Lecture 19

Marginal distribution 边缘概率分布

Joint CDF $F(x,y) = P(X \le x, Y \le y)$

cont. case(joint PDF) : $f(x,y) = \frac{\partial}{\partial x \partial y}F(x,y)$

$P((X,Y) \in A) = \iint_A f(x,y)dxdy$

Marginal PDF of X: $\int f(x,y)dy$

Conditional PDF of $Y|X$ is

$$f_{Y|X}(y|x) = \frac{f_{X,Y}(x,y)}{f_X(x)} = \frac{f_{X|Y}(x|y)f_Y(y)}{f_X(x)}$$

X, Y independent if $f_{X,Y}(x,y) = f_X(x)f_Y(y)$ for all X, Y

2-D LOTUS

Let (X, Y) have joint PDF $f(x,y)$

and let $g(x,y)$ be a real-valued fn of x, y

Then $Eg(X,Y) = \iint g(x,y)f(x,y)dxdy$

Theorm

If X, Y are indep, then $E(XY) = E(X)E(Y)$

Independent implies uncorrelated

Proof (continuous case)

$E(XY) = \iint xyf_X(x)f_Y(y)dxdy = \int yf_Y(y)\int xf_X(x)dxdy = (EX)(EY)$

Example

X, Y i.i.d $Unif(0,1)$ find $E|X-Y|$

LOTUS $\int_0^1\int_0^1 |x-y|dxdy = \iint_{x>y}(x-y)dxdy + \iint_{x \le y}(y-x)dxdy$

$= 2\int_0^1\int_y^1(x-y)dxdy = 2\int_0^1(x^2/2 - yx)|_y^1 dy = 1/3$

Let $M = max(X,Y)$

$L = min(X,Y)$ (L stand for little and less one not large one)

$|X-Y| = M - L$

$E(M-L) = 1/3$

$E(M) - E(L) = 1/3$

$E(M+L) = E(X+Y) = E(M) + E(L) = 1$

=> $E(M) = 2/3, E(L) = 1/3$

Chicken-egg

some hens some hatch some don't hatch, the eggs are independent

$N \sim Pois(\lambda)$ eggs, each hatches with prob. p, indep, Let X = #hatch

so $X|N \sim Bin(N,p)$

Let Y = # don't hatch, so $X + Y = N$

Find joint PMF of X,Y

$P(X=i, Y=j) = \sum P(X=i, Y=j|N=n)P(N=n)$

$= P(X=i, Y=j|N=i+j)P(N=i+j)$

$= P(X=i|N=i+j)P(N=i+j) = \frac{(i+j)!}{(i!j!)} p^i q^j \frac{e^{-\lambda}\lambda^{i+j}}{(i+j)!}$

$= (e^{\lambda p}\frac{(\lambda p)^i}{i!})(e^{\lambda q}\frac{(\lambda q)^j}{j!})$

=> X, Y are indep, $X \sim Pois(\lambda p), Y \sim Pois(\lambda q)$

More Details: [Chicken and Egg (Probability) Problem