Lecture 16

Exponential Distribution

rate parameter λ

$X \sim Expo(\lambda)$ has PDF $\lambda e^{-\lambda x}, x > 0$ 0 otherwise

CDF $F(x) = \int_0^x \lambda e^{-\lambda t} dt = 1 - e^{\lambda x}, x > 0$

Example

Let $Y = \lambda X$ then $Y \sim Expo(1)$

since $P(Y \leq y) = P(X \leq y/\lambda) = 1 - e^{-y}$

Let $Y \sim Expo(1)$ find $E(Y), Var(Y)$

$E(Y) = \int_0^\infty ye^{-y} dy = 1$, $du = dy, dv = -e^{-y}$

$Var(Y) = E(Y^2) - (EY)^2 = 1$ LOTUS

So $X = Y/\lambda$ has $E(X) = 1/\lambda, Var(X) = 1/\lambda^2$

Memoryless Property

$P(X \geq s + t | X \geq s) = P(X \geq t)$

Here $P(X \geq s) = 1 - P(X \leq s) = e^{-\lambda s}$

$P(X \geq s + t | X \geq s) = P(X \geq s + t, X \geq s)/P(X \geq s) = P(X \geq s + t)/P(X \geq s) = e^{-\lambda t} = P(X$

$X \sim Expo(\lambda)$

$E(X|X > a) = a + E(X - a|X > a) = a + q/\lambda$ by memoryless