Lecture 14

$-Z \sim N(0,1)$ (symmetry) Normal

Let $X = \mu + \sigma Z$ - μ (mean, location) $\sigma > 0$ (SD, scale)

Then we say $X \sim N(\mu, \sigma^2)$

$E(X) = \mu, Var(X) = \sigma^2 Var(Z)$

$Var(X + c) = Var(X)$

$Var(cX) = c^2 Var(X)$

$Var(X) \geq 0, Var(X) = 0$, if and only if $P(X = a) = 0$, for some a

$Var(X + Y) \neq Var(X) + Var(Y)$ in general , Var not linear

[equal if X,Y are independent]

$Var(X + X) = Var(2X) = 4Var(X)$

$Z = \frac{X-\mu}{\sigma}$ standard

Find PDF of $X \sim N(\mu, \sigma^2)$

CDF: $P(X \leq x) = P(\frac{X-\mu}{\sigma} \leq \frac{x-\mu}{\sigma}) = \Phi(\frac{X-\mu}{\sigma})$

$-X = -\mu + \sigma(-Z) \sim N(-\mu, \sigma^2)$

Later we'll show if $X_j \sim N(\mu_j, \sigma_j^2)$ indep,

$X_1 + X_2 \sim N(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$

$X_1 - X_2 \sim N(\mu_1 - \mu_2, \sigma_1^2 + \sigma_2^2)$

68-95-99.7% Rule

LOTUS

X: 0, 1, 2, 3...

X^2: 0, 1, 4 ,9...

$E(X) = \sum_x P(X = x)$

$E(X^2) = \sum_x x^2 P(X = x)$

$X \sim Pois(\lambda)$

$E(X^2) = \sum_{k=0}^{\infty} k^2 e^{-\lambda} \frac{\lambda^k}{k!} = \lambda^2 + \lambda$

$Var(X) = \lambda^2 + \lambda - \lambda^2 = \lambda$

$\sum_{k=0}^{\infty} \frac{\lambda^k}{k!} = e^{\lambda}$ always true

$\lambda \sum_{k=1}^{\infty} \frac{k\lambda^{k-1}}{k!} = \lambda e^{\lambda}$

$\sum_{k=1}^{\infty} \frac{k^2\lambda^{k-1}}{k!} = (\lambda+1)e^{\lambda}$

$X \sim Bin(n,p)$ Find Var(X)

$X = I_1 + .. I_n, I_j \sim Bern(p)$

$X^2 = I_1^2 + .. + I_n^2 + 2I_1I_2 + .. + 2I_{n-1}I_n$

$E(X^2) = nE(I_1^2) + 2\binom{n}{2}E(I_1I_2)$ indicator of success on both trials 1,2

$= np + n(n-1)p^2 = np + n^2p^2 - np^2$

$Var(X) = np - np^2 = np(1-q) = npq, q = (1-p)$

Prove LOTUS for discrete sample space

$E(g(x)) = \sum g(x)P(X=x)$

group $\sum_x g(x)P(X=x) = \sum_{s \in S} g(X(s))P(\{s\})$ ungrouped