Lecture 15

Coupon collector(Toy collector)

n toy types, equally likely, find expected time until have complete set

$T = T_1 + T_2 + \dots T_n$

T_1 =(time until 1st new toy) = 1

T_2 = (Addtional time until 2nd new toy)

T_3 = (.....until 3rd)

T_1 = 1

$T_2 - 1 \sim Geom((n-1)/n)$

$T_j - 1 \sim Geom(\frac{n-(j-1)}{n})$

$E(T) = E(T_1) + E(T_2) + \dots E(T_n) = 1 + n/(n-1) + n/(n-2) + n/1 = n(1 + 1/2 + \dots + 1/n)$

Universality

$X \sim F$

$F(x_0) = 1/3$

$P(F(X) \le 1/3) = P(X \le x_0) = F(x_0) = 1/3$

Logistic Distribution

$$F(x) = \frac{e^x}{1+e^x}$$

$U \sim Unif(0,1)$, consider $F^{-1}(U) = \log \frac{U}{1-U}$ is logistic

Let X, Y, Z ,be i.i.d positive r.v.s Find $E(\frac{X}{X+Y+Z})$

$E(\frac{X}{X+Y+Z}) = E(\frac{Y}{X+Y+Z}) = E(\frac{Z}{X+Y+Z})$ by symmetry

$E(\frac{X}{X+Y+Z}) + E(\frac{Y}{X+Y+Z}) + E(\frac{Z}{X+Y+Z}) = E(\frac{X+Y+Z}{X+Y+Z}) = 1$ by linearity

$E(\frac{X}{X+Y+Z}) = 1/3$

LOTUS

$U \sim Unif(0,1)$ $X = U^2, Y = e^x$

Find E(Y) as an integral

$E(Y) = \int_0^1 e^x f(x) dx$ f(x) PDF of x, need more work

$P(U^2 \leq x) = P(U \leq \sqrt{x}) = \sqrt{x}, 0 < x < 1$

Better: $Y = e^{U^2}$ $E(Y) = \int_0^1 e^{U^2} dU$

$X \sim Bin(n, p), q = 1 - p$

find distribution of n-X

$P(n - X = k) = P(X = n - k) = \binom{n}{n-k} p^{n-k} q^k = \binom{n}{k} q^k p^{n-k}$

story: $n - X \sim Bin(n, q)$

by swapping ''success and failure"

Ex #emails I get in time t is $Pois(\lambda t)$

Find PDF of T, time of 1st email.

$P(T > t) = P(N_t = 0) = e^{-\lambda t} (\lambda t)^0 / 0! = e^{-\lambda t}$

with N_* = (#emails in [0, *])

CDF is $1 - e^{-\lambda t}, t > 0$

Distribution is the blueprint, for creating random variable, that was our random house and then don't confuse random variable with a constant

constant would be like a specific house

the random vairable is that the random house

$f(x) = \frac{1}{2} x^{-\frac{1}{2}}, x \in (0, 1)$