The Warehouse Limited: Improving Marketing Decisions

156.762 - Assignment 3

Brett Hawes (96016786)

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Executive Summary

The objectives of this report are to identify optimal managerial decisions that The Warehouse can make to improve profitability by either increasing revenue or decreasing costs and to assess the appropriateness of applying chain level estimates to individual stores.

The previous multiplicative advertising model was modified to remove closed days from the data set rather than utilising a 'closed' indicator variable as the previous model did. The main effect of this was to increase the lambda value for press advertising from 0.4 to 0.6 implying that press advertising has longer term effects.

Using the revised advertising elasticities, it was found that channel marginal returns on investment ranged from 261% to 1711% implying increased sales could be achieve by shifting advertising spend from low return channels to high return channels. Applying optimal allocations resulted in increased allocations to press and digital at the expense of radio, mailers and press and increased The Warehouses sales revenue by 6.6%.

While a multiplicative model already has interaction effects built in, specific effects were also tested for and interaction effects for TV/Mailer, TV/Press and Mailer/Press were found. The presence of interaction effects adjusted advertising elasticities and the resulting optimal allocations. Optimal allocations with interaction effects increases radio allocations at the expense of press advertising with the resulting sales uplift being 1.37% from the base case.

The Warehouse should utilise the elasticities and the derived optimal budget allocations from the interaction model in any decision making due to the confirmed presence of interaction effects.

The interaction model was run against each store to determine whether individual stores had different advertising elasticities. This was done by comparing the confidence interval of each store elasticity against the confidence interval of the chain level elasticity. In general, there was little evidence of store elasticities differing to the chain level with only 2 of the 95 store level elasticity observations appearing to be different to the chain level. Chain level elasticities should not be applied against those two stores however the chain level estimates in general appear to be valid to use.

Advertising effectiveness over time was tested by running the interaction model against each Financial Year and noting the advertising elasticities for each model. Using the same methods as for testing store elasticities found no evidence that elasticities are changing over time meaning the derived elasticities should not need any adjustments to use for The Warehouses' forward focussed decision making.

The lack of evidence of elasticities changing over time or varying by store (except for the two outliers noted) mean The Warehouse should adopt the elasticities and optimal allocations derived from the interaction model across all stores.

The analysis concludes by looking at the possibility of increasing the overall advertising budget and recommendation advertising budgets could be increased by 30% as the indicated marginal returns on investment of c450% on increases in that range should compare favourably to other investment available to The Warehouse.

Model Revision

In assignment 2, an indicator variable was used to identify days where stores were closed. It was noted that this was not ideal in that the variable had the effect of significantly reducing sales but did not drop the sales value to zero.

The model has been revised to exclude closed days instead meaning the remaining model is for trading days only. Full statistical details of the final model can be found in Appendix A.

Table 1 – Previous and Revised Model Estimates

Variable	Previous Estimate	Revised Estimate	Previous Lambda	Revised Lambda
Constant	-1.1605	-1.1249		
Days in December (Indicator)	0.0303	0.0309		
Sunday	0.224	0.225		
Tuesday	0.0463	0.0467		
Wednesday	0.1425	0.1427		
Thursday	0.2103	0.2103		
Friday	0.185	0.185		
Saturday	0.3452	0.3440		
Closed	-1.3432	n/a		
Boxing Day	0.7167	0.6938		
Black Friday	0.5261	0.5239		
Television AdStock (elasticity)	0.0477	0.0492	0.8	0.8
Radio AdStock* (elasticity)	0.0104	0.0072	0.2	0.2
Digital AdStock (elasticity)	0.0749	0.0733	0.8	0.8
Mailer AdStock* (elasticity)	0.0318	0.0335	0.8	0.8
Print AdStock (elasticity)	0.112	0.1078	0.4	0.6

*Variables not significant

Overall parameter estimates are very similar to before with the only notable change being that the Print lambda has increased from 0.4 to 0.6 indicating a longer effect for print advertising.

R-Sq decreased from 89.92% to 88.7% however the revised model has a slightly different dataset.

The final model is:

Daily Sales = exp(-1.1249 + 0.0309 x DaysInDec + 0.225 x Sunday + 0.0467 x Tuesday + 0.1427 x Wednesday + 0.2103 x Thursday + 0.185 x Friday + 0.3440 x Saturday + 0.6938 x BoxingDay + 0.5239 x BlackFriday + 0.0492 x Adstock_Television + 0.0072 x Adstock_Radio + 0.0733 x Adstock_Digital + 0.0335 x Adstock_Mailer + 0.1078 x Adstock_Print

Final Revised Long-Term Elasticities, Short-Term Elasticities and Durations

Variable	Lambda	Long Term Elasticity[1]	Short Term Elasticity[2]	90% Duration[3]
Television AdStock (elasticity)	0.8	0.0492	0.01	10.3 days
Radio AdStock (elasticity)	0.2	0.0072	0.006	1.4 days
Digital AdStock (elasticity)	0.8	0.0733	0.015	10.3 days
Mailer AdStock (elasticity)	0.8	0.0335	0.007	10.3 days
Print AdStock (elasticity)	0.6	0.1078	0.043	4.5 days

Overall long-term elasticities are positive as expected and within expected ranges. Lambda indicate generally long-term effects of advertising, however as this is a daily model 'long-term' effects are still only approximately 10 days.

[1] Defined as the parameter estimate of the model
[2] Defined as (1-lambda) x long term elasticity
[3] Defined as ln(1-.9) / ln(lambda)

Optimising Advertising Budget

Over the observed period, The Warehouse is currently allocating the daily advertising budget as follows:

Channel	Daily Spend - $	Daily Spend - %
Television	$2,476	29.9%
Radio	$767	9.3%
Digital	$1,363	16.5%
Mailer	$2,233	27.0%
Press	$1,433	17.3%
Total	**$8,272**	**100%**

Marginal Return on Investment

The Warehouse's marginal return on investment by channel can be calculated as:

$$\text{Marginal Return on Investment} = Margin\ x\ Channel\ Elasticity\ x\ \frac{Sales}{Advertising\ (Channel)} - 1$$

Utilising this formula requires an assumption around The Warehouse's margins and sales also need to be converted into the same units as the advertising spend. The Warehouses' margin is assumed to be 10%. The daily level of advertising spend generates a modelled sales figure of 2.407 (Appendix B shows the workings for this). It is assumed that this figure is in millions.

Using the above assumptions gives the following marginal returns on investment by channel:

Channel	Marginal ROI
Television	378%
Radio	126%
Digital	1194%
Mailer	261%
Press	1711%

Given that the marginal returns on investment are not all equal, this implies that the allocation can be improved by reallocating advertising expenditure away from low return channels (Radio) to high return channels (Press).

The Dorfman-Steiner rule can be used to determine optimal allocations between advertising channels. The Dorfman-Steiner rule states that the optimal channel allocation should be:

$$Channel\ Allocation = \frac{\beta_c}{\sum_{j=1}^{c} \beta_j} x\ Total\ Budget$$

Applying the Dorfman-Steiner rules gives the following optimal advertising allocations and marginal return on investments:

Channel	Daily Spend - $	Daily Spend - %	Marginal ROI
Television	$1,502	18.2%	743%
Radio	$220	2.7%	743%
Digital	$2,237	27.0%	743%
Mailer	$1,023	12.4%	743%
Press	$3,290	39.8%	743%

As expected, the Dorfman-Steinman allocation has reallocated advertising budget from low return channels to high return channels.

By reallocating the advertising budget, overall sales increase from $2.41m to $2.57m[4] or 6.63% increase.

The Warehouse should adopt optimal budget allocations.

Testing for Interaction Effects

By nature, the current multiplicative sales model already has interaction effects built in. This can be demonstrated through the example below between Television and Radio advertising expenditure.

Table 2 below indicates sales under various combinations of TV/Digital advertising uplift from the current base level of spending. For instance, a 20% increase in TV advertising and 10% increase in Digital advertising would result in sales of 2.4457.

Table 2 – Modelled Sales Under Different Combinations of TV and Digital Advertising Uplift

		Increase in Digital Advertising						
		0%	10%	20%	30%	40%	50%	Marginal Increase - TV[5]
Increase in TV Advertising[6]	0%	2.4070	2.4239	2.4394	2.4537	2.4671	2.4796	
	10%	2.4183	2.4353	2.4508	2.4653	2.4787	2.4912	0.470%
	20%	2.4287	2.4457	2.4613	2.4758	2.4893	2.5019	0.429%
	30%	2.4383	2.4554	2.4711	2.4856	2.4991	2.5118	0.394%
	40%	2.4472	2.4643	2.4801	2.4947	2.5082	2.5210	0.365%
	50%	2.4555	2.4727	2.4885	2.5031	2.5168	2.5295	0.340%
	Marginal Increase - Digital		0.70%	0.64%	0.59%	0.54%	0.51%	

[4] For a Monday outside of December where stores are open.
[5] Defined as the percentage change from one level of TV advertising to another where Digital advertising is zero.
[6] Defined as the percentage change from one level of Digital advertising to another where TV advertising is zero.

From Table 2, we can calculate the difference between each combination and the base case to determine the uplift. For example, the uplift from a 20% increase in TV advertising and 10% increase in Digital advertising would be 2.4457– 2.4070 = 0.0387.

Apply this logic against all values in Table 2 results in Table 3 below, he sales uplifts from the base case.

Table 3 – Sales Uplifts from Base Case

		Digital					
		0%	**10%**	**20%**	**30%**	**40%**	**50%**
TV	**0%**	0.0000	0.0169	0.0324	0.0467	0.0601	0.0726
	10%	0.0113	0.0283	0.0438	0.0582	0.0717	0.0842
	20%	0.0217	0.0387	0.0543	0.0688	0.0823	0.0949
	30%	0.0313	0.0483	0.0640	0.0786	0.0921	0.1048
	40%	0.0402	0.0573	0.0731	0.0877	0.1012	0.1139
	50%	0.0485	0.0657	0.0815	0.0961	0.1098	0.1225

From Table 3 we can then derive the implied interaction effect for each combination by taking the observed uplift and subtracting the two separate uplifts where only one medium was being increased independently. The interaction effect for the 20% TV, 10% Digital example would then be 0.0387 (the uplift of the combination) – 0.0217 (the uplift from TV increase alone) – 0.0169 (the uplift from Digital increase alone) = 0.0002 indicating a small positive interaction effect.

Table 4 below indicates all interaction components of the uplifts observed in Table 2.

Table 4 – Interaction Effects for TV/Digital Uplift Combinations

		Digital					
		0%	**10%**	**20%**	**30%**	**40%**	**50%**
TV	**0%**	n/a	n/a	n/a	n/a	n/a	n/a
	10%	n/a	0.0001	0.0002	0.0002	0.0003	0.0003
	20%	n/a	0.0002	0.0003	0.0004	0.0005	0.0007
	30%	n/a	0.0002	0.0004	0.0006	0.0008	0.0009
	40%	n/a	0.0003	0.0005	0.0008	0.0010	0.0012
	50%	n/a	0.0003	0.0007	0.0009	0.0012	0.0015

Table 3 indicates there are small positive interaction effects already built into the model (at least in the case of the TV/Digital combination).

Explicit Testing for Further Interaction Effects

Explicit testing for interactions can be done through the addition of new variables in the model.

Ten new variables were added to the dataset representing the pairwise combination of the five advertising channels (using Adstock variables). For each pair the interaction variable was defined as (Adstock_Channel A – Avg(Adstock_Channel A) x (Adstock_Channel B – Avg(Adstock_Channel B).

These were then added to the current multiplicative model. The full model is in Appendix C and the interaction effects stated in Table 5 below.

Table 5 – Parameter Estimates for Interaction Effects

Interaction Combination	Estimate
TV / Radio	0.0443
TV / Digital	0.0185
TV / Mailer	-0.1519
TV / Press	0.1679
Radio / Digital	0.0120
Radio / Mailer	-0.0099
Radio / Press	-0.0073
Digital / Mailer	0.0321
Digital / Press	0.1439
Mailer / Press	-0.2559

Interactions in green are significant

The addition of the ten possible interaction pairs indicated the follow pairs were significant at the 5% level:

- TV/Mailer (negative effect)
- TV/Press (positive effect)
- Mailer/Press (large negative effect)

Allowing for these interaction effects results in different advertising elasticities as indicated in Table 6 below. The revised elasticities are still all positive as expected and within expected ranges. Utilising the Dorfman-Sterner rule introduced previously also results in a revised optimal advertising allocation.

Table 6 – Revised Elasticities and Optimal Advertising Allocation with Interaction Effects.

Channel	Previous Elasticity	Revised Elasticity	Original Allocation	Previous Optimal Allocation	Revised Allocation Allowing for Interactions Effects	Marginal Return on Investment – Revised Allocation
TV	0.0492	0.0491	29.9%	18.2%	22.2%	554%
Radio	0.0072	0.0253	9.3%	2.7%	11.4%	554%
Digital	0.0733	0.0566	16.5%	27.0%	25.6%	554%
Mailer	0.0335	0.0299	27.0%	12.4%	13.5%	554%
Press	0.1078	0.0606	17.3%	39.8%	27.4%	554%
Sales			**2.407**	**2.572**	**2.44**	

Elasticities in orange are not significant

Of note, with the addition of interaction effects, is that the radio advertising channel has become significant in its own right with a higher advertising elasticity consequentially increasing the budget allocation at the expense of press advertising.

The full interaction model specification is:

Daily Sales = exp(-0.7613 + 0.0333 x DaysInDec + 0.2253 x Sunday + 0.0464 x Tuesday + 0.144 x Wednesday + 0.2117 x Thursday + 0.1861 x Friday + 0.3452 x Saturday + 0.6968 x BoxingDay + 0.5172 x BlackFriday + 0.0491 x Adstock_Television + 0.0253 x Adstock_Radio + 0.0566 x Adstock_Digital + 0.0299 x Adstock_Mailer + 0.0606 x Adstock_Print + .0443 x TV/Radio Interaction + 0.0185 x TV/Digital Interaction -0.1519 x TV/Mailer Interaction + 0.1679 TV/Press Interaction + 0.012 x Radio/Digital Interaction -0.0099 x Radio/Mailer Interaction - 0.007283 x Radio/Press Interaction + 0.0321 x Digital/Mailer Interaction + 0.1439 x Digital/Press Interaction – 0.2559 x Mailer/Press Interaction)

The overall effect of the interactions is to reduce sales such that the sales increase is now 1.37%. This appears to be explained by the large negative interaction between mailer and press which forces a lower elasticity for press and consequential allocation away from that channel.

Due to the confirmed presence of interaction effects, The Warehouse should use elasticities derived from this interaction model and the resulting optimal allocation.

Reducing Advertising Budget to Achieve Same Sales Level

Rather than using the optimal budget allocations to increase sales The Warehouse could also opt to achieve the same level of sales with a smaller advertising budget if there is a desire to save costs.

By using the solver tool in Microsoft Excel, we can find the spending mix by using the following settings:

- Set the objective to minimise the spend across the five advertising mediums
- As a constraint, set the sales to 2.407, the baseline level of sales
- As a constraint make sure all advertising is greater than $0

Table 7 below indicates the original channel allocations vs the revised allocations which achieve the same sales level (for the interaction model)

Table 7 – Reducing Advertising but Achieving Same Sales Level (Interaction Model)

Channel	Original Advertising Spend	Original Allocation	Revised Minimising Spend	Revised Allocation Minimising Spend	Dorfman-Steiner Optimal Allocations	Marginal Return on Inv. – Reduced Spending
TV	$2,476	29.9%	$1,722	22.2%	22.2%	586%
Radio	$767	9.3%	$887	11.4%	11.4%	586%
Digital	$1,363	16.5%	$1,985	25.6%	25.6%	586%
Mailer	$2,233	27.0%	$1,048	13.5%	13.5%	586%
Press	$1,433	17.3%	$2,123	27.3%	27.4%	586%
Total Spend	**$8,272**		**$7,765**			
Sales	**2.407**		**2.407**			

Table 7 indicates that The Warehouse can reduce spending by 6.12% ($7,765 / $8,272 -1) but still achieve the same sales by reallocating advertising budget from TV and Mailers to Radio, Digital and Press. Table 7 also shows that the optimal allocations determined by the solver tool are effectively the same as the allocations generated under the Dorfman-Steiner rule as we would expect. Reducing advertising spend also increases the marginal return on investment from 554% to 586% as would be expected with diminishing returns on investment.

With such high marginal returns on investment it's not recommended that The Warehouse reduce advertising expenditure.

Store Level Elasticities

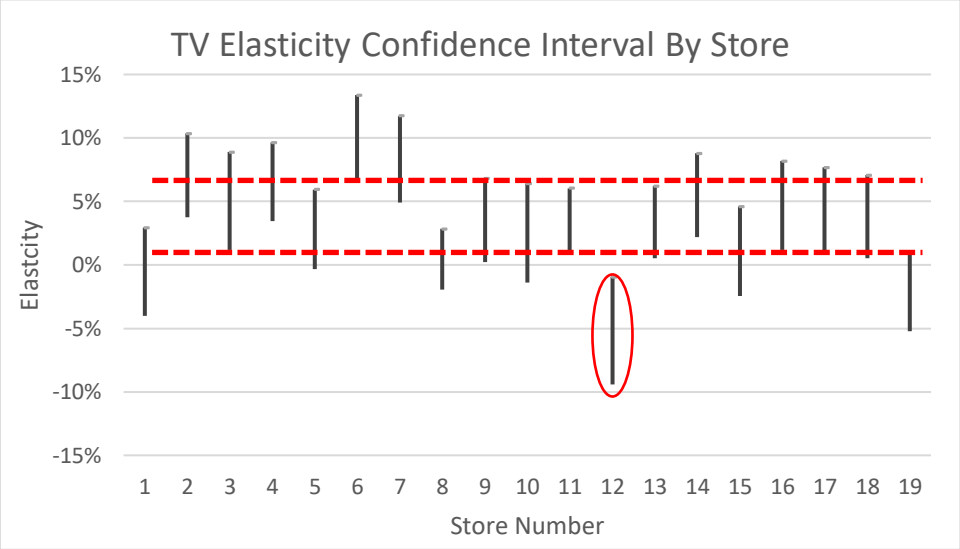
Previously all model estimates have been at the chain level.

In this section we look at advertising elasticities for each store. Assuming the same lambda values that were identified at the chain level apply at the individual store level, 19 individual store level models were run, and their advertising elasticity parameter estimates recorded.

By using the standard errors of each store elasticity, we can create a confidence interval[7] for each store's elasticity and plot those.

Figures 1-5 show those plots of the confidence intervals of store elasticities against each other. Full details are in Appendix D

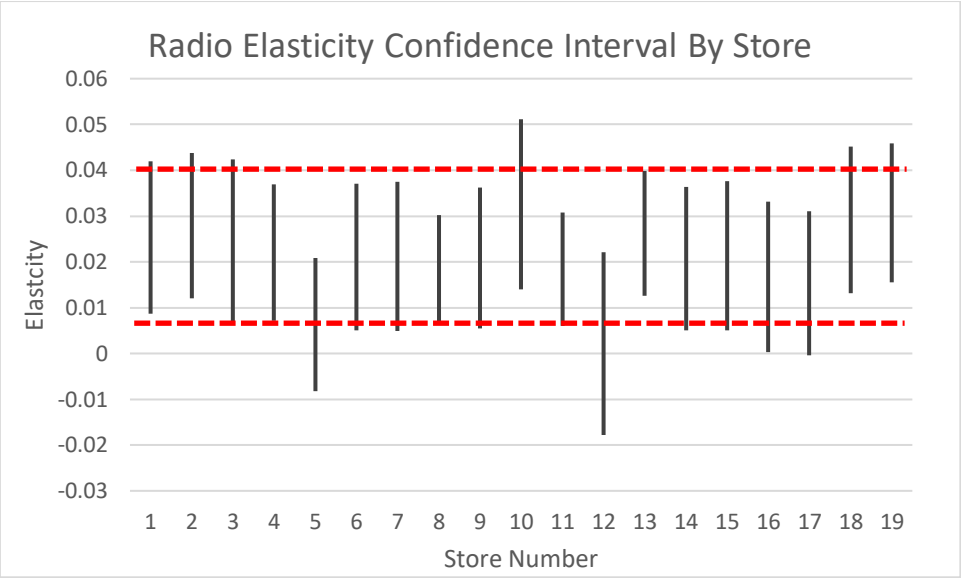
Figure 1 – TV Elasticities by Store Number (Dashed Lines = Chain Elasticity Confidence Interval)



TV elasticities appear to be quite variable. Store 12 appears to be an outlier with a significantly lower elasticity. Care should be taken in using the chain level elasticity for store 12 as it would overstate TV advertising effectiveness, however the negative elasticity suggests this is a true anomaly for store 12.

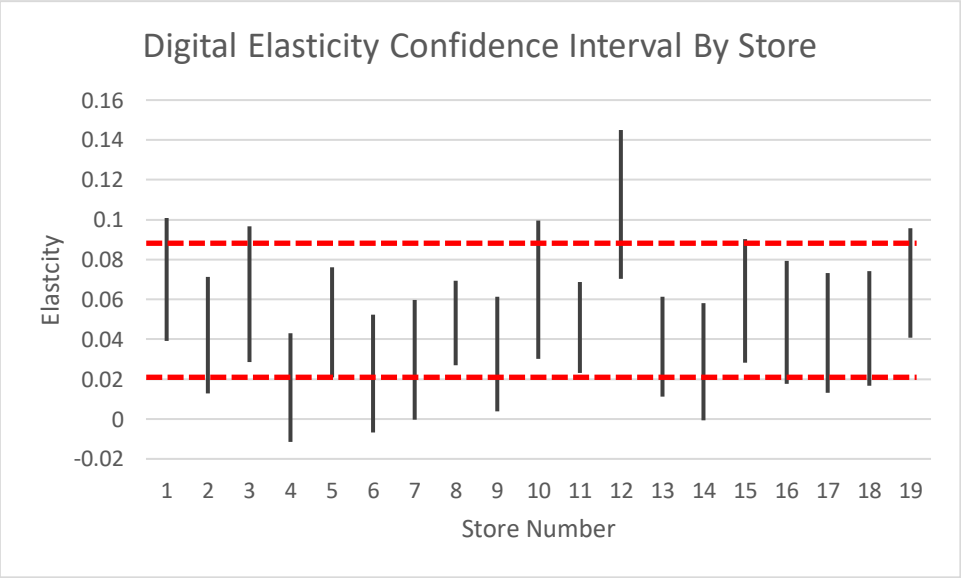
[7] Confidence interval defined as Store Elasticity +/- 1.96 x standard error

Figure 2 – Radio Elasticities by Store Number (Dashed Lines = Chain Elasticity Confidence Interval)



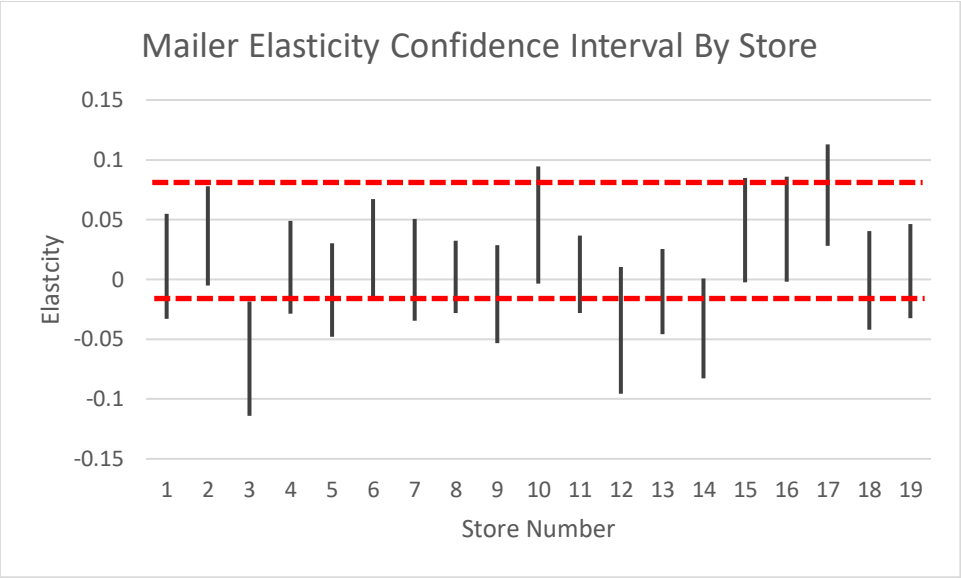
Radio elasticities appear to be similar across the stores with no outliers. Applying the chain level elasticity to each store would appear to be valid.

Figure 3 – Digital Elasticities by Store Number (Dashed Lines = Chain Elasticity Confidence Interval)



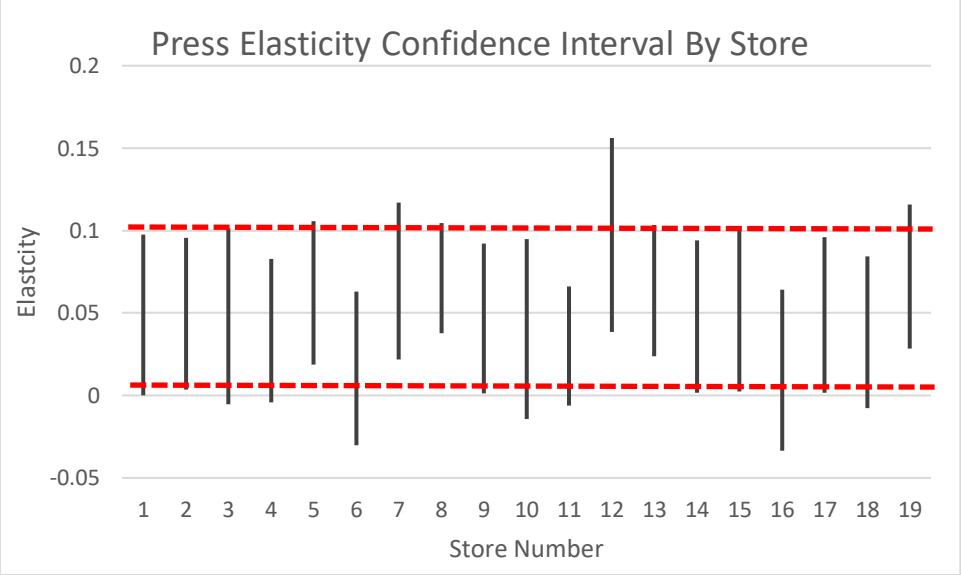
Digital elasticities display a fair amount of variability however no stores appear to be statistical outliers. Applying the chain level elasticity to each store would appear to be valid.

Figure 4 – Mailer Elasticities by Store Number (Dashed Lines = Chain Elasticity Confidence Interval)



Mailer elasticities are quite variable. Stores 3 however is just verging on being a statistical outlier with a lower elasticity than for the chain. The chain level elasticity should not be applied to this store (although again, the negative elasticity could suggest a true anomaly).

Figure 5 – Press Elasticities by Store Number (Dashed Lines = Chain Elasticity Confidence Interval)



Press elasticities are generally the same across stores no apparent store outliers.

The implications of this analysis are that store elasticities appear to be the same as the chain level elasticities and that the optimal allocations previously derived should be appropriate to apply to all stores. The two observed outliers could be anomalies due to their negative estimates.

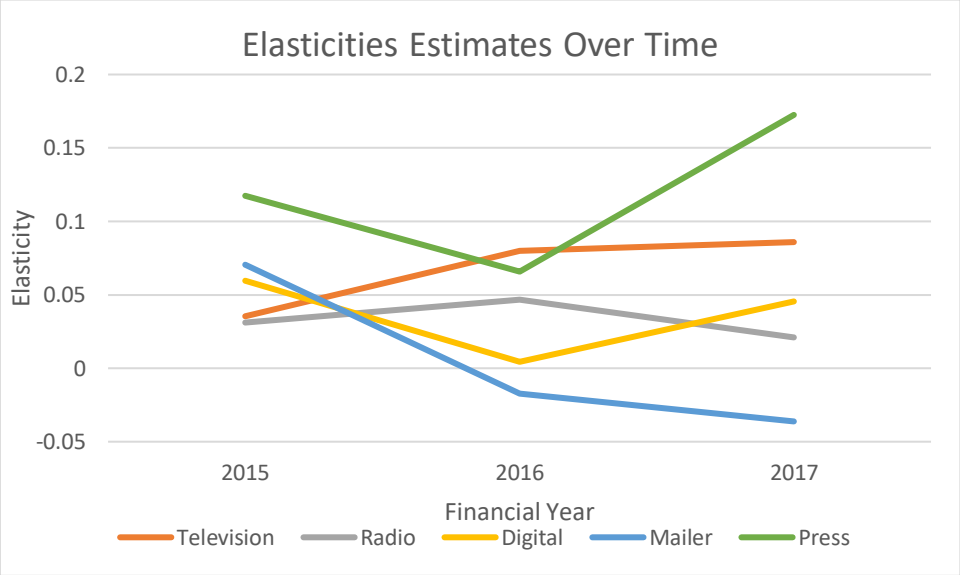
Elasticities Over Time

This section looks at whether there is any evidence that advertising channel elasticities are changing over time. Any proven trends may have an influence on the overall elasticity value utilised for decision making.

This was achieved by running the previous Yule-Walker regression model with interaction effects for each separate Financial Year (and removing the current training/validation data split) giving three observed parameter estimates for each advertising channel over time.

Figure 6 below indicates the point estimates of advertising elasticities over time. Full model details are in Appendix E

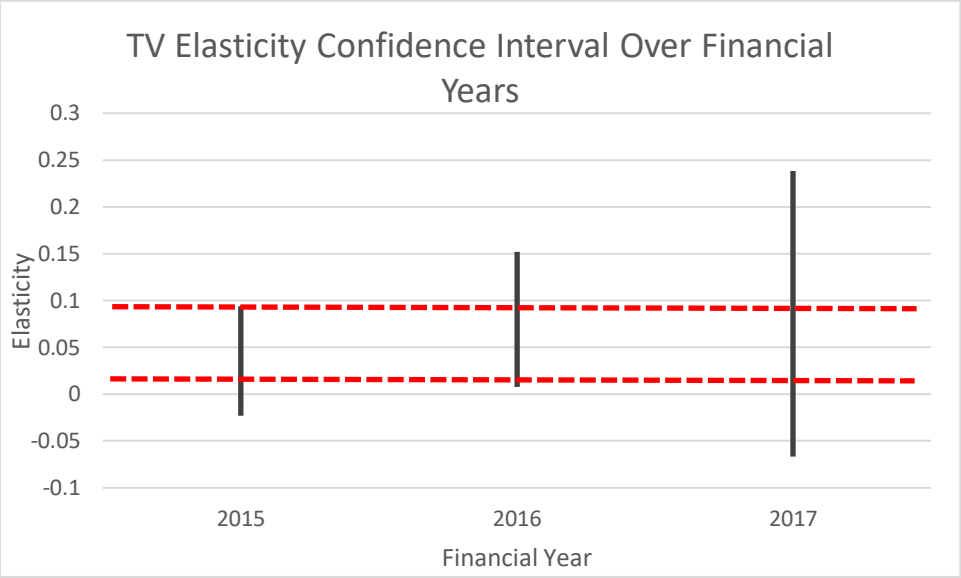
Figure 6 – Elasticity Estimates Over Time



It is difficult to determine trends with mailers possibly decreasing over the period and press increasing.

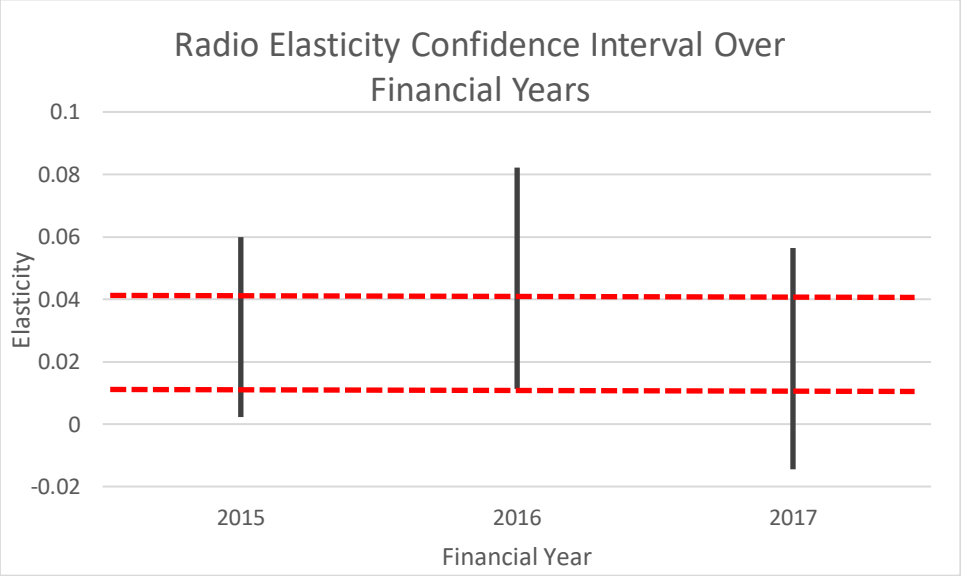
Care needs to be taking around the natural volatility that would occur in estimates from year to year. To allow for this, Figures 7-11 show the confidence intervals for each financial year's estimate for each medium.

Figure 7 – TV Elasticity Confidence Interval Over Time (Dashed Lines = Full Period Elasticity Confidence Interval)



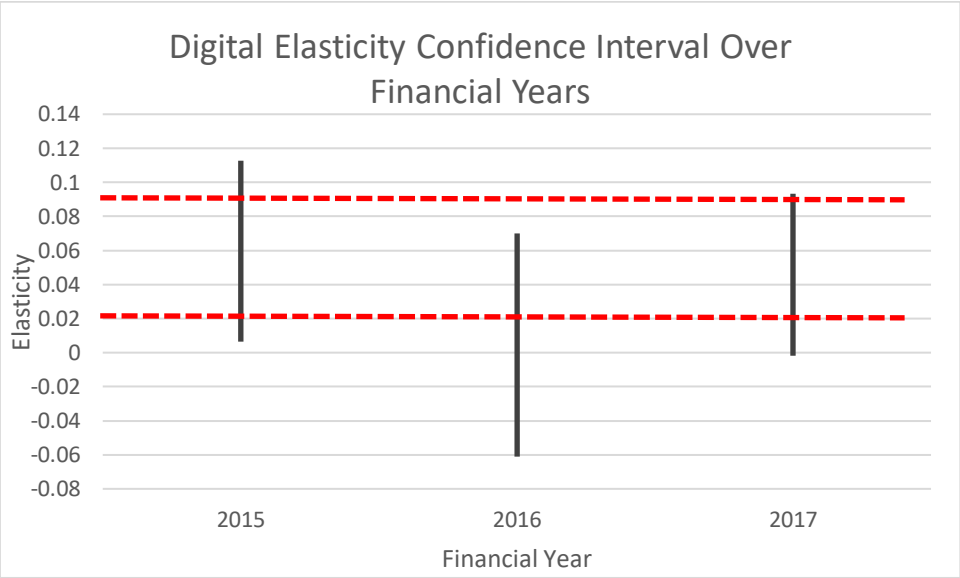
The Financial Year confidence intervals overlapping with the full period confidence interval do not provide evidence that TV elasticity is changing over time.

Figure 8 – Radio Elasticity Confidence Interval Over Time (Dashed Lines = Full Period Elasticity Confidence Interval)



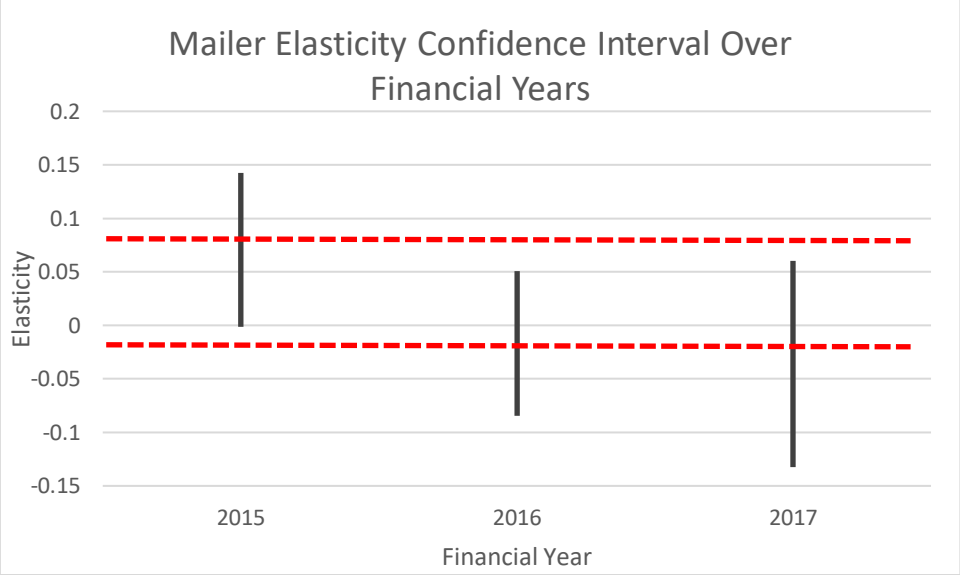
Radio elasticities appear to be constant over time with all confidence intervals overlapping.

Figure 9 – Digital Elasticity Confidence Interval Over Time (Dashed Lines = Full Period Elasticity Confidence Interval)



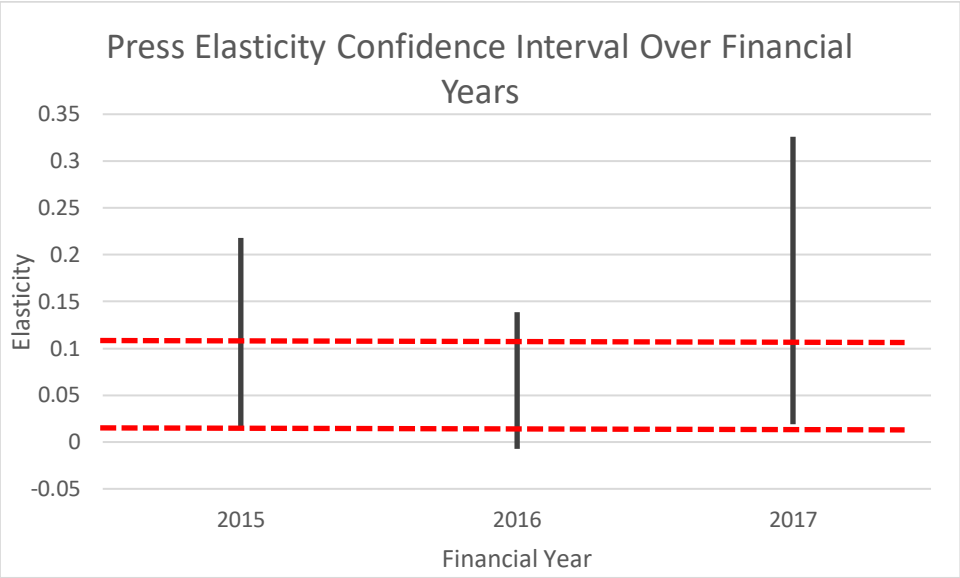
Digital elasticities appear to be constant over time with all intervals overlapping

Figure 10 – Mailer Elasticity Confidence Interval Over Time (Dashed Lines = Full Period Elasticity Confidence Interval)



While the point estimate for mailers suggests the effectiveness could be decreasing over time, the confidence intervals all overlap suggesting that the elasticity is constant over the period.

Figure 11 – Press Elasticity Confidence Interval Over Time (Dashed Lines = Full Period Elasticity Confidence Interval)



All intervals are overlapping suggesting elasticities are constant over time.

There results of this analysis suggest there is no evidence that advertising elasticities are changing over time and therefore the optimal budget allocations derived previously are suitable for The Warehouse going forward and require no adjustments.

Other Considerations - Possibility to Increase Advertising Budget

Another consideration for The Warehouse is to consider whether they should be increasing their overall advertising budget.

Using the interaction model developed previously, Table 8 below indicates the expected sales margin and marginal return on increased advertising expenditure. Increasing the advertising budget by 10% is expected to generate a c500% return on investment while an increase in advertising spend from +90% to +100% increase would generate marginal returns of c300%. The table shows the diminishing returns achieved by increased advertising spending.

Table 8 – Marginal Return on Increase Advertising Expenditure

Advertising Increase	Advertising Budget	Sales	Sales Margin	Marginal Return in Advertising Expenditure[8]
0%	$8,272	$2,440,962	$244,096	
10.0%	$9,099	$2,493,011	$249,301	529%
20.0%	$9,926	$2,541,499	$254,150	486%
30.0%	$10,754	$2,586,938	$258,694	449%
40.0%	$11,581	$2,629,734	$262,973	417%
50.0%	$12,408	$2,670,213	$267,021	389%
60.0%	$13,235	$2,708,643	$270,864	365%
70.0%	$14,062	$2,745,248	$274,525	343%
80.0%	$14,890	$2,780,213	$278,021	323%
90.0%	$15,717	$2,813,698	$281,370	305%
100.0%	$16,544	$2,845,838	$284,584	289%

It should be noted that advertising levels of 15-17k/week are only seen in late November and December so caution would need to be taken in using these modelled results. Advertising levels around the 10k/week mark are common though suggesting it would be reasonable to expect these results if adopted.

Marginal returns of 400%-500% through increased advertising expenditure of around +30% would likely compare favourably to other investment opportunities for The Warehouse and it is recommended that The Warehouse look to increase its marketing budget.

[8] Defined as being (change in sales margin from previous level less change in advertising budget from previous level) / (change in advertising budget from previous level)

Appendix A – Regression Output of Revised Model

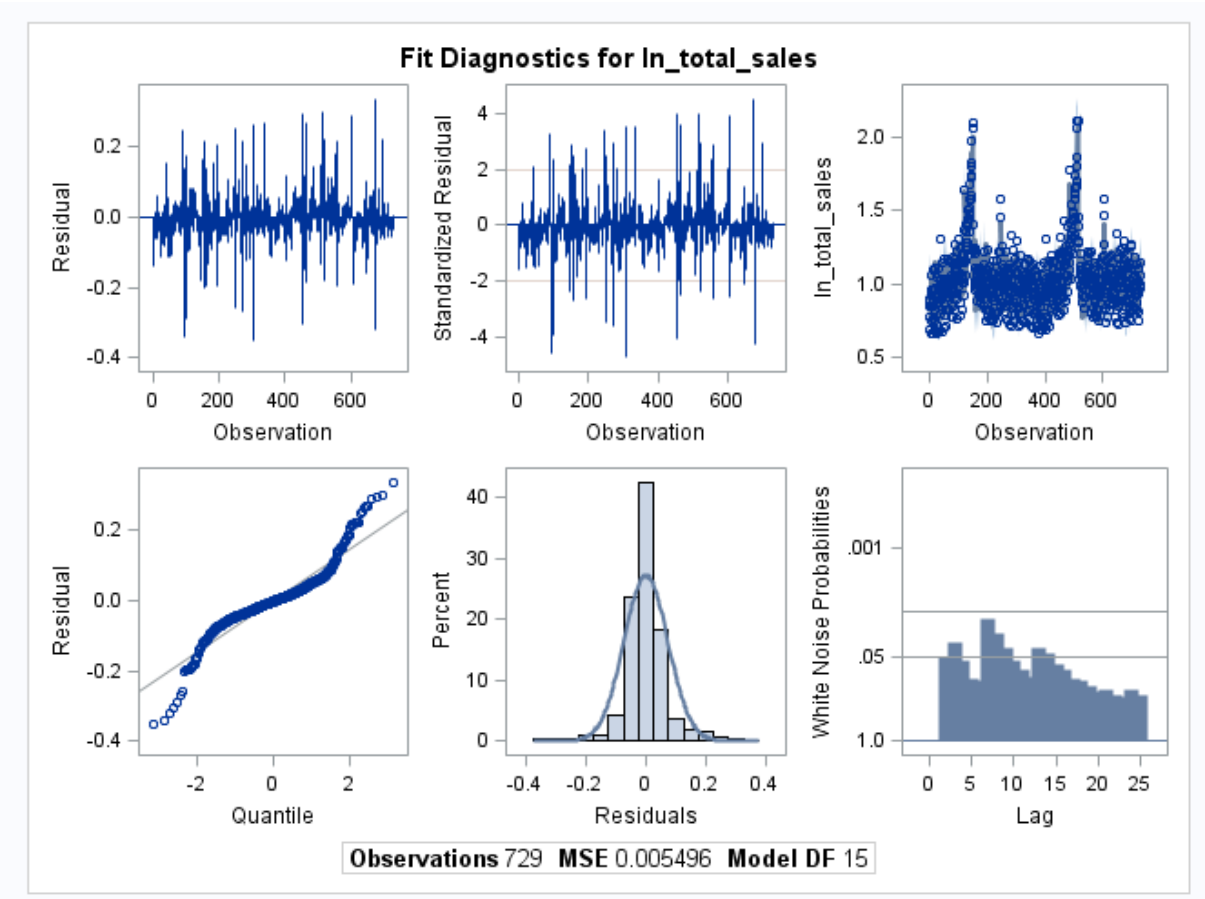
Yule-Walker results were adopted as Ordinary Least Squares showed signs of positive auto-correlation.

Yule Walker Estimates

Yule-Walker Estimates			
SSE	3.91898723	DFE	713
MSE	0.00550	Root MSE	0.07414
SBC	-1635.0043	AIC	-1708.4711
MAE	0.04746576	AICC	-1707.707
MAPE	4.70289057	HQC	-1680.1252
Durbin-Watson	2.0543	Transformed Regression R-Square	0.7681
		Total R-Square	0.8870

Durbin-Watson Statistics	
Order	DW
1	2.0543

Parameter Estimates					
Variable	DF	Estimate	Standard Error	t Value	Approx Pr > |t|
Intercept	1	-1.1249	0.2397	-4.69	<.0001
television_adstock	1	0.0492	0.0199	2.48	0.0135
radio_adstock	1	0.007203	0.006871	1.05	0.2949
digital_adstock	1	0.0733	0.0174	4.21	<.0001
mailer_adstock	1	0.0335	0.0265	1.27	0.2063
press_adstock	1	0.1078	0.0270	3.99	<.0001
DayInDec	1	0.0309	0.001973	15.68	<.0001
Sunday	1	0.2249	0.008232	27.33	<.0001
Tuesday	1	0.0467	0.008173	5.71	<.0001
Wednesday	1	0.1427	0.0100	14.21	<.0001
Thursday	1	0.2103	0.0107	19.59	<.0001
Friday	1	0.1846	0.0108	17.03	<.0001
Saturday	1	0.3440	0.0100	34.34	<.0001
BoxingDay	1	0.6938	0.0508	13.65	<.0001
BlackFriday	1	0.5239	0.0465	11.26	<.0001


Fit Diagnostics for ln_total_sales
Residual
Observation
Standardized Residual
ln_total_sales
Quantile
Percent
Residuals
White Noise Probabilities
Lag
Observations 729 MSE 0.005496 Model DF 15

Appendix B – Calculation of Baseline Sales Using Revised Model

Variable	Parameter Estimate(a)	Input - Advertising Spend (b)	Input - Indicators & Derived Adstock (c)	Calculation Contribution (a x c)
Intercept	-1.1249			-1.125
television_adstock	0.0492	2476	7.81	0.384
radio_adstock	0.0072	767	6.64	0.048
digital_adstock	0.0733	1363	7.22	0.529
mailer_adstock	0.0335	2233	7.71	0.258
press_adstock	0.1078	1433	7.27	0.784
DayInDec	0.0309		0	0.000
Sunday	0.2249		0	0.000
Tuesday	0.0467		0	0.000
Wednesday	0.1427		0	0.000
Thursday	0.2103		0	0.000
Friday	0.1846		0	0.000
Saturday	0.3440		0	0.000
BoxingDay	0.6938		0	0.000
BlackFriday	0.5239		0	0.000
			Total (log)	0.878
			exp Total	2.407

Appendix C – Multiplicative Model with Interaction Effects

Yule-Walker Estimates			
SSE	3.7711168	DFE	703
MSE	0.00536	Root MSE	0.07324
SBC	-1597.2005	AIC	-1716.584
MAE	0.04669117	AICC	-1714.584
MAPE	4.6342044	HQC	-1670.522
Durbin-Watson	2.0215	Transformed Regression R-Square	0.7870
		Total R-Square	0.8913

Durbin-Watson Statistics	
Order	DW
1	2.0215

Parameter Estimates					
Variable	DF	Estimate	Standard Error	t Value	Approx Pr > |t|
Intercept	1	-0.7613	0.2352	-3.24	0.0013
television_adstock	1	0.0491	0.0199	2.47	0.0137
radio_adstock	1	0.0253	0.009254	2.73	0.0065
digital_adstock	1	0.0566	0.0175	3.23	0.0013
mailer_adstock	1	0.0299	0.0247	1.21	0.2277
press_adstock	1	0.0606	0.0276	2.20	0.0283
DayInDec	1	0.0333	0.002666	12.49	<.0001
Sunday	1	0.2253	0.008299	27.15	<.0001
Tuesday	1	0.0464	0.008238	5.63	<.0001
Wednesday	1	0.1440	0.0101	14.27	<.0001
Thursday	1	0.2117	0.0107	19.82	<.0001
Friday	1	0.1861	0.0107	17.31	<.0001
Saturday	1	0.3452	0.009979	34.59	<.0001
BoxingDay	1	0.6968	0.0531	13.11	<.0001
BlackFriday	1	0.5172	0.0471	10.98	<.0001
interact_TV_Radio_adstock	1	0.0443	0.0231	1.92	0.0556
interact_TV_Digital_adstock	1	0.0185	0.0467	0.40	0.6924
interact_TV_Mailer_adstock	1	-0.1519	0.0736	-2.06	0.0394
interact_TV_Press_adstock	1	0.1679	0.0607	2.76	0.0059
interact_Radio_Digital_adstock1	1	0.0120	0.0248	0.48	0.6281
interact_Radio_Mailer_adstock	1	-0.009905	0.0255	-0.39	0.6981
interact_Radio_Press_adstock	1	-0.007283	0.0442	-0.16	0.8690
interact_Digital_Mailer_adstock1	1	0.0321	0.0647	0.50	0.6203
interact_Digital_Press_adstock	1	0.1439	0.0810	1.78	0.0763
interact_Mailer_Press_adstock	1	-0.2559	0.0963	-2.66	0.0081

Appendix D – Store Level Elasticities and Confidence Intervals

Television

Store	Estimate	Standard Error	t-value	Pr > |t|		Lower Bound	Estimate	Upper Bound
1	0.0049	0.0231	0.21	0.8319		-0.0404	0.0049	0.0294
2	0.0804	0.0218	3.69	0.0002		0.0377	0.0804	0.1035
3	0.0618	0.0256	2.41	0.016		0.0116	0.0618	0.0889
4	0.0747	0.0205	3.64	0.0003		0.0345	0.0747	0.0964
5	0.0374	0.0208	1.8	0.0724		-0.0034	0.0374	0.0594
6	0.1104	0.0221	4.99	<.0001		0.0671	0.1104	0.1338
7	0.0933	0.0226	4.13	<.0001		0.0490	0.0933	0.1173
8	0.0117	0.0158	0.74	0.4577		-0.0193	0.0117	0.0284
9	0.0449	0.0217	2.07	0.0392		0.0024	0.0449	0.0679
10	0.0366	0.0259	1.41	0.158		-0.0142	0.0366	0.0641
11	0.0421	0.0171	2.46	0.014		0.0086	0.0421	0.0602
12	-0.0393	0.0280	-1.4	0.1615		-0.0942	-0.0393	-0.0096
13	0.0421	0.0188	2.24	0.0253		0.0053	0.0421	0.0620
14	0.0646	0.0219	2.94	0.0034		0.0217	0.0646	0.0878
15	0.0213	0.0233	0.92	0.3596		-0.0244	0.0213	0.0460
16	0.0571	0.0232	2.46	0.0142		0.0116	0.0571	0.0817
17	0.0526	0.0227	2.32	0.0206		0.0081	0.0526	0.0767
18	0.0478	0.0216	2.21	0.0274		0.0055	0.0478	0.0707
19	-0.0120	0.0206	-0.58	0.5609		-0.0524	-0.0120	0.0098

Radio

Store	Estimate	Standard Error	t-value	Pr > |t|		Lower Bound	Estimate	Upper Bound
1	0.0303	0.0110	2.74	0.0063		0.0087	0.0303	0.0420
2	0.0326	0.0105	3.1	0.002		0.0120	0.0326	0.0437
3	0.0300	0.0117	2.56	0.0106		0.0071	0.0300	0.0424
4	0.0265	0.0098	2.69	0.0074		0.0072	0.0265	0.0369
5	0.0106	0.0096	1.11	0.268		-0.0082	0.0106	0.0208
6	0.0258	0.0106	2.43	0.0155		0.0050	0.0258	0.0370
7	0.0261	0.0108	2.43	0.0155		0.0049	0.0261	0.0375
8	0.0220	0.0078	2.83	0.0048		0.0067	0.0220	0.0303
9	0.0254	0.0102	2.49	0.013		0.0054	0.0254	0.0362
10	0.0381	0.0123	3.09	0.0021		0.0140	0.0381	0.0511
11	0.0221	0.0082	2.68	0.0075		0.0060	0.0221	0.0308
12	0.0081	0.0132	0.61	0.5416		-0.0178	0.0081	0.0221
13	0.0303	0.0091	3.35	0.0009		0.0126	0.0303	0.0399
14	0.0254	0.0104	2.46	0.0143		0.0050	0.0254	0.0364
15	0.0262	0.0108	2.44	0.0151		0.0050	0.0262	0.0376
16	0.0216	0.0109	1.98	0.0478		0.0002	0.0216	0.0332
17	0.0200	0.0104	1.93	0.0542		-0.0004	0.0200	0.0310

18	0.0339	0.0106	3.21	0.0014		0.0131	0.0339	0.0451
19	0.0352	0.0100	3.52	0.0005		0.0156	0.0352	0.0458

Digital

Store	Estimate	Standard Error	t-value	Pr > |t|		Lower Bound	Estimate	Upper Bound
1	0.0792	0.0204	3.88	0.0001		0.0392	0.0792	0.1008
2	0.0508	0.0193	2.64	0.0086		0.0130	0.0508	0.0713
3	0.0727	0.0225	3.23	0.0013		0.0286	0.0727	0.0966
4	0.0240	0.0181	1.32	0.187		-0.0115	0.0240	0.0432
5	0.0568	0.0183	3.11	0.002		0.0209	0.0568	0.0762
6	0.0316	0.0195	1.62	0.1059		-0.0066	0.0316	0.0523
7	0.0386	0.0199	1.94	0.0532		-0.0004	0.0386	0.0597
8	0.0545	0.0140	3.9	0.0001		0.0271	0.0545	0.0693
9	0.0412	0.0191	2.15	0.0317		0.0038	0.0412	0.0614
10	0.0752	0.0229	3.29	0.0011		0.0303	0.0752	0.0995
11	0.0527	0.0151	3.49	0.0005		0.0231	0.0527	0.0687
12	0.1188	0.0247	4.81	<.0001		0.0704	0.1188	0.1450
13	0.0437	0.0166	2.63	0.0086		0.0112	0.0437	0.0613
14	0.0375	0.0194	1.94	0.0533		-0.0005	0.0375	0.0581
15	0.0686	0.0205	3.35	0.0008		0.0284	0.0686	0.0903
16	0.0577	0.0205	2.82	0.0049		0.0175	0.0577	0.0794
17	0.0521	0.0199	2.62	0.0091		0.0131	0.0521	0.0732
18	0.0540	0.0191	2.82	0.0049		0.0166	0.0540	0.0742
19	0.0763	0.0182	4.18	<.0001		0.0406	0.0763	0.0956

Mailer

Store	Estimate	Standard Error	t-value	Pr > |t|		Lower Bound	Estimate	Upper Bound
1	0.0239	0.0290	0.82	0.4107		-0.0329	0.0239	0.0546
2	0.0487	0.0275	1.77	0.0768		-0.0052	0.0487	0.0779
3	-0.0521	0.0316	-1.65	0.0998		-0.1140	-0.0521	-0.0186
4	0.0217	0.0258	0.84	0.4009		-0.0289	0.0217	0.0490
5	0.0027	0.0258	0.11	0.9159		-0.0478	0.0027	0.0301
6	0.0377	0.0278	1.35	0.176		-0.0168	0.0377	0.0672
7	0.0207	0.0283	0.73	0.4661		-0.0348	0.0207	0.0507
8	0.0110	0.0200	0.55	0.5834		-0.0282	0.0110	0.0322
9	-0.0003	0.0271	-0.01	0.9922		-0.0534	-0.0003	0.0285
10	0.0602	0.0325	1.85	0.0644		-0.0035	0.0602	0.0947
11	0.0141	0.0215	0.65	0.5134		-0.0280	0.0141	0.0369
12	-0.0269	0.0350	-0.77	0.4429		-0.0955	-0.0269	0.0102
13	0.0005	0.0237	0.02	0.9846		-0.0460	0.0005	0.0256
14	-0.0286	0.0275	-1.04	0.2973		-0.0825	-0.0286	0.0006
15	0.0541	0.0289	1.87	0.0612		-0.0025	0.0541	0.0847

16	0.0549	0.0290	1.89	0.0586		-0.0019	0.0549	0.0856
17	0.0832	0.0280	2.97	0.0031		0.0283	0.0832	0.1129
18	0.0115	0.0274	0.42	0.6734		-0.0422	0.0115	0.0405
19	0.0188	0.0261	0.72	0.4709		-0.0324	0.0188	0.0465

Press

Store	Estimate	Standard Error	t-value	Pr > |t|		Lower Bound	Estimate	Upper Bound
1	0.0635	0.0323	1.97	0.0495		0.0002	0.0635	0.0977
2	0.0632	0.0305	2.07	0.0385		0.0034	0.0632	0.0955
3	0.0640	0.0353	1.81	0.0705		-0.0052	0.0640	0.1014
4	0.0522	0.0287	1.82	0.0693		-0.0041	0.0522	0.0826
5	0.0750	0.0288	2.61	0.0093		0.0186	0.0750	0.1055
6	0.0304	0.0309	0.98	0.3265		-0.0302	0.0304	0.0632
7	0.0834	0.0315	2.65	0.0083		0.0217	0.0834	0.1168
8	0.0811	0.0222	3.65	0.0003		0.0376	0.0811	0.1046
9	0.0603	0.0302	2	0.0461		0.0011	0.0603	0.0923
10	0.0564	0.0361	1.56	0.1189		-0.0144	0.0564	0.0947
11	0.0409	0.0239	1.71	0.0878		-0.0059	0.0409	0.0662
12	0.1148	0.0390	2.94	0.0033		0.0384	0.1148	0.1561
13	0.0754	0.0263	2.87	0.0043		0.0239	0.0754	0.1033
14	0.0617	0.0306	2.02	0.0439		0.0017	0.0617	0.0941
15	0.0656	0.0322	2.04	0.0421		0.0025	0.0656	0.0997
16	0.0298	0.0323	0.93	0.3552		-0.0335	0.0298	0.0640
17	0.0629	0.0313	2.01	0.0449		0.0016	0.0629	0.0961
18	0.0520	0.0304	1.71	0.0872		-0.0076	0.0520	0.0842
19	0.0851	0.0289	2.94	0.0034		0.0285	0.0851	0.1157

Appendix E – Regression Models for Financial Years

FinancialYear=2015

Yule-Walker Estimates			
SSE	1.95951924	DFE	342
MSE	0.00573	Root MSE	0.07569
SBC	-728.46875	AIC	-830.0789
MAE	0.04820868	AICC	-825.9616
MAPE	4.80025787	HQC	-789.71019
Durbin-Watson	2.0133	Transformed Regression R-Square	0.7924
		Total R-Square	0.8853

Durbin-Watson Statistics	
Order	DW
1	2.0133

Parameter Estimates					
Variable	DF	Estimate	Standard Error	t Value	Approx Pr > |t|
Intercept	1	-1.4493	0.4427	-3.27	0.0012
television_adstock	1	0.0354	0.0298	1.19	0.2346
radio_adstock	1	0.0310	0.0147	2.11	0.0354
digital_adstock	1	0.0596	0.0271	2.20	0.0285
mailer_adstock	1	0.0705	0.0367	1.92	0.0554
press_adstock	1	0.1175	0.0514	2.29	0.0228
DayInDec	1	0.0336	0.004036	8.32	<.0001
Sunday	1	0.2308	0.0124	18.61	<.0001
Tuesday	1	0.0507	0.0123	4.13	<.0001
Wednesday	1	0.1538	0.0149	10.35	<.0001
Thursday	1	0.2102	0.0156	13.49	<.0001
Friday	1	0.1987	0.0156	12.71	<.0001
Saturday	1	0.3480	0.0146	23.82	<.0001
BoxingDay	1	0.7806	0.0796	9.80	<.0001
BlackFriday	1	0.4687	0.0708	6.62	<.0001
interact_TV_Radio_adstock	1	0.0595	0.0435	1.37	0.1717
interact_TV_Digital_adstock	1	-0.0214	0.0770	-0.28	0.7809
interact_TV_Mailer_adstock	1	-0.1728	0.1317	-1.31	0.1903
interact_TV_Press_adstock	1	-0.1687	0.1519	-1.11	0.2675
interact_Radio_Digital_adstock1	1	0.0338	0.0375	0.90	0.3684
interact_Radio_Mailer_adstock	1	-0.008600	0.0330	-0.26	0.7943
interact_Radio_Press_adstock	1	-0.005004	0.1021	-0.05	0.9609
interact_Digital_Mailer_adstock1	1	0.0492	0.0822	0.60	0.5499
interact_Digital_Press_adstock	1	0.0977	0.1379	0.71	0.4794
interact_Mailer_Press_adstock	1	0.3587	0.2574	1.39	0.1644

FinancialYear=2016

Yule-Walker Estimates			
SSE	1.62492738	DFE	335
MSE	0.00485	Root MSE	0.06965
SBC	-772.78342	AIC	-873.89425
MAE	0.04358376	AICC	-869.69065
MAPE	4.28794512	HQC	-833.69484
Durbin-Watson	2.0044	Transformed Regression R-Square	0.8185
		Total R-Square	0.9072

Durbin-Watson Statistics	
Order	DW
1	2.0044

Parameter Estimates					
Variable	DF	Estimate	Standard Error	t Value	Approx Pr > |t|
Intercept	1	-0.4293	0.3627	-1.18	0.2374
television_adstock	1	0.0800	0.0367	2.18	0.0299
radio_adstock	1	0.0467	0.0181	2.57	0.0105
digital_adstock	1	0.004395	0.0334	0.13	0.8953
mailer_adstock	1	-0.0171	0.0345	-0.50	0.6205
press_adstock	1	0.0658	0.0373	1.76	0.0785
DayInDec	1	0.0311	0.004311	7.21	<.0001
Sunday	1	0.2192	0.0113	19.40	<.0001
Tuesday	1	0.0416	0.0112	3.72	0.0002
Wednesday	1	0.1358	0.0136	9.96	<.0001
Thursday	1	0.2139	0.0144	14.82	<.0001
Friday	1	0.1732	0.0145	11.90	<.0001
Saturday	1	0.3453	0.0136	25.44	<.0001
BoxingDay	1	0.5562	0.0766	7.26	<.0001
BlackFriday	1	0.5688	0.0637	8.92	<.0001
interact_TV_Radio_adstock	1	0.0142	0.0496	0.29	0.7742
interact_TV_Digital_adstock	1	0.0145	0.0774	0.19	0.8514
interact_TV_Mailer_adstock	1	-0.2661	0.1216	-2.19	0.0293
interact_TV_Press_adstock	1	0.2227	0.1036	2.15	0.0324
interact_Radio_Digital_adstock1	1	-0.009132	0.0414	-0.22	0.8256
interact_Radio_Mailer_adstock	1	0.0168	0.0726	0.23	0.8174
interact_Radio_Press_adstock	1	0.0489	0.0593	0.82	0.4104
interact_Digital_Mailer_adstock1	1	0.2868	0.1541	1.86	0.0637
interact_Digital_Press_adstock	1	-0.006866	0.1451	-0.05	0.9623
interact_Mailer_Press_adstock	1	-0.2939	0.1199	-2.45	0.0147

FinancialYear=2017

Yule-Walker Estimates			
SSE	0.8123985	DFE	157
MSE	0.00517	Root MSE	0.07193
SBC	-336.50336	AIC	-419.95
MAE	0.04421048	AICC	-410.95
MAPE	4.16718238	HQC	-386.12497
Durbin-Watson	1.8901	Transformed Regression R-Square	0.9030
		Total R-Square	0.9385

Durbin-Watson Statistics	
Order	DW
1	1.8901

Parameter Estimates					
Variable	DF	Estimate	Standard Error	t Value	Approx Pr > |t|
Intercept	1	-1.2426	0.2817	-4.41	<.0001
television_adstock	1	0.0859	0.0778	1.10	0.2716
radio_adstock	1	0.0210	0.0181	1.16	0.2483
digital_adstock	1	0.0457	0.0243	1.88	0.0622
mailer_adstock	1	-0.0362	0.0491	-0.74	0.4618
press_adstock	1	0.1725	0.0783	2.20	0.0291
DayInDec	1	0.0180	0.003246	5.54	<.0001
Sunday	1	0.1940	0.0179	10.82	<.0001
Tuesday	1	0.0531	0.0177	3.01	0.0031
Wednesday	1	0.1468	0.0207	7.07	<.0001
Thursday	1	0.2010	0.0210	9.58	<.0001
Friday	1	0.1905	0.0209	9.11	<.0001
Saturday	1	0.3363	0.0201	16.75	<.0001
BoxingDay	1	0.7361	0.0851	8.65	<.0001
BlackFriday	1	0.6881	0.0716	9.61	<.0001
interact_TV_Radio_adstock	1	-0.0470	0.0994	-0.47	0.6372
interact_TV_Digital_adstock	1	0.2061	0.1372	1.50	0.1351
interact_TV_Mailer_adstock	1	-0.2574	0.2055	-1.25	0.2122
interact_TV_Press_adstock	1	0.1689	0.1974	0.86	0.3935
interact_Radio_Digital_adstock1	1	0.0286	0.0407	0.70	0.4826
interact_Radio_Mailer_adstock	1	-0.1037	0.1050	-0.99	0.3249
interact_Radio_Press_adstock	1	0.1194	0.1561	0.77	0.4454
interact_Digital_Mailer_adstock1	1	-0.0266	0.1374	-0.19	0.8469
interact_Digital_Press_adstock	1	-0.1352	0.1889	-0.72	0.4753
interact_Mailer_Press_adstock	1	0.1695	0.3011	0.56	0.5742