Lecture 8

Binomial Distribution

$X \sim Bin(n, p)$

1. Story: X is #sucess in n **independent** Bern(p) trials
2. sum of indicator : $X = X_1 + X_2 + \ldots + X_n$;

   $X_j = 1$ if jth trial success, 0 otherwise

   i.i.d.Bern(q) => independent identically distributed
3. **PMF** $P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$

CDF

$X \leq x$ is an event

$F(x) = P(X \leq x)$

then F is the CDF of X (**cummulative distribution function**)

PMF(for discrete r.v.s)

Discrete: possible values $a_1, a_2, \ldots, a_n$ could be listed out

$P(X = a_j)$ for all $j = p_j$

$p_j \geq 0$, $\sum p_j = 1$

$X \sim Bin(n, p), Y \sim Bin(m, p) \Rightarrow X + Y \sim Bin(n + m, p)$

Proof:

1. immediate from story
2. $X = x_1 + \ldots + x_n, Y = y_1 + \ldots + y_m \Rightarrow X + Y = \sum x + \sum y$

   sum of n+m i.i.d Bern(p) => Bern
3. $P(X + Y = k) = \sum_{j=0}^{k} P(X + Y = k | X = j) P(X = j)$

   $= \sum_{j=0}^{k} P(Y = k - j | X = j) \binom{n}{j} p^j q^{n-}j$

   $= \sum_{j=0}^{k} \binom{m}{k-j} p^{k-j} q^{m-k-j} \binom{n}{j} p^j q^{n-j}$

   $= p^k q^{m+n-k} \sum_{j=0}^{k} \binom{m}{k-j} \binom{n}{j}$

   VanderMonde $\sum_{j=0}^{k} \binom{m}{k-j} \binom{n}{j} = \binom{m+n}{k}$

   convolution

<u>Ex.</u> 5 card hand find distribution of #aces - PMF(or CDF)

let X = (#aces) find $P(X = k)$,

$$P(X=k)=\frac{\binom{4}{k}\binom{48}{5-k}}{\binom{52}{5}}, for\ k \in \{0,1,2,3,4\}$$

Not Binomial. Like the elk problem(homework)

Hypergeometric

Story: Have b black, w white marbles. Pick simple random sample of size n.

Find distribution of (# white marbles in sample) = X

$$P(X=k)=\frac{\binom{w}{k}\binom{b}{n-k}}{\binom{w+b}{n}}, 0 \le k \le w, 0 \le n-k \le b$$

Hypergeometric sampling without replace

$$\frac{1}{\binom{w+b}{n}}\sum_{k=0}^{w}\binom{w}{k}\binom{b}{n-k}=1$$

CDF $P(X \le x)$