Question 1

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a) Show that $f(n) = 4n^5 + 2n^3 + 3n$ is $O(n^5)$ by specifying c and n_0 values

$$4n^5 + 2n^3 + 3n \leq c \cdot n^5$$

$$n(4n^4 + 2n^2 + 3) \leq c \cdot n^5 \qquad (n > 0)$$

$$4n^4 + 2n^2 + 3 \leq c \cdot n^4$$

If $c = 5$ and $n_0 = 2 \implies 4n^4 + 2n^2 + 3 \leq 5n^4$

$2n^2 + 3 \leq n^4$ for all $n \geq n_0 = 2$

$$11 \leq 16 \qquad 0 \leq n^4 - 2n^2 - 3 \qquad 0 \leq (n^2 - 1)^2 - 4$$

Hence, since there exist constants c and n_0 such that $f(n) \leq c \cdot n^5$ for $n \geq n_0$, $f(n) = 4n^5 + 2n^3 + 3n$ is $O(n^5)$.

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b) Selection sort

swap 65,42 $[40, 25, \underset{\uparrow}{65}, 45, 50, 35, 55, 38, 30, \underset{\uparrow}{42}]$ initial array

swap 55, 30 $[40, 25, 42, 45, 50, 35, \underset{\uparrow}{55}, 38, \underset{\uparrow}{30}, 65]$ after 1st swap

swap 50,38 $[40, 25, 42, 45, \underset{\uparrow}{50}, 35, 30, \underset{\uparrow}{38}, 55, 65]$ after 2nd swap

swap 45,30 $[40, 25, 42, \underset{\uparrow}{45}, 38, 35, \underset{\uparrow}{30}, 50, 55, 65]$ after 3rd swap

swap 42,35 $[40, 25, \underset{\uparrow}{42}, 30, 38, \underset{\uparrow}{35}, 45, 50, 55, 65]$ after 4th swap

swap 40, 38 $[\underset{\uparrow}{40}, 25, 35, 30, \underset{\uparrow}{38}, 42, 45, 50, 55, 65]$ after 5th swap

swap 38,30 $[\underset{\uparrow}{38}, 25, 35, \underset{\uparrow}{30}, 40, 42, 45, 50, 55, 65]$ after 6th swap

swap 35,35 $[30, 25, \underset{\uparrow\uparrow}{35}, 38, 40, 42, 45, 50, 55, 65]$ after 7th swap

swap 30,25 $[\underset{\uparrow}{30}, \underset{\uparrow}{25}, 35, 38, 40, 42, 45, 50, 55, 65]$ after 8th swap

$[25, 30, 35, 38, 40, 42, 45, 50, 55, 65]$ after 9th swap
array is sorted

Insertion Sort

sorted | unsorted
[40 | 25, 65, 45, 50, 35, 55, 38, 40, 42]

1st iteration
sorted | unsorted
[25, 40 | 65, 45, 50, 35, 55, 38, 40, 42]   shift 40 and insert 25

2nd iteration
sorted | unsorted
[25, 40, 65 | 45, 50, 35, 55, 38, 40, 42]   insert 65

3rd iteration
sorted | unsorted
[25, 40, 45, 65 | 50, 35, 55, 38, 40, 42]   shift 65 and insert 45

4th iteration
sorted | unsorted
[25, 40, 45, 50, 65 | 35, 55, 38, 40, 42]   shift 65 and insert 50

5th iteration
sorted | unsorted
[25, 35, 40, 45, 50, 65 | 55, 38, 40, 42]   shift 40 and insert 35

6th iteration
sorted | unsorted
[25, 35, 40, 45, 50, 55, 65 | 38, 40, 42]   shift 65 and insert 55

7th iteration
sorted | unsorted
[25, 35, 38, 40, 45, 50, 55, 65 | 40, 42]   shift 40 and insert 38

8th iteration
sorted | unsorted
[25, 35, 38, 40, 40, 45, 50, 55, 65 | 42]   shift 45 and insert 40

9th iteration
sorted
[25, 35, 38, 40, 40, 42, 45, 50, 55, 65]   shift 45 and insert 42

array is sorted. (unsorted is empty)