Lecture 5

Thinking conditionally is a condition for thinking

How to solve a problem?

1. Try simple and extreme cases
2. Break up problem into simple pieces

   $P(B) = P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + \ldots P(B|A_n)P(A_n)$

   law of total probability

Example 1

Suppose we have 2 random cards from standard deck

Find $P(both\ aces|have\ ace)$, $P(both\ aces|have\ ace\ of\ spade)$

$$P(both\ aces|have\ ace) = P(both\ aces,\ \cancel{have\ ace}\ )/P(have\ ace) = \frac{\binom{4}{2}/\binom{52}{2}}{1-\binom{48}{2}/\binom{52}{2}} = 1/33$$

$$P(both\ aces|have\ ace\ of\ spade) = 3/51 = 1/17$$

Example 2

Patient get tested for disease afflicts 1% of population, tests positve (has disease)

Suppose the test is advertised as "95% accurate" , suppose this means

D: has disease, T: test positive

Trade-off: It's rare that the test is wrong, it's also rare the disease is rare

$$P(T|D) = 0.95 = P(T^c|D^c)$$

$$P(D|T) = \frac{P(T|D)P(D)}{P(T)} = \frac{P(T|D)P(D)}{P(T|D)P(D) + P(T|D^c)P(D^c}$$

Biohazards

1. confusing $P(A|B)$, $P(B|A)$ (procecutor's fallacy)

Ex Sally Clark case, SIDS

want P(innocence |evidence)

1. confusing $P(A) - prior$先验 with $P(A|B) - posterior$后验

P(A|A) = 1

1. confusing independent with conditional independent

Definition:

Events A, B are conditionally independent given C if $P(A \cap B|C) = P(A|C)P(B|C)$

Q:Does conditional indep given C imply indep. ? No

Ex. Chess opponent of unknown strength may be that game outcomes are conditionally independent given strength

Q:Does independent imply conditional independent given C? No

Ex. A: Fire alarm goes off, cause by : F:fire; C:popcorn. suppose F, C independent But $P(F|A, C^c) = 1$ not conditionally indep given A

Recommendations 推荐书籍

Statistical science in the courtroom

Statistics for lawyers