Lecture 4

Definition: Events A, B are independent if $P(A \cap B) = P(A)P(B)$

Note: completely different form disjointness

A, B, C are independent, if $P(A, B) = P(A)P(B)$, $P(A, C) = P(A)P(C)$, $P(B, C) = P(B)P(C)$ and $P(A, B, C) = P(A)P(B)P(C)$

Similarly for events A1,...An

Newton-Pepys Problem(1693)

> The **Newton–Pepys problem** is a probability problem concerning the probability of throwing sixes from a certain number of dice.
>
> In 1693 Samuel Pepys and Isaac Newton corresponded over a problem posed by Pepys in relation to a wager he planned to make. The problem was:
>
> *A. Six fair dice are tossed independently and at least one "6" appears.*
>
> *B. Twelve fair dice are tossed independently and at least two "6"s appear.*
>
> *C. Eighteen fair dice are tossed independently and at least three "6"s appear.*
>
> Pepys initially thought that outcome C had the highest probability, but Newton correctly concluded that outcome A actually has the highest probability.
>
> Quoted from Wikipedia : Newton–Pepys problem

Answer:

$P(A) = 1 - (5/6)^6 \approx 0.665$

$P(B) = 1 - (5/6)^{12} - 12 * (1/6)(5/6)^{11} \approx 0.619$

$P(C) = 1 - \sum_{k=0}^{2} \binom{18}{k} (1/6)^k (5/6)^{(18-k)} \approx 0.597$

Conditional Probability 条件概率

— How should you update probability/beliefs/uncertainty based on new evidence?

> "Conditioning is the soul of statistic"

Definition:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \text{ if } P(B) > 0$$

Intuition:

1. Pebble world , there are finite possible outcomes, each one is represented as a pebble. For example, 9 outcomes, that is 9 pebbles , total mass is 1. B: four pebbles, $P(A|B)$:get rid of pebbles in B^c , renormalize to make total mass again

2. Frequentist world: repeat experiment many times

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   circle repeatitions where B occurred ; among those , what fraction of time did A also occur?

Theorem

1. $P(A \cap B) = P(B)P(A|B) = P(A)P(B|A)$
2. $P(A_1 \dots A_n) = P(A_1)P(A_2|A_1)P(A_3|A_1, A_2) \dots P(A_n|A_1, A_2 \dots A_{n-1})$
3. $P(A|B) = \dfrac{P(B|A)P(A)}{P(B)}$