UTM
UNIVERSITI TEKNOLOGI MALAYSIA

FACULTY OF COMPUTING

SEMESTER 1 2024/2025

SECI 1013 DISCRETE STRUCTURE

SECTION 03

EXERCISE CHAPTER 2

LECTURER: DR. MUHAMMAD ALIIF BIN AHMAD

Student name	Matric Number
TOH SHEE THONG	A24CS0309
NUR ALIA ATHIRAH BINTI SUZUDDIN	A24CS0153
LEE PEI YUAN	A24CS0262
MUHAMMAD ALIFF BIN HELMI	A24CS0272

Exercise Chapter 2

Define a relation R from $\mathbf{Z}$ to $\mathbf{Z}$ as follows: For all integer number m and n, $(m,n) \in \mathbf{Z}\times\mathbf{Z}$,

$$m\ R\ n \leftrightarrow m - n \text{ is even}$$

i) Is 4 R 0?
ii) Is 2 R 6?
iii) Is 3 R (-3)?
iv) Is 5 R 2?
v) List 5 integers that are related by R to 1.

i) Yes
ii) Yes
iii) Yes
iv) No
v) 3, 5, 7, 9, 11

An airline services the five cities c_1, c_2, c_3, c_4 and c_5. Table below gives the cost (in dollars) of going from c_i to c_j. Thus the cost of going from c_1 to c_3 is RM100, while the cost of going from c_4 to c_2 is RM200

To from	c_1	c_2	c_3	c_4	c_5
c_1		140	100	150	~~200~~
c_2	~~190~~		~~200~~	160	~~220~~
c_3	110	180		~~190~~	~~250~~
c_4	~~190~~	~~200~~	120		150
c_5	~~200~~	100	~~200~~	150	

If the relation R on the set of cities $A = \{c_1, c_2, c_3, c_4, c_5\} : c_i\ R\ c_j$ if and only if the cost of going from c_i to c_j is defined and less than or equal to RM180.

i) Find R.
ii) Matrices of relations for R

i) $R = \{(c_1,c_2),(c_1,c_3),(c_1,c_4),(c_2,c_4),(c_3,c_1),(c_3,c_2),(c_4,c_3),(c_4,c_5),(c_5,c_2),(c_5,c_4)\}$

ii) Matrices of Relation R, M_R =

	c_1	c_2	c_3	c_4	c_5
c_1	0	1	1	1	0
c_2	0	0	0	1	0
c_3	1	1	0	0	0
c_4	0	0	1	0	1
c_5	0	1	0	1	0

Let A = {1, 2, 3,4} and R is a relation from A to A.
Suppose R = {(1,2), (1,3), (1,4), (2,3), (2,4) , (3,4)}

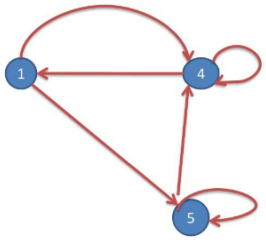
- What is R (represent)?
- What is matrix representation of R?

1. $(m,n) \in R$ if $m<n$ and $m,n \in A$

2. Matrix Representation R =

	1	2	3	4
1	0	1	1	1
2	0	0	1	1
3	0	0	0	1
4	0	0	0	0

Let A = {1, 4, 5} and let R be given by the digraph shown below. list in-degrees and out-degrees of all vertices.



	1	4	5
In-Degree	1	3	2
Out-Degree	2	2	2

---

1. Let A={1,2,3,4} and let R = {(1,2), (2,2), (3,4), (4,1)}

Determine whether R symmetric, asymmetric or antisymmetric.

antisymmetric

Determine which of the relations f are functions from the set **X** to the set **Y**. In case any of these relations are functions, determine if they are one-to-one, onto **Y**, and/or bijection.

a) **X** = { -2, -1, 0, 1, 2 } , **Y** = { -3, 4, 5 } and f = { (-2,-3), (-1,-3), (0,4), (1,5), (2,-3) }

b) **X** = { -2, -1, 0, 1, 2 } , **Y** = { -3, 4, 5 } and f = { (-2,-3), (1,4), (2,5)}

c) **X** = **Y**= { -3, -1, 0, 2 } and f = { (-3,-1), (-3,0), (-1,2), (0,2), (2,-1) }

a) onto Y
b) not a function
c) not a function

Exercise 1

A depositor deposits RM 10,000 in a savings account at a bank yielding 5% per year with interest compounded annually. How much money will be in the account after 30 years? Let P_n denote the amount in the account after n years.

$$P_0 : 10000$$
$$P_1 \cdot 1.05P_0$$
$$P_2 : 1.05P_1 = 1.05\left(1.05P_0\right)$$
$$= (1.05)^2 P_0$$
$$P_3 : 1.05P_2 = 1.05\left[(1,05)^2 P_0\right]$$
$$= (1,05)^3 Po$$
$$\cdots$$
$$p_n = (1.05)^n Po$$
$$p_{30} : (1,05)^{30}(10000)$$
$$= 43219,424$$
$$\therefore RM43219,424$$

Exercise 2

Consider the following sequence:

1, 5, 9, 13, 17

Find the recurrence relation that defines the above sequence.

$P_0 = 1$

$P_1 = 5$

$P_2 = 9$

$\therefore P_n = P_{n-1} + 4 \quad , n \geq 1$

difference : $P_1 - P_0$

: $5 - 1$

: 4

difference : $P_2 - P_1$

: $9 - 5$

: 4

Exercise 3

A basketball is dropped onto the ground from a height of 15 feet. On each bounce, the ball reaches a maximum height 55% of its previous maximum height.

a)Write a recursive formula, a_n, that completely defines the height reached on the n_{th} bounce, where the first term in the sequence is the height reached on the ball's first bounce.

$a_n : (a_{n-1}) \times 0.55$

b)How high does the basketball reach after the 4_{th} bounce? Give your answer to two decimal places.

$a_0 : 15$

$a_1 : (15 \times 0.55)$

$: 8.25$

$a_2 : (8.25 \times 0.55)$

$: 4.5375$

$a_3 : (4.5375 \times 0.55)$

$: 2.4956$

$a_4 : (2.4956 \times 0.55)$

$= 1.37$