# Lecture 6

## Monty Hall

> Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice?
>
> From Wikipedia [Monty Hall problem]

1 door has car, 2 doors have goats, Monty knows which door has car.

Monty always open a goat door. If he has a choice , he picks with equal probability.  Should you switch?

Note: if Monty opens door 2, we know door 2 has a goat, and Monty opened door 2

**LOTP: Law of Total Probability**

wish we knew where the car is

$S$ : succeed (assuming switch)

$D_j$ = Door j has car (j = 1, 2, 3)

You choose door 1 and the host opens door 3

$P(S) = P(S|D_1)\frac{1}{3} + P(S|D_2)\frac{1}{3} + P(S|D_3)\frac{1}{3}$

$= 0 + 1 * \frac{1}{3} + 1 * \frac{1}{3} = \frac{2}{3}$

By symmetry P(S|Monty opens 2) = 2/3

## Simpson's Paradox

| **Dr.Hibbert** | **heart** | **bandaid** |
|---|---|---|
| success | 70 | 10 |
| failure | 20 | 0 |

| **Dr.Nick** | **heart** | **bandaid** |
|---|---|---|
| success | 2 | 81 |
| failure | 8 | 9 |

$A$: successful surgery

$B$: treated by Nick

$C$: heart surgery

$P(A|B) = 81\% > P(A|B^c) = 80\%$

$P(A|B, C) < P(A|B^c, C)$

$P(A|B, C^c) < P(A|B^c, C^c)$

C is a confounder