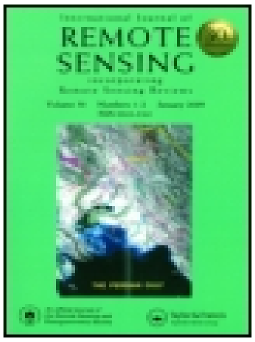

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Reliably estimating the noise in AVIRIS hyperspectral images

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Abstract. A new method is presented for computing the noise affecting each band of an AVIRIS hyperspectral image. Between-band (spectral) and within-band (spatial) correlations are used to decorrelate the image data via linear regression. Each band of the image is divided into small blocks, each of which is independently decorrelated. The decorrelation leaves noise-like residuals whose variance estimates the noise. A homogeneous set of these variances is selected and their values are combined to provide the best estimate of that band's noise. This method provides consistent noise estimates from images with very different land cover types. Its performance is validated by comparing its noise estimates with noise measures provided with two AVIRIS images. The method works well with inhomogeneous images (e.g., of a vegetated area such as Jasper Ridge) unlike a method described recently by Gao. The method is automatic and does not require the intervention of a human operator. Noise estimates are presented for 10 images recorded in the years 1989, 1990 and 1992. We show that the method can be used for both radiance and reflectance (atmospherically corrected) images.

1. Introduction

The Airborne Visible/Infrared Imaging Spectrometer (AVIRIS) produces 224-band hyperspectral images. It has been continually refined from year to year, progressively yielding better data with higher signal-to-noise ratios as the noise contributed by its instrument chain has been reduced. By noise, we mean the fluctuations introduced into the data by the instrument itself and not attributable to variations in the scene. One might expect this noise to be entirely random and uncorrelated from pixel to pixel within an image, but this is not a valid prior assumption for an instrument of the complexity of the AVIRIS. The noise affecting each AVIRIS band varies from band to band and, particularly in the bands covering the visible spectrum, the magnitude of band noise has a wide range. This paper describes a reliable and automatic method for estimating the band noise; it is a revised and extended version of Roger and Arnold (1994). For a good review and assessment of methods that have been proposed to estimate noise affecting image data, we refer readers to Gao (1993).

The AVIRIS is one of a new generation of multispectral remote sensing instruments which are designed to record images in the visible and near-infrared spectrum (Vane *et al.* 1993). Characteristically, imaging spectrometers have many more bands than Landsat and SPOT sensors (e.g., 60–200 as against 5–10). The AVIRIS produces multispectral images in which adjacent bands are very close in wavelength (about 10 nm apart) and in which each band spans a narrow range (also about 10 nm full-width, half-maximum). The spectral responses of adjacent bands therefore overlap significantly (Green 1995). In addition, at this spectral resolution, the solar spectrum, the spectrum of atmospheric attenuation, and the spectral reflectance of land cover types change relatively smoothly from one band to the next. For all these reasons, AVIRIS data have strong between-band (spectral) correlations. For similar reasons relating to the sensor's point spread function (Chrien and Green 1993), the data's within-band (spatial) correlations are quite strong though we have found them to be weaker than the spectral correlations. A partial correlation analysis of AVIRIS images shows convincingly that a band's two adjacent bands provide the best two-parameter linear predictor of it (Roger, in preparation).

2. Method

Our method of estimating band noise exploits the strong spectral correlations between the data in one band and the data in its two adjacent bands as well as the spatial correlation within that band. These correlations are removed using multiple linear regression leaving residuals whose mean square value estimates the variance of the band noise. The method incorporates the data blocking described by Gao (1993) so that many estimates of band noise are produced. A subset of these estimates can be pooled to give the best estimate of band noise. The homogeneity of the subset of estimates can be established using Levene's robust test for the homogeneity of variances (Kotz and Johnson 1983, Green and Margerison 1978, pp. 168–172). There are three major aspects to our method:

(*a*) The computation of the estimates of the noise variance.
(*b*) The selection of a homogeneous set of variances.
(*c*) The calculation of a best estimate of the noise from the homogeneous set of variances.

2.1. *Computing estimates of the noise variance*

Following Gao (1993) and the methods to which he refers, we estimate the noise in each band separately. We also adopt the strategy of dividing the band image into small blocks and independently estimating the noise in each block. We do this by removing significant correlations in the data using a regression model constructed to account for the physical characteristics of the AVIRIS sensor and the statistical characteristics of the data, as described above. This procedure leaves residuals whose variance is used to estimate band noise.

To estimate the noise in band k of the image, we use the data in bands $k-1$, k and $k+1$. The image data for a band are divided into rectangular, contiguous and non-overlapping blocks. Each block is w pixels wide and h pixels high. The noise estimate for each block is computed according to the following procedure.

Let $x_{i,j,k}$ be the pixel in band k at position (i,j) within a block, where $1 \leqslant i \leqslant w$ and $1 \leqslant j \leqslant h$. Residuals, $r_{i,j,k}$, are computed using

$$r_{i,j,k} = x_{i,j,k} - \hat{x}_{i,j,k} \tag{1}$$

where $\hat{x}_{i,j,k}$ is a value predicted for pixel $x_{i,j,k}$ and is given by

$$\hat{x}_{i,j,k} = a x_{i,j,k-1} + b x_{i,j,k+1} + c x_{p,k} + d \tag{2}$$

where

$$x_{p,k} = \begin{cases} x_{i-1,j,k} & i > 1 \\ x_{i,j-1,k} & i = 1, j > 1 \end{cases} \tag{3}$$

The residual $r_{1,1,k}$, is undefined and never needed. The coefficients, a, b, c and d, are computed to minimize S^2, the sum of the squares of these residuals, which is given by

$$S^2 = \sum_{i=1}^{w} \sum_{j=1}^{h} r_{i,j,k}^2 \qquad (i,j) \neq (1,1) \tag{4}$$

By construction, the mean value of these residuals is zero:

$$\sum_{i=1}^{w} \sum_{j=1}^{h} r_{i,j,k} = 0 \qquad (i,j) \neq (1,1) \tag{5}$$

We use the unbiased estimate of the variance of the residuals as the estimate of the noise variance, σ_x^2, for this block:

$$\sigma_x^2 = (M-4)^{-1} S^2, \qquad \text{where} \quad M = w \times h - 1 \tag{6}$$

In this estimate, the degrees of freedom are reduced from M to $M-4$ because four parameters are used in the regression (Johnson and Wichern 1982, Chapter 7). We have tried $w \times h$ block sizes of 16×16 and 614×1 (corresponding to processing each line of the image separately) and we get more consistent results with a 16×16 block size.

2.2. *Selecting a homogeneous set of noise variances*

The above procedure produces many estimates for the noise variance in each band. With our selected block size of 16×16 and an image size of 614×512, we actually get 1216 ($= 38 \times 32$) variance estimates per band. These variances themselves have a statistical distribution. Given their large number, we might expect that some would fall into the upper or lower tails of the distribution. To derive the best estimate of the noise, we need to select a homogeneous subset of them. To do this, we use a statistical test as a guide, and we follow Sachs (1984, pp. 495–500).

Sachs suggests that where there are more than ten estimates of the variance of non-normally distributed data, their homogeneity should be tested using Levene's procedure (see also Green and Margerison (1978), Kotz and Johnson (1983)). Let N be the number of estimates and T be the total number of residuals used in deriving those estimates, i.e., $T = NM$. One postulates the null hypothesis that the variances are equal, $H_0: \sigma_1^2 = \cdots = \sigma_N^2$. This is tested at a selected level of significance, α, against the alternative that not all the variances are the same. One computes a test statistic, W, which is approximately distributed as $F_{N-1,T-N}$ under the null hypothesis; $F_{N-1,T-N}$ is the value of the F distribution (Green and Margerison 1978, p. 60) with degrees of freedom $N-1$ and $T-N$. The null hypothesis is rejected if $W > F_{N-1,T-N}(1-\alpha)$. We have used the form of W given in Kotz and Johnson (1983, pp. 608–610), which can be written as

$$W = \frac{T-N}{N-1} \frac{M^{-1}\left(\sum_{n=1}^{N} S_n^2\right) - T^{-1} S_t^2}{\left(\sum_{n=1}^{N} \sum_{m=1}^{M} r_{nm}^2\right) - M^{-1}\left(\sum_{n=1}^{N} S_n^2\right)} \tag{7}$$

where

$$S_t = \sum_{n=1}^{N} S_n \quad \text{and} \quad S_n = \sum_{m=1}^{M} |r_{nm}| \tag{8}$$

In these equations, the band dependence has been suppressed, and the subscript m indexes the M residuals in each of the N blocks.

We also need to compute the value of F for our values of N, T and $(1-\alpha)$. Given a value of F, we can compute the corresponding probability, $\alpha' = Q(F, N, T)$, using the function described by Press *et al.* (1986, p. 169). For our selected value of $\alpha = 0{\cdot}95$, we find the corresponding value of F by solving the equation, $(1-\alpha) - Q(F, N, T) = 0$, with a root-finding method, e.g., the routine RTBIS in Press *et al.* (1986, p. 246).

2.3. *Best estimate of noise variance from a homogeneous set of variances*

Given a homogeneous set of variances, the best estimate of the variance of the underlying data is obtained by pooling those variances (Green and Margerison 1978, pp. 122–125). In this case, because the number of data points contributing to each estimate is the same, the best estimate of the variance is just the mean value of the variances in the homogeneous set. Gao (1993) averages the estimates of the noise standard deviations in his method. That procedure will underestimate the noise though only by a small amount as the range of the estimates is generally small.

In practice, in estimating the noise for a single band, we have found that the set of 1216 variances is not always homogeneous. It is not then valid to pool all of them and use their mean. We have to select a reasonably homogeneous subset of them and pool these. To do this, we have sorted the variances into order and deleted the top and bottom 15 per cent of them, leaving about 850 values to contribute to the best estimate. Although this does introduce some clipping, we believe it is better to do this rather than use an estimator known to be invalid. In practice, for most bands of most images, the difference between the estimates computed using all variances and only a homogeneous subset of them is seldom greater than 0·1. Nevertheless, this selection procedure is necessary for a small number of bands in some images.

3. **Evaluating the method on 1990 images**

We show that our method gives noise estimates for two test data sets which are consistent between the two images and with corresponding, independent measures of noise. We also compare our noise estimates with those computed using Gao's method (Gao 1993). For purposes of this comparison, we have implemented Gao's method using a block size of 4×4 and 150 bins as specified in the sub-section 'Bin Width Selection' of his paper. In the remainder of this paper and in all the figures, we shall use 'noise estimate' to mean our estimate of the noise standard deviation, i.e., the square root of the best estimate of the noise variance.

3.1. *Test data sets*

We have tested our method on two image data sets collected by the AVIRIS system in 1990. These are

1. A 1990 Moffett Field image (Flt. 900723, Run 013, 07/23/90).
2. A 1990 Jasper Ridge image (Flt. 900727, Run 05, 07/27/90).

The image data in both sets are radiometrically rectified. Figures 1 and 2 show scenes formed from the data in Band 40 of these two images. The Moffett Field image shows a very smooth area of water and a fairly homogeneous urban area.

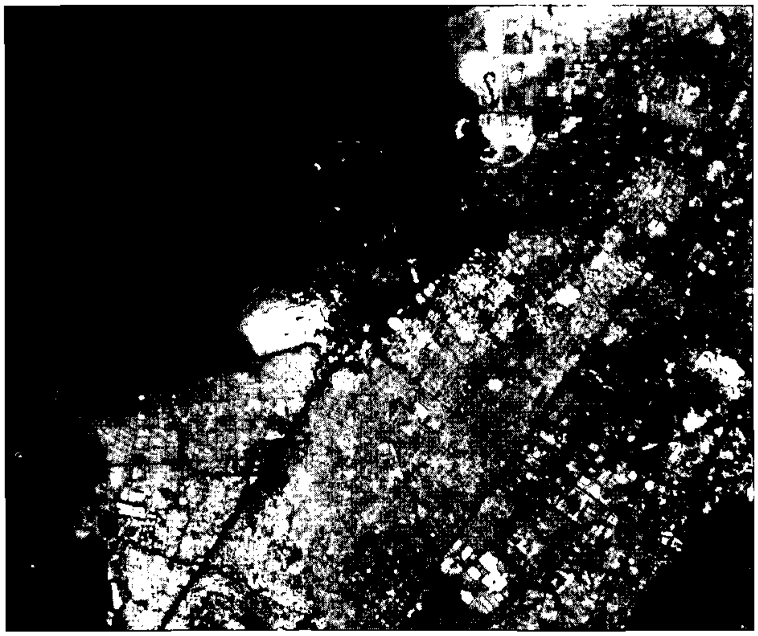

Figure 1. Band 40 of the Moffett Field 1990 AVIRIS image.

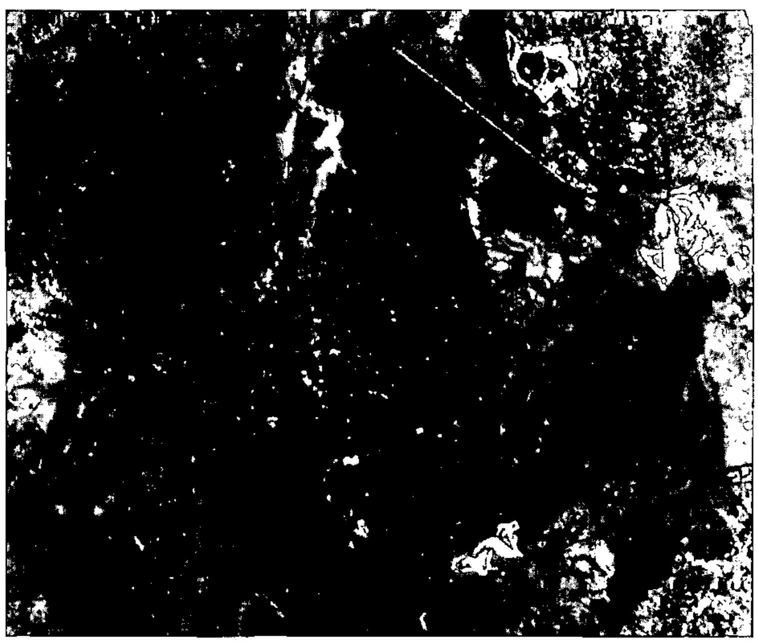

Figure 2. Band 40 of the Jasper Ridge 1990 AVIRIS image.

The Jasper Ridge image has a finer-grained texture within its mainly vegetated areas giving it a less homogeneous appearance.

The Jet Propulsion Laboratory, the supplier of these AVIRIS data sets, also supplies several ancillary files, one of which contains Noise Equivalent Radiances (NERs). The NERs are estimates of the noise in each band derived from sensor measurements. These NERs provide an independent noise measure with which our results may be compared. The NERs for the two data sets are almost indistinguishable at the scales we shall be using, and so we shall only plot the NERs for one of them.

The data for the 224 spectral bands that the AVIRIS sensor has are provided by four grating spectrometers, designated A, B, C and D. The band coverage of the spectrometers in the order A to D is bands 1–32, 33–96, 97–160 and 161–224. There are detector read-out errors associated with the first detector in each spectrometer (i.e., bands 1, 33, 97 and 161) and the data in these bands are unreliable.

3.2. *Characteristics of the residuals for the 1990 data sets*

In developing this method, we have supposed that decorrelation via linear regression removes all significant scene-specific effects, leaving random residuals which characterize the instrument's performance. As a check on this, we have constructed and inspected images of the residuals for about 10 per cent of the bands of each image data set. We chose bands 5, 15, 25,...,215. A band's residual image was computed by linearly mapping its residuals in a range of six standard deviations (centred on zero) to the range 0–255.

For these two data sets, none of the residual images shows any spatial features relating to the original scene content. They all look like random noise except that there are a few broad but indistinct bands running across their full width. As these bands are independent of scene content, it suggests that they are caused by an instrumental effect. These results support our contention that the residuals reflect instrument-specific, rather than scene-specific, characteristics.

3.3. *Comparison of NERs and noise estimates computed using the new method*

Figure 3 shows graphs of the band noise estimated using our new method for both test data sets, and displays a graph of the NERs for the Jasper Ridge data set for comparison. It is clear that our new method provides consistent noise estimates

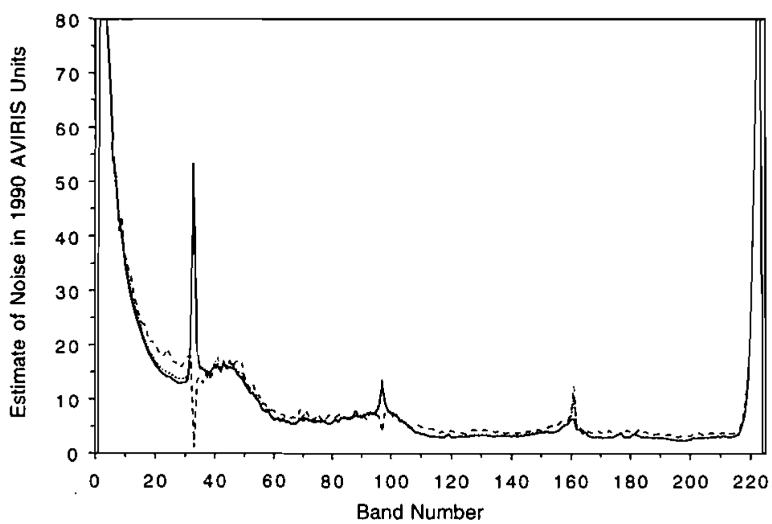

Figure 3. The AVIRIS band noise affecting both 1990 images, estimated with the new method and compared with the Noise Equivalent Radiances for the Jasper Ridge data set. In 1990, 200 AVIRIS units = 1 μW cm^{-2} sr^{-1} nm^{-1}. —— Moffett Field, ····· Jasper Ridge, – – – Noise Equivalent Radiances for Jasper Ridge.

for the two images which closely match the NERs. There are spikes in these graphs at bands 33, 97 and 161 which correspond to the noisy first detectors in three of the grating spectrometers.

3.4. *Comparison of the new method with Gao's method*

Figure 4 displays noise estimates computed using both the method described above and Gao's method. The two methods give very similar estimates for the Moffett Field image with its relatively homogeneous scene content. But, for the Jasper Ridge image, figure 5 shows that Gao's method provides very poor estimates

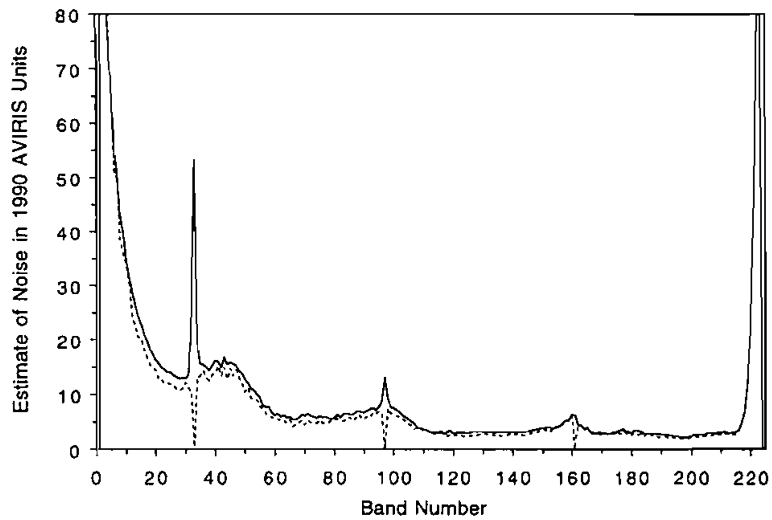

Figure 4. Estimates of the noise standard deviation for the Moffett Field 1990 image made using the new method and Gao's method. In 1990, 200 AVIRIS units = 1 $\mu W\,cm^{-2}\,sr^{-1}\,nm^{-1}$. —— New method, --- Gao's method.

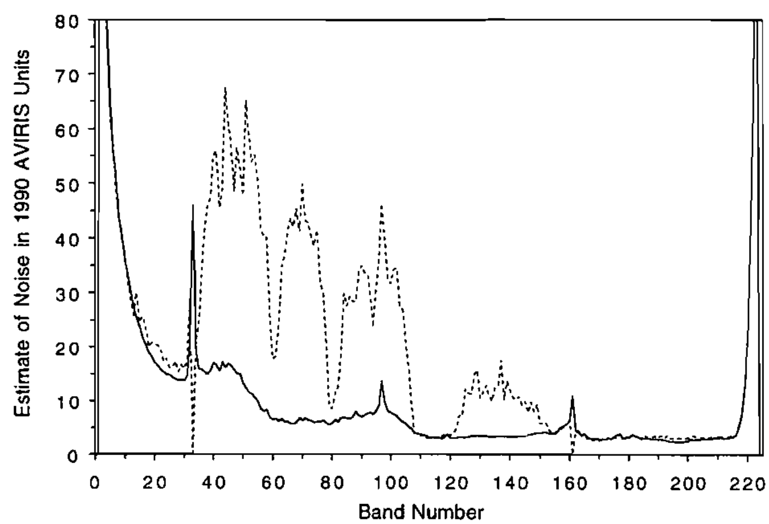

Figure 5. Estimates of the noise standard deviation for the Jasper Ridge 1990 image made using the new method and Gao's method. In 1990, 200 AVIRIS units = 1 $\mu W\,cm^{-2}\,sr^{-1}\,nm^{-1}$. —— New method, --- Gao's method.

of the band noise. Gao's method might provide better estimates with different block sizes but we have not explored this possibility.

4. Noise estimates for 1989 and 1992 AVIRIS images

We have estimated the noise affecting a total of 10 AVIRIS images, all of which are radiometrically rectified. In addition to the two 1990 images, which come with corresponding sets of NERs, we have processed four 1989 images and four 1992 images, for which sets of NERs are not supplied. These eight images are:

1. A 1989 Moffett Field image (Flt. 02, Run 005, 13 April 1989).
2. A 1989 Jasper Ridge image (Flt. 15, Run 011, 20 September 1989).
3. A 1989 Cuprite image (Flt. 20, Run 005, 29 September 1989).
4. A 1989 Rogers Dry Lake image (Flt. 15, Run 011, 20 September 1989).
5. A 1992 Moffett Field image (Flt. 920820B, Run 11, Scene 05).
6. A 1992 Jasper Ridge image (Flt. 920602A, Run 08, Scene 02).
7. A 1992 Cuprite image (Flt. 920603B, Run 02, Scene 01).
8. A 1992 Rogers Dry Lake image (Flt. 920530B, Run 07, Scene 02).

4.1. *Characteristics of the residuals for the 1989 and 1992 data sets*

As with the two test data sets, we have generated and viewed residual images for these eight data sets. There are differences between the 1989 and 1992 images which we attribute to the improvements made to the AVIRIS during the intervening years.

1989 data sets: The residual images for Jasper Ridge nearly all look like random noise. However, most of them show definite horizontal striations running their full width. Those for Rogers Dry Lake also look random though some bands show a few faint linear features, on which we comment below. They do not show horizontal striations but the residual images for the bands in spectrometer B show patterning which is repeated across the image and unrelated to the scene content. Similar patterning appears in the residual images for the same bands of Moffett Field. Cuprite's residual images do not show any strong patterning or striation in the noise though many show faint, narrow linear features (see below).

1992 data sets: In general, the residual images for all four data sets look random. They mostly show no spatial features at all and the noise exhibits no striations nor banding. Each image does have a few bands which show indistinct narrow linear features. Of all the 1992 residual images, only those for Bands 25 and 35 of Moffett Field and Band 25 of Rogers Dry Lake show any strong spatial features. These features are localized and occupy a small proportion of the image; the variances for these areas are untypically high and they are rejected when the homogeneous subset of variances is selected.

Linear features: Some residual images show sparse, faint and narrow linear features which correspond to sharp edges in the original images. Despite their weakness, these features are discernible to the eye as the visual system is very sensitive to edges. As linear features, they contribute only weakly to the estimates of variance which are computed over areas. For this reason, they have little effect on our procedure for noise estimation.

4.2. *Estimates for 1989 images*

Figure 6 shows our noise estimates for the 1989 data sets. There are obvious differences in the band noises for the different grating spectrometers; these are particularly clear at the ends of the spectrometers (bands 33 and 97, in particular).

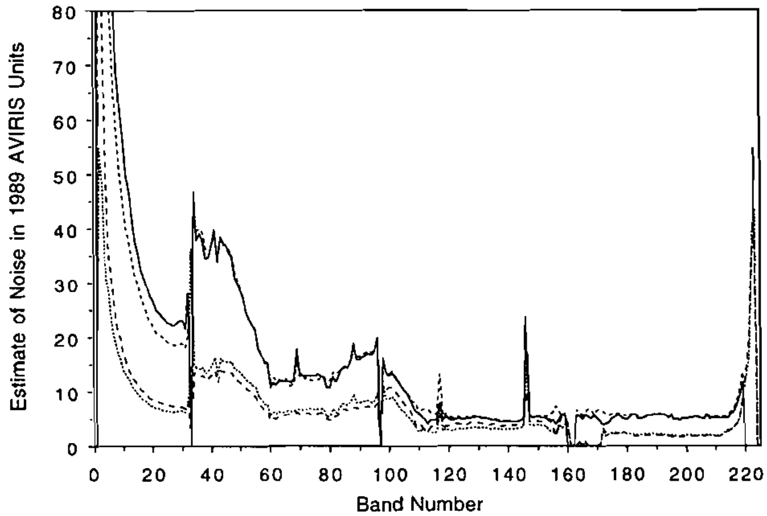

Figure 6. Estimates of the noise standard deviation for the four 1989 images made using the new method. In 1989, 200 AVIRIS units = 1 μW cm^{-2} sr^{-1} nm^{-1}, but see text for further comment. —— Moffett Field, ----- Rogers Dry Lake, ····· Jasper Ridge, – – – Cuprite.

The 1989 estimates for the Jasper Ridge and Cuprite images are very similar, but they differ from those for the Moffett Field and Rogers Dry Lake images which are alike. We are not sure why the estimates differ and we have been unable to ascertain a reason from JPL. It may be a true reflection of the performance of the AVIRIS instrument, though we suspect not. This is because the Jasper Ridge and Rogers Dry Lake images were recorded on the same day and their noise estimates differ considerably, whereas the estimates for the Jasper Ridge/Cuprite and Moffett Field/Rogers Dry Lake image pairs are very similar, even though the images in each pair were recorded on different days. The differences may be caused by an unrecorded difference in the scaling of the radiometrically rectified data as they were written to file; that is, the calibration of 200 AVIRIS units = 1 μW cm^{-2} sr^{-1} nm^{-1} given in the caption to figure 6 may not be correct for all four 1989 images.

4.3. *Estimates for 1992 images*

Figure 7 shows the noise estimates for the 1992 data sets. As the caption notes, the calibration constant for these images is not the same as those for the 1989 and 1990 images. One can see that our new method provides consistent noise estimates for the four images. The estimates of the noises in the bands recorded by grating spectrometers A, C and D are virtually indistinguishable, though there are differences for the bands covered by grating spectrometer B. We consider that these differences are not an artefact of our method but truly reflect changes in the behaviour of the AVIRIS. As these differences are limited exactly to spectrometer B, this gives confidence that our method captures instrument-specific, rather than scene-specific, information.

Gao's method has also been used to estimate the band noise for these eight images. It performs reasonably well on three of the 1989 images though it gives bad estimates for the Jasper Ridge image, as it does with the corresponding 1990 image.

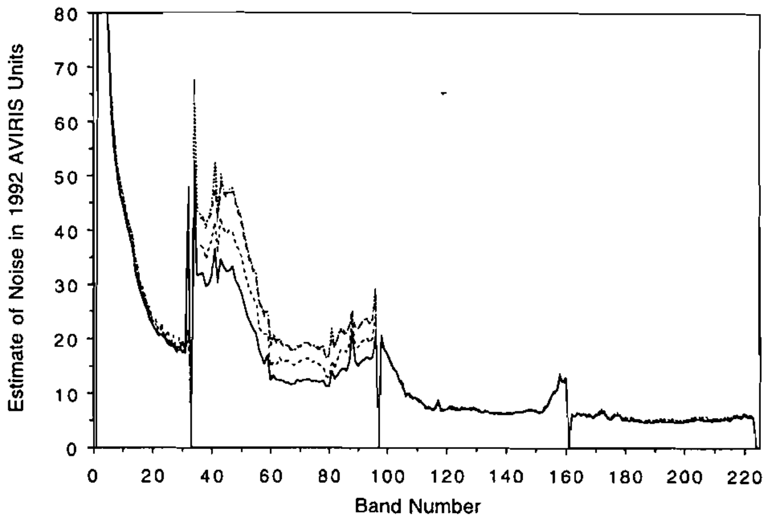

Figure 7. Estimates of the noise standard deviation for the four 1992 images, made using the new method. In 1992, 500 AVIRIS units = 1 μW cm^{-2} sr^{-1} nm^{-1}. —— Moffett Field, ----- Rogers Dry Lake, ····· Jasper Ridge, – – – Cuprite.

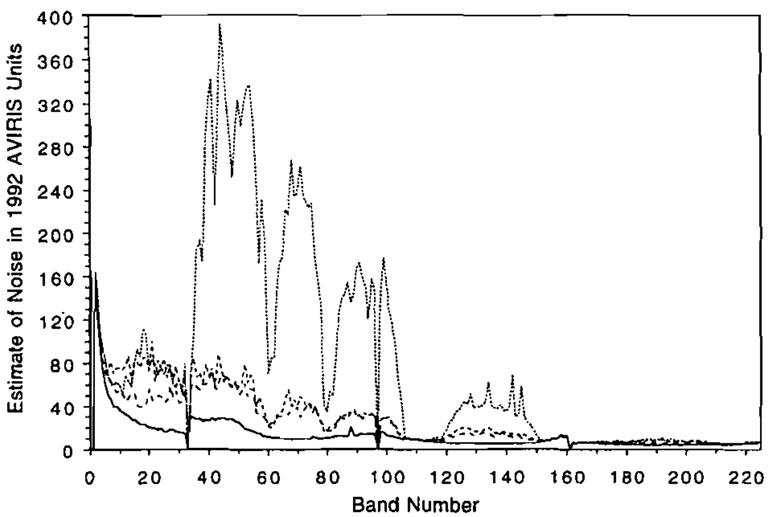

Figure 8. Estimates of the noise standard deviation for the four 1992 images, made using Gao's method. In 1992, 500 AVIRIS units = 1 μW cm^{-2} sr^{-1} nm^{-1}. (Note the different scale for the y-axis.) —— Moffett Field, ----- Rogers Dry Lake, ····· Jasper Ridge, – – – Cuprite.

With the 1992 images, it performs well on only one of the four images (Moffett Field), as figure 8 shows; note the change in vertical scale for this figure.

5. Discussion

The predictor we have used to decorrelate the image data is quite simple. As our comments have indicated, the residual images it produces show weak or no spatial features. Nevertheless, the residuals themselves still show some correlations, though, given the structure observed in the residual images, it is quite likely that this means the noise is correlated and not white. We have investigated three ways of reducing the correlations in the residuals and how they change our noise estimates.

Firstly, more sophisticated predictors have been tried, e.g.,

$$\hat{x}_{i,j,k} = ax_{i,j,k-1} + bx_{i-1,j,k-1} + cx_{i,j,k+1} + dx_{i-1,j,k+1} + ex_{i-1,j,k} + f \tag{9}$$

Regression which includes pixels from the previous line offers no clear gain over regression on pixels from just the one line. We have also tried predictors with more independent parameters but they are complicated to implement, slower to execute, and make little difference to the overall results. Secondly, the residuals in one band can themselves be further decorrelated by linear regression, e.g., of one residual on two adjacent residuals; we call this 'secondary decorrelation'. Thirdly, we can combine these two methods serially.

The use of the more sophisticated predictor shown above reduces the noise estimates by 6·6 per cent on average. The use of secondary decorrelation alone reduces them by 11 per cent. The serial combination of the two gives very little reduction over secondary decorrelation alone: 0·9 per cent on average though only about 5 per cent of bands have their noise estimates changed by more than 0·5 units. It is a moot point as to whether this more sophisticated processing is eliminating correlations due to scene effects or due to the noise itself being correlated.

6. Estimating noise in reflectance images

In the above, we have shown that our method works well for estimating noise affecting AVIRIS radiance images. In some applications, scientists prefer to work with reflectance images from which the effects of the solar radiation spectrum and the atmosphere have been removed by the process of atmospheric correction. Several procedures to do this are in common use and Hoffbeck and Landgrebe (1994) discuss them. They identify the important fact that some of these procedures are just affine transformations, and show that such procedures have no effect on the Maximum Likelihood Classification of image data. This is because affine transformations do not change the correlation structure of the data. They also report that other atmospheric-correction procedures which are not affine transformations have little effect on image classification. For precisely the same reason, namely, the correlation structure of the data is invariant under affine transformation, our method of noise estimation will work equally well on reflectance data generated by that type of atmospheric correction procedure. Given the experimental results of Hoffbeck and Landgrebe (1994) concerning the non-affine correction procedures, we are confident that our method will work just as reliably on reflectance images generated by them.

7. Conclusions

A reliable, automatic means of estimating the noise affecting the bands of an AVIRIS image has been described. We have shown it to provide consistent estimates and to work for images whose scene content is inhomogeneous. Our method works well on images for which another recent method fails. The method can be used for both radiance and reflectance images.

Acknowledgments

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References

CHRIEN, T. G., and GREEN, R. O., 1993, Instantaneous field of view and spatial sampling of the Airborne Visible/Infrared Imaging Spectrometer. *Fourth Annual JPL Airborne Earth Science Workshop, 25–29 October 1993*, JPL Publication 93-26, **Vol. 1** (Pasadena, CA: Jet Propulsion Laboratory), pp. 23–26.

GAO, B.-C., 1993, An operational method for estimating signal to noise ratios from data acquired with imaging spectrometers. *Remote Sensing of Environment*, **43,** 23–33.

GREEN, R. O., 1995, An improved spectral calibration requirement for AVIRIS. *Fifth Annual JPL Airborne Earth Science Workshop, 23–26 January 1995*, JPL Publication 95-1, **Vol. 1** (Pasadena, CA: Jet Propulsion Laboratory), pp. 75–80.

GREEN, J. R., and MARGERISON, D., 1978, *Statistical Treatment of Experimental Data* (Amsterdam: Elsevier).

HOFFBECK, J. P., and LANDGREBE, D. A., 1994, Effect of radiance-to-reflectance transformation and atmosphere removal on maximum likelihood classification accuracy of high-dimensional remote sensing data. *Proceedings of IGARSS'94 held in Pasadena, CA, USA, on 8–12 August 1994* (Piscataway, NJ: I.E.E.E.), pp. 2538–2540.

JOHNSON, R. A., and WICHERN, D. W., 1982, *Applied Multivariate Statistical Analysis* (Englewood Cliffs, NJ: Prentice-Hall).

KOTZ, S., and JOHNSON, N. L., editors, 1983, *Encyclopedia of Statistical Sciences*, **Vol. 4** (New York: John Wiley & Sons).

PRESS, W. H., FLANNERY, B. P., TEUKOLSKY, S. A., and VETTERLING, W. T., 1986, *Numerical Recipes, The Art of Scientific Computing* (New York: Cambridge University Press).

ROGER, R. E., and ARNOLD, J. F., 1994, A reliable method for estimating the noise in AVIRIS hyperspectral images. In *Proceedings of the 7th Australasian Remote Sensing Conference, Melbourne, Australia, 1–4 March 1994* (Floreat, WA: Remote Sensing and Photogrammetry Association Australia Ltd.), pp. 527–533.

SACHS, L., 1984, *Applied Statistics: A Handbook of Techniques*, Springer Series in Statistics, 2nd edition (New York: Springer-Verlag).

VANE, G., GREEN, R. O., CHRIEN, T. G., ENMARK, H. T., HANSEN, E. G., and PORTER, W. M., 1993, The Airborne Visible/Infrared Imaging Spectrometer (AVIRIS). *Remote Sensing of Environment*, **44,** 127–143.