Grundlagen der künstlichen Intelligenz: Hausaufgabe 6

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Aufgabe 1 - Hidden Markov-Prozess

Die Anfangsbedingung wird mit $P(X_0 = w) = 0.5$ angenommen.

(a)

$$P(X_1 = \cdots = X_k = w, X_{k+1} = f \mid X_0 = f) = P(X_{k+1} = f \mid X_k = w) \cdot \prod_{i=1}^{k} P(X_{i+1} = w \mid X_i = w) \cdot P(X_1 = w \mid X_0 = f)$$

$$= 0.1 \cdot 0.8^{k-1} \cdot 0.2$$

(b)

$$p_t = P(X_t = w \mid Y_{1:t})$$
$$p_{t-1} = P(X_{t-1} = w \mid Y_{1:t-1})$$
$$P(X_t = w \mid Y_1, ..., Y_t = g) = \alpha P(Y_t = g \mid X_t = w) \sum_{X_{t-1}} P(X_t = w \mid X_{t-1}) P(X_{t-1} \mid Y_{1:t-1})$$

$$= \alpha * 0,3 * (0,8 * p_{t-1} + 0,1 * (1 - p_{t-1}))$$
$$= \alpha * 0,24 * p_{t-1} + \alpha * 0,03 - \alpha * 0,03 * p_{t-1}$$
$$= \alpha * (0,21 * p_{t-1} + 0,03)$$

$$\alpha = \frac{1}{P(Y_t = g \mid Y_{1:t-1})}$$

(c)

$$P(X_1 = w) = P(X_1 = w|X_0 = f)P(x_0 = f) + P(X_1 = w|x_0 = w)P(X_0 = w)$$
$$= 0.1 \cdot 0.5 + 0.2 \cdot 0.5) = 0.45$$
$$P(X_1 = f) = P(X_1 = f|X_0 = f)P(x_0 = f) + P(X_1 = f|x_0 = w)P(X_0 = w)$$
$$= 0.9 \cdot 0.5 + 0.2 \cdot 0.5 = 0.45$$

$$P(X_1 = w|Y_1 = c) = \frac{P(X_1 = w, Y_1 = c)}{P(Y_1 = c)}$$
$$= \frac{P(X_1 = w|Y_1 = c)P(Y_1 = c)}{P(Y_1 = c|X_1 = w)P(X_1 = w) + P(Y_1 = c|X_1 = f)P(X_1 = f)}$$
$$= \frac{0.3 \cdot 0.45}{0.3 \cdot 0.45 + 0.2 \cdot 0.55} = \frac{0.135}{0.245}$$

$$P(X_1 = f|Y_1 = c) = \frac{P(X_1 = f, y_1 = c)}{P(Y_1 = c)} = \frac{0.2 \cdot 0.55}{0.3 \cdot 0.45 + 0.2 \cdot 0.55} = \frac{0.11}{0.245}$$

$$P(Y_2 = g|Y_1 = c) = \sum_{X_2}[P(Y_2 = g|X_2)P(X_2|Y_1 = c)]$$
$$= \sum_{X_2}[P(Y_2 = g)\sum_{X_1}[P(X_2|X_1)P(X_1|Y_1 = c)]]$$
$$= 0.3(0.8\frac{0.135}{0.245} + 0.1\frac{0.11}{0.245}) + 0.2(0.2\frac{0.135}{0.245} + 0.9\frac{0.11}{0.245}) = \frac{0.0609}{0.245}$$

$$P(X_2 = w|Y_2 = g, Y_1 = c) = \frac{P(X_2 = w, Y_2 = g, Y_1 = c)}{P(Y_2 = g, Y_1 = c)}$$
$$= \frac{P(Y_2 = g|X_2 = w)P(X_2 = w|Y_1 = c)\cancel{P(Y_1 = c)}}{P(Y_2 = g|Y_1 = c)\cancel{P(Y_1 = c)}}$$
$$= \frac{P(Y_2 = g|X_2 = w)\sum_{X_1}[P(X_2 = w|X_1)P(X_1|Y_1 = c)]}{P(Y_2 = g|Y_1 = c)}$$
$$= \frac{0.3(0.8\frac{0.135}{0.245} + 0.1\frac{0.11}{0.245})}{\frac{0.0609}{0.245}}$$
$$= \frac{\frac{0.0357}{\cancel{0.245}}}{\frac{0.0609}{\cancel{0.245}}} = \frac{0.0357}{0.0609}$$

$$P(X_2 = f|Y_2 = g, Y_1 = c) = \frac{0.2(0.2\frac{0.135}{0.245} + 0.9\frac{0.11}{0.245})}{\frac{0.0609}{0.245}}$$
$$= \frac{\frac{0.0252}{\cancel{0.245}}}{\frac{0.0609}{\cancel{0.245}}} = \frac{0.0252}{0.0609}$$

$$P(X_3 = w|Y_2 = g, Y_1 = c) = \sum_{X_2}[P(X_3 = w|X_2)P(X_2|Y_2 = g, Y_1 = c)]$$
$$= 0.8\frac{0.0357}{0.0609} + 0.1\frac{0.0252}{0.0609} = \frac{0.03108}{0.0609} \approx 0.51$$

$$P(X_3 = f|Y_2 = g, Y_1 = c) = \sum_{X_2}[P(X_3 = f|X_2)P(X_2|Y_2 = g, Y_1 = c)]$$
$$= 0.2\frac{0.0357}{0.0609} + 0.9\frac{0.0252}{0.0609} = \frac{0.02982}{0.0609}$$

$$P(X_4 = w|Y_2 = g, Y_1 = c) = \sum_{X_3}[P(X_4 = w|X_3)P(X_3|Y_2 = g, Y_1 = c)]$$
$$= 0.8\frac{0.03108}{0.0609} + 0.1\frac{0.02982}{0.0609} = \frac{0.027846}{0.0609} \approx 0.46$$

(d)

(e)

$$m(w,0) = P(X_0 = w) = 0,5$$
$$m(f,0) = P(X_0 = f) = 0,5$$

$$m(w,1) = P(Y_1 = a \mid X_1 = w) * max_v(P(X_1 = w \mid X_0 = v) * m(v,0))$$
$$= 0,2 * max[0,8 * 0,5; 0,1 * 0,5]$$
$$= 0,08 (mit X_0 = w)$$
$$m(f,1) = P(Y_1 = a \mid X_1 = f) * max_v(P(X_1 = f \mid X_0 = v) * m(v,0))$$
$$= 0,3 * max[0,2 * 0,5; 0,9 * 0,5]$$
$$= 0,135 (mit X_0 = f)$$

$$m(w,2) = P(Y_2 = c \mid X_2 = w) * max_v(P(X_2 = w \mid X_1 = v) * m(v,1))$$
$$= 0,3 * max[0,8 * 0,08; 0,1 * 0,135]$$
$$= 0,0192 (mit X_1 = w)$$
$$m(f,2) = P(Y_2 = a \mid X_2 = f) * max_v(P(X_2 = f \mid X_1 = v) * m(v,1))$$
$$= 0,2 * max[0,2 * 0,08; 0,9 * 0,135]$$
$$= 0,0243 (mit X_1 = f)$$

$$m(w,3) = P(Y_3 = g \mid X_3 = w) * max_v(P(X_3 = w \mid X_2 = v) * m(v,2))$$
$$= 0,3 * max[0,8 * 0,0192; 0,1 * 0,0243]$$
$$= 0,004608 (mit X_2 = w)$$
$$m(f,3) = P(Y_3 = g \mid X_3 = f) * max_v(P(X_3 = f \mid X_2 = v) * m(v,2))$$
$$= 0,2 * max[0,2 * 0,0192; 0,9 * 0,0243]$$
$$= 0,004374 (mit X_2 = f)$$

$$m(w,4) = P(Y_4 = t \mid X_4 = w) * max_v(P(X_4 = w \mid X_3 = v) * m(v,3))$$
$$= 0,3 * max[0,8 * 0,004608; 0,1 * 0,004374]$$
$$= 0,00073728 (mit X_3 = w)$$
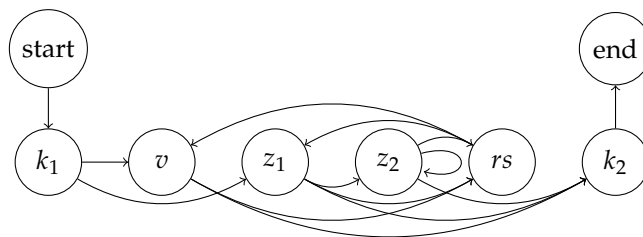$$m(f,4) = P(Y_4 = t \mid X_4 = f) * max_v(P(X_4 = f \mid X_3 = v) * m(v,3))$$
$$= 0,2 * max[0,2 * 0,004608; 0,9 * 0,004374]$$
$$= 0,00118098 (mit X_3 = f)$$

Es folgt also: $X_4 = f, X_3 = f, X_2 = f, X_1 = f, X_0 = f$

Aufgabe 2 - Hidden Markov-Modell

(a)

(b)

$$P(x_{0:6} \mid y_{1:6} = (x + 30)) = P(x_0) \prod_{t=1}^{6} P(y_t \mid x_t) P(x_t \mid x_{t-1})$$
$$= 1.0 \cdot (0.6 \cdot 0.6) \cdot (0.8 \cdot 0.7) \cdot (0.7 \cdot 0.2) \cdot (0.5 \cdot 0.4) \cdot (0.3 \cdot 1.0) \cdot 1.0$$
$$= 0.00203$$

(c) Keine Ahnung wie formal das muss: Ich nehme an, zwei-stellige Zahlen sind ausgeschlossen. Es gibt folgende Möglichkeiten:

- ... $rs \{x, y, z\}$ rs ...
- ... $rs \{1, 2, 3\}$ rs ...

$$P(\ rs\ x\ rs\ ) = P(v \mid rs) \cdot P(x \mid v) \cdot P(rs \mid v) = 0.3 \cdot 0.6 \cdot 0.8 = 0.144$$
$$P(\ rs\ y\ rs\ ) = 0.3 \cdot 0.3 \cdot 0.8 = 0.72$$
$$P(\ rs\ z\ rs\ ) = 0.3 \cdot 0.3 \cdot 0.8 = 0.24$$
$$P(\ rs\ 1\ rs\ ) = 0.7 \cdot 0.5 \cdot 0.3 = 0.105$$
$$P(\ rs\ 2\ rs\ ) = 0.7 \cdot 0.3 \cdot 0.3 = 0.063$$
$$P(\ rs\ 3\ rs\ ) = 0.7 \cdot 0.2 \cdot 0.3 = 0.042$$

Somit ist die Variable x am wahrscheinlichsten. Die Aussage sollte mit einer Wahrscheinlichkeit von $\frac{0.144}{0.144 + 0.72 + 0.24 + 0.105 + 0.063 + 0.042} = 0.1095$ zutreffen.

(d) Die Möglichkeiten wären $(z_1, z_2) z_1 \in \{0, 1, 2\}, z_2 \in \{0, 1, 2, 3\}$ bzw. *start $k_1 z_1 z_2 k_2$ stop*

$$P(x_5 = sto, x_4 = k_2, x_3 = z_2, x_2 = z_1, x_1 = k_1, x_0 = sta) = 1.0 \cdot 0.4 \cdot 0.5 \cdot 0.3 \cdot 1.0 = 0.08$$

8% ist die Wahrscheinlichkeit das ein mathematischer Ausdruck mit genau 4 Zeichen auftritt.