UNIVERSITI TEKNOLOGI MALAYSIA

FACULTY OF COMPUTING

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SECI 1013 DISCRETE STRUCTURE

SECTION 03

EXERCISE CHAPTER 1

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Exercise

Determine whether each pair of sets is equal

{1, 2, 2, 3}, {1, 3, 2}

$A = \{1, 2, 2, 3\}$, 1, 2, 3

$B = \{1, 3, 2\}$, 1, 2, 3

$\therefore A = B$

Exercise

- If *M* is finite, determine the |*M*|
  - If *M*= {1, 2, 3, 4}
  - If *M* = {4, 4, 4}
  - If *M*= {}
  - If *M*= {ø,{ø}, {ø, {ø}}}

$M = \{1, 2, 3, 4\} \rightarrow |M| = 4$

$M = \{4, 4, 4,\} \rightarrow |M| = 1$

$M = \{\} \longrightarrow |M| = 0$

$M = \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\} \rightarrow |M| = 3$

- Let,

  $U = \{ a, b, c, d, e, f, g, h, i, j, k, l, m \}$

  $A = \{ a, c, f, m\}$

  $B = \{ b, c, g, h, m \}$

- Find:

  $| A \cup B|$, $A - B$ dan A'.

$|A \cup B| = \{a, b, c, f, g, h, m\}$

$A - B = \{a, f\}$

$A' = \{b, d, e, g, h, i, j, k, l\}$

- Let A, B and C be sets such that
  $A \cap B = A \cap C$ and $A \cup B = A \cup C$

- Prove that $B = C$

$A \cap B = A \cap C$ } $A \cup B = A \cup C$

Prove $B = C$

$B = B$

$B = B \cap (A \cup B)$ → Absorption law

$B = B \cap (A \cup C)$ → condition

$B = (B \cap A) \cup (B \cap C)$ → distributie law

$B = (A \cap C) \cup (B \cap C)$ → condition

$B = C \cap (A \cup B)$ → distributie law

$B = C \cap (A \cup C)$ → condition

$B = C$ ✱ (proven) → absorption law

- $A = \{a, b\}$, $B = \{1, 2\}$, $C = \{x, y\}$

- Determine the following set nad their cardinality,
  a) $B \times C$
  b) $A \times B \times C$,

a) $B \times C = \{(1,x), (1,y), (2,x)$
$(2,y)\}$

$|B \times C| = 2 \times 2 = 4$

b) $A \times B \times C = \{(a,1,x), (a,1,y)$
$(a,2,x), (a,2,y)$
$(b,1,x), (b,1,y),$
$(b,2,x), (b,2,y)\}$

$|A \times B \times C| = 2 \times 2 \times 2 = 12$

Part 3 : Fundamental and Elements of Logic

Exercise

Suppose x is a particular real number. Let ***p***, ***q*** and ***r*** symbolize "$0 < x$", "$x < 3$" and "$x = 3$", respectively. Write the following inequalities symbolically:

a) $x \leq 3$

b) $0 < x < 3$

c) $0 < x \leq 3$

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a) q ^ r

b) p v q

c) p v (q^r)

Exercise

Propositional functions ***p***, ***q*** and ***r*** are defined as follows:

p is "$n = 7$"
q is "$a > 5$"
r is "$x = 0$"

Write the following expressions in terms of ***p***, ***q*** and ***r***, and show that each pair of expressions is **logically equivalent**. State carefully which of the above laws are used at each stage.

(a) $((n = 7)$ or $(a > 5))$ *and* $(x = 0)$
$((n = 7)$ *and* $(x = 0))$ or $((a > 5)$ *and* $(x = 0))$
(b) $\neg((n = 7)$ *and* $(a \le 5))$
$(n \ne 7)$ or $(a > 5)$
(c) $(n = 7)$ or $(\neg((a \le 5)$ *and* $(x = 0)))$
$((n = 7)$ or $(a > 5))$ or $(x \ne 0)$

a) (p v q) ^ r

(p ^ r) v (q ^ r)

p	q	r	p v q	p ^ r	q^r	(p v q) ^ r	(p ^ r) v (q ^ r)
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	T	T	F	T	T
T	F	F	T	F	F	F	F
F	T	T	T	F	T	T	T
F	T	F	T	F	F	F	F
F	F	T	F	F	F	F	F
F	F	F	F	F	F	F	F

(p v q) ^ r ≡ (p ^ r) v (q ^ r)

-Distributive Laws

b) $\neg(p \wedge \neg q)$

$\neg p \vee q$

p	q	$\neg p$	$\neg q$	$p \wedge \neg q$	$\neg(p \wedge \neg q)$	$\neg p \vee q$
T	T	F	F	F	T	T
T	F	F	T	T	F	F
F	T	T	F	F	T	T
F	F	T	T	F	T	T

$\neg(p \wedge \neg q) \equiv \neg p \vee q$

-De Morgan's Laws

c) $p \vee (\neg(\neg q \wedge r))$

$(p \vee q) \vee \neg r$

p	q	r	$\neg q$	$\neg r$	$\neg q \wedge r$	$\neg(\neg q \wedge r)$	$p \vee (\neg(\neg q \wedge r))$	$p \vee q$	$(p \vee q) \vee \neg r$
T	T	T	F	F	F	T	T	T	T
T	T	F	F	T	F	T	T	T	T
T	F	T	T	F	T	F	T	T	T
T	F	F	T	T	F	T	T	T	T
F	T	T	F	F	F	T	T	T	T
F	T	F	F	T	F	T	T	T	T
F	F	T	T	F	T	F	F	F	F
F	F	F	T	T	F	T	T	F	T

$p \vee (\neg(\neg q \wedge r)) \equiv (p \vee q) \vee \neg r$

-Associative Laws

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Exercise

Propositions ***p***, ***q***, ***r*** and ***s*** are defined as follows:
p is "I shall finish my Coursework Assignment"
q is "I shall work for forty hours this week"
r is "I shall pass Maths"
s is "I like Maths"

Write each sentence in symbols:
(a) I shall not finish my Coursework Assignment.
(b) I don't like Maths, but I shall finish my Coursework Assignment.
(c) If I finish my Coursework Assignment, I shall pass Maths.
(d) I shall pass Maths only if I work for forty hours this week and finish my Coursework Assignment.

Write each expression as a sensible (if untrue!) English sentence:
(e) $q \lor p$
(f) $\neg p \rightarrow \neg r$

a) $\neg p$

b) $\neg s \wedge p$

c) $p \rightarrow r$

d) $r \leftrightarrow (q \wedge p)$

e) I shall work for forty hours this week or I shall finish my coursework assignment

f) If I shall not finish my coursework assignment , then I shall not pass maths

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Exercise

For each pair of expressions, construct truth tables to see if the two compound propositions are logically equivalent:

(a) $p \vee (q \wedge \neg p)$
$p \vee q$

(b) $(\neg p \wedge q) \vee (p \wedge \neg q)$
$(\neg p \wedge \neg q) \vee (p \wedge q)$

a)

p	q	$\neg p$	$q \wedge \neg p$	$p \vee (q \wedge \neg p)$	$p \vee q$
T	T	F	F	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	F	T	F	F	F

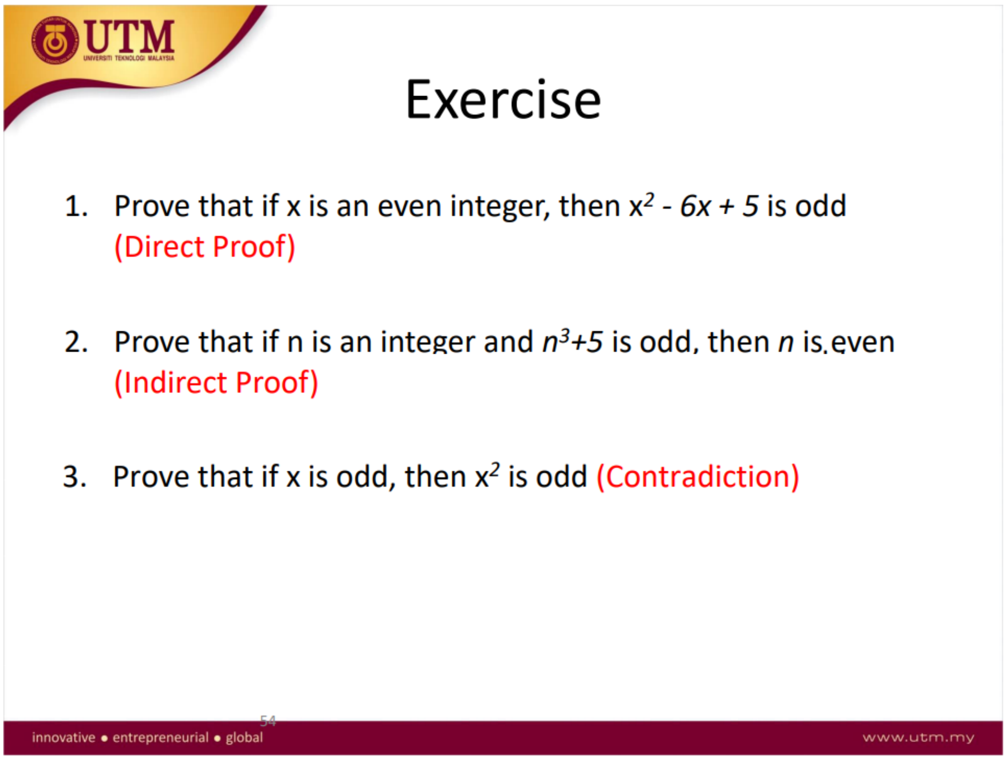
$p \vee (q \wedge \neg p) \equiv p \vee q$

b)

p	q	┐p	┐q	┐p ^ q	p ^ ┐q	┐p ^ ┐q	p ^ q	(┐p ^ q) v (p ^ ┐q)	(┐p ^ ┐q) v (p ^ q)
T	T	F	F	F	F	F	T	F	T
T	F	F	T	F	T	F	F	T	F
F	T	T	F	T	F	F	F	T	F
F	F	T	T	F	F	T	F	F	T

(┐p ^ q) v (p ^ ┐q) ≠ (┐p ^ ┐q) v (p ^ q)

PART 4 : Quantifiers and Proof Technique



1. P(x)= x is an even integer

   Q(x)=$x^2 – 6x + 5$ is odd

   $x = 2n$

   $x^2 – 6x + 5 = (2n)^2 – 6(2n)+5$

   $=4n^2 -12n + 5$

   $= 2(2n^2-6n) + 5$ $\quad$ $m=2n^2-6n$ is an integer

   $=2m+5$

   $x^2 – 6x + 5$ is odd

2.P(x)=n is an integer and $n^3 + 5$ is odd

Q(x)=n is even

$\neg$ P(x) = $n^3 + 5$ is even

$\neg$ Q(x) = n is odd

n=2m+1

$n^3+5 = (2m+1)^3 + 5$

$= 8m^3 + 12m^2 + 6m + 6$

$= 2(4m^3 + 6m^2 + 3m + 3)$ $t = 4m^3 + 6m^2 + 3m + 3$

$= 2t$

$n^3+5 = 2t$

n^3+5 is even integer .

n is odd

3.P(x)= x is odd

Q(x) = x^2 is odd

Contradiction : x is odd , x^2 is even

x = 2m + 1 (odd)

$x^2 = (2m + 1)^2$

$= 4m^2 + 4m + 1$

$=2(2m^2 + 2m) + 1$ $\quad$ $t=2m^2 + 2m$

$= 2t + 1$ (odd)