Lecture 3

Birthday Problem

(Exclude Feb 29, assume 365 days equally likely, assume indep. of birth)

k people , find prob. that two have same birthday

If k > 365, prob. is 1

Let k <= 365, $P(nomatch) = \frac{365*364*...(365-k+1)}{365^k}$

P(match) ~ 50.7%, if k = 23; 97% if k = 50; 99.9999%, if k = 100

$\binom{k}{2} = \frac{k(k-1)}{2}$ $\binom{23}{2} = 253$

Properties of Probability

1. $P(A^c) = 1 - P(A)$
2. If $A \subseteq B$, then $P(A) \subseteq P(B)$
3. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

Proof:

1. $1 = P(S) = P(A \cap A^c) = P(A) + P(A^c)$
2. $B = A \cup (B \cap A^c)$ $P(B) = P(A) + P(B \cap A^c)$
3. $P(A \cup B) = P(A \cap (B \cap A^c)) = P(A) + P(B \cap A^c)$

General case:

deMontmort's Problem(1713)

matching problem

n cards labeled 1 to n, flipping cards over one by one, you win if the card that you name is the card that appears.

Let A_j be the event, ''jth card matches"

$P(A_j) = 1/n$ since all position equally likely for card labeled j

$P(A_1 \cap A_2) = (n-2)!/n! = 1/n(n-1)$

...

$P(A_1 \cap \ldots A_k) = (n-k)!/n!$

$P(A_1 \cup \ldots A_n) = n * 1/n - n(n-1)/2 * 1/n(n-1) + \ldots$

$= 1 - 1/2! + 1/3! - 1/4! \ldots (-1)^n 1/n! \approx 1 - 1/e$