MACHINE LEARNING 1: ASSIGNMENT 1

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Exercise 1

$$P(error) = \int P(error \mid x)p(x)dx \tag{1}$$

$$P(error \mid x) = \min(P(w_1 \mid x), P(w_2 \mid x)) \tag{2}$$

With these equations, we want to show that

$$P(error) \leq \int \frac{2}{\frac{1}{P(w_1|x)} + \frac{1}{P(w_2|x)}} p(x)dx \tag{3}$$

At first, without restricting the general case, we assume that $P(w_1 \mid x) \geq P(w_2 \mid x)$, that is the function $P(error \mid x) = P(w_2 \mid x)$. Now with $(1), (2) and (3)$ we have:

$$\int P(w_2 \mid x)p(x)dx \leq \int \frac{2}{\frac{1}{P(w_1|x)} + \frac{1}{P(w_2|x)}} p(x)dx \tag{4}$$

Because both sides are integrating over the same variable we can simplify the term to:

$$P(w_2 \mid x)p(x) \leq \frac{2}{\frac{1}{P(w_1|x)} + \frac{1}{P(w_2|x)}} p(x) \tag{5}$$

$$\Leftrightarrow \frac{1}{P(w_2 \mid x)p(x)} \geq \frac{\frac{1}{P(w_1|x)} + \frac{1}{P(w_2|x)}}{2} p(x) \tag{6}$$

$$\Leftrightarrow \frac{1}{P(w_2 \mid x)} \geq \frac{\frac{1}{P(w_1|x)} + \frac{1}{P(w_2|x)}}{2} \tag{7}$$

$$\Leftrightarrow \frac{2}{P(w_2 \mid x)} \geq \frac{1}{P(w_1 \mid x)} + \frac{1}{P(w_2 \mid x)} \tag{8}$$

$$\Leftrightarrow \frac{1}{P(w_2 \mid x)} \geq \frac{1}{P(w_1 \mid x)} \tag{9}$$

$$\Leftrightarrow P(w_2 \mid x) \leq P(w_1 \mid x) \tag{10}$$

This holds true with the assumptions we made earlier.