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7-DOF Redundant Manipulator Control using Quaternion Feedback based on Virtual Spring-Damper Hypothesis

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Abstract:
In this paper, redundant manipulator control method of position and orientation for human-like movements is investigated. For orientation control, singularity-free quaternion representation is applied, and computation of orientation error with coordinate transformation is inspected. Control law for set-point regulation of position/orientation is proposed, which is based on the virtual spring-damper hypothesis, and the stability is studied using a Lyapunov-like energy function. Dynamic simulation is conducted and analyzed for validation of the proposed method.

1. Introduction

A seven degree-of-freedom (DOF) manipulator is a redundant system in 3D space as a human arm which has high capability in various tasks. Control of redundant manipulators has been an issue for more than thirty years. There have been variety of methods to handle this problem, and this is still in concern for many researchers.

The conventional method is inverse kinematics approach using pseudo-inverse of Jacobian matrix.[(1)~(3)] Given the end-effector velocity, joint angle velocity is achieved, and then a control signal is generated using computed torque method.

This approach is extended to closed-loop inverse kinematics method.[(4)~(6)] Tracking error feedback is augmented in the computation of joint angle velocity to eliminate the steady-state error. Both of Jacobian pseudo-inverse or Jacobian transpose guarantee the stability due to the closed-loop feedback scheme.

Another method is inverse dynamics approach proposed by Khatib.[(7)] This method generates a control signal directly using the inertia matrix and compensation of Coriolis and centrifugal forces.

For above methods, redundant motion in Jacobian null-space should is generated according to certain artificial performance indices, such as torque and kinetic energy. In order to obtain human-like movements, Arimoto, *et al.* proposed a natural control method based on a virtual spring-damper hypothesis.[(8)~(10)] Control signal is composed of two terms: angular velocity damping and task-space (position) error feedback with stiffness/damping parameters.

$$\mathbf{u} = -\mathbf{C}\dot{\mathbf{q}} - \mathbf{J}^{\mathrm{T}}(\mathbf{q})(c\,\dot{\mathbf{x}} + k\,\Delta\mathbf{x}) \tag{1}$$

In this quite simple structure, the control signal is directly computed for reaching motion. In addition to the experimental result, the virtual spring-damper hypothesis is regarded to offer human-like movement in the viewpoint of physiology.

In this paper, this method is extended to simultaneous position/orientation control. Within the basic control scheme, task-space orientation error is obtained, and a corresponding control signal is computed.

There are various representations for orientation. The most frequently used representation is the Euler angle convention which is composed of three parameters, but it inevitably contains singular points.[(11)]

Quaternion (also known as Euler parameters) which is composed of four parameters with one holonomic constraint is rather popular for orientation control. It has been applied to a spherical wrist, a manipulator, an underwater vehicle, etc. for set-point regulation.[(12)~(14)] There are papers on redundant manipulator control using quaternion. Xian, *et al.* applied quaternion feedback to model-based tracking control with Jacobian pseudo-inverse,[(15)] and Caccavale, *et al.* adopted it to set-point regulation based on closed-loop inverse kinematics.[(16)]

This paper applies quaternion feedback control to human-like position/orientation set-point regulation of a redundant manipulator. Quaternion-based orientation control scheme is introduced in section 2, and set-point regulation control of a redundant manipulator is investigated in section 3. To validate the proposed method, simulation result is presented in section 4, and section 5 concludes this paper.

2. Quaternion-based Orientation Control

2·1 Orientation Representations

There are several representations for orientation of a rigid body. The minimal number of parameters to represent orientation is three. Representations with more than three parameters have holonomic constraints as many as the number of parameters that exceeds three. The followings are the typical representation of orientation, where S^n denotes n-sphere in (n+1)-dimensional Euclidean space.

- Rotation matrix, $SO(3) \subset \mathbb{R}^9$
- Euler angle, $(\phi, \psi, \theta) \subset \mathbb{R}^3$
- Quaternion, $(\eta, \boldsymbol{\varepsilon}) \subset S^3$
- Angle/Axis Parameters, $(\theta, \mathbf{u}) \subset \mathbb{R} \times S^2$

Our concern is to inspect the representation adequate for orientation control. The requirement is that the representation should be nonsingular for any attitude and efficient to calculate.

Rotation matrix is in 3-by-3 matrix, i.e. composed of nine real numbers. The set of rotation matrices is also called as the *Special Orthogonal Group*, SO(3), which has six holonomic constraints. Since rotation matrix has six redundant parameters, this representation is computationally expensive. Moreover, orientation error in rotation matrix form is difficult to be applied to control loop directly.

Euler angles are very representation, which is defined by three successive rotations about three principal axes in a certain sequence. As noted in section 1, however, it contains singular points. For example, ZYX Euler angles has a singularity for rotation of 90° about Y-axis.

Angle/axis parameters and quaternion have four parameters, thus these can represent orientation globally and nonsingularily. Angle/axis parameters are composed of unit rotation axis and corresponding rotation angle. Euler's rotation theorem establishes that just one rotation about an axis can cover all of orientation transformations.

Quaternion is used for orientation control in this paper. Its parameters are a scalar and a three dimensional vector. Equation (2) is the relationship with quaternion and angle/axis parameters.

$$\begin{cases} \eta = \cos(\theta/2) \\ \boldsymbol{\varepsilon} = \sin(\theta/2)\,\mathbf{u} \end{cases} \tag{2}$$

where $\theta \in \mathbb{R}^1$ and $\mathbf{u} \in S^2$ are rotation angle and axis, respectively. The holonomic constraint for $\boldsymbol{\xi} = \{\eta, \boldsymbol{\varepsilon}\} \in S^3$ is as follows.

$$\eta^2 + \boldsymbol{\varepsilon}^T\boldsymbol{\varepsilon} = 1 \tag{3}$$

Note that $\{\eta, \boldsymbol{\varepsilon}\}$ and $\{-\eta, -\boldsymbol{\varepsilon}\}$ indicates the same orientation, so that quaternion space, S^3, is a double cover of SO(3).

Several formulae for quaternion are introduced here. Equation (4) is for singular-free extraction of quaternion from a rotation matrix.

$$\begin{bmatrix} \eta^2 \\ \varepsilon_1^2 \\ \varepsilon_2^2 \\ \varepsilon_3^2 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ -1 & 1 & -1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} r_{11} \\ r_{22} \\ r_{33} \\ r_{44} \end{bmatrix} \tag{4}$$

$$\text{where} \begin{cases} \mathrm{sgn}(\eta) = \pm 1 \\ \mathrm{sgn}(\varepsilon_1) = \mathrm{sgn}(\eta)\cdot\mathrm{sgn}(r_{32} - r_{23}) \\ \mathrm{sgn}(\varepsilon_2) = \mathrm{sgn}(\eta)\cdot\mathrm{sgn}(r_{13} - r_{31}) \\ \mathrm{sgn}(\varepsilon_3) = \mathrm{sgn}(\eta)\cdot\mathrm{sgn}(r_{21} - r_{12}) \end{cases}$$

r_{ij} stands for the element in i-th row and j-th column in rotation matrix, $\mathbf{R}$, and $\mathrm{sgn}(\cdot)$ implies the signum function that returns the sign of the given value.

Accordingly, quaternion for $\mathbf{R}^T$ is obtained by conjugate of quaternion for $\mathbf{R}$.

$$\boldsymbol{\xi}^* = \{\eta, -\boldsymbol{\varepsilon}\} \tag{5}$$

Note that the quaternion vector part, $\boldsymbol{\varepsilon}$, is the eigenvector of $\mathbf{R}$ with unit eigenvalue of 1 or -1, satisfying Eq. (6).

$$\mathbf{R}\boldsymbol{\varepsilon} = \mathbf{R}^T\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon} \tag{6}$$

Quaternion multiplication, which means successive rotations, is defined as follows.

$$\boldsymbol{\xi}_1\boldsymbol{\xi}_2 = \begin{bmatrix} \eta_1 & -\boldsymbol{\varepsilon}_1^T \\ \boldsymbol{\varepsilon}_1 & \eta_1\mathbf{I} + [\boldsymbol{\varepsilon}_1] \end{bmatrix} \begin{bmatrix} \eta_2 \\ \boldsymbol{\varepsilon}_2 \end{bmatrix} \tag{7}$$

where $[\cdot]$ stands for skew-symmetric matrix operation.

$$[\mathbf{x}] = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix} \quad \text{for } \mathbf{x} = [x_1 \;\; x_2 \;\; x_3] \in \mathbb{R}^3 \tag{8}$$

2·2 Computation of Orientation Error

Orientation error in rotation matrix can be represented as follows, which is difficult to be used in control loop directly.

$$\widetilde{\mathbf{R}} = (\mathbf{R}_d)^T\,\mathbf{R} \tag{9}$$

where subscript d implies *desired value*.

Equation (9) can be identically represented in quaternion using formulae in section 2·1.

$$\widetilde{\boldsymbol{\xi}} = \boldsymbol{\xi}_d^*\boldsymbol{\xi} \tag{10}$$

$$\text{or} \begin{cases} \widetilde{\eta} = \eta\eta_d + \boldsymbol{\varepsilon}^T\boldsymbol{\varepsilon}_d \\ \widetilde{\boldsymbol{\varepsilon}} = -\eta\boldsymbol{\varepsilon}_d + \eta_d\boldsymbol{\varepsilon} - [\boldsymbol{\varepsilon}_d]\boldsymbol{\varepsilon} \end{cases} \tag{11}$$

The control objective is formulated as follows.

$$\boldsymbol{\xi} = \boldsymbol{\xi}_d \Leftrightarrow \widetilde{\boldsymbol{\xi}} = \begin{bmatrix} \pm 1 \\ \mathbf{0} \end{bmatrix} \tag{12}$$

The time derivative of quaternion is obtained from quaternion propagation rule, introducing the coordinate transformation matrix, $\mathbf{U}(\mathbf{q})$.

$$\dot{\widetilde{\boldsymbol{\xi}}} = \frac{1}{2}\mathbf{U}(\widetilde{\mathbf{q}})\widetilde{\boldsymbol{\omega}} = \frac{1}{2}\begin{bmatrix} -\widetilde{\boldsymbol{\varepsilon}}^T \\ \widetilde{\eta}\mathbf{I}_3 + [\widetilde{\boldsymbol{\varepsilon}}] \end{bmatrix}\widetilde{\boldsymbol{\omega}} = \frac{1}{2}\begin{bmatrix} -\widetilde{\boldsymbol{\varepsilon}}^T \\ \mathbf{T}(\widetilde{\boldsymbol{\xi}}) \end{bmatrix}\widetilde{\boldsymbol{\omega}} \tag{13}$$

where $\widetilde{\boldsymbol{\omega}} = \boldsymbol{\omega} - \boldsymbol{\omega}_d$ denotes the angular velocity error.

Orientation error in this paper is defined using quaternion and the coordinate transformation matrix. The stability and equilibrium points are discussed in section 3·2.

$$\mathbf{e}_o = 2\mathbf{T}^T(\widetilde{\boldsymbol{\xi}})\,\widetilde{\boldsymbol{\varepsilon}} = 2\widetilde{\eta}\widetilde{\boldsymbol{\varepsilon}} \tag{14}$$

Note that the orientation error in quaternion is represented in angle/axis parameters using Eq. (2) as follows, giving us the physical interpretation.

$$\mathbf{e}_o = 2\widetilde{\eta}\widetilde{\boldsymbol{\varepsilon}} = (\sin\widetilde{\theta})\,\widetilde{\mathbf{u}} \tag{15}$$

3. Set-point Regulation of a Redundant Manipulator

3·1 Controller for Set-point Regulation

Set-point regulation for position and orientation of the end-effector is the goal of this paper. A Controller based on the virtual spring-damper hypothesis for a redundant manipulator is proposed here.

Task-space error for position and orientation is defined as:

$$\mathbf{e} = \begin{bmatrix} \mathbf{e}_p \\ \mathbf{e}_o \end{bmatrix} = \begin{bmatrix} \tilde{\mathbf{x}} \\ \mathbf{T}^{\mathrm{T}}(\tilde{\boldsymbol{\xi}})\,\tilde{\boldsymbol{\varepsilon}} \end{bmatrix} = \begin{bmatrix} \mathbf{x} - \mathbf{x}_d \\ 2\tilde{\eta}\tilde{\boldsymbol{\varepsilon}} \end{bmatrix} \tag{16}$$

Let us consider a potential function, $V(\mathbf{e})$, corresponding to task-space error, where $\mathbf{K}$ is a positive definite diagonal matrix.

$$V(\mathbf{e}) = \frac{1}{2}\mathbf{e}^{\mathrm{T}}\mathbf{K}\mathbf{e} \tag{17}$$

Time derivative of $V(\mathbf{e})$ gives an analogy of power and torque - joint velocity.

$$\dot{V}(\mathbf{e}) = \mathbf{e}^{\mathrm{T}}\mathbf{K}\dot{\mathbf{e}} = \mathbf{e}^{\mathrm{T}}\mathbf{K}\,\mathbf{J}(\mathbf{q})\dot{\mathbf{q}} = -\boldsymbol{\tau}^{\mathrm{T}}\dot{\mathbf{q}} \tag{18}$$

$$\dot{P}(\mathbf{x}) = \left(\frac{\partial P}{\partial \mathbf{x}}\right)^{\mathrm{T}} \dot{\mathbf{x}} = -\mathbf{f}^{\mathrm{T}}\dot{\mathbf{x}} \tag{19}$$

where $P(\mathbf{x})$ is a potential function, and $\mathbf{f}$ is a conservative force. Thus, joint torque can be assigned using this result.

$$\boldsymbol{\tau} = -\mathbf{J}^{\mathrm{T}}(\mathbf{q})\mathbf{K}\mathbf{e} \tag{20}$$

This is the basic scheme of the virtual spring hypothesis. A virtual spring at the end-effector imposes the manipulator to get to the target pose at which the task-space error vanishes.

It is extended to the virtual spring-damper hypothesis including a virtual damper at the end-effector.

$$\boldsymbol{\tau} = -\mathbf{J}^{\mathrm{T}}(\mathbf{q})(\mathbf{C}\boldsymbol{\nu} + \mathbf{K}\mathbf{e}) \tag{21}$$

where $\boldsymbol{\nu} = [\,\dot{\mathbf{x}}^{\mathrm{T}} \;\; \boldsymbol{\omega}^{\mathrm{T}}\,]^{\mathrm{T}}$ and $\mathbf{C} = \mathrm{diag}[\,c_p\mathbf{I}_3,\ c_o\mathbf{I}_3\,]$ for $c_p > 0$, $c_o > 0$.

As a consequence, a set-point regulation controller including joint damping and gravity compensation is proposed as follows.

$$\begin{aligned} \boldsymbol{\tau} &= -\mathbf{J}^{\mathrm{T}}(\mathbf{q})(\mathbf{C}\boldsymbol{\nu} + \mathbf{K}\mathbf{e}) - \mathbf{C}_0\dot{\mathbf{q}} + g(\mathbf{q}) \\ &= -\mathbf{J}_p^{\mathrm{T}}(\mathbf{q})(c_p\dot{\mathbf{x}} + k_p\mathbf{e}_p) - \mathbf{J}_o^{\mathrm{T}}(\mathbf{q})(c_o\boldsymbol{\omega} + k_o\mathbf{e}_o) \\ &\qquad - \mathbf{C}_0\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) \end{aligned} \tag{22}$$

where $\mathbf{K} = \mathrm{diag}[\,k_p\mathbf{I}_3,\ k_o\mathbf{I}_3\,]$ and $\mathbf{C}_0 = \mathrm{diag}[\,c_1, \cdots, c_n\,]$ for $k_p > 0$, $k_o > 0$, $c_i > 0$.

3·2 Stability Analysis on Orientation

The closed-loop system with the proposed controller is described as follows.

$$\mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \left\{\frac{1}{2}\dot{\mathbf{H}}(\mathbf{q}) + \mathbf{S}(\mathbf{q}, \dot{\mathbf{q}})\right\}\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \boldsymbol{\tau} \tag{23}$$

$$\begin{aligned} &\mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \left\{\frac{1}{2}\dot{\mathbf{H}}(\mathbf{q}) + \mathbf{S}(\mathbf{q}, \dot{\mathbf{q}})\right\}\dot{\mathbf{q}} + \mathbf{C}_0\dot{\mathbf{q}} \\ &\quad + \mathbf{J}_p^{\mathrm{T}}(\mathbf{q})(c_p\dot{\mathbf{x}} + k_p\mathbf{e}_p) + \mathbf{J}_o^{\mathrm{T}}(\mathbf{q})(c_o\boldsymbol{\omega} + k_o\mathbf{e}_o) = 0 \end{aligned} \tag{24}$$

where $\mathbf{H}(\mathbf{q})$ denotes the inertia matrix, and $\mathbf{S}(\mathbf{q}, \dot{\mathbf{q}})$ stands for centrifugal and Coriolis forces.

By the inner product of Eq. (24) with $\dot{\mathbf{q}}$, we obtain the time derivative of Lyapunov-like energy function, $E(\mathbf{q}, \dot{\mathbf{q}})$. Note that $\dot{\mathbf{x}} = \mathbf{J}_p(\mathbf{q})\dot{\mathbf{q}}$ and $\boldsymbol{\omega} = \mathbf{J}_o(\mathbf{q})\dot{\mathbf{q}}$.

$$\frac{\mathrm{d}}{\mathrm{d}t}E = \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\left\{\dot{\mathbf{q}}^{\mathrm{T}}\mathbf{H}(\mathbf{q})\dot{\mathbf{q}}\right\} + k_p\dot{\mathbf{x}}^{\mathrm{T}}\mathbf{e}_p + k_o\boldsymbol{\omega}^{\mathrm{T}}\mathbf{e}_o \tag{25a}$$

$$= -\dot{\mathbf{q}}^{\mathrm{T}}\mathbf{C}_0\dot{\mathbf{q}} - c_p\|\dot{\mathbf{x}}\|^2 - c_o\|\boldsymbol{\omega}\|^2 \tag{25b}$$

For $\dot{\mathbf{x}}_d = \boldsymbol{\omega}_d = \mathbf{0}$, Eq. (26) and Eq. (27) are satisfied.

$$\frac{\mathrm{d}}{\mathrm{d}t}\|\tilde{\mathbf{x}}\|^2 = \frac{\mathrm{d}}{\mathrm{d}t}\|\mathbf{x} - \mathbf{x}_d\|^2 = 2\dot{\mathbf{x}}^{\mathrm{T}}\mathbf{e}_p \tag{26}$$

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}(1 - \tilde{\eta}^2) &= \frac{\mathrm{d}}{\mathrm{d}\tilde{\eta}}(1 - \tilde{\eta}^2)\cdot\dot{\tilde{\eta}} \\ &= 2\tilde{\eta}\tilde{\boldsymbol{\varepsilon}}^{\mathrm{T}}\tilde{\boldsymbol{\omega}} = 2\boldsymbol{\omega}^{\mathrm{T}}\mathbf{e}_o \end{aligned} \tag{27}$$

Thus, $E(\mathbf{q}, \dot{\mathbf{q}})$ is obtained as follows.

$$E(\mathbf{q}, \dot{\mathbf{q}}) = \frac{1}{2}\left\{\dot{\mathbf{q}}^{\mathrm{T}}\mathbf{H}(\mathbf{q})\dot{\mathbf{q}} + k_p\|\tilde{\mathbf{x}}\|^2 + k_o(1 - \tilde{\eta}^2)\right\} \tag{28}$$

The first term on the right-hand side in Eq. (28) is the kinetic energy in joint-space, and the second and the third terms are the artificial potentials in task-space. The formulation of $E(\mathbf{q}, \dot{\mathbf{q}})$ reminds of Lyapunov stability analysis.

The time derivative, $\mathrm{d}E/\mathrm{d}t$, is negative semi-definite. Equation (25b) implies the passivity of the closed-loop system dynamics. The first term on the right-hand side is energy dissipation in joint-space, and the rest two terms are that in task-space, i.e. position and orientation respectively. According to Barbalat's lemma, the system satisfies:

$$0 \le E(t) = E(\mathbf{q}(t), \dot{\mathbf{q}}(t)) \le E(0) \tag{29}$$

$$0 \le \int_0^{\infty} \frac{\mathrm{d}}{\mathrm{d}t}E(t)\mathrm{d}t \le E(0) \tag{30}$$

which lead to the convergence of $\dot{\mathbf{q}}(t)$ to zero as $t \to \infty$.

However, $E(\mathbf{q}, \dot{\mathbf{q}})$ is not positive definite, due to the system redundancy. As shown in Eq. (28), the two artificial potentials are positive definite quadratic functions, but the kinetic energy has an infinite number of equilibrium points with respect to the state vector $(\mathbf{q}, \dot{\mathbf{q}}) \in \mathbb{R}^{14}$ for a 7-DOF manipulator. This equilibrium points are defined by a one-dimensional manifold in $\mathbb{R}^{14}$.

$$\begin{aligned} M_1 = \{(\mathbf{q}, \dot{\mathbf{q}}) : E(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{0}, \\ \text{i.e. } \dot{\mathbf{q}} = \mathbf{0},\ \mathbf{x}(\mathbf{q}) = \mathbf{x}_{\mathbf{d}},\ \boldsymbol{\xi}(\mathbf{q}) = \boldsymbol{\xi}_{\mathbf{d}}\} \end{aligned} \tag{31}$$

In other words, any posture of $(\mathbf{q}, \dot{\mathbf{q}})$ in static state with the end-effector at the desired position and orientation is in M_1.

The stability of set-point position control for a redundant manipulator is theoretically proved using the concepts of *stability on manifold* and *transferability to a submanifold*.(10) This paper extends the proof to the position/orientation control by augmenting orientation artificial potential to the total potential.

Let us denote the artificial potential for position and orientation as $P_p(\widetilde{\mathbf{x}}) = \frac{1}{2}k_p\|\widetilde{\mathbf{x}}\|^2$ and $P_o(\widetilde{\eta}) = \frac{1}{2}k_o(1-\widetilde{\eta}^2)$, respectively. The total potential is denoted as $P_t(\widetilde{\mathbf{x}}, \widetilde{\eta}) = P_p(\widetilde{\mathbf{x}}) + P_o(\widetilde{\eta})$.

According to the *stability on manifold* analysis, the artificial potential is positive definite. $P_p(\widetilde{\mathbf{x}})$ is already known to be positive definite. For orientation stability, it is easy to show that $P_o(\widetilde{\eta})$ is positive semi-definite for $\widetilde{\eta} \in [-1, 1]$, and the asymptotically stable equilibrium point is $\widetilde{\eta} = \pm 1$.

On the other hand, there exists an unstable equilibrium point, $\widetilde{\eta} = 0$. However, provided that the system is perturbed to $\widetilde{\eta} = 0 \pm \varepsilon$ for $\varepsilon > 0$, the followings are satisfied.

$$P_o(\widetilde{\eta}) = \frac{1}{2}k_o(1-\varepsilon^2) < P_{o,ss} \tag{32}$$

where $P_{o,ss} = P_o(0)$ is the potential value at steady-state. Since $\mathrm{d}E/\mathrm{d}t$ is negative semi-definite, the orientation state never returns to the unstable equilibrium point.

Thus, the sum of position and orientation potential does not violate the condition for artificial potential in *stability on manifold* analysis. Notice that in a case of position control, the stability manifold is of 4-dimension for a 7-DOF manipulator. For position/orientation control, however, the dimension of stability manifold reduces to one, while the stability analysis scheme is identical.

4. Simulation Study

4·1 Simulation Model

Dynamic simulation is conducted in order to validate the proposed controller. *Robotics Lab* of *Simulation Studio Co.*, is used as the simulator. Torque control is supposed for every joint, and gravity effect is applied. The kinematics of the 7-DOF redundant manipulator model is presented in Fig. 1 and Table 1. Note that the coordinate frame (X, Y, Z) is stands for the global coordinate frame, while (x_0, y_0, z_0) denotes the robot's base frame.

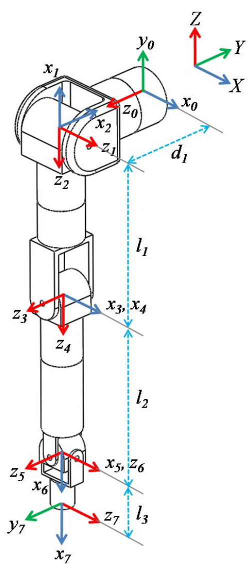


Fig. 1 Kinematics of 7-DOF manipulator model

Table 1 Denavit-Hartenberg parameters of the manipulator ($d_1 = 0.2$ m, $l_1 = l_2 = 0.3$ m, $l_3 = 0.155$ m)

i	θ_{i-1}	d_{i-1}	a_i	α_i
1	$q_1+\pi/2$	d_1	0	$\pi/2$
2	$q_2-\pi/2$	0	0	$\pi/2$
3	$q_3+\pi/2$	l_1	0	$-\pi/2$
4	q_4	0	0	$\pi/2$
5	q_5	l_2	0	$-\pi/2$
6	$q_6-\pi/2$	0	0	$-\pi/2$
7	q_7	0	l_3	0

4·2 Simulation Results

Two simulation results are presented in this section. One is the set-point regulation control of position and orientation. The other is the comparison of quaternion and Euler angle representation for singularity poses.

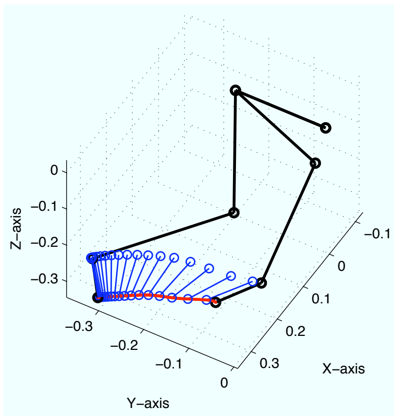
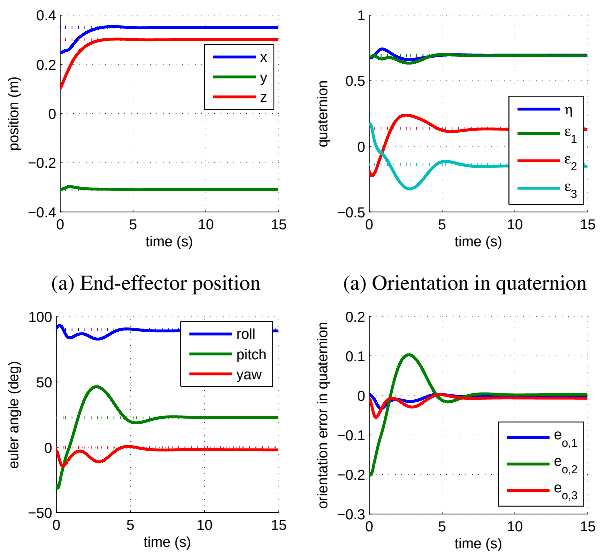


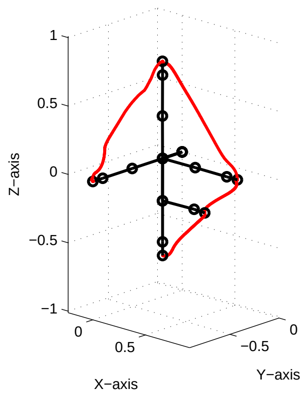
Fig. 2 Set-point regulation for position and orientation



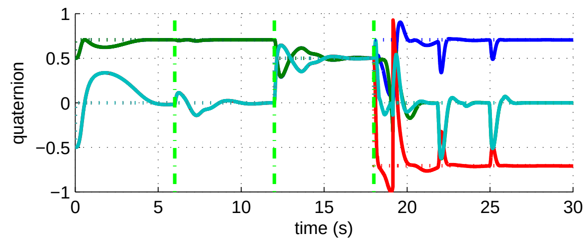
(a) End-effector position (a) Orientation in quaternion

(a) Orientation in Euler angles (a) Orientation error in quaternion

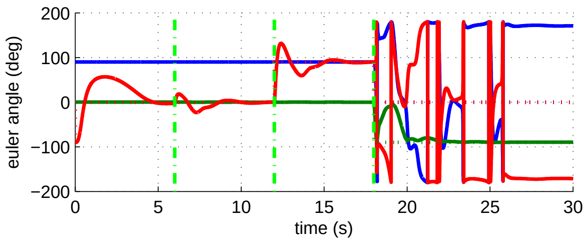
Fig. 3 Set-point regulation for position and orientation



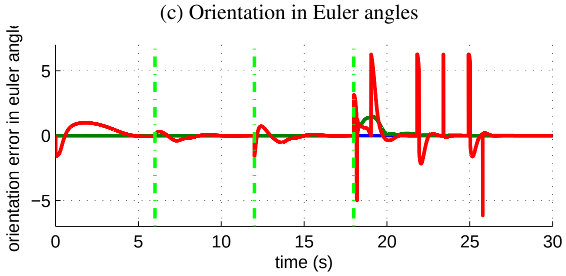
(a) Schematic representation



(b) Orientation in quaternion

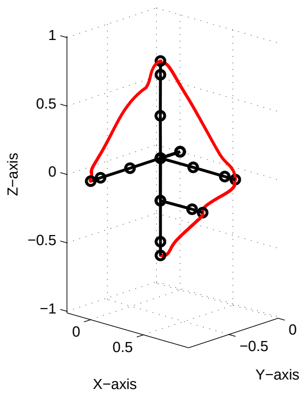


(c) Orientation in Euler angles

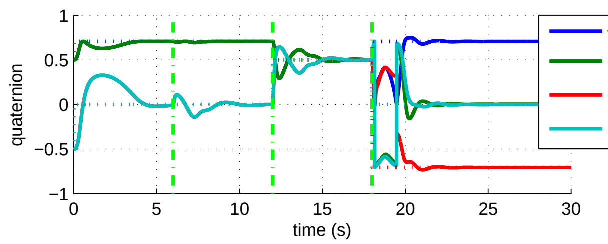


(d) Orientation error in Euler angles

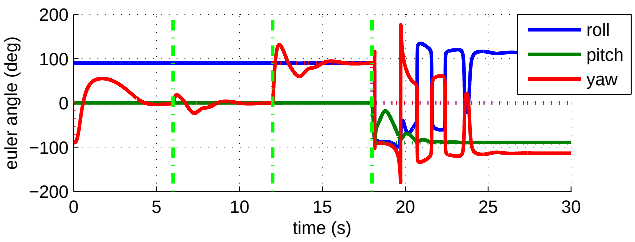
Fig. 4 Euler angle-based orientation control



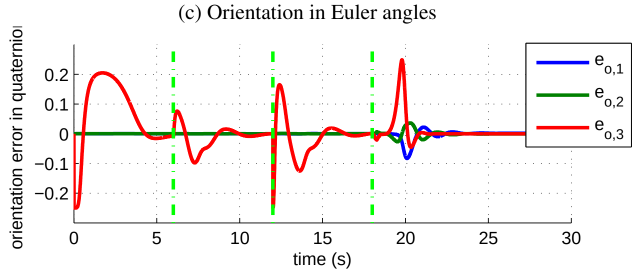
(a) Schematic representation



(b) Orientation in quaternion



(c) Orientation in Euler angles



(d) Orientation error in quaternion

Fig. 5 Quaternion-based orientation control

1) Set-point Regulation of Position and Orientation: Fig. 2 and Fig. 3 describe the trajectory of position and orientation in task-space and end-effector motion with respect to time, respectively. Orientation is controlled based on quaternion feedback, and ZXY Euler angles are presented as a reference. The initial pose is $\{(0.25\,\mathrm{m}, -0.31\,\mathrm{m}, 0.1\,\mathrm{m}), (90^\circ, -30^\circ, 0^\circ)\}$, and the desired pose is $\{(0.35\,\mathrm{m}, -0.31\,\mathrm{m}, 0.3\,\mathrm{m}), (90^\circ, 25^\circ, 0^\circ)\}$ with respect to the robot's base frame.

It is shown that the simultaneous control of position and orientation of the redundant manipulator is successfully achieved. The motion of the end-effector is quite smooth and natural without unnecessary motions. However, it should be noted that there exists a conflict between the gains for position and orientation control. In this simulation, the gain for the position control is prior to that of the orientation control, so that an overshoot is induced in orientation only. Therefore the gains should be selected carefully in regard of the specific control purpose.

2) Comparison of Quaternion and Euler Angles: Simulation of successive set-point regulation for singular poses is conducted. Four different set-points are assigned at 0, 6, 12, 18 sec, and two different orientation representations are used for orientation control: ZYX Euler angles and quaternion. The results are depicted in Fig. 4 and Fig. 5, respectively.

Both are successful in the end-effector position control, while the orientation control based on Euler angles is not stable. In Fig. 4(d), the orientation error computed from Euler angles is described. For the set-point after 18 sec, the computed orientation error shows abrupt change as the pitch angle crosses the value of 90°, i.e. the singular point.

Orientation based on quaternion representation is stable for any poses. From 18 sec to 20 sec in Fig. 5(a), the value of η crosses zero axis which is the unstable equilibrium point, and $\boldsymbol{\varepsilon}$ shows abrupt change. However, as shown in Fig. 5(d), the orientation error does not show any abnormalities.

5. Conclusion

In this paper, quaternion-based orientation control is augmented into the set-point position regulation scheme based on virtual spring-damper hypothesis. Among the several representations of orientation, quaternion is applied in the computation of orientation error feedback. Set-point regulation controller for position and orientation is proposed using virtual spring-damper hypothesis, which offers human-like reaching motion with a simple structure. Simulation with 7-DOF redundant manipulator model gave quite complacent results for successive set-point regulation without any singularities.

The relationship between the gains for position and orientation should be investigated further. Since the stiffness/damping parameters are directly related to the system stability, the guideline to find the adequate parameter values would be an important issue.

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