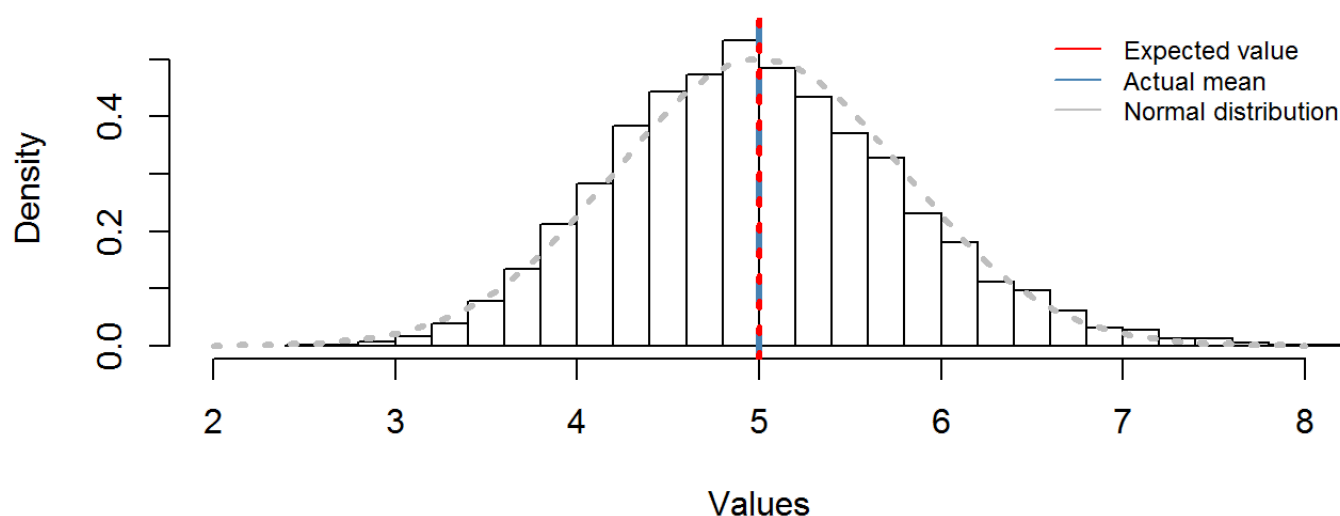
Each simulation used a rate parameter $\lambda = 0.2$ used to produce $n = 40$ random variables. The mean & standard deviation of the 40 values were then calculated 10,000 times.

```
set.seed(1230); lambda <- .2; n <- 40; no_sim <- 10000
mean_values <- NULL; mean_sds <- NULL
for (i in 1:no_sim){
  values <- rexp(n, lambda); means <- mean(values); sds <- sd(values)
  mean_values <- c(mean_values, means); mean_sds <- c(mean_sds, sds)
}
avg <- mean(mean_values); s <- sd(mean_values)
```

The expected value for an exponential distribution is $\mathbb{E}[X] = 1/\lambda$ which is equal to 5 in this case. The mean for the experiment was calculated to be 5.00. As expected, due to the CLT, the expected value of the sample mean is equal to the mean it's trying to estimate. The distribution of the sample mean is gaussian, centered at 5 & concentrated at the center as shown below.

```
## [1] 5.003
```

Probability density function for 10000 simulations

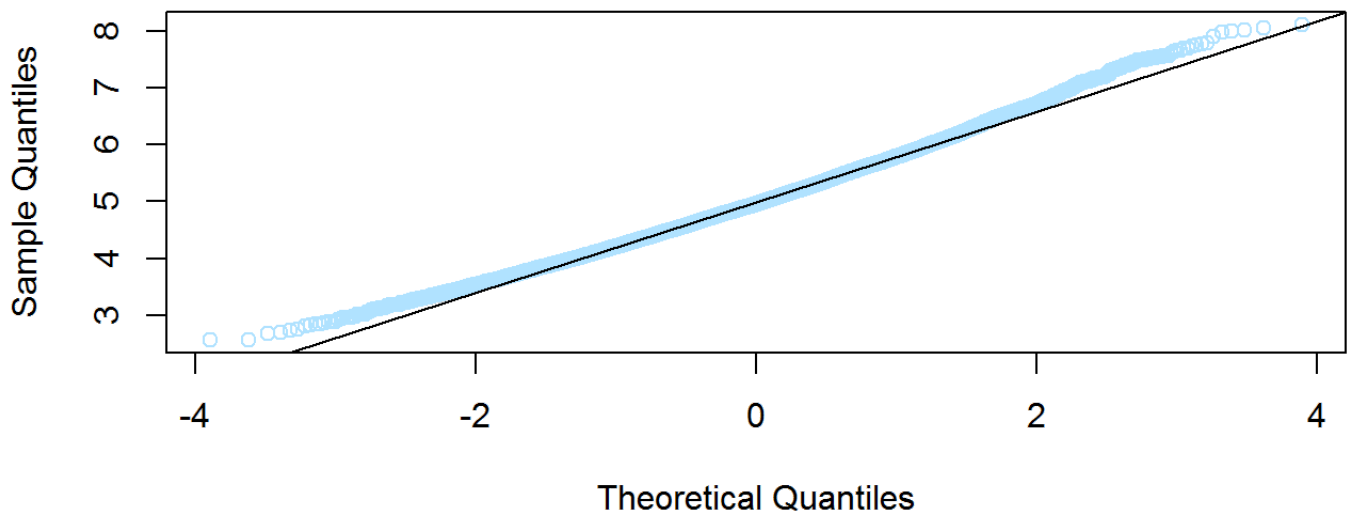


Next, the variance of the sample mean was worked out to be 0.80. This corresponds with the standard error of the mean (i.e. $SE = \sigma/\sqrt{n}$) which is equal to 0.79 for the 40 observations.

```
## [1] 0.7969
```

A Q-Q plot of the mean values was plotted below. There is little deviation between the actual quantile values & the theoretical; this indicates aggregated sample distribution is indeed normal.

Normal Q-Q Plot



The 95% confidence interval of each simulation was worked out using each simulation's own standard deviation and mean according to the equation $\bar{X} \pm 1.96\sigma/\sqrt{n}$. The coverage was computed as the percent of times the true mean fell within each interval's confidence interval.

```
upper <- mean_values + 1.96 * (mean_sds/sqrt(n))  
lower <- mean_values - 1.96 * (mean_sds/sqrt(n))  
sum(lower < 5 & 5 < upper)/no_sim * 100
```

```
## [1] 92.23
```

For visualization purposes, an extra experiment for 100 simulations was run & the confidence interval for each simulation was plotted.

Plot of confidence interval coverage for 100 simulations

