Grundlagen der künstlichen Intelligenz: Hausaufgabe 6

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Aufgabe 1 - Hidden Markov-Prozess

Die Anfangsbedingung wird mit $P(X_0 = w) = 0.5$ angenommen.

(a)

$$P(X_1 = \cdots = X_k = w, X_{k+1} = f \mid X_0 = f) = P(X_{k+1} = f \mid X_k = w) \cdot \prod_{i=1}^{k} P(X_{i+1} = w \mid X_i = w) \cdot P(X_1 = w \mid X_0 = f)$$
$$= 0.2 \cdot 0.8^{k-1} \cdot 0.1$$

(b)

$$p_t = P(X_t \mid Y_{1:t})$$
$$= \alpha p(X_t, Y_t \mid Y_{1:t-1}) = \alpha p(Y_t \mid X_t, Y_{1:t-1}) p(x_t, e_{1:t-1})$$
$$= \alpha p(Y_t, X_t) p(X_t, Y_{1:t-1})$$
$$p(X_t \mid Y_{1:t-1}) = \sum_{x_t} p(X_t, X_t \mid Y_{1:t-1}) = \sum_{x_t} p(X_t \mid X_{t-1}) p(X_t \mid Y_{1:t})$$
$$p(X_t \mid Y_{1:t}) = \alpha p(Y_t \mid X_t) \sum_{x_t} p(X_t \mid X_{t-1}) p(X_t \mid Y_{1:t-1})$$
$$= \alpha p(Y_t = g \mid X_t = w) \sum_{x_t} p(X_t \mid X_{t-1}) p(X_t \mid Y_{1:t-1})$$
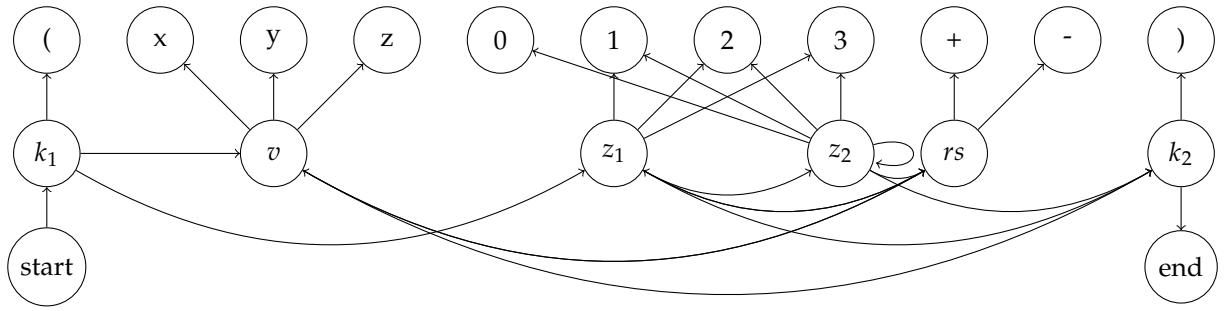$$= \alpha 0.3 \cdot \sum_{x_t} p(X_t \mid X_{t-1}) p(X_t \mid Y_{1:t-1})$$

(c)

$$P(X_{2+1} = w, Y_1 = c, Y_2 = g) = \sum_{X_2} p(X_{2+1} = w \mid X_2) p(X_2 \mid Y_{1:2})$$
$$= p(X_3 = w \mid X_2 = w) p(X_2 = w \mid Y_{1:2}) + p(X_3 = w \mid X_2 = f) p(X_2 = f \mid Y_{1:2})$$
$$= p(X_3 = w \mid X_2 = w) \sum_{X_1} p(X_2 = w \mid X_1) p(X_1 \mid Y_1) + p(X_3 = w \mid X_2 = f) p(X_2 = f \mid Y_{1:2})$$
$$= p(X_3 = w \mid X_2 = w)(p(X_2 = w \mid X_1 = w) p(X_1 = w \mid Y_1) + p(X_2 = w \mid X_1 = f) p(X_1 = f \mid Y_1)) +$$
$$+ p(X_3 = w \mid X_2 = f) p(X_2 = f \mid Y_{1:2})$$
$$= \ldots$$

(d)

(e)

Aufgabe 2 - Hidden Markov-Modell

(a)

(b)

$$P(x_{0:6} \mid y_{1:6} = (x + 30)) = \frac{P(x_0) \prod_{t=1}^{6} P(y_t \mid x_t) P(x_t \mid x_{t-1})}{P(y_{1:6} = (x + 30))}$$

$$= \frac{1.0 \cdot (0.6 \cdot 0.6) \cdot (0.8 \cdot 0.7) \cdot (0.7 \cdot 0.2) \cdot (0.5 \cdot 0.4) \cdot (0.3 \cdot 1.0) \cdot 1.0}{P(y_{1:6} = (x + 30))}$$

$$= \frac{0.00203}{P(y_{1:6} = (x + 30))}$$

(c)

(d)