1.1) $V = \{ v \in H^1(\Omega) : v|_{\Gamma_0} = 0 \}$

$$u = u_0 + R_g \quad , \quad R_g : \Omega \to \mathbb{R} \quad s.t. \quad R_g = g \text{ on } \Gamma_D$$

(PW) Find $u_0 \in V$ s.t

$$\Rightarrow \int_\Omega -\mu \Delta u \cdot v \, dx + \int_\Omega \bar{b} \cdot \nabla u \cdot v \, dx + \int_\Omega \sigma u v \, dx = \int_\Omega f v \, dx$$

$$\Rightarrow \int_\Omega \mu \nabla u \cdot \nabla v \, dx - \int_{\Gamma_N} \mu \nabla u \bar{n} \cdot v \, dx - \int_\Omega \bar{b} u \cdot \nabla v \, dx + \int_{\Gamma_N} b u \bar{n} v \, dx + \int_\Omega \sigma u v \, dx = \int_\Omega f v \, dx$$

$$\Rightarrow \int_\Omega \mu \nabla u \nabla v \, dx - \int_{\Gamma_N} (\mu \nabla u \bar{n} - \bar{b} \bar{n} u) v \, dx - \int_\Omega \bar{b} u \cdot \nabla v \, dx + \int_\Omega \sigma u v \, dx = \int_\Omega f v \, dx$$

$$\underset{\phi}{\curvearrowleft}$$

$$\Rightarrow \underbrace{\int_\Omega \mu \nabla u \nabla v - \int_\Omega \bar{b} u \cdot \nabla v \, dx + \int_\Omega \sigma u v \, dx}_{a(u,v)} = \underbrace{\int_\Omega f v \, dx + \int_{\Gamma_N} \phi v \, dx}_{F(v)}$$

$$\Rightarrow a(u,v) = F(v) \xrightarrow[u = u_0 + R_g]{} a(u_0, v) = F(v) - a(R_g, v) = \tilde{F}(v)$$

1.6) $\Phi_1 = \phi|_{\Gamma_1}$, $n = \vec{x}$

$$\mu \frac{\partial u}{\partial x} - b \cdot \binom{1}{0} u = \Phi_1 \quad \text{on } \Gamma_1$$

$$\mu - b_1(x+y) = \phi_1$$

$$\phi_1 = \mu - b_1(1+y)$$

$$\Phi_3 = \phi|_{\Gamma_3}$$

$$\mu \frac{\partial u}{\partial y} - b \cdot \binom{0}{1} u = \phi_3$$

$$\mu - b_2(x+y) = \phi_3$$

$$\phi_3 = \mu - b_2(x+1)$$

$$F = b_1 + b_2 + \sigma(x+y)$$

$$g(x,y) = u_{ex}(x,y) \quad \text{on } \Gamma_D$$

$$= x + y$$

$$\text{div}(\nabla u) = \frac{\partial^2 u}{\partial x} + \frac{\partial^2 u}{\partial y} = 0 \; ; \; -\text{div}(\mu \nabla u) = 0$$

$$\nabla u = \binom{1}{1} \qquad ; \; \text{div}(bu) = \bar{b} \cdot \nabla u = b_1 + b_2$$

$$u = x + y \qquad ; \; \sigma u = \sigma(x+y)$$