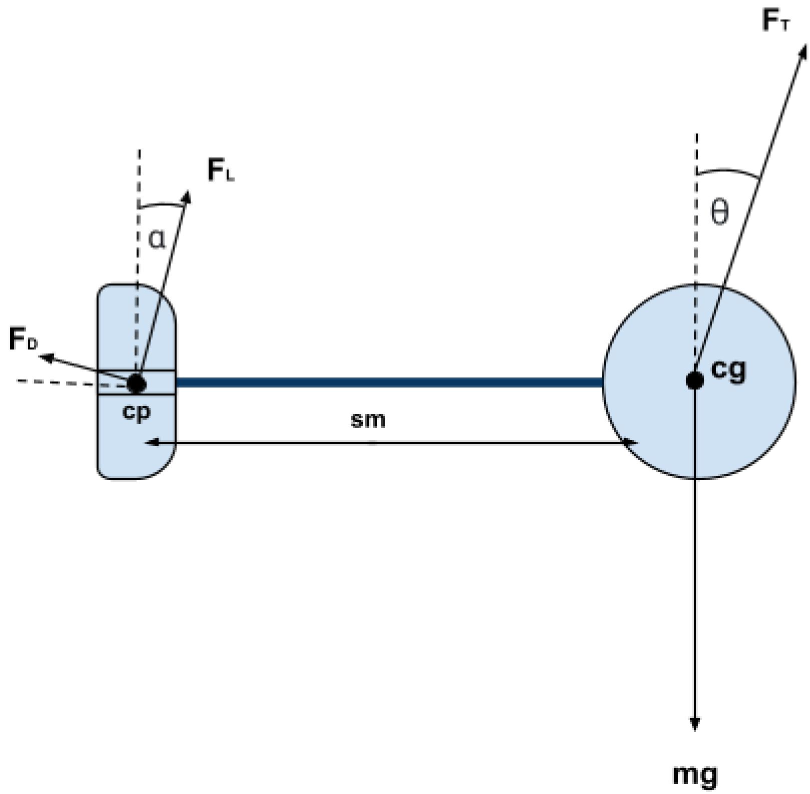
# TOBIAS Pitch Axis Stabilty Analysis




Pitch axis stability equations about center of gravity:

$\Sigma M_0$: $F_T cos(\theta)d - F_D sin(\alpha)(sm) - F_L cos(\alpha)(sm) = 0$

$F_Y$: $F_T cos(\theta) + F_D sin(\alpha) + F_L cos(\alpha) - mg = 0$

$F_X$: $F_T sin(\theta) + F_L sin(\alpha) - F_D cos(\alpha) = 0$

Rearrange moment equation & substitute for vertical component of tension using Y force balance:

$$d = \frac{F_D sin(\alpha)(sm) + F_L cos(\alpha)(sm)}{F_T cos(\theta)} \Rightarrow d = \frac{F_D sin(\alpha)(sm) + F_L cos(\alpha)(sm)}{mg - F_D sin(\alpha) - F_L cos(\alpha)} = sm\frac{F_D sin(\alpha) + F_L cos(\alpha)}{mg - F_D sin(\alpha) - F_L cos(\alpha)}$$

Solving for d, method 1: Lift-to-drag ratio (approximation, as this assumes cruise condition)

$$d = (sm)\frac{F_L(\frac{sin(\alpha)}{(L/D)} + cos(\alpha))}{mg - F_L(\frac{sin(\alpha)}{(L/D)} - cos(\alpha)} = (sm)\frac{(\frac{1}{2}C_L\rho U_\infty^2 S)(\frac{sin(\alpha)}{(L/D)} + cos(\alpha))}{mg - (\frac{1}{2}C_L\rho U_\infty^2 S)(\frac{sin(\alpha)}{(L/D)} - cos(\alpha)}$$

Solving for d, method 2: Lift and Drag formulae

$$d = sm\frac{F_D sin(\alpha)+F_L cos(\alpha)}{mg - F_D sin(\alpha) - F_L cos(\alpha)} = sm\frac{(\frac{1}{2}\rho_\infty U_\infty{}^2 S_{wetted} C_f + \frac{2L^2}{e\,\rho_\infty U_\infty^2 \pi b^2})sin(\alpha)+(\frac{1}{2}C_L \rho U_\infty^2 S)cos(\alpha)}{mg - (\frac{1}{2}\rho_\infty U_\infty{}^2 S_{wetted} C_f + \frac{2L^2}{e\,\rho_\infty U_\infty^2 \pi b^2})sin(\alpha) - (\frac{1}{2}C_L \rho U_\infty^2 S)cos(\alpha)}$$

$$\Rightarrow d = sm\frac{(\rho_\infty U_\infty{}^2 S)(\,(C_f + \frac{\frac{1}{2}(C_L cos(\alpha))(C_L S cos(\alpha))}{e\pi b^2})sin(\alpha)+(\frac{1}{2}C_L)cos(\alpha)\,)}{mg - (\rho_\infty U_\infty{}^2 S)((C_f + \frac{\frac{1}{2}(C_L cos(\alpha))(C_L S cos(\alpha))}{e\pi b^2})sin(\alpha)+(\frac{1}{2}C_L)cos(\alpha))}$$

**Conclusion:** the tether must attach to the towbody a distance d in front of the center of gravity, where d is of the form $sm\frac{aU_\infty^2}{b - aU_\infty^2}$. The faster the wind speed, the further forward the tether attachment point. If we have any lift-generating surface behind the center of gravity, the tether attachment point must lie in front of the center of gravity, in order to balance the moments.

As the windspeed changes, we should shift the attachment point, the center of lift, or even the center of gravity - any of the three would resolve the moment imbalance.

Potential fix - a variable-sweep tail: Changing the sweep of the tail changes the magnitude of the lift, the position of the center of pressure, and slightly changes the position of the center of gravity.

Alternative fix proposed by Luis: a system that passively changes the attachment point as the windspeed changes.