

TIME-DEP. STOKES

removed for $Re = \frac{c \cdot h}{\nu} \ll 1$ ($\nu \gg 1$)

$$(P) : \begin{cases} \frac{\partial u}{\partial t} - \nu \Delta u + (u \cdot \nabla) u + \nabla p = F & \text{in } \Omega \\ \operatorname{div} u = 0 & \text{in } \Omega \\ u(x, t) = g & \text{on } \Gamma_D \\ \nu \frac{\partial u}{\partial n} - p \bar{n} = h & \text{on } \Gamma_N \end{cases}$$

$$(PW) : \begin{cases} \int_{\Omega} \frac{\partial u}{\partial t} + \int_{\Omega} \nu \nabla u \nabla v + \int_{\Omega} p \cdot \operatorname{div}(v) = \int_{\Omega} f v + \int_{\Omega} \phi v & \forall v \in V \\ \int_{\Omega} q \operatorname{div}(u) = 0 & \forall q \in Q \end{cases}$$

$$a(u, v) = \int_{\Omega} \nu \nabla u \nabla v \quad F(v) = \int_{\Omega} f v + \int_{\Omega} \phi v$$

$$b(q, u) = \int_{\Omega} q \operatorname{div}(u)$$

$$B^T(v, p) = \int_{\Omega} p \operatorname{div}(v)$$

(semi-linearisation)

$$(\theta\text{-method}) \begin{cases} \left(\frac{1}{\Delta t} M + \theta A \right) u^n + \theta B^T p^n = \theta F(t_n) + (1-\theta) F(t_{n-1}) + \left(\frac{1}{\Delta t} M - (1-\theta) A \right) u^{n-1} - (1-\theta) B^T p^{n-1} - (u^{n-1} \cdot \nabla) u^{n-1} \cdot \bar{v} \\ B u = 0 \end{cases}$$