Lecture 11

Sympathetic magic

Dont' confuse r,v with its distribution

~~P(X=x) + P(Y=y)~~

> Word is not the thing, the map is not the territory.

r.v -> random house distribution -> blueprint

Poisson Distribution - 泊松分布

$X \sim Pois(\lambda)$

PMF:

$P(X=k)=e^{-\lambda}\frac{\lambda^k}{k!}$ λ is the rate parameter >0

Valid: $\sum_{k=0}^{\infty} e^{-\lambda}\frac{\lambda^k}{k!}=1$

E(X)

$E(X)=\lambda e^{-\lambda}\sum_{k=1}^{\infty}\frac{\lambda^{k-1}}{(k-1)!}=\lambda$

often used for applications where counting # of "successes" where there are a large # trials each with small prob of success

Examples:

1. #emails in an hour
2. #chips in choc chip cookies
3. #earthquakes in a year in some area

Pois Paradigm (Pois Approximation)

Events $A_1, A_2, \ldots A_n$, $P(A_j)=p_j$, n large, p_j's small

events independent or "weakly dependent"

# of Aj's that occure is approx $Pois(\lambda)$, $\lambda=\sum p_j$

Binomial converges to Poisson

<u>Example.</u>

Have n people, find approx prob that there are 3 people with same birthday.

$\binom{n}{3}$ triplets of people , indicator r,v for each, I_{ijk} , i<j<k

$E(triple\ matches)=\binom{n}{3}1/365^2$

X = #triple matches Approx $Pois(\lambda)$, $\lambda = \binom{n}{3} 1/365^2$

I_{123}, I_{124} are not independent

$P(X \geq 1) = 1 - P(X = 0) \approx 1 - e^{-\lambda}$