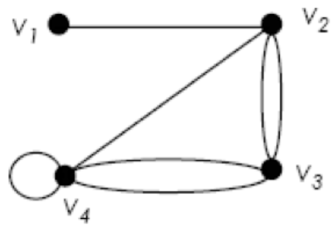
4.1

Exercise 1

- Find the degree of each vertex in the graph.



$d(v_1) = 1$

$d(v_2) = 4$

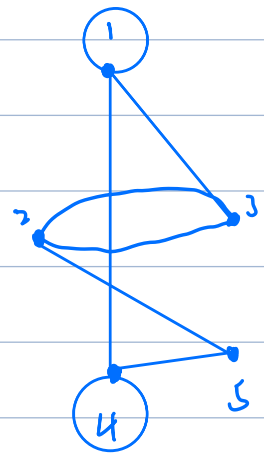
$d(v_3) = 4$

$d(v_4) = 5$

Exercise 2

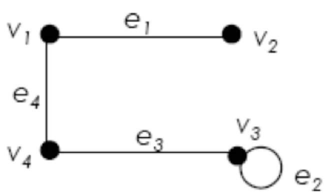
Draw the graph based on the following matrix:

$$A_G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 2 & 0 & 1 \\ 1 & 2 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$



Exercise 3

- Find the adjacency matrix and the incidence matrix of the graph.



adjacent

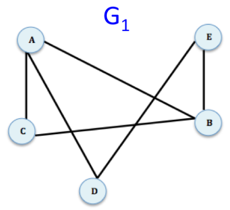
$$\begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

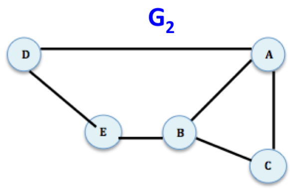
incidence

$$\begin{array}{c|cccc} & e_1 & e_2 & e_3 & e_4 \\ \hline v_1 & 0 & 1 & 0 & 1 \\ v_2 & 1 & 0 & 0 & 0 \\ v_3 & 0 & 2 & 1 & 0 \\ v_4 & 0 & 0 & 1 & 1 \end{array}$$

Exercise 4

Q: Show that the following two graphs are isomorphic.





number of Vertices of G_1 = number of Vertices of G_2

$f(A_{G_1}) = A_{G_2}$ $\quad f(E_{G_1}) = E_{G_2}$

$f(B_{G_1}) = B_{G_2}$ $\quad f(F_{G_1}) = F_{G_2}$

$f(C_{G_1}) = C_{G_2}$

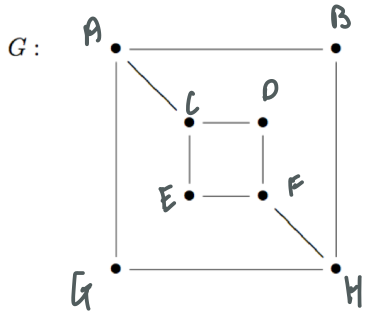
$f(D_{G_1}) = D_{G_2}$

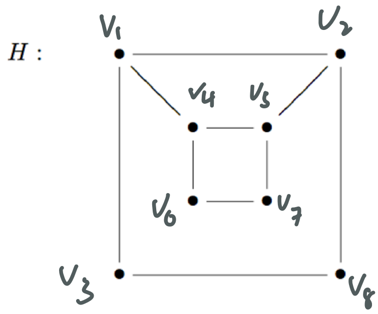
$$A_{G_1} = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad A_{G_2} = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$A_{G_1} = A_{G_2}$, so G_1 and G_2 are isomorphic.

Exercise 5

Q: Is these two graphs are isomorphic?





$f(A) = V_1$ $\quad f(F) \neq V_7$

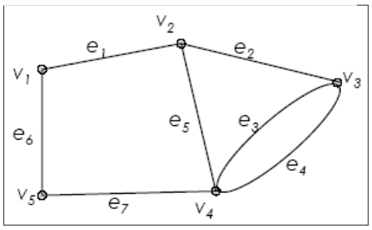
$f(C) = V_4$ $\quad \therefore$ G and H are not isomorphic

$f(D) = V_6$

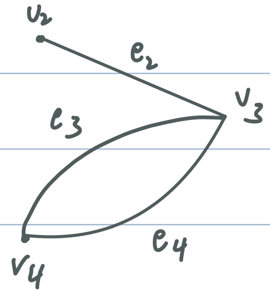
Exercise 6

Tell whether the following is either a trail, path, circuit, simple circuit or none of these.

- $(v_2, e_2, v_3, e_3, v_4, e_4, v_3)$ (trail)
- $(v_4, e_7, v_5, e_6, v_1, e_1, v_2, e_2, v_3, e_3, v_4)$ (Simply circuit)
- $(v_4, e_4, v_3, e_3, v_4, e_5, v_2, e_1, v_1, e_6, v_5, e_7, v_4)$ (Circui-



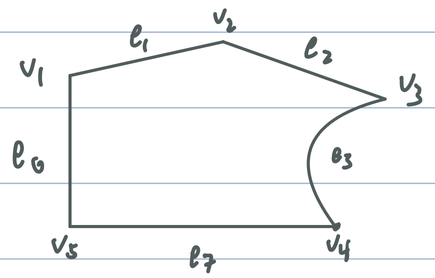
• $(v_2, e_2, v_3, e_3, v_4, e_4, v_3)$



∴ Does not have repeated edge, starting point and ending point are difference.

Trail

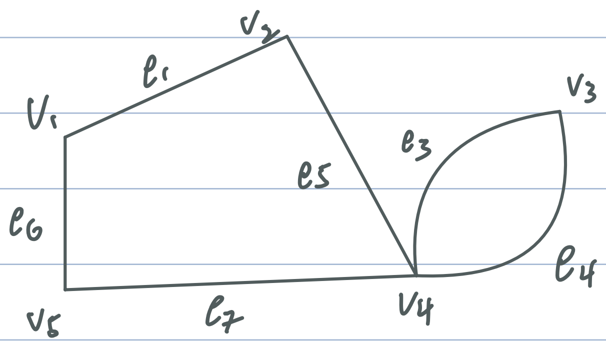
• $(v_4, e_7, v_5, e_6, v_1, e_1, v_2, e_2, v_3, e_3, v_4)$



∴ Starting point and ending point are the same, no repeated edge and vertex except the first and last.

Simple circuit

• $(v_4, e_4, v_3, e_3, v_4, e_5, v_2, e_1, v_1, e_6, v_5, e_7, v_4)$

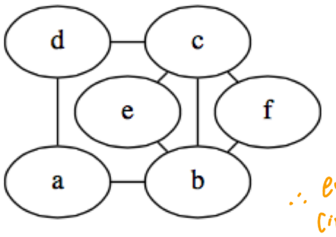


∴ Circuit, because it has repeated vertex but not repeated edge.

Exercise 7

Q: Which of the following graphs has Euler circuit?
Justify your answer.

G_1

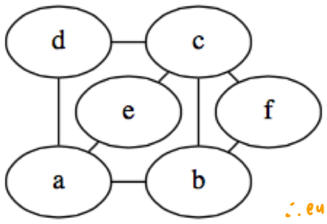


∴ euler circuit

vertex	a	b	c	d	e	f
edge	2	2	2	2	2	2

∴ because all the vertex does not have odd degree

G_2

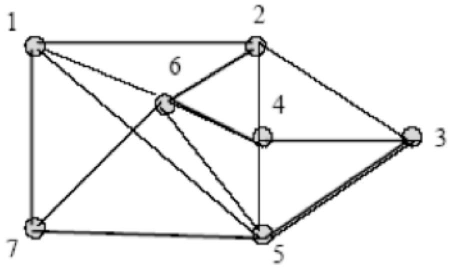


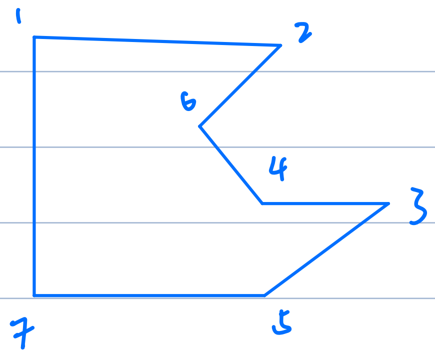
∴ euler trail , because a and b have odd degree

vertex	a	b	c	d	e	f
edge	3	3	4	2	2	2

Exercise 8

Question: Is this graph has Hamiltonian cycle?

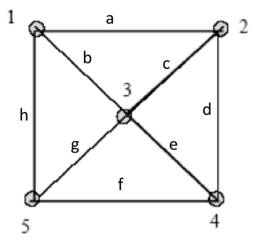




∴ Yes , (1, 2, 6, 4, 3, 5, 7)

Exercise 9

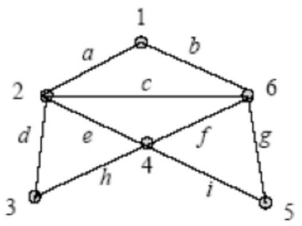
Question: Prove that this graph has Hamiltonian circuit.



∴ (1, a, 2, d, 4, f, 5, g, 3, b, 1) , all the vertex are visited exactly once except the first and the last.

Exercise 10

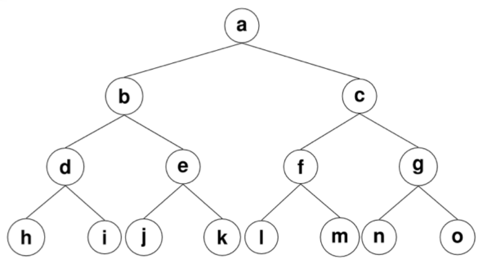
Find a Hamiltonian circuit in this graph.



(1, a, 2, d, 3, h, 4, i, 5, g, 6, b, 1)

4.2

Exercise 1



Find:

- Ancestors of m
- Descendents of g
- Parent of *e*
- Children of *e*
- Sibling of *h*

- Ancestors of m : {a, c, f, m}
- Descendents of g : { g, o }
- Parent of e : b
- Sibling of h : { i }

Exercise 2

- How many matches are played in a tennis tournament of 27 players?

$m = 2$

$l = 27$

$i = \frac{27-1}{2-1}$

$= 26$

Exercise 3

Suppose 1000 people enter a chess tournament. Use a rooted tree model of the tournament to determine how many games must be played to determine a champion, if a player is eliminated after one loss and games are played until only one entrant has not lost. (Assume there are no ties.)

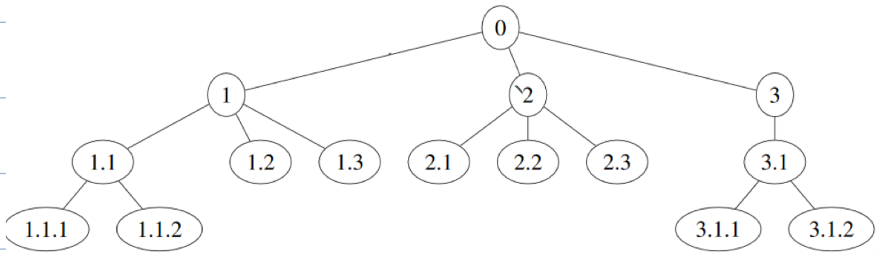
$l = 1000$

$m = 2$

$i = \frac{1000-1}{2-1}$

$= 99$

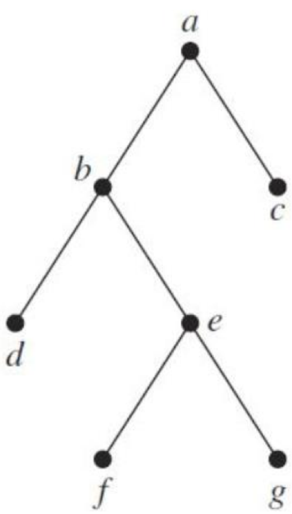
Exercise 4



Find the lexicographic ordering of the above tree.

$0 < 1 < 1.1 < 1.1.1 < 1.1.2 < 1.2 < 1.3 < 2 < 2.1 < 2.2 < 2.3 < 3 < 3.1 <$
$3.1.1 < 3.1.2$

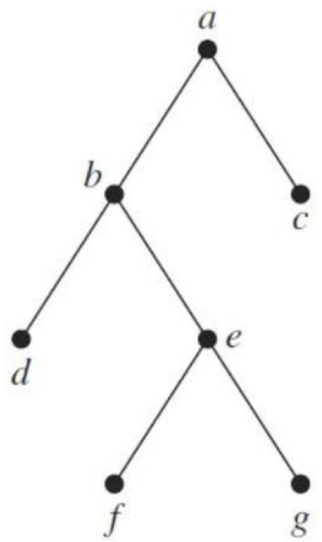
Exercise 5



Determine the order in which a preorder traversal visits the vertices of the given ordered rooted tree.

a, b, d, e, f, g, c

Exercise 6



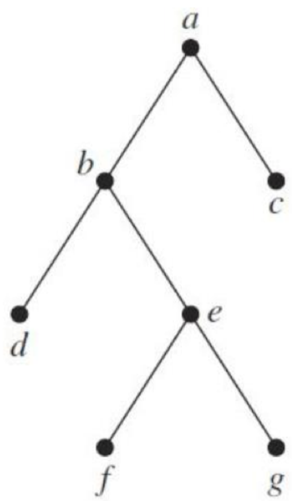
Determine the order in which a inorder traversal visits the vertices of the given ordered rooted tree.

b a c
d e
f g

d b e a c
f g

∴ d b f e g a c

Exercise 7



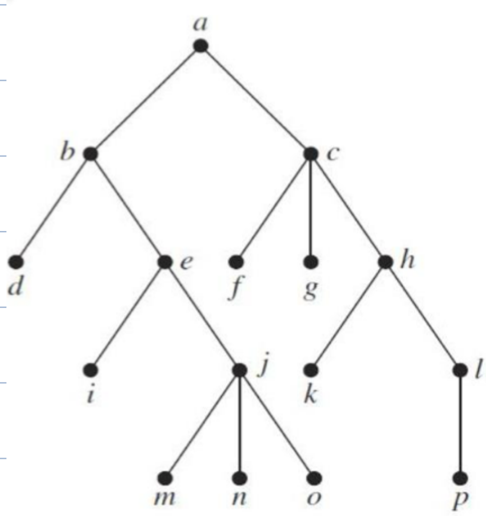
Determine the order in which a postorder traversal visits the vertices of the given ordered rooted tree.

b c a
d e
f g

d e b c a
f g

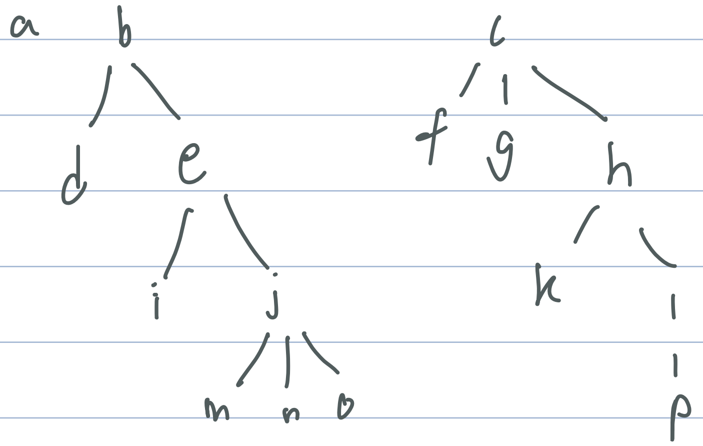
d f g e b c a

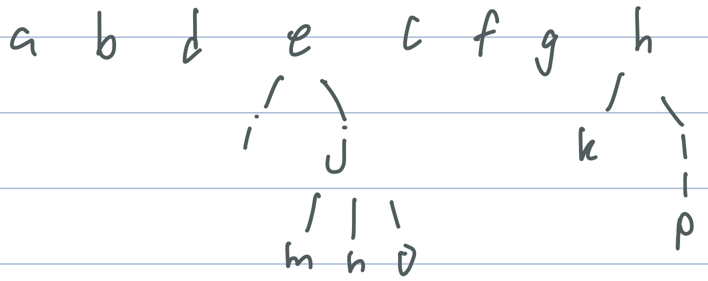
Exercise 8, 9, 10

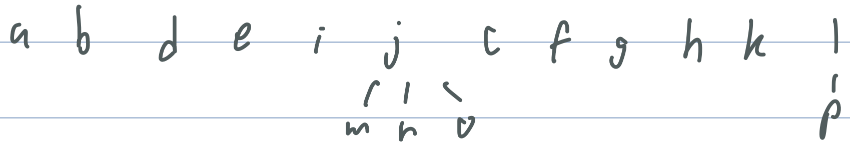


Determine the order of preorder (8), inorder (9) and postorder (10) of the given rooted tree.

Exercise 8

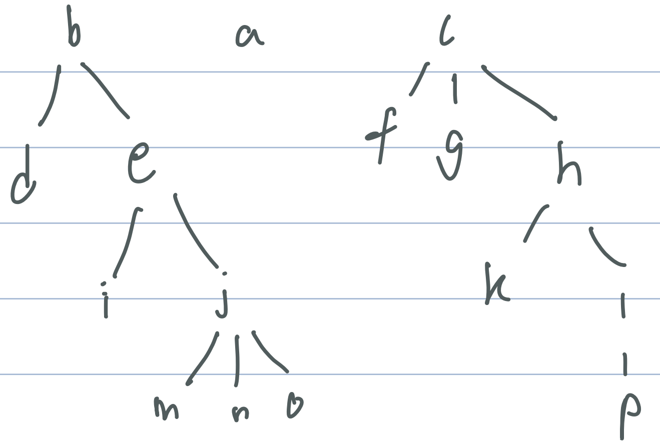


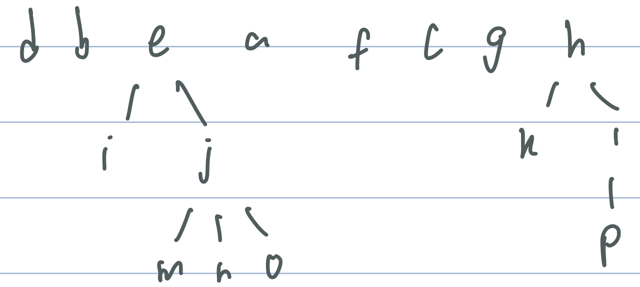


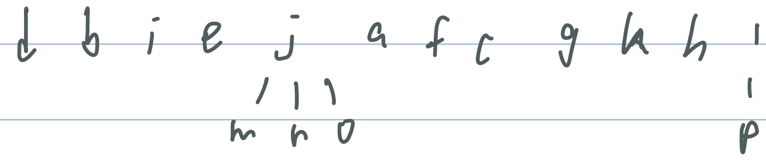


a b d e i j m n o c f g h k l p

Exercise 9

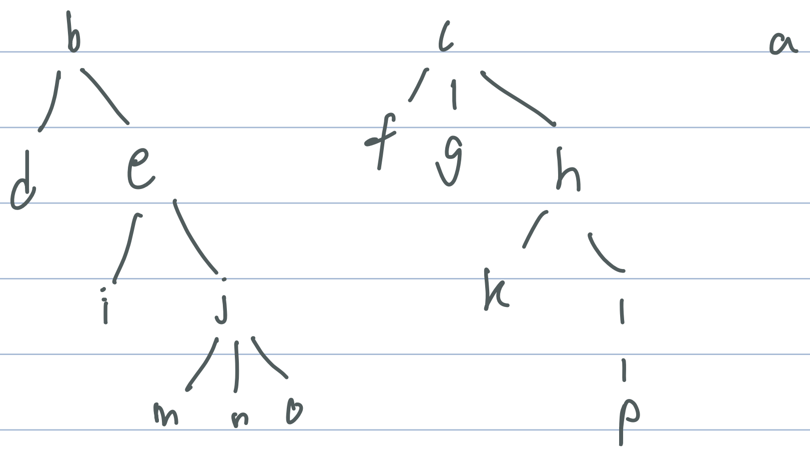


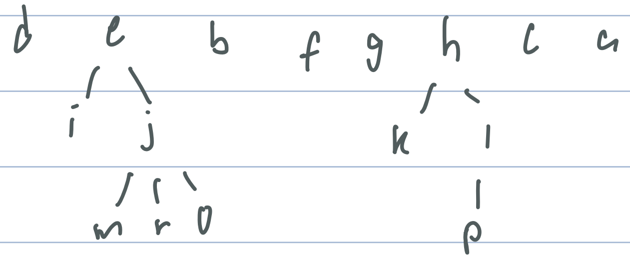


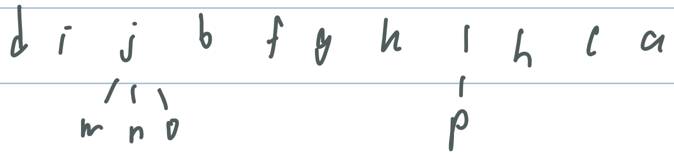


d b i e m j n o a f c g k h p l

Exercise 10

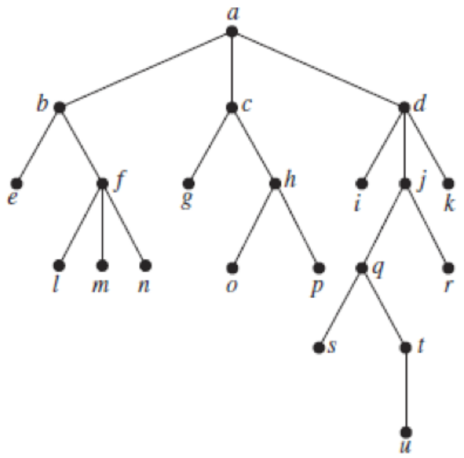






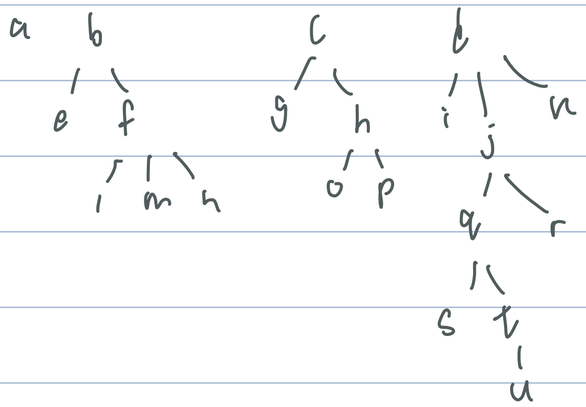
d i m n o j b f g k p l h c a

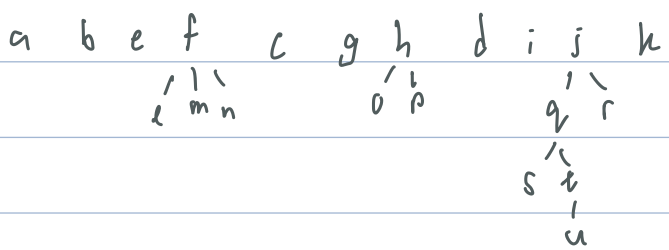
Exercise 11, 12, 13

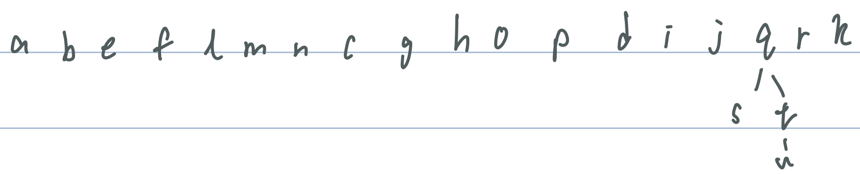


Determine the order of preorder (11), inorder (12) and postorder (13) of the given rooted tree.

Exercise 11



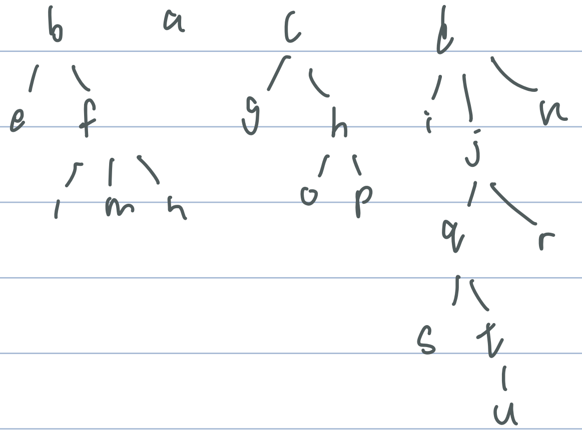


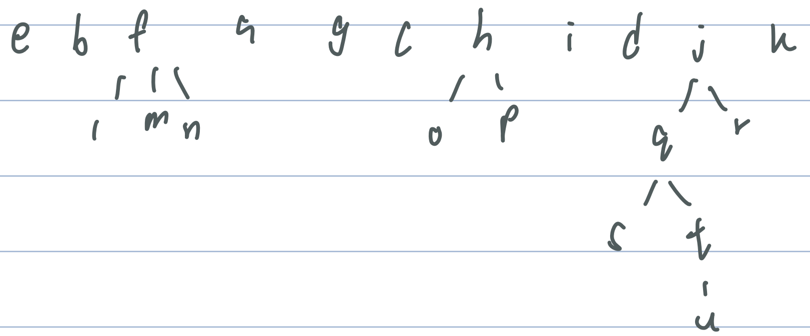


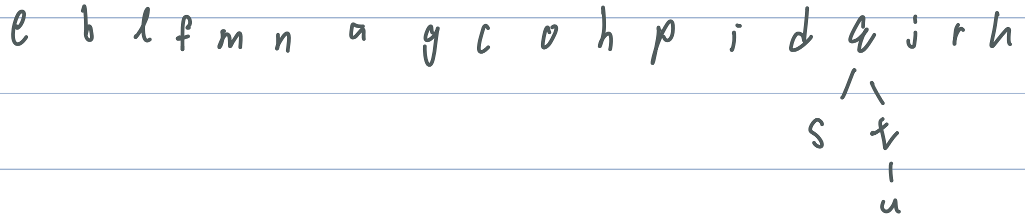
a b e f l m n c g h o p d i j q s t r k
u

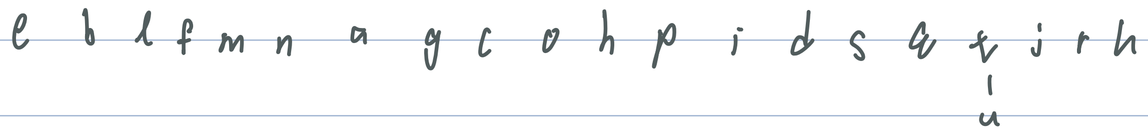
a b e f l m n c g h o p d i j q s t u r k

Exercise 12



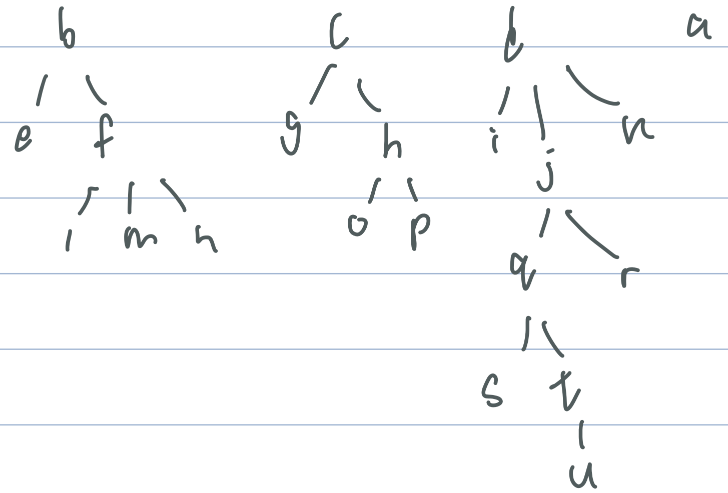


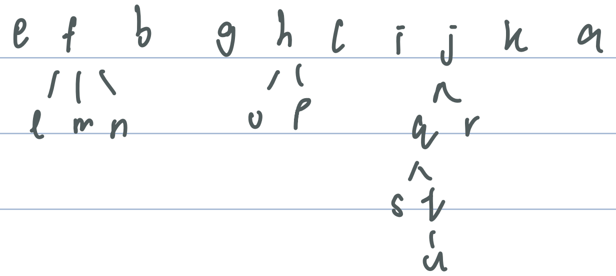


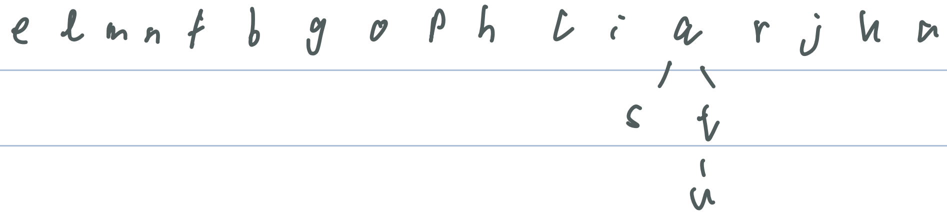


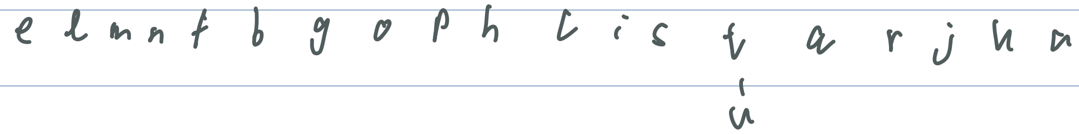
e b l f m n a g c o h p i d s q u t j r h

Exercise 13







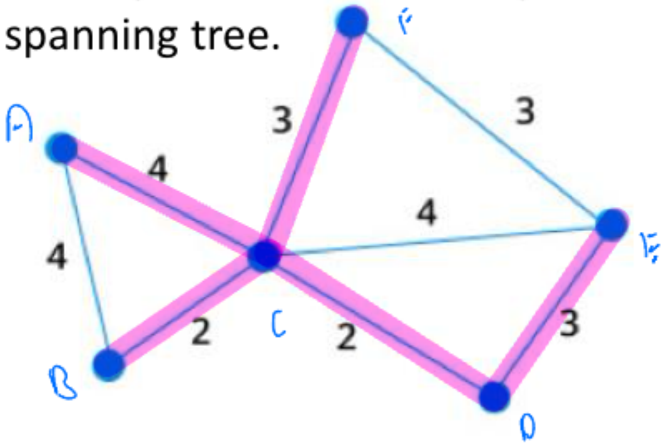


e l m n f b g o p h c i s u t a r j k a

Exercise 14

Find the minimum spanning tree using Kraskal's Algorithm and give the total weight for the minimum spanning tree.

AB - 4
AC - 4
~~BC - 2~~
~~CD - 2~~
~~DE - 3~~
CE - 4
FE - 3
~~CF - 3~~



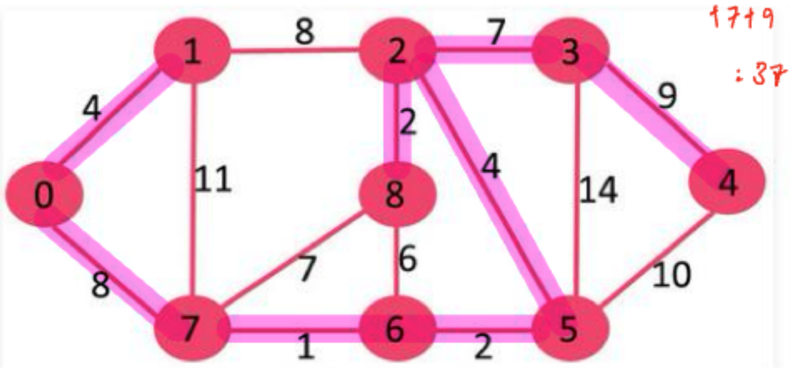
UTM

Exercise 15

~~76 - 1~~
~~28 - 2~~
~~65 - 2~~
~~01 - 4~~
~~25 - 4~~
~~65 - 6~~
~~78 - 7~~
~~23 - 7~~
~~12 - 8~~
~~07 - 8~~
~~34 - 9~~
34 - 10
17 - 11
35 - 14

Find the minimum spanning tree using Kraskal's Algorithm and give the total weight for the minimum spanning tree.

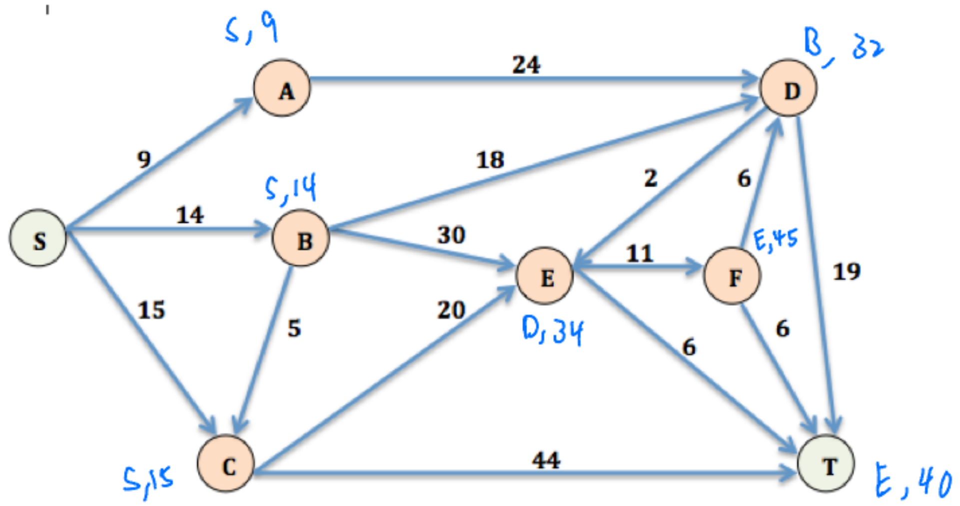
total : 4 + 8 + 1 + 2 + 4 + 2 + 7 + 9
: 37



4.3

Exercise 1

Q: Given a weighted digraph, find the shortest path from **S** to **T**, using Djikstra Algorithm.



Note: Weights are arbitrary numbers (i.e., not necessarily distances).

	S	N	L(S)	L(A)	L(B)	L(C)	L(D)	L(E)	L(F)	L(T)
1	{ }	{S, A, B, C, D, E, F, T}	0	∞	∞	∞	∞	∞	∞	∞
2	{S}	{A, B, C, D, E, F, T}	0	9	14	15	∞	∞	∞	∞
3	{S, A}	{B, C, D, E, F, T}	0	9	14	15	33	∞	∞	∞
4	{S, A, B}	{C, D, E, F, T}	0	9	14	15	32	44	∞	∞
5	{S, A, B, C}	{D, E, F, T}	0	9	14	15	32	35	∞	59
6	{S, A, B, C, D}	{E, F, T}	0	9	14	15	32	34	∞	51
7	{S, A, B, C, D, E}	{F, T}	0	9	14	15	32	34	45	40
8	{S, A, B, C, D, E, T}	{F}	0	9	14	15	32	34	45	40

Shortest path : S → B → D → E → T

Exercise 2

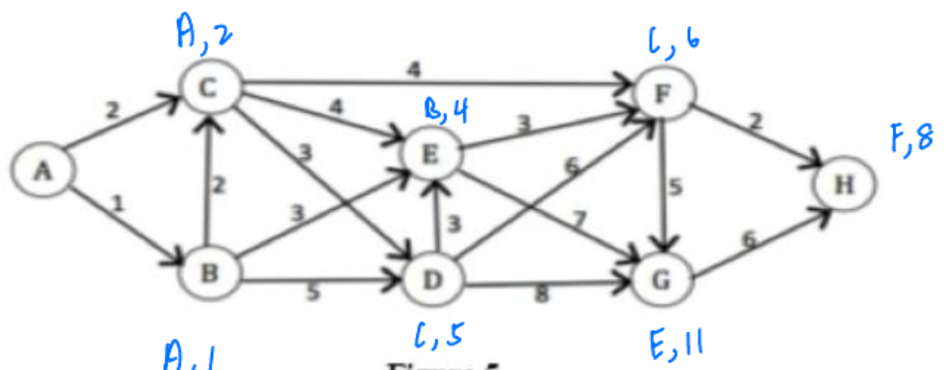


Figure 5

Based on Dijkstra's algorithm, complete Table 1 to find the shortest path from city A to city H. (Note: Copy Table 1 into your answer booklet).

(8 marks)

	S	N	L(A)	L(B)	L(C)	L(D)	L(E)	L(F)	L(G)	L(H)
1	{ }	{A,B,C,D,E,F,G,H}	0	∞	∞	∞	∞	∞	∞	∞
2	{A}	{B,C,D,E,F,G,H}	0	1	2	∞	∞	∞	∞	∞
3	{A,B}	{C,D,E,F,G,H}	0	1	2	6	4	∞	∞	∞
4	{A,B,C}	{D,E,F,G,H}	0	1	2	5	4	6	∞	∞
5	{A,B,C,E}	{D,F,G,H}	0	1	2	5	4	6	11	∞
6	{A,B,C,E,D}	{F,G,H}	0	1	2	5	4	6	11	∞
7	{A,B,C,E,D,F}	{G,H}	0	1	2	5	4	6	11	8
8	{A,B,C,E,D,F,H}	{G}	0	1	2	5	4	6	11	8

Shortest path : A → C → F → H