Grundlagen der künstlichen Intelligenz: Hausaufgabe 6

Tom Nick - 340528
Niklas Gebauer - 340942
Leonard Witte - 341457
Johannes Herrmann - 341091

Aufgabe 1 - Hidden Markov-Prozess

Die Anfangsbedingung wird mit $P(X_0 = w) = 0.5$ angenommen.

(a)

$$\begin{aligned} P(X_1 = \dots = X_k = w, X_{k+1} = f \mid X_0 = f) &= P(X_{k+1} = f \mid X_k = w) \cdot \prod_{i=1}^{k} P(X_{i+1} = w \mid X_i = w) \cdot P(X_1 = w \mid X_0 = f) \\ &= 0.2 \cdot 0.8^{k-1} \cdot 0.1 \end{aligned}$$

(b)

$$\begin{aligned} p_t &= P(X_t \mid Y_{1:t}) \\ &= \alpha p(X_t, Y_t \mid Y_{1:t-1}) = \alpha p(Y_t \mid X_t, Y_{1:t-1}) p(x_t, e_{1:t-1}) \\ &= \alpha p(Y_t, X_t) p(X_t, Y_{1:t-1}) \\ p(X_t \mid Y_{1:t-1}) &= \sum_{x_t} p(X_t, X_t \mid Y_{1:t-1}) = \sum_{x_t} p(X_t \mid X_{t-1}) p(X_t \mid Y_{1:t}) \\ p(X_t \mid Y_{1:t}) &= \alpha p(Y_t \mid X_t) \sum_{x_t} p(X_t \mid X_{t-1}) p(X_t \mid Y_{1:t-1}) \\ &= \alpha p(Y_t = g \mid X_t = w) \sum_{x_t} p(X_t \mid X_{t-1}) p(X_t \mid Y_{1:t-1}) \\ &= \alpha 0.3 \cdot \sum_{x_t} p(X_t \mid X_{t-1}) p(X_t \mid Y_{1:t-1}) \end{aligned}$$

(c)

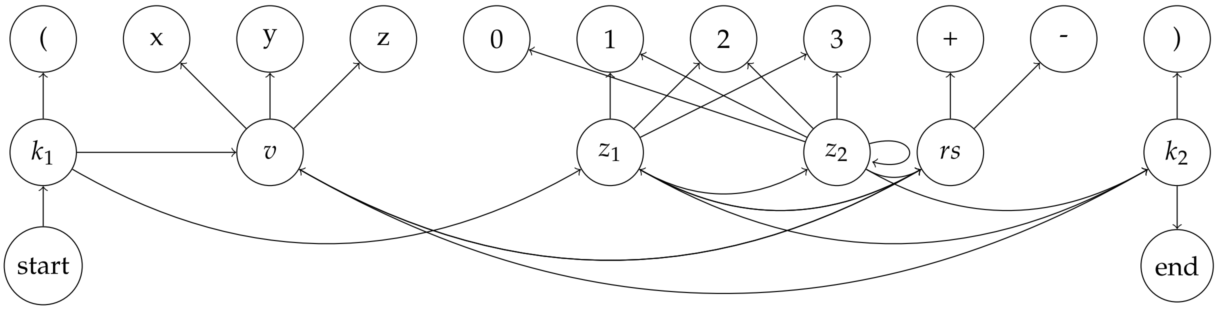
$$\begin{aligned} P(X_{2+1} = w, Y_1 = c, Y_2 = g) &= \sum_{X_2} p(X_{2+1} = w \mid X_2) p(X_2 \mid Y_{1:2}) \\ &= p(X_3 = w \mid X_2 = w) p(X_2 = w \mid Y_{1:2}) + p(X_3 = w \mid X_2 = f) p(X_2 = f \mid Y_{1:2}) \\ &= p(X_3 = w \mid X_2 = w) \sum_{X_1} p(X_2 = w \mid X_1) p(X_1 \mid Y_1) + p(X_3 = w \mid X_2 = f) p(X_2 = f \mid Y_{1:2}) \\ &= p(X_3 = w \mid X_2 = w)(p(X_2 = w \mid X_1 = w) p(X_1 = w \mid Y_1) + p(X_2 = w \mid X_1 = f) p(X_1 = f \mid Y_1)) + \\ &\quad + p(X_3 = w \mid X_2 = f) p(X_2 = f \mid Y_{1:2}) \\ &= \dots \end{aligned}$$

(d)

(e)

Aufgabe 2 - Hidden Markov-Modell

(a)



(b)

$$
\begin{aligned}
P(x_{0:6} \mid y_{1:6} = (x+30)) &= P(x_0) \prod_{t=1}^{6} P(y_t \mid x_t) P(x_t \mid x_{t-1}) \\
&= 1.0 \cdot (0.6 \cdot 0.6) \cdot (0.8 \cdot 0.7) \cdot (0.7 \cdot 0.2) \cdot (0.5 \cdot 0.4) \cdot (0.3 \cdot 1.0) \cdot 1.0 \\
&= 0.00203
\end{aligned}
$$

(c)

(d)