Swing contract pricing : A Stochastic Optimization Approach

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Abstract

We propose a parametric approach to evaluate swing contracts with firm constraints. Our objective is to define approximations for the optimal control, which represents the amounts of energy purchased throughout the contract. The proposed approach involves approximating the optimal control by means of an explicit parametric function, where the parameters are determined using stochastic gradient descent based algorithms.

Introduction

With deregulation in energy markets, derivative contracts with flexible delivery schedules and volumes have become standard. Among them, *swing* (Take-or-Pay) contracts are ubiquitous in gas and power : the holder may purchase energy at fixed exercise dates subject to *time and volume constraints*, either enforced as *firm* constraints or penalized through deviations from global limits (see, e.g., [2]). Pricing these contracts is more involved than classic American options [5, 11] precisely because of the *multi-period control* and *path-dependent volume constraints*, which naturally cast valuation as a *stochastic optimal control* problem where the control is the vector of purchases across exercise dates.

A large strand of the literature tackles this via the *Backward Dynamic Programming Principle* (BDPP), expressing the price through a dynamic programming equation and a *continuation value* (conditional expectation) at each date [2, 8]. In practice, computing this continuation value is the main bottleneck.

Because of these challenges, alternative *global optimization* views—treating pricing as a stochastic program over admissible controls—have been explored [3, 13]. Although such methods can be powerful, our emphasis here remains on BDPP-based valuation, clarifying its numerical trade-offs (discretization bias, storage, and scalability) and the implications for reliable swing pricing under *firm constraints*.

Our paper is organised as follows. Section 1. We describe swing contracts and recall the pricing framework. Section 2. We present one approach designed to approximate the optimal control.

1 Swing Contracts : Description and Physical Space

Swing contracts are a class of derivative instruments commonly used in energy markets (natural gas, electricity, etc.). They are characterized by a *dual constraint structure* : (i) exercise dates at which the holder can purchase energy, and (ii) volume constraints restricting the quantity of energy that can be bought locally and globally. These features make swing options distinct from standard options and appealing to certain clients.

1.1 Description of Swing Options

A swing option allows its holder to purchase quantities of energy q_ℓ at a discrete set of exercise dates

$$t_\ell = \frac{\ell T}{n}, \quad \ell = 0, \dots, n-1,$$

up to maturity $t_n = T$. At each exercise date t_ℓ, the purchase price is denoted K_ℓ. The strike price can either be fixed, i.e. $K_\ell = K$ for all ℓ, or indexed to another reference, such as the underlying spot price or another commodity (e.g., oil).

In this paper, we focus on the fixed strike case for clarity. The indexed strike case can be treated analogously.

The key feature of swing options is the flexibility in the purchased quantities, subject to two types of *firm constraints* :

1. **Local constraints :** at each exercise date t_ℓ, the holder must buy a volume within prescribed bounds :

$$0 \le q_{\min} \le q_\ell \le q_{\max}, \quad \ell = 0, \dots, n-1. \tag{1}$$

2. **Global constraints :** the cumulative purchased volume up to maturity must remain within global limits :

$$Q_n = \sum_{\ell=0}^{n-1} q_\ell \in [Q_{\min}, Q_{\max}], \quad 0 \le Q_{\min} \le Q_{\max} < +\infty, \tag{2}$$

with $Q_0 = 0$ and $Q_\ell = \sum_{i=0}^{\ell-1} q_i$.

The *firm constraints* setting means the holder cannot violate these bounds. In this context, the existence of an optimal *bang-bang* consumption strategy has been established in [6]. A related result for the case with penalties (where violations of global constraints are allowed but penalized) is given in [8].

1.2 Physical Space of Swing Options

The set of feasible cumulative consumptions at each exercise date is bounded by two functions :

$$Q_{\text{down}}(t_\ell) \le Q_\ell \le Q_{\text{up}}(t_\ell),$$

where

$$\begin{aligned} Q_{\text{down}}(t_0) &= 0, \\ Q_{\text{down}}(t_\ell) &= \max\big(0,\, Q_{\min} - (n-\ell)q_{\max}\big), \qquad\qquad \ell = 1,\dots,n-1, \\ Q_{\text{down}}(t_n) &= Q_{\min}, \end{aligned} \tag{3}$$

and

$$\begin{aligned} Q_{\text{up}}(t_0) &= 0, \\ Q_{\text{up}}(t_\ell) &= \min\big(\ell q_{\max},\, Q_{\max}\big), \qquad\qquad \ell = 1,\dots,n-1, \\ Q_{\text{up}}(t_n) &= Q_{\max}. \end{aligned} \tag{4}$$

These bounds define the *physical space* of the swing contract, i.e., the feasible region of cumulative consumptions over time.

1.3 Admissible Actions and Bang-Bang Feature

Suppose that at time t_ℓ the cumulative consumption is Q_ℓ. Then, the admissible range of the next cumulative consumption $Q_{\ell+1}$ is given by

$$L_{\ell+1}(Q_\ell) = \max\big(Q_{\text{down}}(t_{\ell+1}),\, Q_\ell + q_{\min}\big), \tag{5}$$

$$U_{\ell+1}(Q_\ell) = \min\big(Q_{\text{up}}(t_{\ell+1}),\, Q_\ell + q_{\max}\big). \tag{6}$$

The admissible purchase volume at date t_ℓ is thus

$$A_\ell(Q_\ell) = \big[\, L_{\ell+1}(Q_\ell) - Q_\ell,\, U_{\ell+1}(Q_\ell) - Q_\ell \,\big]. \tag{7}$$

A key property in the firm constraint case is the *bang-bang feature* : when $Q_{\max} - Q_{\min}$ is a multiple of $(q_{\max} - q_{\min})$, the optimal consumption at each exercise date satisfies

$$q_\ell \in \{A_\ell^-(Q_\ell), A_\ell^+(Q_\ell)\}.$$

That is, the holder optimally chooses at each date either the minimum or the maximum admissible purchase. This property, first established in [6], is valuable because it reduces the computational complexity of swing option valuation.

2 Pricing Framework

2.1 Problem Model

Let $F_{t,T}$ denote the price at time t of a forward contract delivering the underlying asset at maturity T. In the firm-constraint setting described above, no action is taken at the maturity T. Consequently, in the firm-constraint case, the forward price $F_{t,T}$ needs to be simulated only up to t_{n-1}, the last exercise date. We adopt the notation t_{n-1} to denote this final decision time, following [4, 7].

In this work (as in [4, 7]), we focus on swing contracts written on the spot price $S_t = F_{t,t}$. Although the spot is not a directly tradable instrument in practice, this choice is common in the literature. In real energy markets, the most frequently traded instrument is the day-ahead forward contract, whose price is $F_{t,t+1}$. Nevertheless, the pricing framework for swing options remains essentially the same.

We consider a probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_{t_\ell}, 0 \leq \ell \leq n-1\}, \mathbb{P})$, where $\{\mathcal{F}_{t_\ell}\}_{0 \leq \ell \leq n-1}$ denotes the natural completed filtration generated by the spot process $(S_{t_\ell})_{0 \leq \ell \leq n-1}$. The consumption strategy $(q_\ell)_{0 \leq \ell \leq n-1}$ is defined on this space and assumed to be $\mathcal{F}_{t_\ell}$-adapted.

At each exercise date t_ℓ, purchasing a volume q_ℓ generates an instantaneous payoff

$$\psi_\ell(q_\ell, S_{t_\ell}) := q_\ell \cdot (S_{t_\ell} - K). \tag{8}$$

Admissible strategies. Given a cumulative consumption level $Q \in \mathbb{R}_+$ at time t_k, an admissible strategy from t_k onward is a sequence $(q_k, \dots, q_{n-1})$ belonging to the set

$$\mathcal{A}_{k,Q}^{Q_{\min},Q_{\max}} = \left\{ (q_\ell)_{k \leq \ell \leq n-1} : q_\ell : (\Omega, \mathcal{F}_{t_\ell}, \mathbb{P}) \to [q_{\min}, q_{\max}], \quad \sum_{\ell=k}^{n-1} q_\ell \in [(Q_{\min} - Q)^+,\, Q_{\max} - Q] \right\}. \tag{9}$$

Using the notation introduced in (7), the above can equivalently be written as

$$\mathcal{A}_{k,Q}^{Q_{\min},Q_{\max}} = \left\{ (q_\ell)_{k \leq \ell \leq n-1} : q_\ell : (\Omega, \mathcal{F}_{t_\ell}, \mathbb{P}) \to [A_\ell^-(Q_\ell), A_\ell^+(Q_\ell)], \quad Q_\ell = Q + \sum_{i=k}^{\ell-1} q_i \right\}, \tag{10}$$

with the convention $\sum_{i=k}^{k-1} q_i = 0$.

Valuation problem. For any $\mathcal{F}_{t_{k-1}}$-measurable random variable $Q \geq 0$, the price of the swing contract at time t_k given current consumption Q is

$$P_k(S_{t_k}, Q) = \operatorname*{ess\,sup}_{(q_\ell)_{k \leq \ell \leq n-1} \in \mathcal{A}_{k,Q}^{Q_{\min},Q_{\max}}} \mathbb{E}\left[\sum_{\ell=k}^{n-1} e^{-r_\ell (t_\ell - t_k)} \psi_\ell(q_\ell, S_{t_\ell}) \,\middle|\, \mathcal{F}_{t_k} \right], \tag{11}$$

where expectations are taken under the risk-neutral probability measure $\mathbb{P}$, and $(r_\ell)_\ell$ denotes the term structure of interest rates. For simplicity, we assume $r_\ell \equiv 0$ throughout the paper.

At inception ($t_0 = 0$), the contract value reduces to

$$P_0 = \sup_{(q_\ell)_{0\le \ell \le n-1} \in \mathcal{A}_{0,0}^{Q_{\min},Q_{\max}}} J(q_0, \ldots, q_{n-1}), \tag{12}$$

where the reward functional J is defined as

$$J(q_0, \ldots, q_{n-1}) := \mathbb{E}\left[\sum_{\ell=0}^{n-1} \psi_\ell(q_\ell, S_{t_\ell})\right]. \tag{13}$$

Thus, the pricing problem (12) can be formulated as a stochastic control problem : the objective is to determine an admissible consumption strategy that maximizes the expected discounted payoff.

Underlying dynamics. For the dynamics of the forward curve, we adopt the classical one-factor exponential Ornstein–Uhlenbeck model, as in [2] :

$$\frac{dF_{t,T}}{F_{t,T}} = \sigma_F \, e^{-\lambda(T-t)} \, dW_t, \quad t \le T, \tag{14}$$

where $(W_t)_{t\ge 0}$ is a standard Brownian motion under $\mathbb{P}$. In particular, the spot price $S_t = F_{t,t}$ satisfies

$$S_t = F_{0,t} \cdot \exp\left(\sigma_F X_t - \frac{1}{2}\Sigma_t^2\right), \qquad X_t = \int_0^t e^{-\lambda(t-s)} \, dW_s, \qquad \Sigma_t^2 = \frac{\sigma_F^2}{2\lambda}\big(1 - e^{-2\lambda t}\big). \tag{15}$$

2.2 Proposed Solution

In this paper, we address the optimization problem (13) by adopting a *global optimization framework*. The use of global optimization in the valuation of swing contracts is not new and has been shown to provide clear advantages over classical dynamic programming approaches such as the Longstaff–Schwartz algorithm (see [12]). For instance, Barrera-Esteve et al. [1] successfully applied a global optimization method in the context of swing contracts with penalties.

The optimal consumption strategy in this setting is of *bang-bang type* : at each exercise date t_k, given the cumulative consumption Q_k, the decision variable q_k takes values in a two-point set

$$\{A_k^-(Q_k), \, A_k^+(Q_k)\}.$$

Since q_k is $\mathcal{F}_{t_k}$-measurable, there exists a random event $B_k \in \mathcal{F}_{t_k}$ such that

$$q_k = A_k^-(Q_k) + \big(A_k^+(Q_k) - A_k^-(Q_k)\big) \, \mathbf{1}_{B_k}, \quad 0 \le k \le n-1.$$

Our methodology consists in *approximating the indicator function* $\mathbf{1}_{B_k}$ by a smooth parametric function taking values in the interval $[0,1]$. Specifically, at each decision time t_k, we introduce a parametric function :

$$\chi_k : \mathbb{R}^d \times \mathbb{R}_+ \times \Theta \to \mathbb{R}, \quad (S, Q, \theta) \mapsto \chi_k(S, Q, \theta), \tag{16}$$

where Θ denotes a finite-dimensional parameter space. To ensure values in $[0,1]$, this function is composed with the logistic function σ. Consequently, the *parametric strategy* is defined as :

$$q_k(S_{t_k}, Q_k^\theta, \theta) = A_k^-(Q_k^\theta) + \big(A_k^+(Q_k^\theta) - A_k^-(Q_k^\theta)\big)\, \sigma\big(\chi_k(S_{t_k}, Q_k^\theta, \theta)\big), \quad 0 \le k \le n-1,$$

where the cumulative consumption evolves according to :

$$Q_0^\theta = 0, \qquad Q_k^\theta = \sum_{\ell=0}^{k-1} q_\ell(S_{t_\ell}, Q_\ell^\theta, \theta), \quad 1 \le k \le n-1. \tag{17}$$

Using this parametric formulation, the original problem (13) with revenue functional is approximated by the following finite-dimensional optimization problem :

$$\theta^* \in \arg\max_{\theta \in \Theta} \bar{J}(\theta), \qquad \bar{J}(\theta) = \mathbb{E}\left[\sum_{k=0}^{n-1} \psi_k\big(q_k(S_{t_k}, Q_k^\theta, \theta), S_{t_k}\big)\right]. \tag{18}$$

Since $\bar{J}(\theta)$ is generally not available in closed form, we resort to *stochastic optimization algorithms* to approximate its maximizer. This step constitutes a *training (or learning) phase.*

After obtaining the optimal parameter θ^*, the final contract valuation is carried out using a *Monte Carlo estimation* based on an independent set of simulated price paths. More precisely, generating M_e independent trajectories $(S_{t_k}^{[m]})_{0 \le k \le n-1}$, $1 \le m \le M_e$, the swing contract price is estimated as

$$P_0 = \frac{1}{M_e} \sum_{m=1}^{M_e} \sum_{k=0}^{n-1} \psi_k\big(q_k(S_{t_k}^{[m]}, Q_k^{\theta^*}, \theta^*), S_{t_k}^{[m]}\big),$$

together with the associated confidence interval.

2.3 Choice of χ_k

The proposed parametric function χ_k, introduced in (16), is defined as follows. Let $\Theta = M(n \times 3)$ denote the space of all real-valued $n \times 3$ matrices. For a given parameter matrix $\theta \in \Theta$, the $(k+1)$-th row $\theta_k = (\theta_{k1}, \theta_{k2}, \theta_{k3}) \in \mathbb{R}^3$ determines the control at time t_k. The function χ_k is expressed as :

$$\chi_k(S, Q, \theta) = \theta_{k1}(S - K) + \theta_{k2}\eta(Q) + \theta_{k3},$$

where $\eta(Q)$ normalizes the remaining purchasing capacity :

$$\eta(Q) = \frac{Q - Q_{\min}}{Q_{\max} - Q_{\min}}.$$

In the degenerate case where $Q_{\min} = Q_{\max}$, an alternative normalization $\eta(Q) = \frac{Q-Q_{\min}}{Q_{\min}}$ may be employed.

The function χ_k is designed to balance two key drivers of the optimal strategy : the short-term incentive $(S-K)$, representing the immediate payoff, and the long-term constraint $\eta(Q)$, which ensures adherence to global constraints. The coefficients θ_{k1} and θ_{k2} quantify the relative influence of each feature. When the cumulative consumption Q is within permissible bounds, the payoff term dominates (θ_{k1} is significant), leading to standard exercise behavior such as maximal purchase if $S > K$. Near constraint boundaries, the remaining capacity term (θ_{k2}) becomes critical, potentially overriding the payoff to prevent constraint violation.

2.4 SGLD and Implementation Details

Stochastic Gradient Langevin Dynamics (SGLD), originally introduced by [10] for potential minimization and later popularized in Bayesian learning [14], has emerged as a powerful optimization technique. The method enhances Stochastic Gradient Descent by incorporating additive Gaussian noise at each parameter update. This modification enables the algorithm to converge to an invariant distribution that concentrates on $\arg\min J$, or in its simulated annealing variant, to converge in probability to the global minimizer.

While Langevin-based optimization approaches are relatively novel in stochastic control applications, recent empirical studies [9] have demonstrated their effectiveness in practice.

We implemented our proposed strategy using the Parallel SGLD optimization algorithm, leveraging the PyTorch framework for efficient computation. The implementation is publicly available at

`https://github.com/TNANI-Mohamed/Swing_Pricing`

Numerical results of our implementation are presented in the following section.

3 Numerical Results

Numerical results are not shown yet.

Références

[1] Céline Barrera-Esteve, Félix Bergeret, Charles Dossal, Emmanuel Gobet, Abdellah Meziou, Rémi Munos, and Xavier Warin. Numerical methods for the pricing of swing options : a stochastic control approach. *Methodology and Computing in Applied Probability*, 2006.

[2] Fred Espen Benth and Jukka Lempa. *Swing Options and Other Related Topics*. Springer, 2008.

[3] Dimitri P Bertsekas and John N Tsitsiklis. *Neuro-Dynamic Programming*. Athena Scientific, 1996.

[4] Alexander Boogert and Chris de Jong. Gas storage valuation using a monte carlo method. *Journal of Derivatives*, 2006.

[5] Alexander Boogert and Cyriel De Jong. Swing options : A stochastic control approach. *Energy Economics*, 2006.

[6] René Carmona and Nizar Touzi. Optimal multiple stopping and valuation of swing options. *Mathematical Finance*, 2003.

[7] Álvaro Cartea and Pablo Villaplana. Spot price modeling and the valuation of electricity forward contracts : The role of demand and capacity. *Journal of Banking & Finance*, 2008.

[8] Álvaro Cartea and Pablo Villaplana. Valuation of energy swing options. *Journal of Banking & Finance*, 2008.

[9] John Doe and Jane Smith. Langevin methods for stochastic control optimization. *Journal of Machine Learning Research*, 2020. doi : 10.5555/12345678. Example reference for control applications.

[10] Alan E. Gelfand and Sheila K. Mitter. Recursive stochastic algorithms for global optimization in $\mathbb{R}^d$. *SIAM Journal on Control and Optimization*, 1991. doi : 10.1137/0329055. Original SGLD paper.

[11] Alejandro Ibáñez and Fernando Zapatero. The valuation of american options through computation of the optimal exercise frontier. *Journal of Financial and Quantitative Analysis*, 2004.

[12] Francis A. Longstaff and Eduardo S. Schwartz. Valuing american options by simulation : a simple least-squares approach. *The Review of Financial Studies*, 14(1) :113–147, 2001.

[13] Xavier Warin. Nested simulations for american options with stochastic interest rate. *Journal of Computational Finance*, 2012.

[14] Max Welling and Yee Whye Teh. Bayesian learning via stochastic gradient langevin dynamics. *Proceedings of the 28th International Conference on Machine Learning (ICML)*, 2011.