Introduction: Bayesian vs frequentist data analysis

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A bit about myself

1. Professor of Linguistics at Potsdam
2. Background in Japanese, Computer Science, Statistics
3. Current research interests
   - Computational models of language processing
   - Understanding comprehension deficits in aphasia
   - Applications of Bayesian methods to data analysis
   - Teaching Bayesian methods to non-experts

The main points of this lecture

1. Frequentist methods work well when power is high
2. When power is low, frequentist methods break down
3. Bayesian methods are useful when power is low
4. Why are Bayesian methods to be preferred?
   - answer the question directly
   - focus on uncertainty quantification
   - are more robust and intuitive
5. I illustrate these points with simple examples

The frequentist procedure

Imagine that you have some independent and identically distributed data: $x_1, x_2, \ldots, x_n$

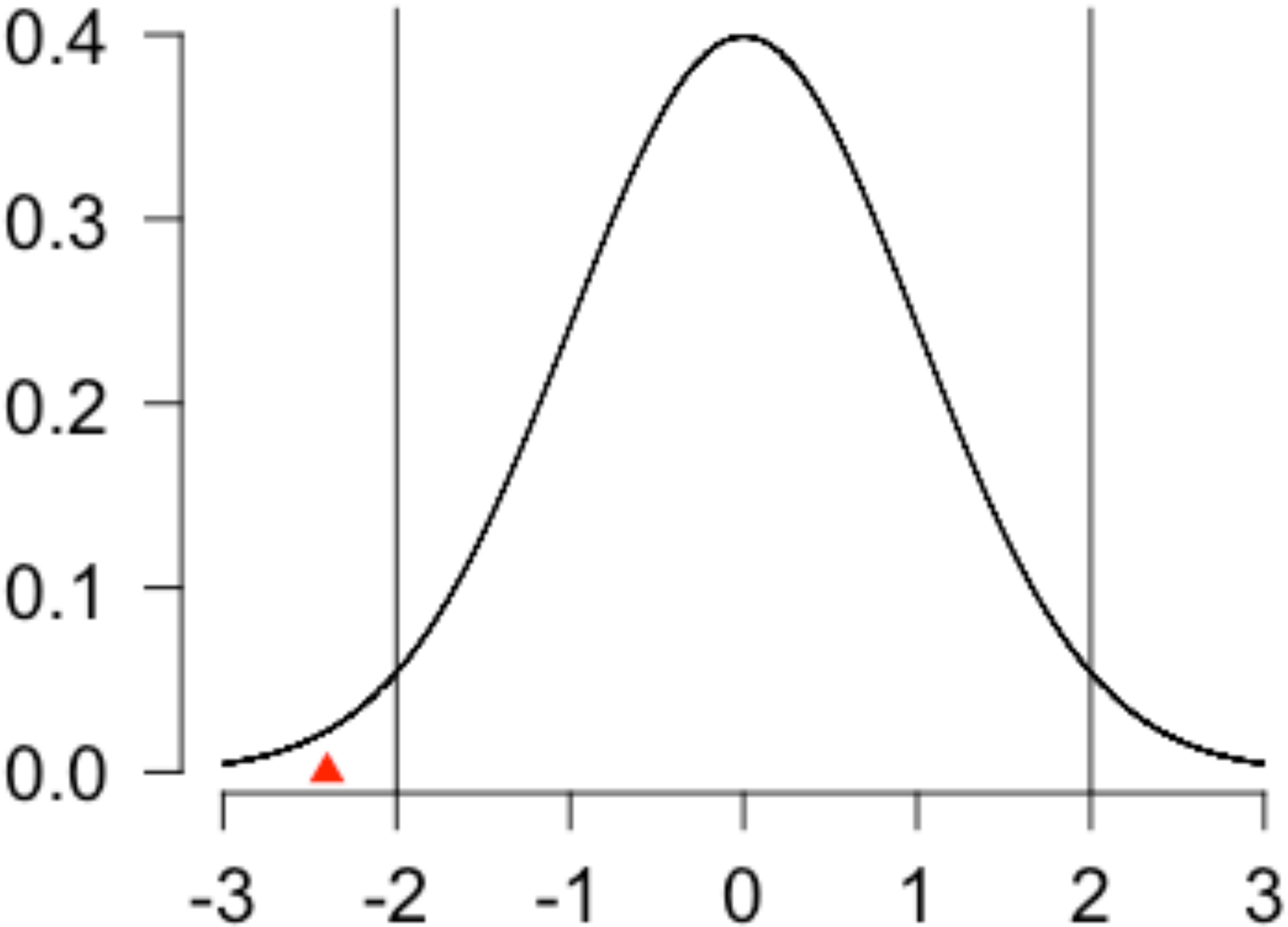
$$X \sim Normal(\mu, \sigma)$$

1. Set up a null hypothesis: $H_0 : \mu = 0$
2. Check if sample mean $\bar{x}$ is consistent with null
3. If inconsistent with null, **accept specific alternative**

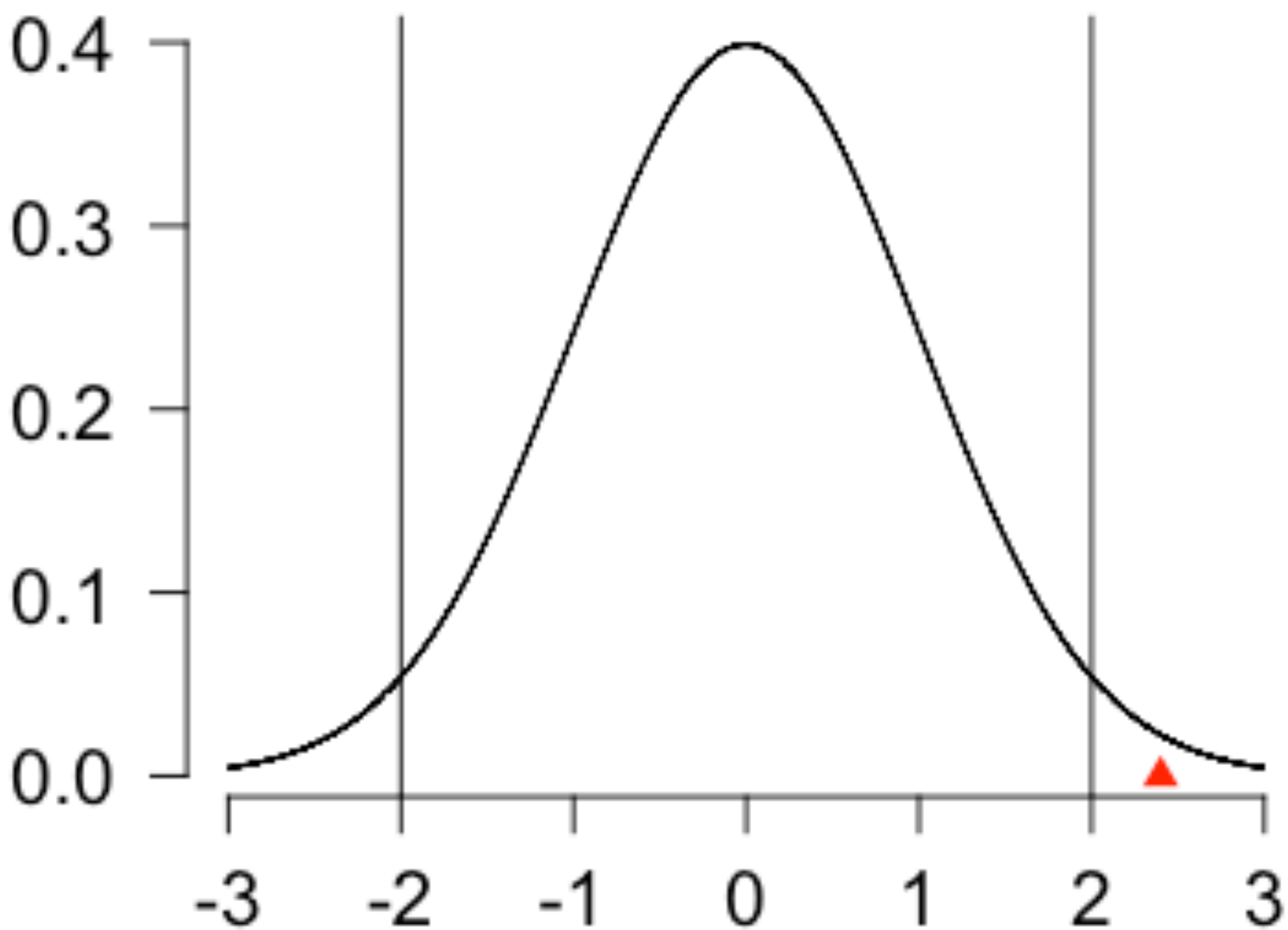
Statistical data analysis is reduced to checking for significance (is $p<0.05$?)

The frequentist procedure

Decision: Reject null and publish

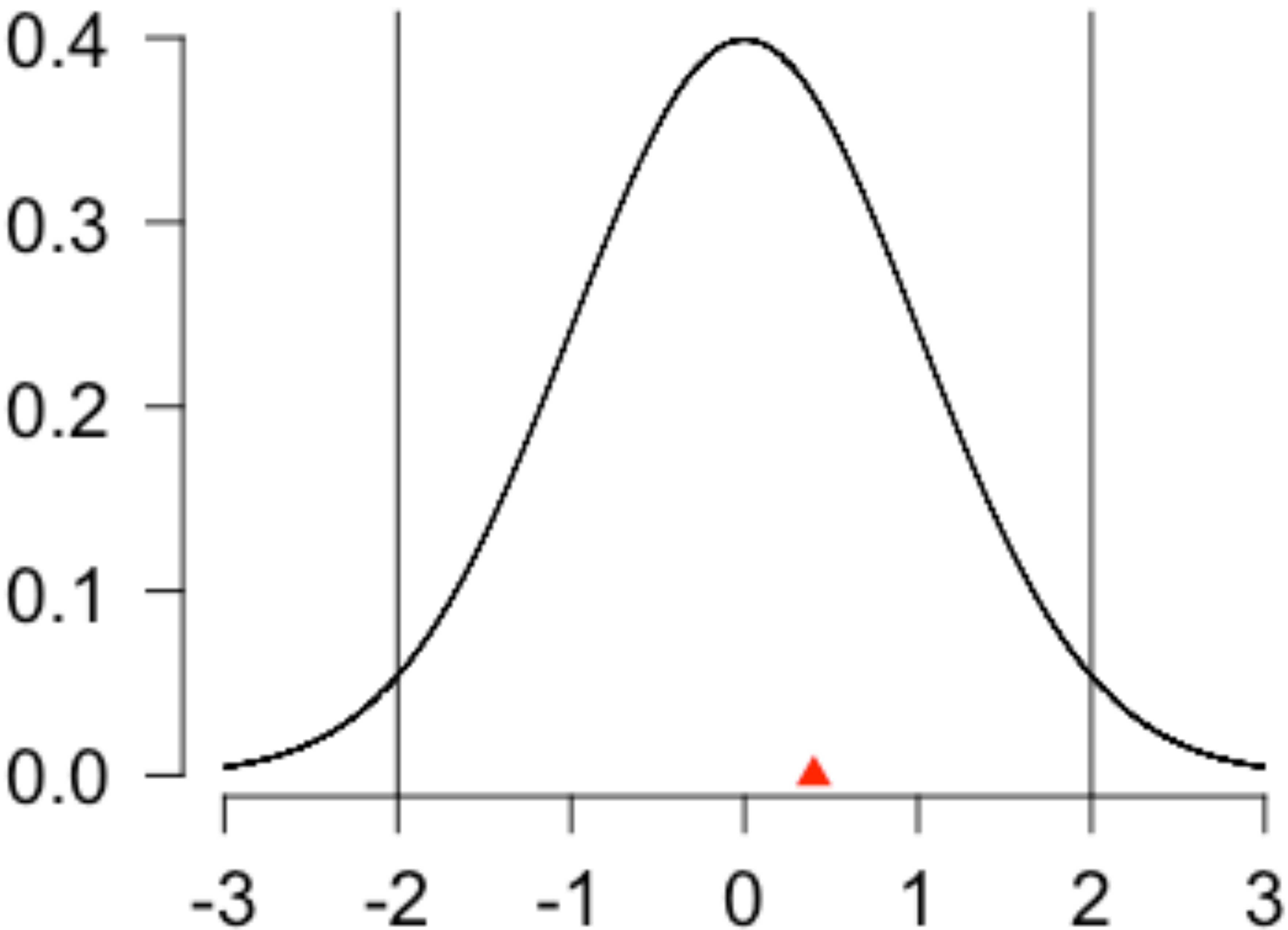


Decision: Reject null and publish



The frequentist procedure

Accept null? Publish or (more likely) put into file drawer



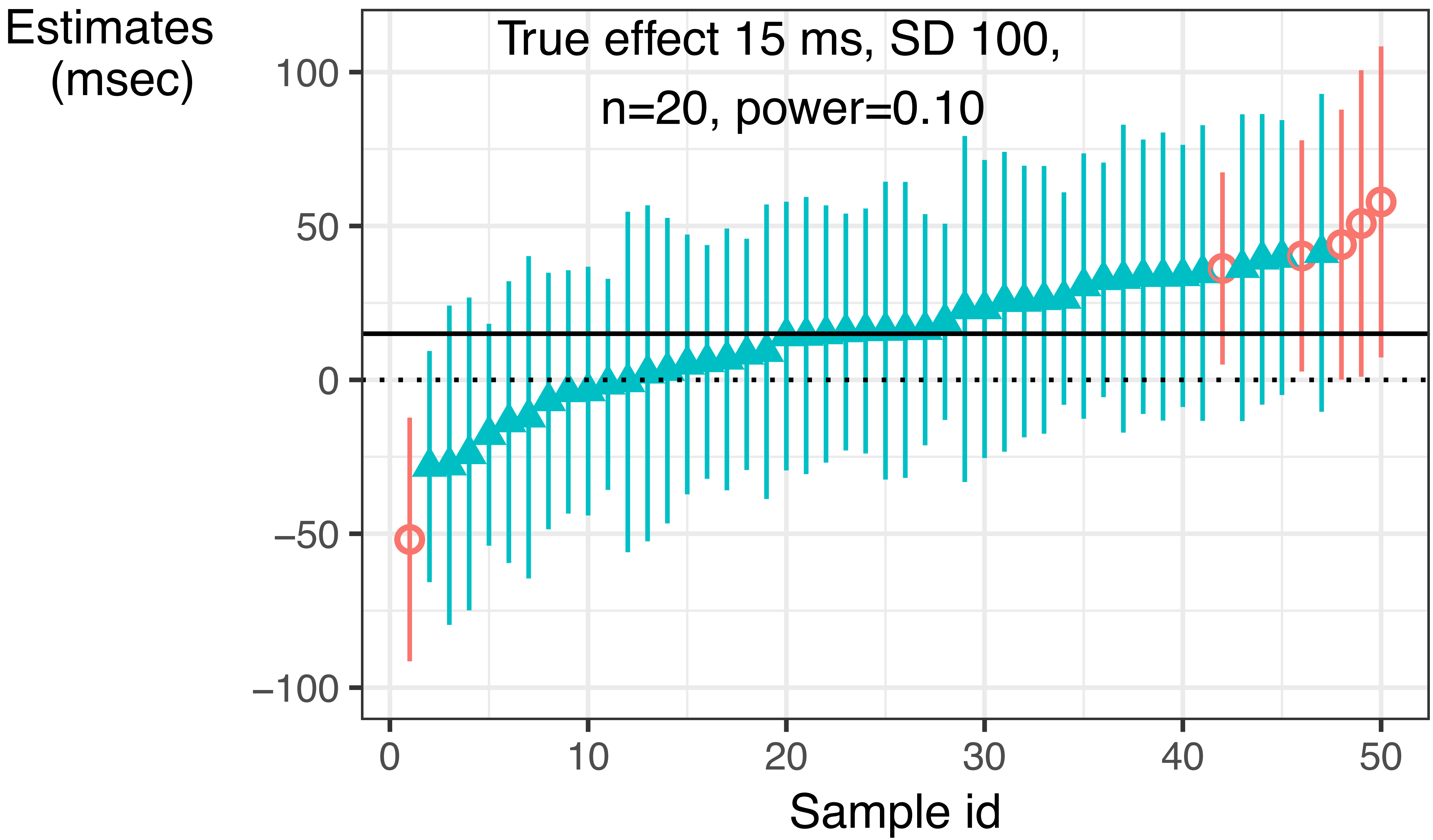
The frequentist procedure

Power: the probability of detecting a particular effect (simplifying a bit)

The frequentist paradigm works when power is high (80% or higher).

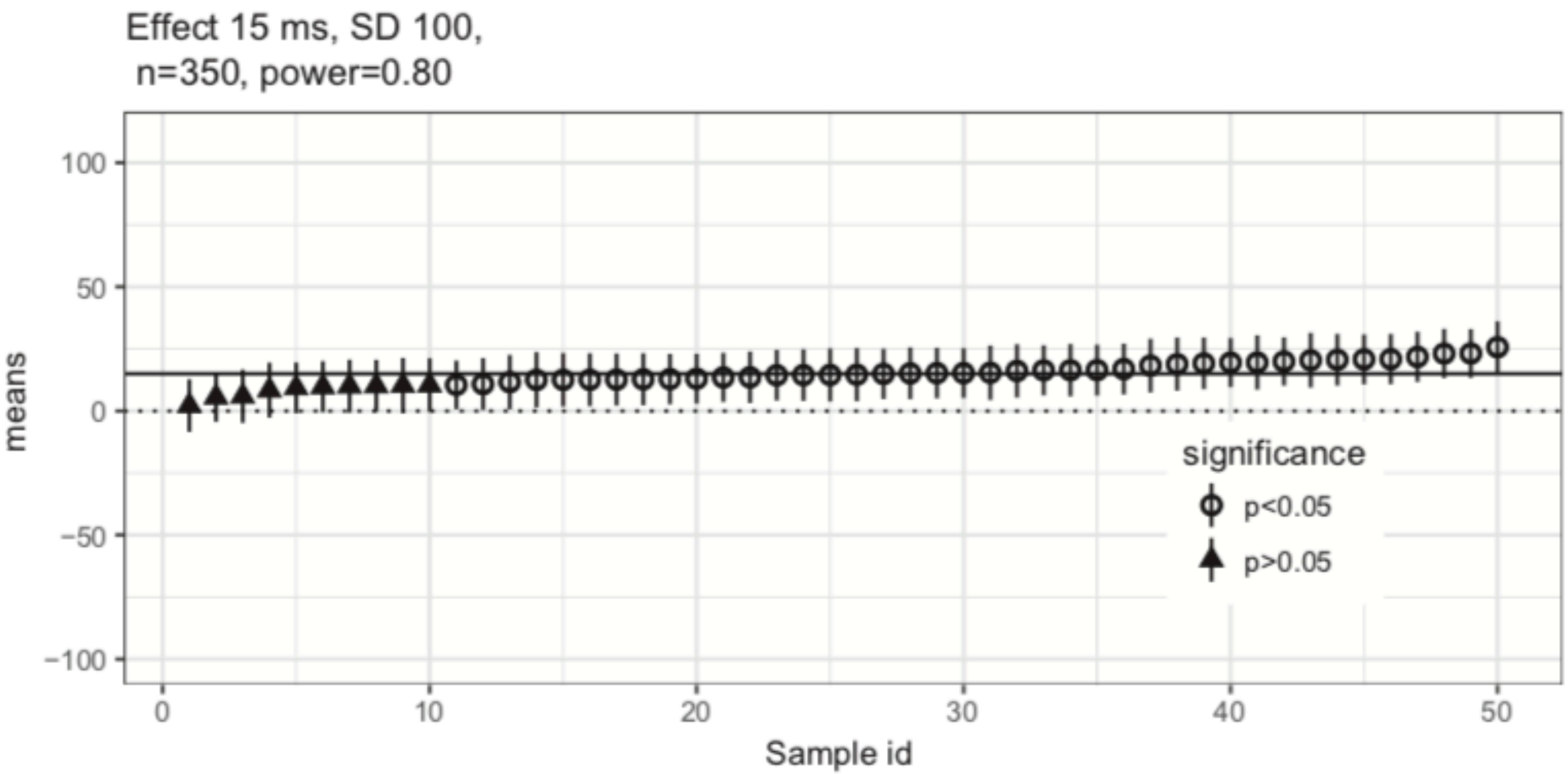
The frequentist paradigm is not designed to be used in low power situations.

Low power leads to exaggerated estimates: Type M error (simulated data)



Gelman & Carlin, 2014

Compare with a high power situation



The frequentist paradigm breaks down when power is low

1. Null results are inconclusive
2. Significant results are based on biased estimates (Type M error)

Consequences:

1. Non-replicable results
2. Incorrect inferences

The frequentist paradigm breaks down when power is low

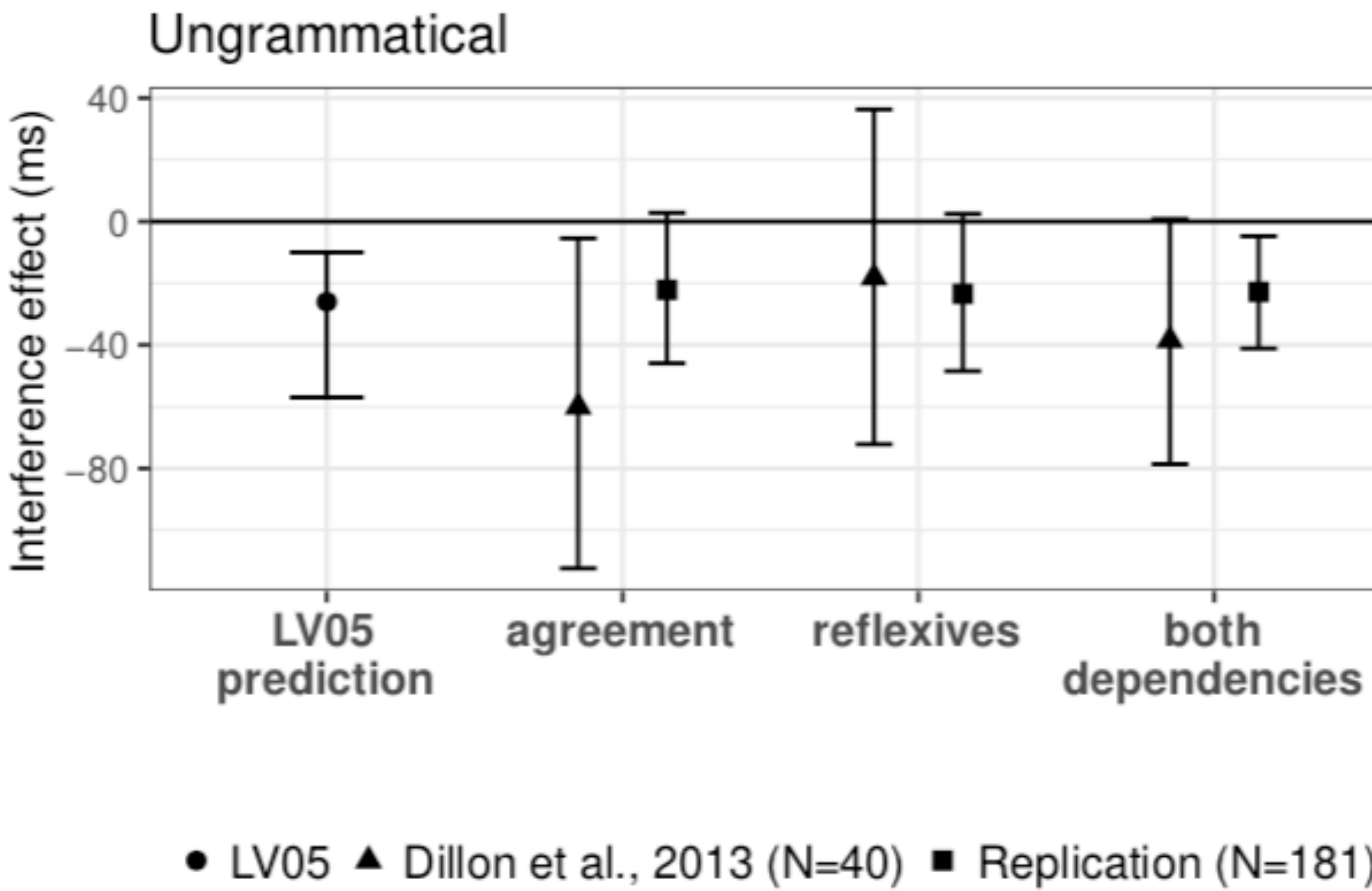
A widely held but incorrect belief:

"A significant result ($p<0.05$) reduces the probability of the null being true"

[switch to shiny app by Daniel Schad]
https://danielschad.shinyapps.io/probnull/

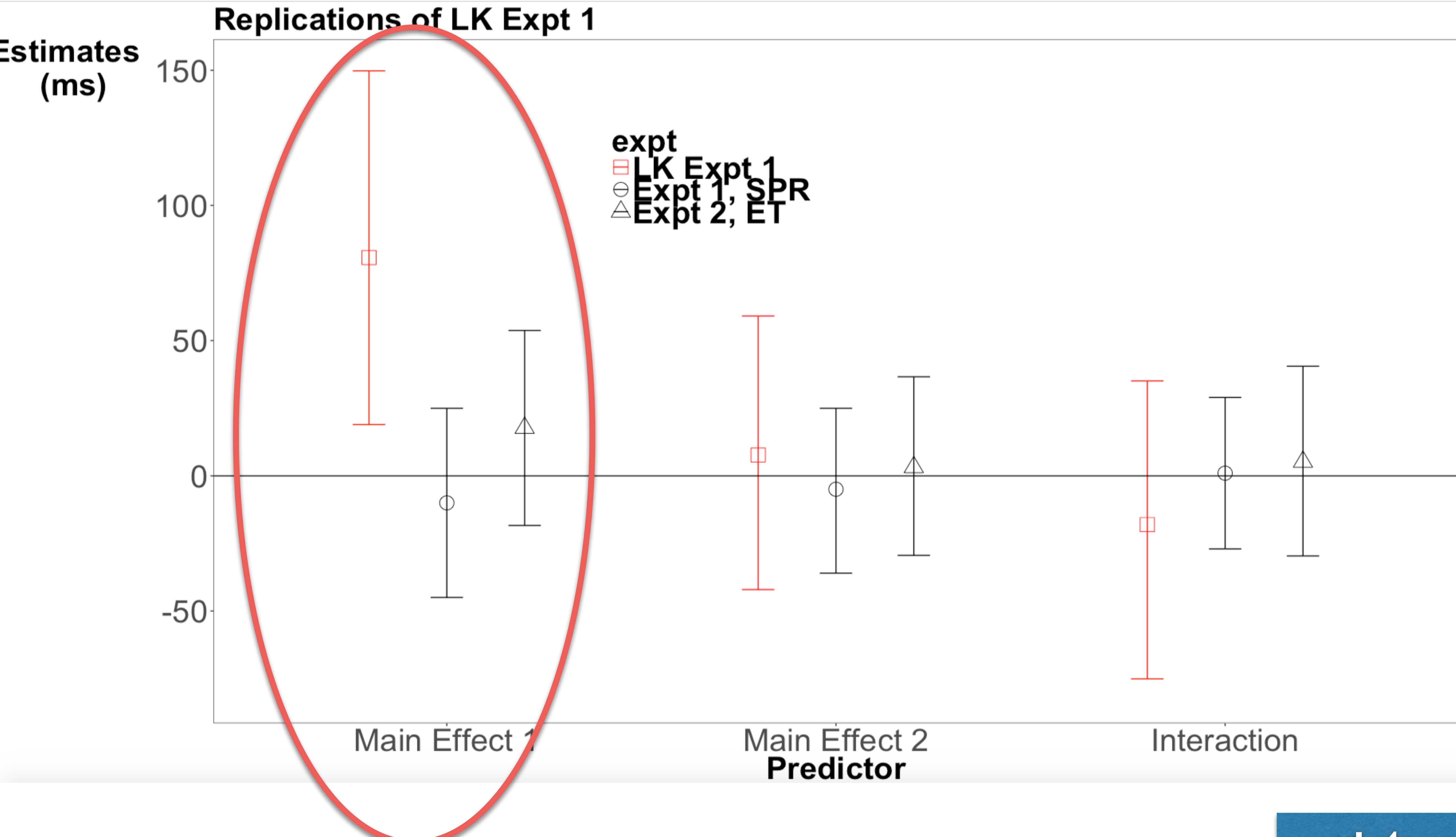
Under low power, even if we get a significant effect, our belief about the null hypothesis should not change much!

Example 1 of a replication of a low-powered study



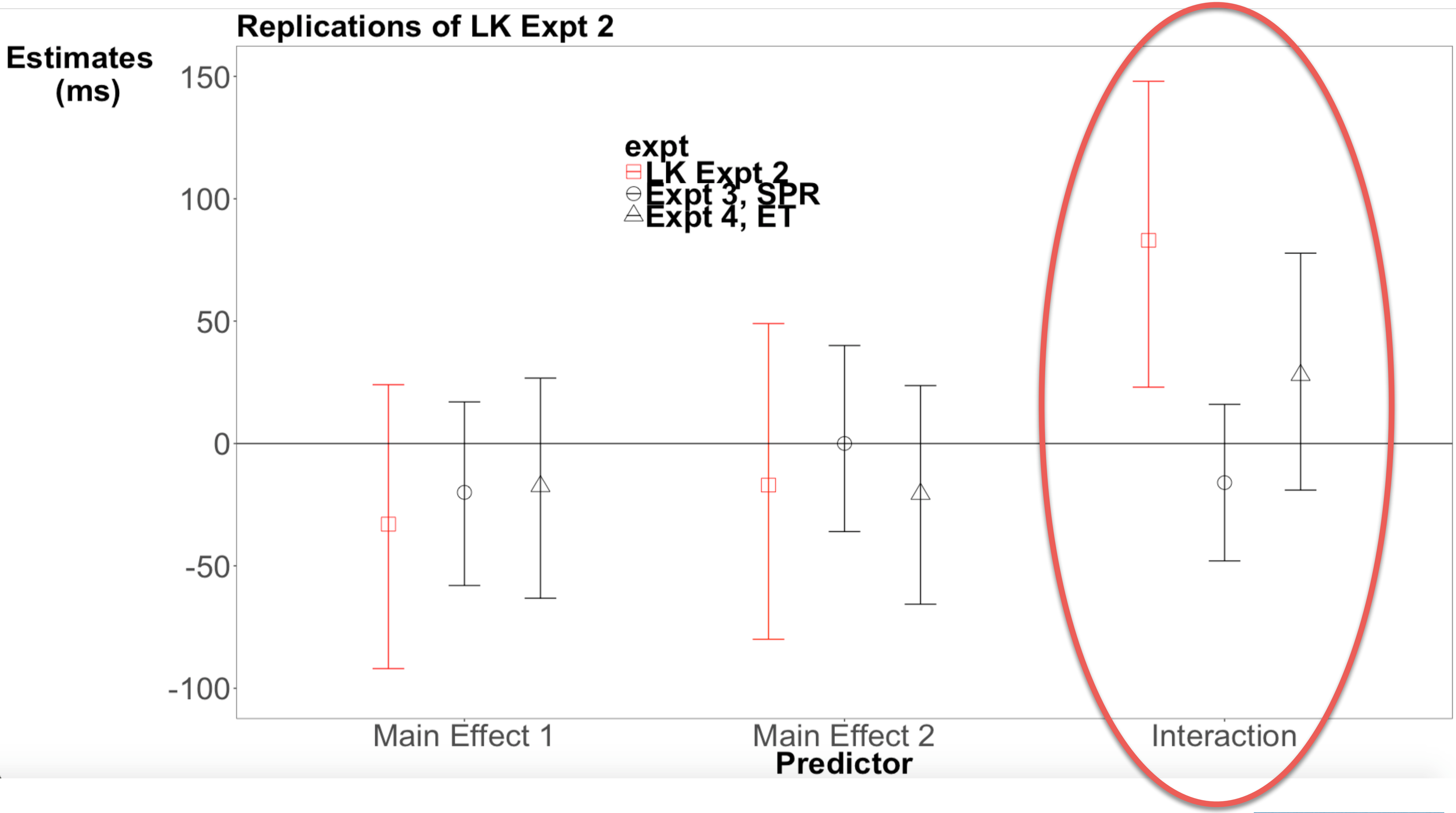
Jäger, Mertzen, Van Dyke & Vasishth, MS, 2018

Example 2 of a replication of a low-powered study



Vasishth, Mertzen, Jäger, Gelman, JML 2018

Example 3 of a replication attempt of a low-powered study



Vasishth, Mertzen, Jäger, Gelman, JML 2018

The Bayesian approach

Imagine again that you have some independent and identically distributed data: $x_1, x_2, \ldots, x_n$

$$X \sim Normal(\mu, \sigma)$$

1. Define **prior distributions** for the parameters μ, σ
2. Derive **posterior distribution** of the parameter(s) of interest using Bayes' rule:

$$\underbrace{f(\mu \mid data)}_{\text{posterior}} \propto \underbrace{f(data \mid \mu)}_{\text{likelihood}} \times \underbrace{f(\mu)}_{\text{prior}}$$

3. Carry out inference based on the posterior

Example: Modeling mortality after surgery

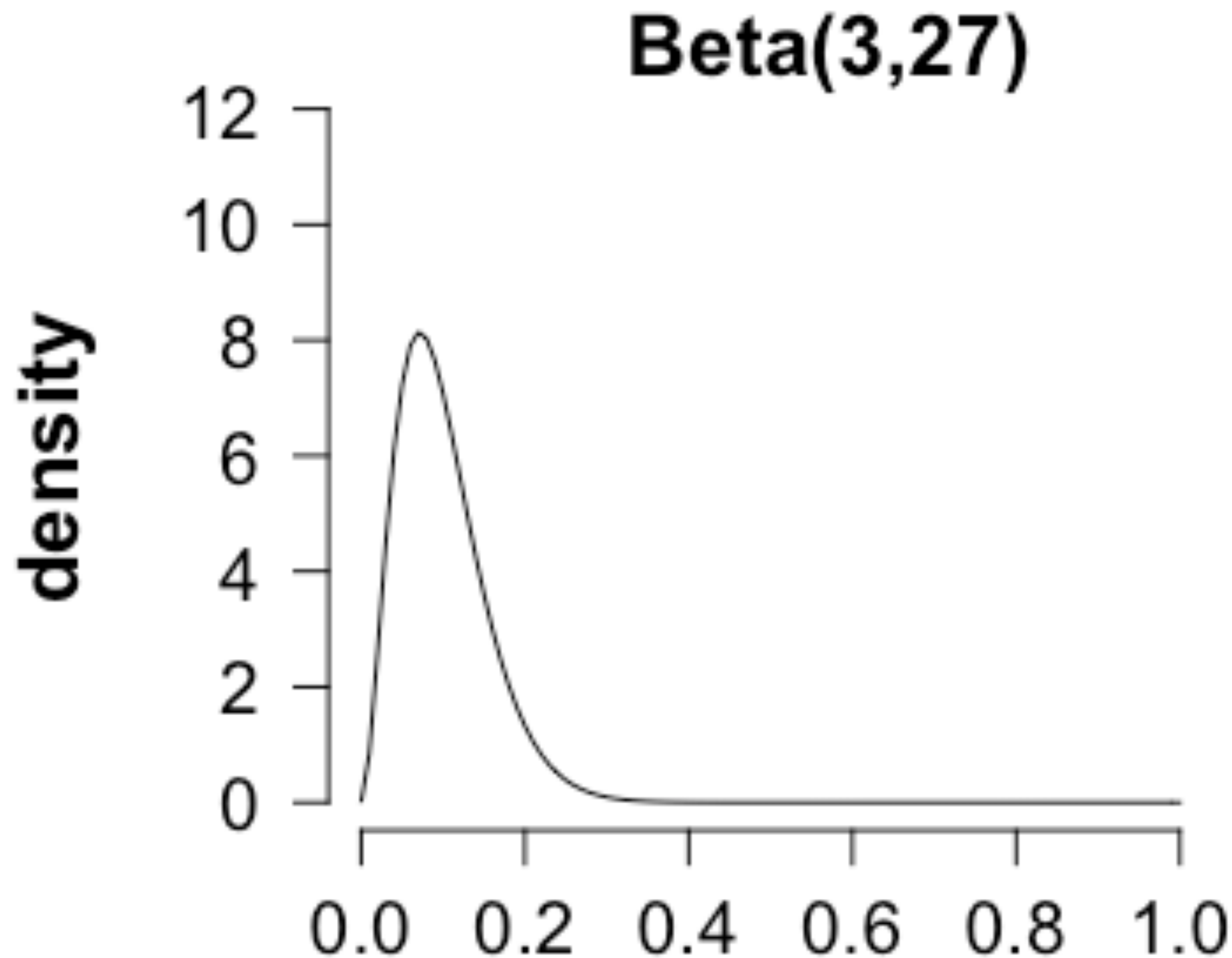
Modeling prior knowledge:

- Suppose we know that 3 out of 30 patients will die after a particular operation
- This prior knowledge can be represented as a Beta(3,27) distribution

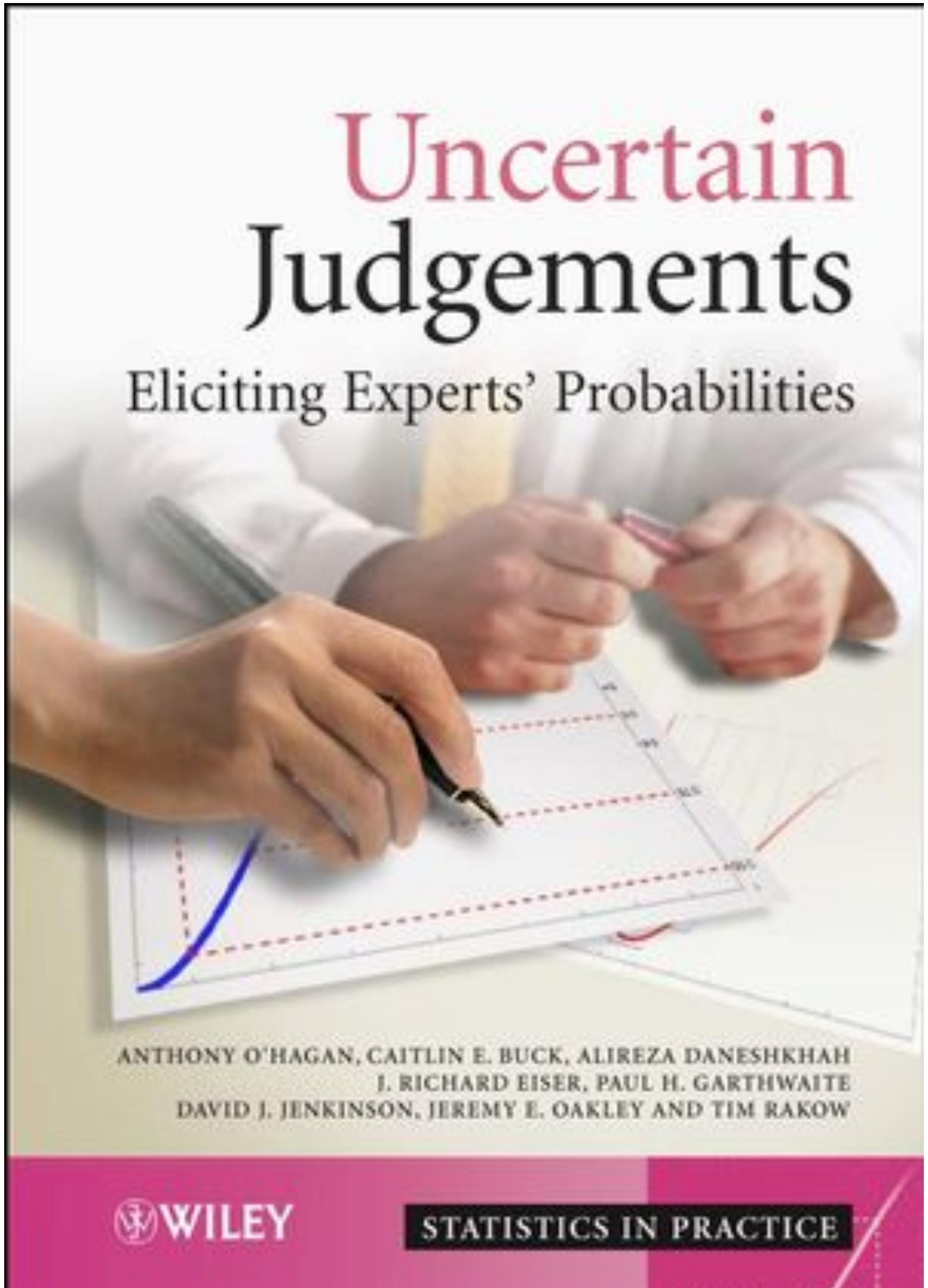
Example: Modeling mortality after surgery

Modeling prior knowledge:

Prior probability of death:



Example: Modeling mortality after surgery

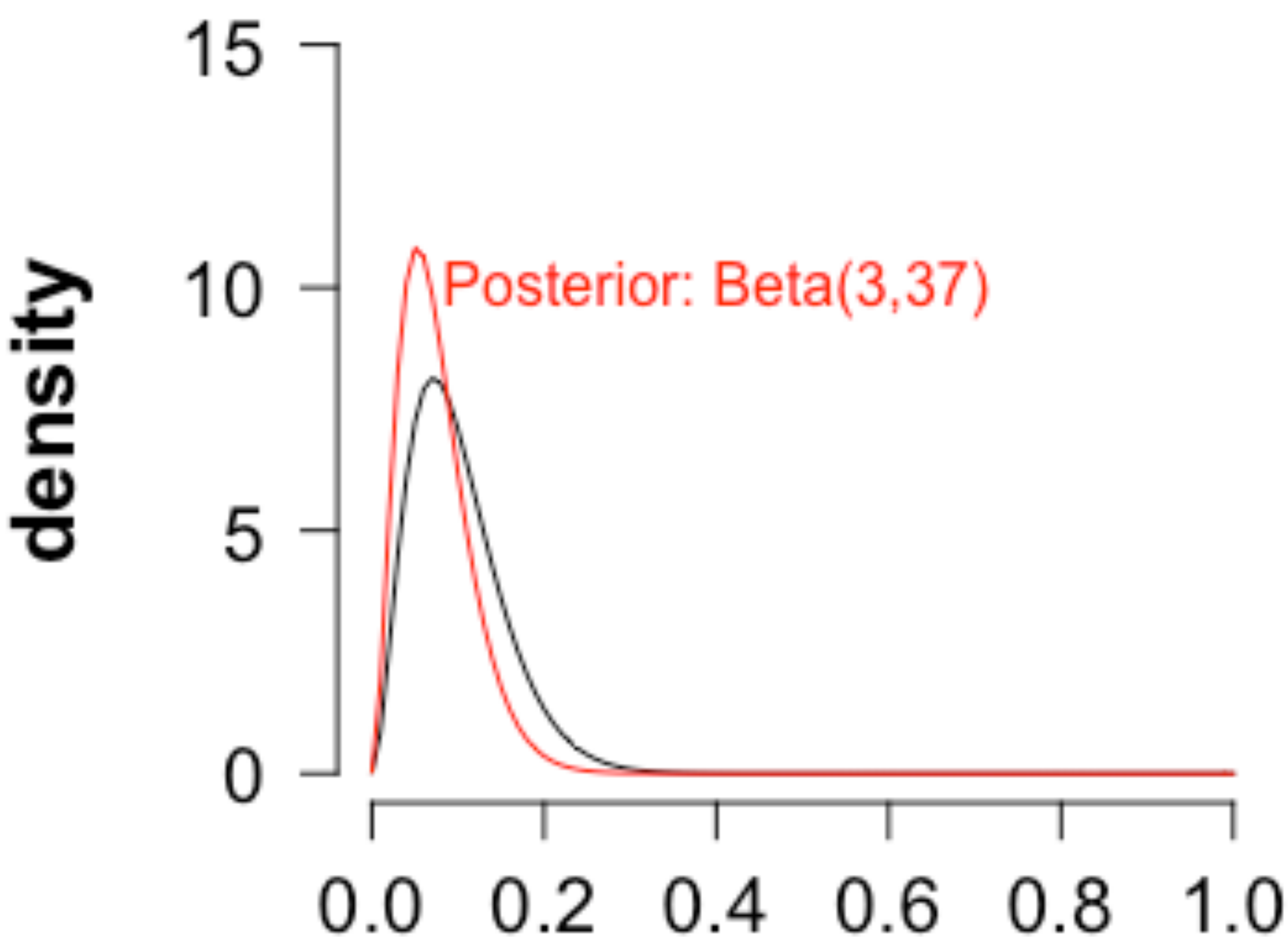


Example: Modeling mortality after surgery

The data: 0 deaths in the next 10 operations.

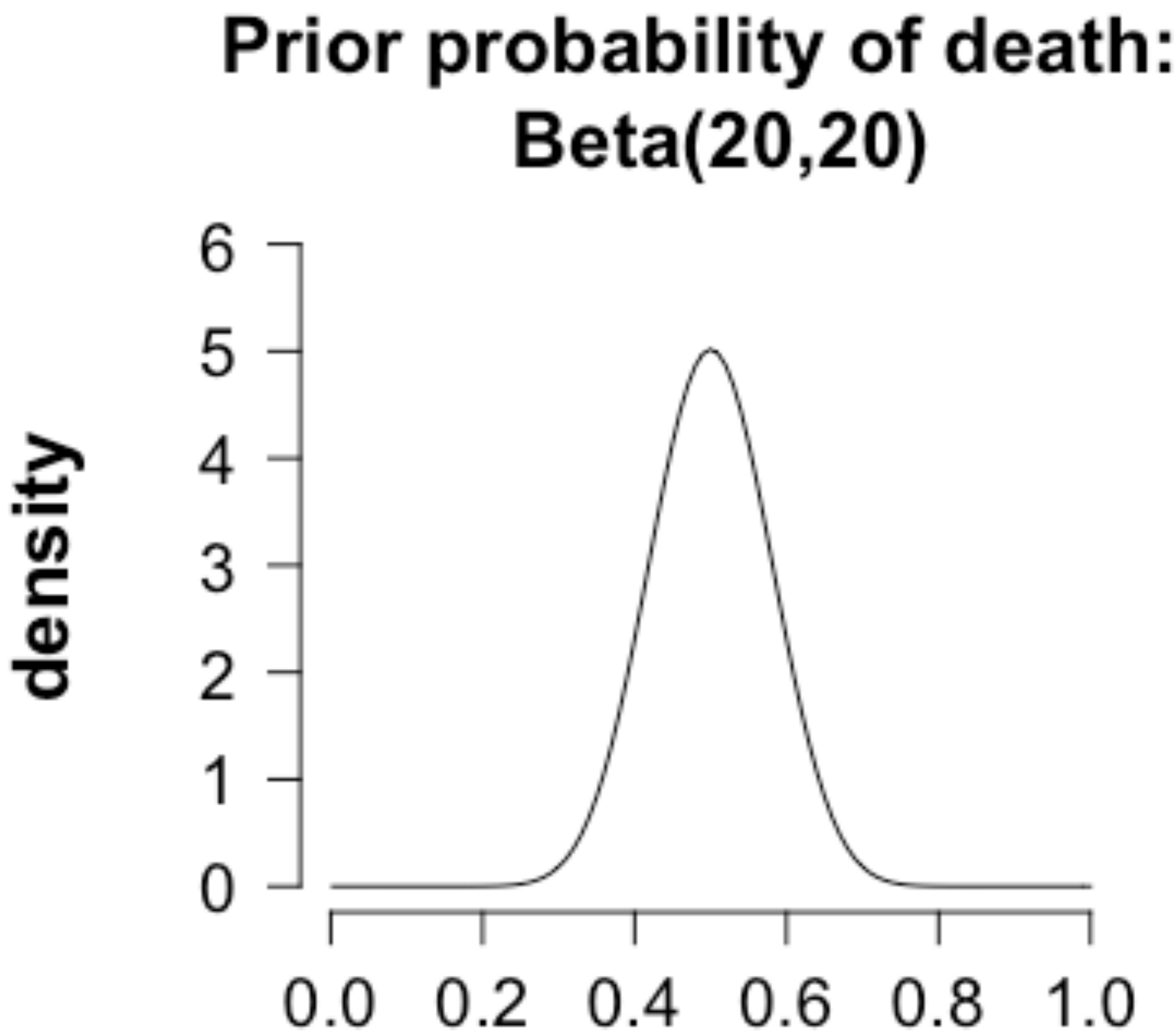
The **posterior distribution** of the probability of death:

$$Posterior \propto Likelihood \times Prior$$



Example: Modeling mortality after surgery

Suppose that Prior probability of death was higher:



Example: Modeling mortality after surgery

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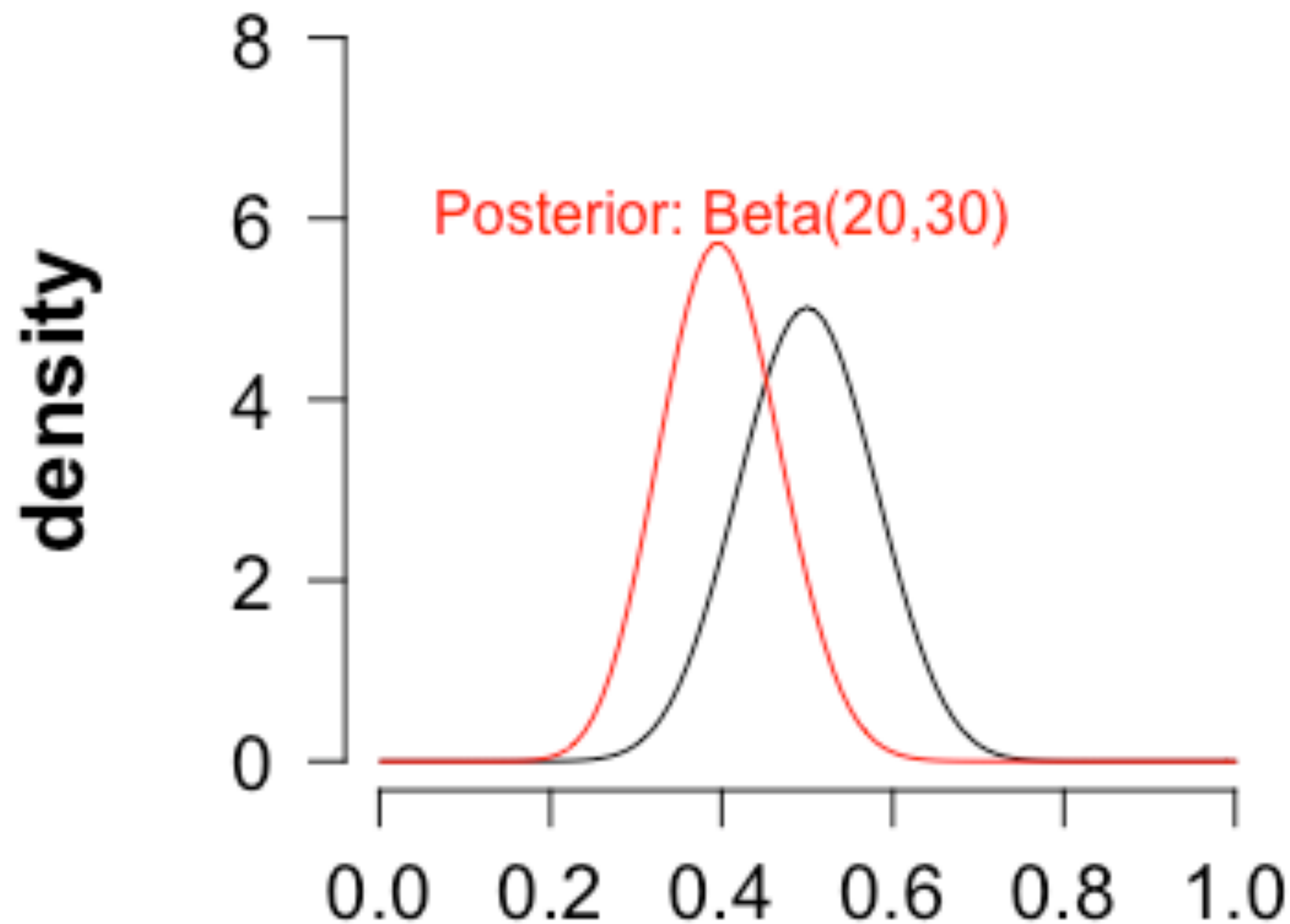
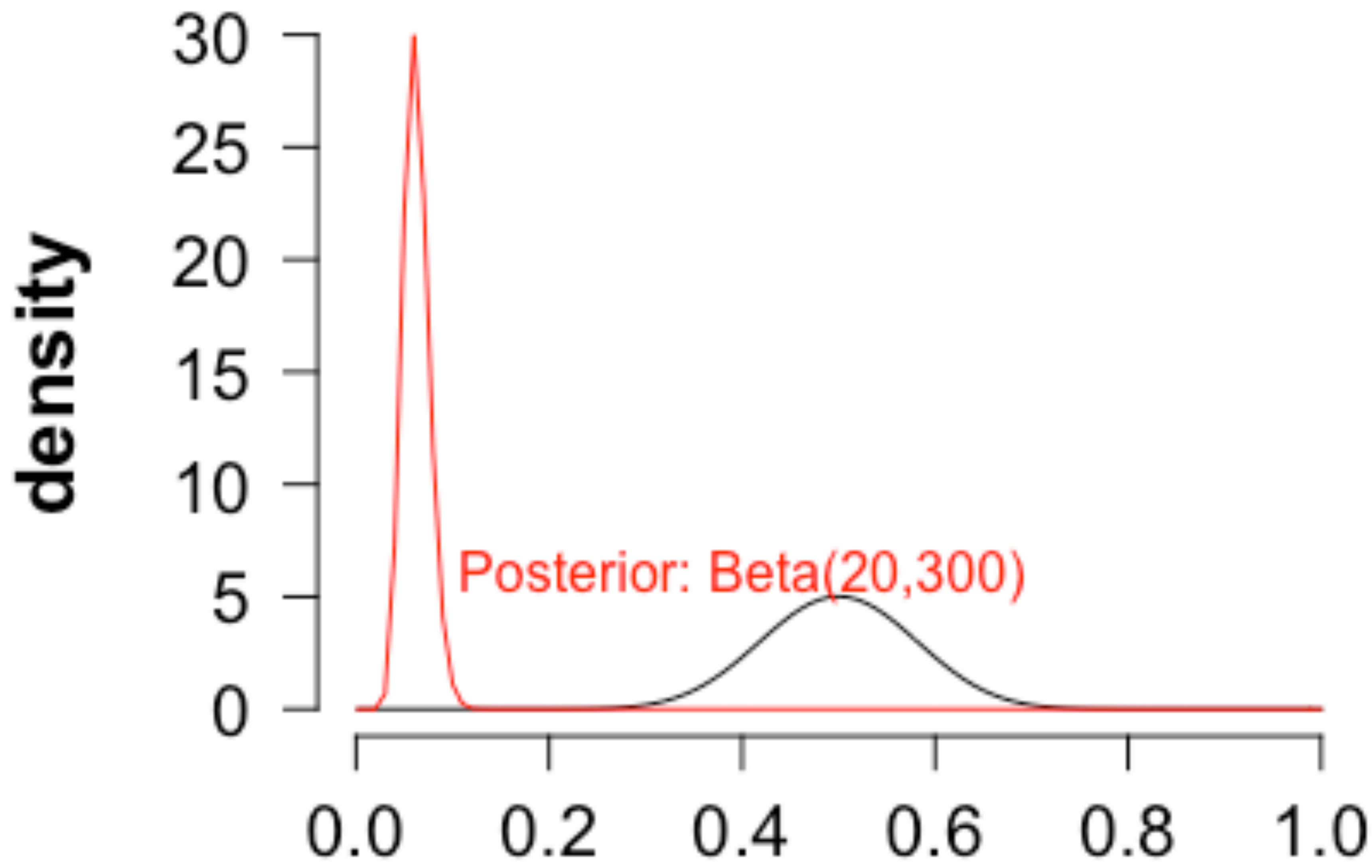


Example: Modeling mortality after surgery

The data: 0 deaths in the next **300** operations.

The **posterior distribution** of the probability of death:



Summary

The posterior is a compromise between
the prior and the data

When data are sparse, the posterior reflects the prior

When a lot of data is available, the posterior reflects
the likelihood

Hypothesis testing using the Bayes factor

We may want to compare two alternative models:

$$Model\ 1 : \text{Probability of death} = 0.5$$

$$Model\ 2 : \text{Probability of death} \sim Beta(1,1)$$

Bayes factor: $$BF_{12} = \frac{Prob(Data\,|\,Model\ 1)}{Prob(Data\,|\,Model\ 2)}$$

Hypothesis testing using the Bayes factor

$Model\ 1$: Probability of death $= 0.5$

$$\binom{n}{k}\theta^0(1-\theta)^{10} = \binom{10}{0}0.5^{10} = 0.000977$$

$Model\ 2$: Probability of death $\sim Beta(1,1)$

(Some calculus needed here) $\dfrac{1}{11}$

$$BF_{12} = \frac{Prob(Data\,|\,Model\ 1)}{Prob(Data\,|\,Model\ 2)} = \frac{0.000977}{1/11} = 0.01$$

Model 2 is 10 times more likely than Model 1

Comparison of Frequentist vs Bayesian approaches

	Frequentist	Bayesian
Parameters	Fixed	Random
Data	Random	Fixed
Prior knowledge used	No	Yes
Type I, II error	relevant	irrelevant
Hypothesis testing	reject null	Bayes factor
Uncertainty quantification	No	Yes

Some advantages of the Bayesian approach

1. Handles sparse data without any problems
2. Highly customised models can be defined
3. The focus is on uncertainty quantification
4. Answers the research question directly

Some disadvantages of the Bayesian approach

1. **You have to understand what you are doing**
   - Distribution theory
   - Random variable theory
   - Maximum likelihood estimation
   - Linear modeling theory
2. **Requires programming ability**
   - Statistical computing using Stan (mc-stan.org)
3. **Computational cost**
   - Cluster computing is sometimes needed
   - GPU based computing is coming in 2019
4. **Priors require thought**
   - Eliciting priors from experts
   - Adversarial analyses