Lecture 10

Linearity

Let $T = X + Y$, show $E(T) = E(X) + E(Y)$

$\sum_t P(T = t) = \sum_x xP(X = x) + \sum_y yP(Y = y)$

Extreme dependent $X = Y$

$E(X + Y) = E(2X) = 2E(X)$

Negetive Binomial

parameters r, p

story: independent $Bern(p)$ trials #failures before the rth success

PMF: $P(X = n) = \binom{n+r-1}{r-1} p^r (1-p)^n$

E(X): $E(X) = E(X*1+\dots +X_r) = E(X*1) +\dots +E(X_r) = rq/p

X_j is #failures between $(j-1)$th and jth success, $X_j \sim Geom(p)$

Geom

$X \sim FS(p)$ time until 1st success , counting the success

Let $Y = X - 1$, Then $Y \sim Geom(p)$

$E(X) = E(Y + 1) = E(Y) + 1 = q/p + 1 = 1/p$

Putnam

Random permutation of $1, 2, ..n$, where $n \geq 2$

Find expected # of local maxima. Ex. **3**214**7**5**6**

Let I_j be indecator r.v of position j having a local max, $1 \leq j \leq n$

$E(I_1 + \cdots + I_n) = E(I_1) + \cdots + E(I_n) = \frac{n-2}{3} + 2/2 = \frac{n+1}{3}$

St.Petersburg Paradox

Get 2^x where X is #filps of fair coin until first H, including the success

$Y = 2^x$ find $E(Y)$

$E(Y) = \sum_{k=1}^{\infty} 2^k \frac{1}{2^k} = \sum 1 = \infty$

bound at 2^{40} . Then $\sum_{k=1}^{40} 2^k \frac{1}{2^k} = 40$

$E(2^x) = \infty$ not $q = 2^{E(x)} = 4$