Time-Dep. Stokes

$$(P):\begin{cases} \dfrac{du}{dt} - \nu\Delta u + \overbrace{(u\cdot\nabla)u}^{\text{removed for } Re=\frac{c\cdot h}{\nu}\ll 1\ (\nu\gg 1)} + \nabla p = F & \text{in } \Omega \\ \operatorname{div} u = 0 & \text{in } \Omega \\ u(x,t) = g & \text{on } \Gamma_D \\ \nu \dfrac{du}{d\bar n} - p\bar n = h & \text{on } \Gamma_N \end{cases}$$

$$(PW):\begin{cases} \displaystyle\int_\Omega \frac{du}{dt} + \int_\Omega \nu\,\nabla u\nabla v + \int_\Omega p\cdot \operatorname{div}(v) = \int_\Omega f v + \int_\Omega \phi v & \forall v\in V \\ \displaystyle\int_\Omega q\, \operatorname{div}(u) = 0 & \forall q\in Q \end{cases}$$

$$a(u,v) = \int_\Omega \nu\,\nabla u\nabla v \qquad F(v) = \int_\Omega f v + \int_\Omega \phi v$$

$$b(q,u) = \int_\Omega q\,\operatorname{div}(u)$$

$$b^T(v,p) = \int_\Omega p\,\operatorname{div}(v)$$

$$(\theta\text{-method})\begin{cases} \left(\frac{1}{\Delta t}M + \theta A\right)u^n + \theta B^T p^n = \theta F(t_n) + (1-\theta)F(t_{n-1}) + \left(\frac{1}{\Delta t}M - (1-\theta)A\right)u^{n-1} - (1-\theta)B^T p^{n-1} \\ Bu = 0 \end{cases}$$