Lecture 7

> Conditioning : the soul of statistics
>
> Random variables and their distribution

Gambler's Ruin

Two gamblers, A and B, sequence of rounds bet $1

$p = P(A\ wins\ a\ certain\ round), q = 1 - p$, the game goes until one bankrupt

what's the probability that A wins entire gmae(so B is "ruined")?

Assume: A start with $\$i$, B starts with $\$(N-i)$

> **Random walk:**
>
> A particle in the i position, move right 1 step or left 1 step
>
> p = probability of going right. Absorbing states: reach 0 or N

A either wins the 1st round or lose it

Strategy: condition on first step

Let $P_i = (A\ wins\ game | A\ start\ at\ \$i)$

$P_i = pp_{i+1} + qp_{i-1}, 1 \leq i \leq N-1$

(difference equation)

$P_0 = 0, P_N = 1$ (boundary)

Solve difference equation

Guess $P_i = x^i$

$x^i = px^{i+1} + qx^{i-1}$

$px^2 - x + q = 0$

$x = \{1, q/p\}$

$p_i = A1^i + B(q/p)^i$

$p_0 = 0, B = -A, P_n = 1 \Rightarrow 1 = A(1 - q/p)^n$

$P_i = \frac{1-(q/p)^i}{1-(q/p)^N}, if\ p \neq q$

$P_i = i/N, if\ p = q$

Random Variable

It's a function from sample space S to R think of a as numerical "summary" of an aspect of the experiment.

Bernoulli

X is said to have Bern Distribution, if X has only 2 possible values , 0 and 1.

$P(X=1)=p, P(X=0)=1-p.$

X = 1 is an event S:X(S) = 1

Binomial (n,p)

The distribution of #success X in n indep Bern(p) trials is called Bin(n, p)

its distribution is given by

$P(X=k)=\binom{n}{k}p^k(1-p)^{n-k}$

$X \sim Bin(n,p), Y \sim Bin(m,p)$ independent

Then $X+Y \sim Bin(n+m,p)$

Proof: consider n trials then m more trials