LAPLACE TRANSFORM AND ITS APPLICATION

A SEMINAR 2 PRESENTATION

BY

OLUJOBI, ADEOLA ESTHER

(MATRIC NUMBER: 20172595)

DEPARTMENT OF MATHEMATICS

COLLEGE OF PHYSICAL SCIENCES

IN PARTIAL FULFILMENT OF THE REQUIREMENTS

FOR THE AWARD OF BACHELOR OF SCIENCE DEGREE

IN MATHEMATICS,

FEDERAL UNIVERSITY OF AGRICULTURE, ABEOKUTA.

SUPERVISOR: DR. E. ILOJIDE

FEBRUARY 2023

CERTIFICATION

This is to certify that this report was undertaken and submitted by **OLUJOBI ADEOLA ESTHER** with matriculation number **20172595**, a student of the department of Mathematics, College of Physical Sciences, Federal University of Agriculture, Abeokuta, for SEMINAR 2.

....

DR. E. ILOJIDE **Date**

(Supervisor)

....

DR. E.O. ADELEKE **Date**

(Head of Department)

Table of Contents

Chapter 1

INTRODUCTION

A differential equation is an equation of a function that relates the values of the function to the values of its derivatives.

An Ordinary Differential Equation (ODE) is a differential equation for a function of a single variable, e.g., $x(t)$ while a Partial Differential Equation (PDE) is a differential equation for a function of several variable, e.g $v(x, y, z, t)$.

An Ordinary differential equation contains ordinary derivatives and a partial differential equation contains partial derivatives, differential equations be it ordinary differential equations or Partial differential equations. Find great applications in many fields of study as most real-life physical problems are involved considering changes in variables with respect to time or location. An Equation involving derivatives of one or more dependent variables with respect to one or more independent variables is called a differential equation.

This study shall therefore be aimed at considering the Laplace transform as one way of solving differential equation,and the Laplace transform to be used shall depend on the type of equation and the possibility of reducing the differential or integral expression to a more simpler expression.

The differential equations can however be considered as initial value problem or boundary value problem depending on the underlying information available about the differential equations. Though at times, physical problems come in integral equation form.

- Its an equation in which the unknown function or the vector function appears under the sign of the derivative.
- It can also be an equation of that relates in a non-trivial way an unknown function and one or more of the derivatives or differentials of an unknown function with respect to one or more independent variables.
- A differential equation is an equation containing the derivatives of one or more dependent variables, with respect to one or more independent variables.

1.1 MOTIVATION OF STUDY

Various work has been carried out on the methods of solving Laplace transform to Ordinary Differential Equation (ODE) and Partial Differential Equations (PDE). What motivated this study is to show how the Laplace Transformation can be applied to various Electric circuits and mechanics equations, the Laplace transform is derived from Lerch's Cancellation Law. In the Laplace transform method, the function in the time domain is transformed to a Laplace function in the frequency domain. This Laplace function will be in the form of an algebraic equation and it can be solved easily. The solution can be again transformed back, transform is most commonly used for control systems, as briefly mentioned above. The transforms are used to study and analyze systems such as ventilation, heating and air conditions etc. These systems are used in every single modern-day construction and building.

1.2 STATEMENT OF PROBLEM

(i) Laplace transformation in mathematics deals with the conversion of complex function to a simpler function.

(ii) The Laplace transform method finds its application in those problems which can't be solved directly.

(iii) Laplace transforms can only be used to solve complex differential equations and like other methods, it does have a disadvantage that is the method can only be used to solve differential equations with known constants.

1.3 OBJECTIVES OF THE STUDY

This Project work is aimed at studying the concept of Laplace Transform. This project is also aimed at examining the Applications of Laplace Transform to Ordinary differential equation. My Objectives are to:

(i) Review and study the methods of solving LAPLACE TRANSFORMATION.

(ii) Review some properties of Laplace Transform.

(iii) Show and study the practical applications of the Laplace transformation.

Chapter 2

LITERATURE REVIEW

2.1 REVIEW OF LAPLACE TRANSFORM

The theory of Laplace transforms or Laplace transformation, also referred to as operational calculus, has in recent years become an essential part of the mathematical background required of engineers, physicists, mathematicians and other scientists. This is because, in addition to being of great theoretical interest in itself, Laplace transform methods provide easy and effective means for the solution of many problems arising in various fields of science and engineering. The subject originated in many attempts to justify rigorously certain 'operational rules' used by Heaviside in the latter part of the 19th century for solving equations in electromagnetic theory. These attempts finally proved successful in the early part of the 20th century through the efforts of BROMWICH, CARSON, VAN DER POL and other mathematicians who employed complex variable theory. Integral transformations have been successfully used for almost two centuries in solving many problems in applied mathematics, mathematical physics, and engineering science. Historically, the origin of integral transformations including the Laplace and Fourier transforms can be traced back to celebrated work of P.S Laplace (1749-1827) on probability theory in the 1780s and to monumental treatise of Joseph Fourier (1768-1830) on La Theorie Analytique de la chaleur published in 1822. In fact, Laplace classic book on La Theorie Analytique des probabilities includes some basic results of the Laplace transform which is one of the oldest and most commonly used integral transformation available in the mathematical literature. This has effectively been used in finding the solution of linear differential equations and integral equations. On the other hand, Fourier's treatise provided the modern mathematical theory of heat conduction, Fourier series, and fourier integrals with applications. In his treatise, Fourier stated a remarkable result that is universally known as the Fourier Integral Theorem. He gave a series of examples before stating that an arbitrary function defined on a finite interval can be expanded in terms of trigonometric series which is now universally known as the Fourier series. In an attempt to extend his new ideas to functions defined on an infinite interval, Fourier discovered an integral transform and its inversion formula which are now well known as the Fourier transform and the inverse Fourier transform. However, this celebrated idea of Fourier was known to Laplace and A.L Cauchy (1789-1857) as some of their earlier work involved this transformation. On the other hand, S.D Poisson (1781-1840) also independently used the method of transform in his research on the propagation of water waves. However, it was G.W Leibniz (1646-1716) who first introduced the idea of a symbolic method in calculus. Subsequently, both J.L Lagrange (1736-1813) and Laplace made considerable contribution to symbolic methods which became known as operational calculus. Although both the Laplace and the Fourier transforms have been discovered in the nineteenth century, it was the British electrical engineer Oliver Heaviside (1850-1925) who made the Laplace transform very popular by using it to solve ordinary differential equations of electrical circuits and systems, and then to develop modern operational calculus. It may be relevant to point out that the Laplace transform is essentially a special case of the Fourier transform for a class of functions defined on the positive real axis, but it is simpler than the Fourier transform for the following reasons. First, the questions of convergence of the Laplace transform is much less delicate because of its exponentially decaying kernel. Secondly, the Laplace transform is an analytic function of the complex variable and its properties can easily be studied with the knowledge of the theory of complex variable. Third, the Fourier integral formula provided the definitions the Laplace transform and the inverse Laplace transforms in terms of a complex contour integral that can be evaluated with the help of the Cauchy residue theory and deformation of contour in a complex plane. The complete history of the Laplace transforms can also be tracked a little more to the past, more specifically 1744. This is when another great mathematician called Leonard Euler was researching on other types of integral. Euler however did not pursue it very far and left it, an admirer of Euler called Joseph Lagrange, made some modifications to Euler's work and did further work. Lagrange's work got Laplace's attention 38 years later, in 1782 where he continued to pick up where Euler left off. But it was not 3 years later, in 1785 where Laplace had a stroke of genius and changed the way we solve differential equation forever. He continued to work on it and continued to unlock the true power of the Laplace transform until 1809, where he started to use infinity as an integral condition.

Chapter 3

DISCUSSION

3.1 SOME METHODS OF SOLVING LAPLACE TRANSFORMATION

There are various means of solving laplace transforms, but four (4) methods of solving laplace transformation is being discussed in this project work. They include

(1) **The Direct Method:** Let $f(t)$ be a function of t specified for $t > 0$. Then the Laplace transforms of $f(t)$ denoted by $L\{f(t)\}$ is defined by:

$$L\{f(t)\} = F(s) = \int_0^\infty \mathrm{e}^{-st} f(t)\ dt$$

where we assume at present that the parameter is real, Later it will be found useful to consider s complex. The Laplace transform of $f(t)$ is said to exist. For sufficient conditions under which the Laplace transform does exist.

(2) **Series Method:** The method involves the use of power series expansion in deriving various laplace transform for a given function. $F(t)$ has a power series expansion given by;

$$F(t) = a_0 + a_1 t + a_2 t^2 + \cdots = \sum_{n=0}^{\infty} a_n t^n$$

Its Laplace transform can be obtained by taking the sum of the laplace transforms of each terms in the series. Thus

$$L\{F(t)\} = \frac{a_0}{s} + \frac{a_1}{s^2} + \frac{2!a_2}{s^3} + \cdots = \sum_{n=0}^{\infty} \frac{n!a_n}{s^{n+1}}$$

A condition under which the result is valid is that the series above is convergent for $s > \gamma$

(3) **Method of Solving differential Equation:** This involves finding a differential equation satisfied by $F(t)$ and then using the above theorems in getting the transform. Making use if the differential method in solving the function $J_0(t)$ The function $J_0(t)$ satisfies the differential equation if:

$$tJ"_0(t) + J'_0 + tJ_0(t) = 0$$

from the modified Bessel function of differential equation which states that if $J_n(t)$ satisfies Bessel differential equation.

$$t^2Y"(t) + tY'(t) + (t^2 - n^2)Y(t) = 0$$

with $n = 0$ taking the Laplace transform of both with $J_0(0) = 1$, $J_0'(0) = 0$, $y = LJ_0(t)$ we have

$$\frac{d}{ds}s^2 \cdot y - s(1) - 0 + sy - 1 - \frac{dy}{ds} = 0$$

from which

$$\frac{dy}{ds} = \frac{-sy}{s^2+1}$$

Thus

$$\frac{dy}{y} = \frac{-sds}{s^2+1}$$

and by Integration

$$y = \frac{c}{\sqrt{s^2+1}}$$

Now

$$\lim_{s\to\infty} Sy(c) = \frac{cs}{\sqrt{s^2+1}} = c$$

and $\lim_{t\to 0} J_0(t) = 1$ Thus by initial value theorem, we have $c = 1$ and so

$$L\{J_0(t)\} = \frac{1}{\sqrt{s^2+1}}$$

(4) **The use of Tables method:** This method involves the use of Laplace table in carrying out the transformation of various differential equations.

	f(t)	$G_\alpha(f)$
1	1	$u^{\alpha+1}$
1	t	$u^{\alpha+2}$
3	t^n	$n! \cdot u^{n+\alpha+1}$
4	e^{at}	$\frac{u^{\alpha+1}}{1-au}$
5	$\sin at$	$\frac{au^{\alpha+2}}{1+u^2a^2}$
6	$\cos at$	$\frac{u^{\alpha+1}}{1+u^2a^2}$
7	$\sinh at$	$\frac{au^{\alpha+2}}{1-u^2a^2}$
8	$\cosh at$	$\frac{u^{\alpha+1}}{1-u^2a^2}$
9	$e^{at}\cos bt$	$\frac{u^{\alpha}(\frac{1}{u}-a)}{(\frac{1}{u}-a)^2+b^2}$
10	$e^{at}\sin bt$	$\frac{bu^{\alpha}}{(\frac{1}{u}-a)^2+b^2}$

Figure 3.1: **Table of Laplace**

3.2 SOLUTION TO DIRECT METHOD OF LAPLACE TRANSFORM

Let $f(t)$ be a function; $t > 0$

$$Lf(t) = \int_0^\infty \mathrm{e}^{-st} f(t)\ dt$$

The integral is evaluated with respect to t hence once the limits are substituted, what is left are in terms of s. Example: Find the Laplace transform of the constant function

$$f(t) = 1, 0 \leq t < \infty$$

$$L\{1\} = \int_0^\infty \mathrm{e}^{-st}.1dt = \int_0^\infty \mathrm{e}^{-st}dt$$

$$[-\frac{1}{s}\mathrm{e}^{-st}] = -\frac{1}{s}[\mathrm{e}^{-s(\infty)} - \mathrm{e}^{-s(0)}]$$

$$= -\frac{1}{s}[\mathrm{e}^{-\infty} - \mathrm{e}^{0}] = -\frac{1}{s}[0 - 1] = \frac{1}{s}$$

$$L\{1\} = \frac{1}{s}$$

Also,

$$L\{a\} = \mathrm{e}^{-st}.adt = \frac{a}{s}$$

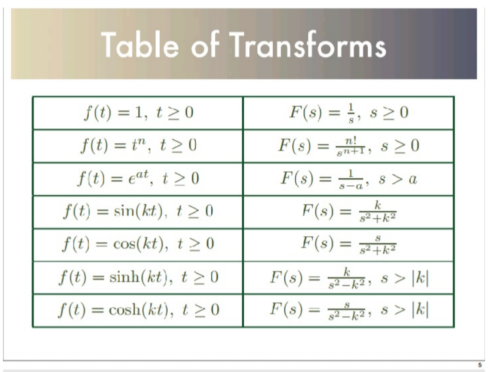
Table of Transforms

$f(t) = 1,\ t \geq 0$	$F(s) = \frac{1}{s},\ s \geq 0$
$f(t) = t^n,\ t \geq 0$	$F(s) = \frac{n!}{s^{n+1}},\ s \geq 0$
$f(t) = e^{at},\ t \geq 0$	$F(s) = \frac{1}{s-a},\ s > a$
$f(t) = \sin(kt),\ t \geq 0$	$F(s) = \frac{k}{s^2+k^2}$
$f(t) = \cos(kt),\ t \geq 0$	$F(s) = \frac{s}{s^2+k^2}$
$f(t) = \sinh(kt),\ t \geq 0$	$F(s) = \frac{k}{s^2-k^2},\ s > \lvert k \rvert$
$f(t) = \cosh(kt),\ t \geq 0$	$F(s) = \frac{s}{s^2-k^2},\ s > \lvert k \rvert$

5

Figure 3.2: **Direct Method Transform Table**

3.3 SOLUTIONS TO SERIES METHOD OF LAPLACE TRANSFORMATION

In this section the procedures or process of using the series method to solve problems is explained in details below:

Example

- Find $L\{J_0(t)\}$ where $J_0(t)$ is the Bessel function of Order zero.
- Use the result of (a) to find $L\{J_0(t)\}$ using the series method, Letting $n = 0$ in the equation of a Bessel function of order n which is given as

$$J_n(t) = \frac{t^n}{2^n\Gamma(n+1)}\{1 - \frac{t^2}{2(2n+2)} + \frac{t^4}{2.4(2n+2)(2n+4)} - \cdots\}$$

setting $n = 0$, we get

$$J_n(t) = 1 - \frac{t^2}{2^2} + \frac{t^4}{2^2 4^2} - \frac{t^6}{2^2 4^2 6^2} + \cdots$$

Then

$$L\{J_0\} = \frac{1}{s} + \frac{1}{2^2}\frac{2!}{s^3} + \frac{1}{2^2 4^2}\frac{4!}{s^5} - \frac{1}{2^2 4^2 6^2} \cdots$$

$$= \frac{1}{s}\{1 - \frac{1}{2}(\frac{1}{s^2} + \frac{1}{2}.\frac{3}{4}(\frac{1}{s^4})) - \frac{1.3.5}{2.4.6}(\frac{1}{s^6}) + \cdots\}$$

$$= \frac{1}{s}\{(1 + \frac{1}{s^2})^- \frac{1}{2}\}$$

$$= \frac{1}{\sqrt{s^2+1}}$$

Example Find $L\{sin\sqrt{t}\}$ using the series

$$sin\sqrt{t} = \sqrt{t} - \frac{(\sqrt{t})^3}{3!} + \frac{(\sqrt{t})^5}{5!} - \frac{(\sqrt{t})^7}{7!} + \cdots$$

$$= t^{\frac{1}{2}} - \frac{t^{\frac{3}{2}}}{3!} + \frac{t^{\frac{5}{2}}}{5!} - \frac{t^{\frac{7}{2}}}{7!} + \cdots$$

Then the Laplace transform is

$$L\{sin\sqrt{t}\} = \frac{\Gamma(\frac{3}{2})}{S^{\frac{3}{2}}} - \frac{\Gamma(\frac{5}{2})}{S^{\frac{5}{2}}3!} + \frac{\Gamma(\frac{7}{2})}{S^{\frac{7}{2}}5!} + \cdots$$

$$= \frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}\{1 - (\frac{1}{2^2 s}) + \frac{(\frac{1}{2^2 s})^2}{2!} - \frac{(\frac{1}{2^2 s})^3}{3!} + \cdots$$

$$= \frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}\mathrm{e}^{-\frac{1}{2s}} = \frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}\mathrm{e}^{-\frac{1}{4s}}$$

3.4 SOLUTIONS TO SYSTEMS OF DIFFERENTIAL EQUATIONS BY LAPLACE TRANSFORM

The Function $J - 0(t)$ satisfies the differential equation if;

$$tJ"_0(t) + J'_0 + tJ_0(t) = 0$$

From the modified Bessel function of differential equation which states that if $J_n(t)$ satisfies Bessel differential equation.

$$t^2Y"(t) + tY'(t) + (t^2 - n^2)Y(t) = 0$$

with $n = 0$ taking the Laplace transform of both with $J_0(0) = 1, J_0'(0) = 0, y = L\{J_0(t)\}$, we have

$$\frac{d}{ds}s^2y - s(1) - 0 + sy - 1 - \frac{dy}{ds} = 0$$

from which

$$\frac{dy}{ds} = \frac{-sy}{s^2 + 1}$$

Thus

$$\frac{dy}{y} = \frac{-sds}{s^2 + 1}$$

and by integration

$$y = \frac{c}{\sqrt{s^2 + 1}}$$

Now $\lim\limits_{s \to \infty} Sy(c) = \frac{cs}{\sqrt{s^2+1}} = c$ and $\lim\limits_{t \to 0} J_0(t) = 1$. Thus by the initial-value theorem we have $c = 1$ and so $L\{J_0(t)\} = \frac{1}{\sqrt{s^2+1}}$.

Example 2:

Derive the Laplace transform of the function

$$4tY'' + 2Y' + Y = 0$$

Taking the Laplace transform, we have if $y = L\{Y(t)\}$

$$-4\frac{d}{ds}(s^2y - sY(0) - Y'(0) + 2\{sy - Y(0)\} + y) = 0$$

$$4s^2y' + (6s - 1)y = 0$$

$$y = \frac{c}{s^{\frac{3}{2}}}\mathrm{e}^{-\frac{1}{4s}}$$

3.5 SOLUTION OF SYSTEMS OF LINEAR FIRST ORDER EQUATIONS BY THE LAPLACE TRANSFORM

The Laplace transform can be used to solve initial value problems for systems of linear first order differential equations by introducing the Laplace transform of each dependent variable that is involved, Solving the resulting algebraic equations for each transformed dependent variable, and then inverting the results. As a System of linear higher Order differential equations can always be reduced to a system of first Order equation by Introducing higher order derivatives as new dependent variables. The Solution of a system of linear first order equations can be considered to be the most general case. The Example that follows involving two simultaneous first Order equations, equations illustrate the approach to be used in all cases, but by restricting the number of equations and using simple non homogeneous terms (forcing functions) the algebra is kept to a minimum.
Example: Solve the initial value problem

$$x' - 2x + y = sint$$

$$y' + 2x - y = 1$$

, with $x(0) = 1, y(0) = 1$. Solution: We define the transforms of the dependent variable $x(t)$ and $y(t)$ to be

$$L\{x(t)\} = X(s), L\{y(t)\} = Y(s).$$

Transforming the system of equations in the usual way leads to the following system of linear algebraic equations for $X(s)$ and $Y(s)$:

$$sX(s) - 1 - 2X(s) + Y(s) = \frac{1}{s^2 + 1}$$

$$sY(s) + 1 + 2X(s) - Y(s) = \frac{1}{s}$$

Solving these for $X(s)$ and $Y(s)$ gives:

$$X(s) = \frac{(s-1)(s^3+s^2+2s+1)}{s^2(s-3)(s^2+1)}$$

Expressing these results in terms of partial fractions, we find that

$$X(s) = \frac{4}{9}\frac{1}{s} + \frac{1}{3}\frac{1}{s^2} - \frac{1}{5}\frac{1}{s^2+1} - \frac{2}{5}\frac{s}{s^2+1} + \frac{43}{45}\frac{1}{s-3}$$

and

$$Y(s) = \frac{5}{9}\frac{1}{s} + \frac{2}{3}\frac{1}{s^2} - \frac{1}{5}\frac{1}{s^2+1} - \frac{3}{5}\frac{s}{s^2+1} + \frac{43}{45}$$

Finally, taking the inverse transform gives the solution

$$x(t) = \frac{4}{9} + \frac{1}{3t} - \frac{1}{5}sint - \frac{2}{5}cost + \frac{43}{45}\mathrm{e}^{3t}$$

and

$$y(t) = \frac{5}{9} + \frac{2}{3t} - \frac{1}{5}sint - \frac{3}{5}cost + \frac{43}{45}\mathrm{e}^{3t}$$

for $t > 0$. This method can be used for any number of simultaneous linear differential equations, through the complexity of both the algebraic manipulation and the associated inversion problem increase rapidly when more than two equations are involved.

Chapter 4

SOME APPLICATIONS

4.1 Electric Circuits

The Laplace transformation method can be applied in various fields such as Physical sciences and Engineering but in this project work, we shall be considering it's applications to Electrical circuits and Mechanics. A simple electrical circuit [Fig 4.2] consists of the following circuit elements connected in series with a switch or key K:

(i) A generator or battery, supplying an electromotive force or e.m.f E(volts)

(ii) A resistor having resistance R (Ohms)

(iii) An inductor having inductance L(henrys)

(iv) A capacitor having capacitance C(farads) These circuit elements are represented symbolically as in figure 4.2

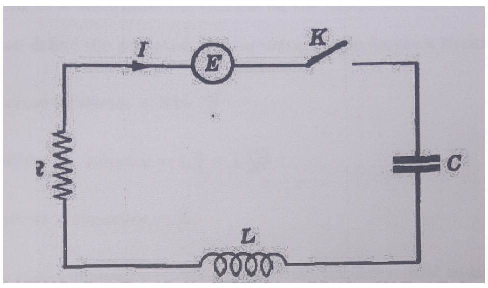

Figure 4.1: **Fig 4.2**

When the switch or key K is closed, so that the circuits completed, a charge Q(coulombs) will flow to the capacitor plates. The time rate of flow of charge given by $\frac{dQ}{dT} = I$, is called the current and is measured in amperes () when time t is measured in seconds. More complex electrical circuits, as shown for example in fig 4.3, can occur in practice.

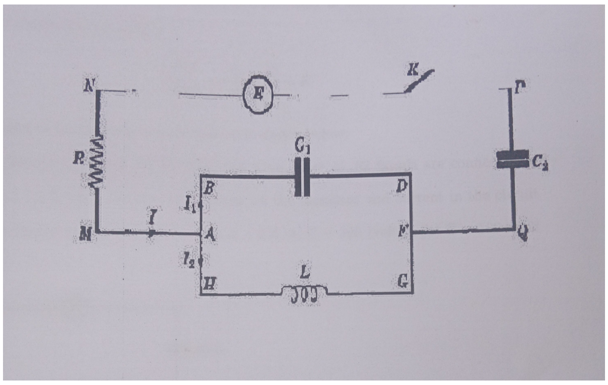

Figure 4.2: **Fig 4.3**

An important problem is to determine the charges on the capacitors and currents as functions of time. To do this we define the potential drop or voltage drop across a circuit element.

(i) Voltage drop across a resistor $=RI=\frac{dQ}{dt}$

(ii) Voltage drop across an inductor $=L\frac{dl}{dt}=L\frac{d^2Q}{dt^2}$

(iii) Voltage drop across a capacitor $=\frac{Q}{C}$

(iv) Voltage drop across a generator = -Voltage rise = -E, the differential equation can bed found by using the Kirchhoff's law:

1. The Kirchhoff law states that the algebraic sum of the currents flowing towards any junction point [for example A in Fig 4.3 is equal to zero].

2. The algebraic sum of the potential drops, or voltage drops, around any closed loop [such as ABDFGHA or ABDFQPNMA in Fig 4.3] is equal to zero.

3. For the simple circuit of Fig 4.2 application of these laws is particularly easy [the first law is actually not necessary in this case]. We find that the equation for

determination of Q is

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = E$$

A typical application of the Laplace transformation is shown below: An inductor of 2 henrys, a resistor of 16 ohms and a capacitor of 0.02 farads are connected in series with an e.m.f of E volts. At $t = 0$ the charge on the capacitor and current in the circuit are zero. Find the charge and current at time $t > 0$ if (a) E = 300 (volts), (b) E = 100sin3t (volts).

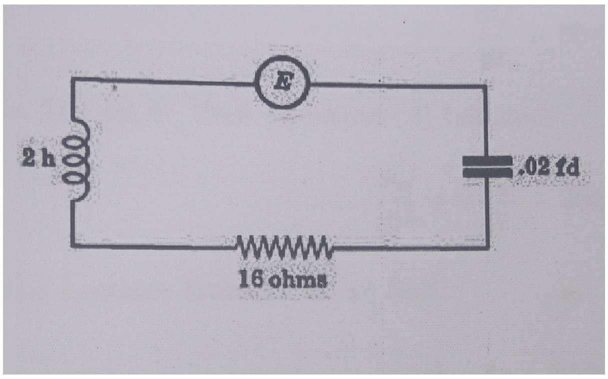

Figure 4.3: **Fig 4.4**

Let Q and I be the instantaneous charge and current respectively at time t. By Kirchhoff's laws, we have

$$2\frac{dI}{dt} + 16I + \frac{Q}{0.02} = E$$

Since $I = \frac{dQ}{dt}$

$$2\frac{d^2Q}{dt^2} + 16\frac{dQ}{dt} + 50Q = E$$

With the initial conditions $Q(0) = 0, I(0) = Q'(0) = 0$. (a)If $E = 300$, then equation (2) becomes

$$\frac{d^2Q}{dt^2} + 8\frac{dQ}{dt} + 25Q = 150$$

Transforming to Laplace we have

$$s^2q - sQ(0) - Q'(0) + 8sq - Q(0) + 25q = \frac{150}{s}$$

$$q = \frac{150}{s(s^2 + 8s + 25)} = \frac{6}{s} - \frac{6s + 48}{s^2 + 8s + 25}$$

$$\frac{6}{s} - \frac{6(s+4)+24}{(s+4)^2+9}$$

$$\frac{6}{s} - \frac{6(s+4)}{(s+4)^2+9} - \frac{24}{(s+4)^2+9}$$

$$Q = 6 - 6\mathrm{e}^{-4t} cos3t - 8\mathrm{e}^{-4t} sin3t$$

$$I = \frac{dQ}{dt} = 50\mathrm{e}^{-4t} sin3t$$

(b)If $E = 100sin3t$, then equation 2 becomes

$$\frac{dQ}{dt^2} + 8\frac{dQ}{dt} + 25Q = 50sin3t$$

Taking the Laplace transform, we find

$$(s^2 + 8s + 25)q = \frac{150}{s^2+9}$$

and

$$q = \frac{150}{(s^2+9)(s^2+8s+25)}$$

$$= \frac{75}{26}\frac{1}{s^2+9} - \frac{75}{52}\frac{s}{s^2+9} + \frac{75}{26}\frac{1}{(s+4)^2+9} + \frac{75}{52}\frac{s+4}{(s+4)^2+9}$$

$$Q = \frac{25}{26} sin3t - \frac{75}{52} sin3t + \frac{25}{26}\mathrm{e}^{-4t} sin3t + \frac{75}{52}\mathrm{e}^{-4t} cos3t$$

$$\frac{25}{52}(2sin3t - 3cos3t) + \frac{25}{52}\mathrm{e}^{-4t}(3cos3t + 2sin3t)$$

and

$$I = \frac{dQ}{dt} = \frac{75}{52}(2cos3t + 3sin3t) - \frac{25}{52}\mathrm{e}^{-4t}(17sin3t + 6cos3t)$$

For large t, those terms of Q or I which involves e^{-4t} are negligible and these are called the transient terms or transient part of the solution, the other terms or steady-state part of the solution.

Chapter 5

Conclusion and Recommendation

5.1 CONCLUSION

The Laplace transform is an important integral transform with many applications in Mathematics, Physics and Engineering. It has been discovered that Electronic circuits are governed by linear polynomial differential equations with constant coefficients. This is the same problem that Laplace transform solves with ease as the speed of light. In Addition to these, this transform is very attractive in solving differential equations and therefore play an important Role in computational problems. i.e. Solution and Analysis of Time invariant systems such as Mechanical system, Electric circuits, Optical devices and Harmonic Oscillators.

5.2 RECOMMENDATIONS

Mathematics and Engineering Students should be well exposed in the study of Laplace transformation and its applications to various fields be it differential equations and physical sciences, and in Engineering which has a wide variety of choice. The Laplace transformation is applicable in Mathematics, Physics, Engineering and other Areas of Science.

References

[1] Arendt W. Batty (2002), *Vector-Valued Laplace Transforms and Cauchy Problems, Birkhäuser Basel*, ISBN 978-3-7643-6549-3.

[2] Davies, Brian (2002), *Integral transforms and their applications (Third ed.), New York: Springer*, ISBN 978-0-387-95314-4

[3] Deakin, M. A. B. (1981), *The development of the Laplace transform", Archive for History of Exact Sciences*, 25 (4): 343–390, doi:10.1007/BF01395660, S2CID 117913073

[4] Deakin, M. A. B. (1982), *The development of the Laplace transform, Archive for History of Exact Sciences*, 26 (4): 351–381, doi:10.1007/BF00418754, S2CID 123071842

[5] Dodson C.I.J. (2002), *Introduction to Laplace Transforms for Engineers, School of Mathematics, The University of Manchester, London, England .*

[6] Murray R. Spiegel (1965), *Schaum outlines on Laplace transforms*, byvMcGraw-Hill publishers.

[7] Zalman Rubinstein clark (2017), *A course in Ordinary and practical differential equations www.gen.lib.rus.ecla course in ordinary and practical differential equations.*