Lecture 9

CDF

$F(x) = P(X \leq x)$, as a function of real x

$P(a < X < b) = F(b) - F(a)$

Properties of CDF

1. increasing
2. right continuous
3. $F(x) \rightarrow 0 \; as \; x \rightarrow -\infty, \; F(x) \rightarrow 1 \; as \; x \rightarrow \infty$

This is "only if"

Independence of r.v.s

X, Y are independent r.v.s if $P(X \leq x, Y \leq y) = P(X \leq x)P(Y \leq y)$ for all x, y

Discrete case : $P(X = x, Y = y) = P(X = x)P(Y = y)$

Average(Means, Expected Values)

<u>Example</u>

1,2,3,4,5,6, -> 1+2+..+6 / 6 = 3.5

1,1,1,1,1,3,3,5

two ways:

1. add, divide

1. 5/8 * 1 +2/8 * 3 + 1/8 * 5

Average of a discrete r.v.s

$E(X) = \sum xP(X = x)$

summed over x with $P(X = x) > 0$

X~ Bern(p)

$E(x) = 1P(X = 1) + 0P(X = 0) = p$

X = 1 if A occurs , 0 otherwise (indicator r.v.s)

Then $E(x) = P(A)$ **fundamental bridge** between E and P

X~Bin(n,p)

$E(X) = \sum_{k=0}^{n} k\binom{n}{k}p^k q^{n-k}$

$= \sum_{k=1}^{n} n\binom{n-1}{k-1} p^k q^{n-k}$

$= np\sum_{k=0}^{n} \binom{n-1}{k-1} p^{k-1} q^{n-k}$

$= np\sum_{j=0}^{n-1} \binom{n-1}{j} p^j q^{n-j-1}$

$= np$

Linearity

1. $E(X+Y) = E(X) + E(Y)$ even if X, Y are dependent
2. $E(cX) = cE(X)$

Redo Bin

$E(X) = np$ by linearity since $X = x_1 + .. + x_n$

Ex. 5 Card hand ,$X = \#aces$ let X_j be indecator of jth card being as ace, $1 \leq j \leq 5$

$E(X) = (indicator)E(X_1 + .. + X_5) = (linearity)E(X_1) + .. + E(X_5) = (symmetry)5E(X_1)$

$= (fundamental\ bridge)5P(1st\ card\ ace) = 5/13$

even though X_j's are dependent

This gives expected value of any Hypergeometric

Geometric

$Geom(p)$: independent $Bern(p)$ trials, count # failures before 1st success.

Let $X \sim Geom(p)$, $q = 1 - p$

PMF:

$P(X = k) = q^k p$ valid since $\sum_{k=0}^{\infty} pq^k = p/1 - q = 1$

$E(x) = \sum_{k=0} kpq^k$

$= p\sum_{k=1} kq^k$

$= q/p$

E(X) Story Proof:

Let $c = E(X)$,

$c = 0 * p + (1 + c)q$

$= q + cq => c = q/p$