Grundlagen der künstlichen Intelligenz: Hausaufgabe 6

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Aufgabe 1 - Hidden Markov-Prozess

Die Anfangsbedingung wird mit $P(X_0 = w) = 0.5$ angenommen.

(a)

$$\begin{aligned} P(X_1 = \cdots = X_k = w, X_{k+1} = f \mid X_0 = f) &= P(X_{k+1} = f \mid X_k = w) \cdot \prod_{i=1}^{k} P(X_{i+1} = w \mid X_i = w) \cdot P(X_1 = w \mid X_0 = f) \\ &= 0.1 \cdot 0.8^{k-1} \cdot 0.2 \end{aligned}$$

(b)

$$\begin{aligned} p_t &= P(X_t = w \mid Y_{1:t}) \\ p_{t-1} &= P(X_{t-1} = w \mid Y_{1:t-1}) \\ P(X_t = w \mid Y_1, ..., Y_t = g) &= \alpha P(Y_t = g \mid X_t = w) \sum_{X_{t-1}} P(X_t = w \mid X_{t-1}) P(X_{t-1} \mid Y_{1:t-1}) \\ &= \alpha * 0,3 * (0,8 * p_{t-1} + 0,1 * (1 - p_{t-1})) \\ &= \alpha * 0,24 * p_{t-1} + \alpha * 0,03 - \alpha * 0,03 * p_{t-1} \\ &= \alpha * (0,21 * p_{t-1} + 0,03) \end{aligned}$$

$$\alpha = \frac{1}{P(Y_t = g \mid Y_{1:t-1})}$$

(c)

$$
\begin{aligned}
P(X_1 = w) &= P(X_1 = w|X_0 = f)P(x_0 = f) + P(X_1 = w|x_0 = w)P(X_0 = w) \\
&= 0.1 \cdot 0.5 + 0.2 \cdot 0.5) = 0.45 \\
P(X_1 = f) &= P(X_1 = f|X_0 = f)P(x_0 = f) + P(X_1 = f|x_0 = w)P(X_0 = w) \\
&= 0.9 \cdot 0.5 + 0.2 \cdot 0.5 = 0.45 \\
P(X_1 = w|Y_1 = c) &= \frac{P(X_1 = w, Y_1 = c)}{P(Y_1 = c)} \\
&= \frac{P(X_1 = w|Y_1 = c)P(Y_1 = c)}{P(Y_1 = c|X_1 = w)P(X_1 = w) + P(Y_1 = c|X_1 = f)P(X_1 = f)} \\
&= \frac{0.3 \cdot 0.45}{0.3 \cdot 0.45 + 0.2 \cdot 0.55} = \frac{0.135}{0.245} \\
P(X_1 = f|Y_1 = c) &= \frac{P(X_1 = f, y_1 = c)}{P(Y_1 = c)} = \frac{0.2 \cdot 0.55}{0.3 \cdot 0.45 + 0.2 \cdot 0.55} = \frac{0.11}{0.245} \\
P(Y_2 = g|Y_1 = c) &= \sum_{X_2}[P(Y_2 = g|X_2)P(X_2|Y_1 = c)] \\
&= \sum_{X_2}[P(Y_2 = g)\sum_{X_1}[P(X_2|X_1)P(X_1|Y_1 = c)]] \\
&= 0.3(0.8\frac{0.135}{0.245} + 0.1\frac{0.11}{0.245}) + 0.2(0.2\frac{0.135}{0.245} + 0.9\frac{0.11}{0.245}) = \frac{0.0609}{0.245} \\
P(X_2 = w|Y_2 = g, Y_1 = c) &= \frac{P(X_2 = w, Y_2 = g, Y_1 = c)}{P(Y_2 = g, Y_1 = c)} \\
&= \frac{P(Y_2 = g|X_2 = w)P(X_2 = w|Y_1 = c)\cancel{P(Y_1 = c)}}{P(Y_2 = g|Y_1 = c)\cancel{P(Y_1 = c)}} \\
&= \frac{P(Y_2 = g|X_2 = w)\sum_{X_1}[P(X_2 = w|X_1)P(X_1|Y_1 = c)]}{P(Y_2 = g|Y_1 = c)} \\
&= \frac{0.3(0.8\frac{0.135}{0.245} + 0.1\frac{0.11}{0.245})}{\frac{0.0609}{0.245}} \\
&= \frac{\frac{0.0357}{\cancel{0.245}}}{\frac{0.0609}{\cancel{0.245}}} = \frac{0.0357}{0.0609} \\
P(X_2 = f|Y_2 = g, Y_1 = c) &= \frac{0.2(0.2\frac{0.135}{0.245} + 0.9\frac{0.11}{0.245})}{\frac{0.0609}{0.245}} \\
&= \frac{\frac{0.0252}{\cancel{0.245}}}{\frac{0.0609}{\cancel{0.245}}} = \frac{0.0252}{0.0609} \\
P(X_3 = w|Y_2 = g, Y_1 = c) &= \sum_{X_2}[P(X_3 = w|X_2)P(X_2|Y_2 = g, Y_1 = c)] \\
&= 0.8\frac{0.0357}{0.0609} + 0.1\frac{0.0252}{0.0609} = \frac{0.03108}{0.0609} \approx 0.51 \\
P(X_3 = f|Y_2 = g, Y_1 = c) &= \sum_{X_2}[P(X_3 = f|X_2)P(X_2|Y_2 = g, Y_1 = c)] \\
&= 0.2\frac{0.0357}{0.0609} + 0.9\frac{0.0252}{0.0609} = \frac{0.02982}{0.0609} \\
P(X_4 = w|Y_2 = g, Y_1 = c) &= \sum_{X_3}[P(X_4 = w|X_3)P(X_3|Y_2 = g, Y_1 = c)] \\
&= 0.8\frac{0.03108}{0.0609} + 0.1\frac{0.02982}{0.0609} = \frac{0.027846}{0.0609} \approx 0.46
\end{aligned}
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(d)

$$\begin{aligned}
P(X_1 = w|Y_1 = c, Y_2 = g) &= \frac{P(Y_2 = g|X_1 = w)P(X_1 = w|Y_1 = c)}{\sum\limits_{X_1}[P(Y_2 = g|X_1)P(X_1|Y_1 = c)]} \\
&= \frac{\sum\limits_{X_2}[P(X_2|X_1 = w)P(Y_2 = g|X_2))P(X_1 = w|y_1 = c)]}{\sum\limits_{X_1}[\sum\limits_{X_2}[P(X_2|X_1)P(Y_2 = g|X_2)]P(X_1|Y_1 = c)]} \\
&= \frac{(0.8 \cdot 0.3 + 0.2 \cdot 0.2)\frac{0.135}{0.245}}{((0.8 \cdot 0.3 + 0.2 \cdot 0.2)\frac{0.135}{0.245}) + ((0.1 \cdot 0.3 + 0.9 \cdot 0.2)\frac{0.11}{0.245})} \\
&\approx 0.62
\end{aligned}$$

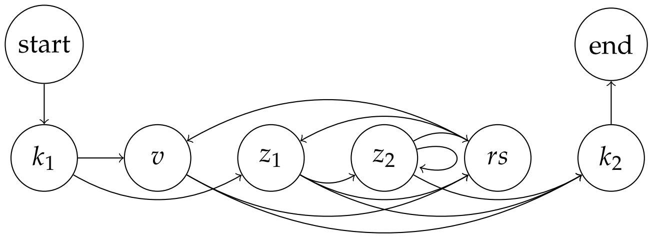
(e)

$$\begin{aligned}
m(w, 0) &= P(X_0 = w) = 0,5 \\
m(f, 0) &= P(X_0 = f) = 0,5 \\
\\
m(w, 1) &= P(Y_1 = a \mid X_1 = w) * max_v(P(X_1 = w \mid X_0 = v) * m(v, 0)) \\
&= 0,2 * max[0,8 * 0,5; 0,1 * 0,5] \\
&= 0,08(mit X_0 = w) \\
m(f, 1) &= P(Y_1 = a \mid X_1 = f) * max_v(P(X_1 = f \mid X_0 = v) * m(v, 0)) \\
&= 0,3 * max[0,2 * 0,5; 0,9 * 0,5] \\
&= 0,135(mit X_0 = f) \\
\\
m(w, 2) &= P(Y_2 = c \mid X_2 = w) * max_v(P(X_2 = w \mid X_1 = v) * m(v, 1)) \\
&= 0,3 * max[0,8 * 0,08; 0,1 * 0,135] \\
&= 0,0192(mit X_1 = w) \\
m(f, 2) &= P(Y_2 = a \mid X_2 = f) * max_v(P(X_2 = f \mid X_1 = v) * m(v, 1)) \\
&= 0,2 * max[0,2 * 0,08; 0,9 * 0,135] \\
&= 0,0243(mit X_1 = f) \\
\\
m(w, 3) &= P(Y_3 = g \mid X_3 = w) * max_v(P(X_3 = w \mid X_2 = v) * m(v, 2)) \\
&= 0,3 * max[0,8 * 0,0192; 0,1 * 0,0243] \\
&= 0,004608(mit X_2 = w) \\
m(f, 3) &= P(Y_3 = g \mid X_3 = f) * max_v(P(X_3 = f \mid X_2 = v) * m(v, 2)) \\
&= 0,2 * max[0,2 * 0,0192; 0,9 * 0,0243] \\
&= 0,004374(mit X_2 = f) \\
\\
m(w, 4) &= P(Y_4 = t \mid X_4 = w) * max_v(P(X_4 = w \mid X_3 = v) * m(v, 3)) \\
&= 0,3 * max[0,8 * 0,004608; 0,1 * 0,004374] \\
&= 0,00073728(mit X_3 = w) \\
m(f, 4) &= P(Y_4 = t \mid X_4 = f) * max_v(P(X_4 = f \mid X_3 = v) * m(v, 3)) \\
&= 0,2 * max[0,2 * 0,004608; 0,9 * 0,004374] \\
&= 0,00118098(mit X_3 = f)
\end{aligned}$$

Es folgt also: $X_4 = f, X_3 = f, X_2 = f, X_1 = f, X_0 = f$

Aufgabe 2 - Hidden Markov-Modell

(a)



(b)

$$
\begin{aligned}
P(x_{0:6} \mid y_{1:6} = (x+30)) &= P(x_0) \prod_{t=1}^{6} P(y_t \mid x_t) P(x_t \mid x_{t-1}) \\
&= 1.0 \cdot (0.6 \cdot 0.6) \cdot (0.8 \cdot 0.7) \cdot (0.7 \cdot 0.2) \cdot (0.5 \cdot 0.4) \cdot (0.3 \cdot 1.0) \cdot 1.0 \\
&= 0.00203
\end{aligned}
$$

(c) Keine Ahnung wie formal das muss: Ich nehme an, zwei-stellige Zahlen sind ausgeschlossen. Es gibt folgende Möglichkeiten:

- $\ldots\ rs\ \{x, y, z\}\ rs\ \ldots$
- $\ldots\ rs\ \{1, 2, 3\}\ rs\ \ldots$

$$
\begin{aligned}
P(\ rs\ x\ rs\ ) &= P(v \mid rs) \cdot P(x \mid v) \cdot P(rs \mid v) = 0.3 \cdot 0.6 \cdot 0.8 = 0.144 \\
P(\ rs\ y\ rs\ ) &= 0.3 \cdot 0.3 \cdot 0.8 = 0.72 \\
P(\ rs\ z\ rs\ ) &= 0.3 \cdot 0.3 \cdot 0.8 = 0.24 \\
P(\ rs\ 1\ rs\ ) &= 0.7 \cdot 0.5 \cdot 0.3 = 0.105 \\
P(\ rs\ 2\ rs\ ) &= 0.7 \cdot 0.3 \cdot 0.3 = 0.063 \\
P(\ rs\ 3\ rs\ ) &= 0.7 \cdot 0.2 \cdot 0.3 = 0.042
\end{aligned}
$$

Somit ist die Variable x am wahrscheinlichsten. Die Aussage sollte mit einer Wahrscheinlichkeit von $\frac{0.144}{0.144+0.72+0.24+0.105+0.063+0.042} = 0.1095$ zutreffen.

(d) Die Möglichkeiten wären $(z_1, z_2) z_1 \in \{0, 1, 2\}, z_2 \in \{0, 1, 2, 3\}$ bzw. $start\ k_1 z_1 z_2 k_2\ stop$

$$
P(x_5 = sto, x_4 = k_2, x_3 = z_2, x_2 = z_1, x_1 = k_1, x_0 = sta) = 1.0 \cdot 0.4 \cdot 0.5 \cdot 0.3 \cdot 1.0 = 0.08
$$

8% ist die Wahrscheinlichkeit das ein mathematischer Ausdruck mit genau 4 Zeichen auftritt.