Nonlinear Diffusion

$$R(u,v) = \int_\Omega \left(\mu_0 + \mu_1 u^2\right) \nabla u \nabla v + \int_\Omega \bar{b} \cdot \nabla u v + \int_\Omega \sigma u v - F(v)$$

$$R(u+\delta, v) = \int_\Omega \left(\mu_0 + \mu_1 (u+\delta)^2\right) \nabla(u+\delta) \nabla v + \int_\Omega \bar{b} \cdot \nabla u v + \bar{b} \cdot \nabla \delta v + \int_\Omega \sigma u v + \sigma \delta v - F(v)$$

$$R(u+\delta, v) - R(u,v) = \int_\Omega (2\mu_1 u \delta) \nabla u \nabla v + \int_\Omega (\mu_0 + \mu_1 u^2) \nabla \delta \nabla v + \underbrace{\int_\Omega \bar{b} \nabla \delta v + \int_\Omega \sigma \delta v}_{\text{Normal addition to matrix as in standard elliptic.}} + o\left(\|\delta\|^2\right)$$

NOTE: other non-linearities lead to diff. result!