Lecture 13

Universality of Unif

let F be a cont. strictly increasing CDF

Then $X = F^{-1}(U) \sim F$ if $U \sim unif(0,1)$

Also: if $X \sim F$, then $F(X) \sim Unif(0,1)$

$F(x) = P(X \leq x)$ ~~F(X) = P(X \leq X) = 1~~

Ex. Let $F(x) = 1 - e^{-x}, x > 0$ (Expo(1)), $U \sim Unif(0,1)$

simulate X~F. $F^{-1}(u) = -\ln(1-u)$ => $F^{-1}(U) = -\ln(1-U) \sim F$

$1 - U \sim Unif(0,1)$ symmetry of Unif

$a + bU$ is Unif on some interval. Nonlinear usually -> Non Unif.

Independent of r.v.s

$X_1, \ldots X_n$

Definiton:

$X_1, \ldots X_n$ independent if $P(X_1 \leq x_1, \ldots X_n \leq x_n) = P(X_1 \leq x_1) \ldots P(X_n \leq x_n)$

for all $x_1, \ldots x_n$

Discrete case

joint PMF $P(X_1 = x_1, \ldots X_n = x_n) = P(X_1 = x_1) \ldots P(X_n = x_n)$

Example

$X_1, X_2 \sim Bern(1/2)$ i.i.d, $X_3 = 1$ if $X_1 = X_2$; 0 otherwise

These are pairwise indep, not indep

Normal Distribution 正态分布

(Central Limit Therom: sum of a lot of i.i.d r.v.s looks Normal)

$N(0,1)$ - mean = 0, var = 1

has PDF

$$f(z) = ce^{\frac{z^2}{2}}$$

c - normalizing const, $c = 1/\sqrt{2\pi}$

$Z \sim N(0,1)$

$EZ = 0$ by symmetry odd function

$E(Z^3) = 0$ "3rd moment"

$Var(Z) = E(Z^2) - (EZ)^2 = E(Z^2) = 1$ LOTUS

Notation : Φ is the standard Normal CDF

$$\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{t^2}{x}} \, dt$$

$\Phi(-z) = 1 - \Phi(z)$ by symmetry