UTM
UNIVERSITI TEKNOLOGI MALAYSIA

FACULTY OF COMPUTING

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SECI 1013 DISCRETE STRUCTURE

SECTION 03

EXERCISE CHAPTER 3

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Part 1

- There are 8 male students and 21 female students in Discrete Structure class. Among all of them, 7 students are Chinese and the rest are Malay.

a) In how many ways can we select 1 student - a boy or a girl?

b) In how many ways can we select 1 student - a Chinese or a Malay?

a) A boy = 8 ways
A girl = 21 ways
(8+21) ways = 29 ways

b) A chinese = 7 ways
A malay = 22 ways
(7+22) ways = 29 ways

How to find the number of integers between 5 and 50 that end with 1 or 7.

(4+5) ways = 9 ways

End with 1 : 11, 21, 31, 41 → 4 ways
End with 7 : 7, 17, 27, 37, 47 → 5 ways

The letters A, B, C, D, and E are to be used to form strings of length 4. How many strings begin with A, if repetitions are not allowed? For example: $ADEC$, $ACBD$, $AEBC$ ..

5x4x3x2 = 120 strings

- There are 8 male students and 21 female students in Discrete Structure class. Among all of them, 7 students are Chinese and the rest are Malay.

a) In how many ways can we select 2 students - a boy and a girl?

b) In how many ways can we select 2 students - a Chinese and a Malay?

a) A boy = 8 ways
A girl = 21 ways
(8x21) ways = 168 ways

b) A chinese = 7 ways
A malay = 22 ways
(7x22) ways = 154 ways

A six-person committee composed of Aina, Wan, Chan, Tan, Syed and Helmi is to be selected to hold as a chairperson, secretary, and treasurer.

3. In how many ways can this be done if Syed must hold one of the position?
4. In how many ways can this be done if Tan and Helmi must hold any position?

3. If Syed chairperson : 5x4 = 20 ways
If Syed secretary : 5x4 = 20 ways
If Syed treasurer : 5x4 = 20 ways
20+20+20 = 60 ways

4. Tan chairperson, Helmi secretary : 4
Tan chairperson, Helmi treasurer : 4
Tan secretary, Helmi chairperson : 4
Tan secretary, Helmi treasurer : 4
Tan treasurer, Helmi chairperson : 4
Tan treasurer, Helmi secretary : 4
4+4+4+4+4+4 = 24 ways

How many ways can we select three books each from a different subject from a set of six distinct history books, nine distinct classics books, seven distinct law books, and four distinct education books?

(378+216+252+168) ways = 1014 ways

6H, 9C, 7L, 4E
HCL = 6x9x7 = 378 ways
HCE = 6x9x4 = 216 ways
CLE = 9x7x4 = 252 ways
HLE = 6x7x4 = 168 ways

Exercise 1

In how many dance pairs (dance pairs means a pair (W,M), where W stands for a women and M for man), can be formed from a group of 6 women and 10 men?

$W = P(6,6) = 6! = 720$

$M = P(10,6) = \frac{10!}{(10-6)!} = 151200$

$= 151200 \times 720$

$= 108864000$

Exercise 2

- In how many ways can 10 distinct books be divided among 3 students if Khairin gets 4 books and Nurina and Sarah each get 3 books.

Find the answer.

$P(10) = \frac{10!}{(4!\,3!\,3!)} = 4200$

Exercise 3

- In how many ways can five people A, B, C, D, and E be seated around a circular table if

  a) A and B must sit next to each other

  b) A and B must not sit next to each other

  c) A and B must be together and CD must be together

a) $(4-1)! = 3! = 6$ ways

AB, BA

$\therefore 6 \times 2 = 12$ ways

b) $(5-1)! = 4! = 24$ ways

AB, BA ✕

$\therefore 24 - 12 = 12$ ways

↑ sit together

c) $(3-1)! = 2! = 2$ ways

AB, BA, CD, DC = 4

$2 \times 4 = 8$

Exercise 4

A student is required to answer 7 out of 12 questions, which are divided into two groups, each containing 6 questions. The student is not permitted to answer more than 5 questions from either group. In how many different ways can the student choose the 7 questions?

Case 1: group A $= C(6,5) = \frac{6!}{5!\,1!} = 6$

group B $= C(6,2) = \frac{6!}{2!\,4!} = 15$

Case 2: group A $= C(6,4) = \frac{6!}{4!\,2!} = 15$

group B $= C(6,3) = \frac{6!}{3!\,3!} = 20$

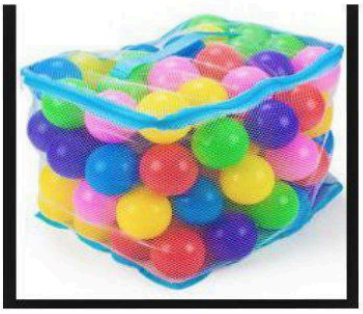
Case 3: group A $= C(6,3) = \frac{6!}{3!\,3!} = 20$

group B $= C(6,4) = \frac{6!}{3!\,3!} = 15$

$(6 \times 15) + (15 \times 20) + (20 \times 15) = 690$

Exercise 5

There is a box containing identical blue, green, pink, yellow, red and dark blue balls. In how many ways we can select 4 balls?



$n = 6$
$r = 4$

$$C(6+4-1, 4) = \frac{(6+4-1)!}{4!(6-1)!}$$

$$C(9, 4) = \frac{9!}{4!5!} = 126 \text{ ways...}$$

Exercise 1

- How many students must be in a course to guarantee at least two students receive same score in the test, if the test is graded on a scale from 0 to 100. Explain your answer.

Pigeon (n) - students in a course

Pigeonhole (m)- score from 0 to 100

$m = 101$

$k = 2$

$n = m(k-1)+1$

$= 101(2-1)+1$

$= 102$

If every student get a different score, so they must be have at least 101 students to obtain the scores from 0 to 100.They must add more one person to make sure they have at least two students receive a same score.

Exercise 2

- Show that every set of 15 socks chosen among 14 pairs of socks contains at least one matched pair.

pigeon - 14 pair of sock which is 28 socks

pigeonhole - 15 socks.

pigeonhole < Pigeon, so there must have at least one matched pair.

* prove by pp- 1st form

CHAPTER 3 PART 4

Exercise 1

There are exactly 3 red balls in a bucket of 15 balls. If we choose 4 balls at random, what is the probability that we do not choose a red ball?

Exercise 1

red balls $= \frac{3}{15}$

probability of choosing 4 balls $= \frac{4}{15}$

There are $C(15,4)$ ways to choose 4 random balls.

There are $C(12,4)$ ways to choose not a red ball.

The probability : $\frac{C(12,4)}{C(15,4)} = \frac{495}{1365} = \frac{33}{91}$

Exercise 2

What is the probability that if a fair coin is tossed 6 times you will get,

i) Less than 2 heads

ii) At least 2 heads

Exercise 2

$2^6 = 64$

i) A = less than 2 heads

A = { TTTTTT TTTTTH TTTTHT TTTHTT TTHTTT THTTTT HTTTTT }

P(A) = $\frac{7}{64}$

ii) B = at least 2 heads

P(B) = $1 - \frac{7}{64}$

= $\frac{57}{64}$

Exercise 3

On New Year's Eve, the probability of a person having a car accident is 0.09. The probability of a person driving while intoxicated is 0.32 and probability of a person having a car accident while intoxicated is 0.15. What is the probability of a person driving while intoxicated or having a car accident?

Exercise 3

having car accident = 0.09 (C)

driving while intoxicated = 0.32 (D)

having car accident while intoxicated = 0.15 (E)

$P(D \text{ or } C) = P(D) + P(C) - P(D \cap C)$

$= 0.32 + 0.09 - 0.15$

$= 0.26$

Exercise 4

Weather records show that the probability of high barometric pressure is 0.85 and the probability of rain and high barometric pressure is 0.15. What is the probability of rain given high barometric pressure?

Exercise 4

$P(A|B) = P($

$P(\text{high BP}) = 0.85$ $\frac{17}{20}$

$P(\text{rain and high BP}) = 0.15$ $\frac{3}{20}$

A = high BP

B = rain ~~and high BP~~

C = rain and high BP

$P(B|A) = \frac{P(B \cap A)}{P(A)}$

$= \frac{0.15}{0.85}$

$= \frac{3}{17} = 0.1765$

Exercise 6

- Hana, Amir and Dani write a program that schedule tasks for manufacturing toys.
- The table shows the percentage of code written by each person and the percentage of buggy code for each person.

	Coder		
	Hana	Amir	Dani
% of code	30	45	25
% of bugs	3	2	5

- Given that a bug was found, **find the probability that it was in the program code written by Dani.**

Exercise 5

A = Hana write the code — P(A) = 0.30 — P(D|A) = 0.03

B = Amir write the code — P(B) = 0.45 — P(D|B) = 0.02

C = Dani write the code — P(C) = 0.25 — P(D|C) = 0.05

D = a bug was found — ~~P(D)~~

$$P(C|D) = \frac{P(D|C)P(C)}{P(D|C)P(C) + P(D|A)P(A) + P(D|B)P(B)}$$

$$= \frac{(0.05)(0.25)}{(0.05)(0.25) + (0.03)(0.30) + (0.02)(0.45)}$$

$$= \frac{25}{61}$$

$$= 0.4098$$

Exercise 7

- Halim and Aina take a final examination in Fortran.
- The probability that Halim passes is 0.85, and the probability that Aina passes is 0.70.
- Assume that the events "Halim passes the final exam" and "Aina passes the final exam" are independent.

- Find the probability that Halim does not pass.
- Find the probability that both pass.
- Find the probability that both fail.
- Find the probability that at least one passes.

Exercise 6

P (Halim passes) = 0.85 P(H)
P (Aina passes) = 0.70 P(A)

i) P (Halim does not pass) = 1 - 0.85
= 0.15

ii) P (Halim and Aina passes) = P(H ∩ A)
= P(A) · P(B)
= 0.70 · 0.85
= 0.595

iii) P(Halim and Aina failed) = P(H') < P(A')
= (1 - 0.85) (1 - 0.70)
= 0.045

iv) P (at least one passes) = 1 - P (Halim and Aina failed)
= 1 - 0.045
= 0.955