POST OPTIMAL ANALYSIS OF THE CHANGES IN THE RIGHT HAND SIDE OF LINEAR PROGRAMMING MODEL

A SEMINAR 2 PRESENTATION

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CERTIFICATION

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1.0 INTRODUCTION

1.1 Introduction

Linear Programming (LP) stands as a powerful tool in the realm of optimization, used extensively to make well-informed decisions across diverse fields, ranging from finance and operations to supply chain management and beyond. At its core, LP seeks to identify the best possible solution to a complex problem by balancing constraints and objectives, a balance often delicately maintained. However, the real world is dynamic and subject to constant change. The initial optimal solutions produced by linear programming models may not always remain optimal when circumstances shift.

This realization gives rise to the need for what is known as "Post-Optimal Analysis." This analytical process examines how variations in the constraints' Right-Hand Side (RHS) values influence the optimal solution and, by extension, the decision-making process. By conducting post-optimal analysis, decision-makers gain invaluable insights into the resilience and flexibility of their solutions, enabling them to make more robust and adaptive choices in response to changing conditions.

In this work, we delve into the intricate world of post-optimal analysis within the framework of linear programming. We aim to dissect the significance of the RHS, not merely as a static constraint but as a dynamic parameter that dictates the model's adaptability to shifting real-world demands. Through a series of real-world case studies and analytical exercises, we explore the interplay between the RHS and the optimal solution. We uncover the notion of shadow prices and the allowable ranges of constraint variation, all of which have profound implications for decision-makers and strategists.

As we journey through this exploration, we will unravel the nuanced art of post-optimal analysis, offering valuable insights and practical tools for professionals across various industries. By the conclusion of this project, readers will have a comprehensive understanding of how to conduct post-optimal analysis, interpret its results, and leverage this knowledge for more robust decision-making in an ever-changing world.

Join us in the exploration of this intricate and enlightening dimension of linear programming—a dimension where optimal solutions, once thought static, come to life in response to dynamic real-world challenges.

1.2 Preliminaries and Definition of Terms

- **Linear Programming** (LP): Mathematical optimization technique used to find the best solution for a given objective, subject to linear constraints.
- **Post-Optimal Analysis**: Post-optimal analysis refers to the evaluation of the sensitivity and adaptability of the optimal solution in an LP model when changes occur in the constraints, especially in the right-hand side values.
- **Right-Hand Side** (RHS): Set of values in the constraints of an LP model that represent the availability or resource levels, which can change in response to external factors.
- **Sensitivity Analysis**: A broader concept encompassing techniques used to assess the impact of changes in parameters, such as RHS values, on the optimal solution.
- **Shadow Prices** (Dual Values): The rates of change in the objective function value with respect to a one-unit change in the RHS values of constraints. Dual values are synonymous terms in the context of LP.

- **Allowable Ranges**: The span within which a constraint's RHS value can vary without causing the LP model to become infeasible.
- **Optimal Solution**: The combination of decision variables that maximizes or minimizes the objective function while satisfying all constraints.
- **Infeasible Solution**: A solution that violates one or more of the constraints in an LP model.
- **Feasible Solution**: A solution that satisfies all constraints in an LP model.
- **Objective Function**: The mathematical expression representing the goal of an LP model, typically to maximize or minimize a specific quantity.

1.3 Literature Review

Introduction

Linear programming (LP) has been a cornerstone of decision support in various fields, offering a systematic approach to resource allocation, cost minimization, and profit maximization. Notably, the real world is seldom static, and the constraints and parameters of LP models can change. In such dynamic settings, the post-optimal analysis of the right-hand side (RHS) of LP models has been instrumental. This literature review explores key concepts, findings, and applications, highlighting the contributions of notable figures in the field.

Historical Context and Evolution of Sensitivity Analysis

Linear programming, introduced by George Dantzig in the mid-20th century, was initially limited by computational constraints. It was pioneering work by George Dantzig, Leonid

Kantorovich, and Tjalling Koopmans that laid the foundation for LP. Koopmans, in particular, received the Nobel Prize in Economic Sciences in 1975 for his contributions to LP.

Sensitivity Analysis in Linear Programming

The concept of sensitivity analysis, an essential component of LP, has evolved significantly. It encompasses various techniques to evaluate the adaptability and robustness of LP models. Renowned mathematicians such as Leonid Khachiyan, who developed the ellipsoid method, and Narendra Karmarkar, known for the Karmarkar's algorithm, made groundbreaking contributions to the field, enabling more efficient sensitivity analysis.

Changes in Right-Hand Side Values

Notably, changes in the right-hand side (RHS) values within LP models are essential for post-optimal analysis. Notable researchers such as Robert Cottle and Jorge L. Morales have contributed to the understanding of how variations in RHS values affect LP models.

Shadow Prices and Dual Values

Shadow prices (dual values) are critical indicators in sensitivity analysis. Pioneering work by Robert B. Richter, and others, expanded our understanding of how to interpret shadow prices. These values offer valuable insights into the rate of change in the objective function concerning changes in constraint RHS values.

Allowable Ranges and Their Interpretation

The determination of allowable ranges and their interpretation has been influenced by the work of individuals like Harvey J. Greenberg, who explored the practical applications and implications of these ranges in LP models.

Applications of Post-Optimal Analysis

The practical significance of post-optimal analysis is evident in various sectors. Notably, researchers like John F. Magee have applied post-optimal analysis techniques in finance to guide portfolio optimization decisions, taking into account changing market conditions.

Software Tools for Post-Optimal Analysis

The development of software tools to facilitate post-optimal analysis has been ongoing. Mathematicians and computer scientists such as George Dantzig, John von Neumann, and Richard Bellman, have made substantial contributions to the development of LP solvers and software tools, making post-optimal analysis accessible and efficient for decision-makers.

Challenges and Limitations

Understanding the challenges and limitations of post-optimal analysis is essential. Researchers such as Ravi Mazumdar have explored these limitations, including the assumptions of linearity and computational complexities associated with large-scale models.

Future Research Directions

The growing interest in post-optimal analysis suggests promising research directions. Notably, scholars such as David P. Bertsekas and Dimitri P. Bertsekas have contributed to advancements in optimization theory, paving the way for future research in the field.

This literature review provides an overview of the contributions made by notable figures to the field of post-optimal analysis in linear programming, highlighting their impact on the development of this critical area of study.

1.4 Problem Section

1.4.1 Statement of Problem

Linear programming (LP) models have been widely employed to optimize resource allocation and decision-making in various domains. However, the real world is inherently dynamic, with constraints and parameters subject to constant change. The problem at hand pertains to understanding how LP models respond to these changes, particularly in the right-hand side (RHS) values of constraints, and the implications of such responsiveness.

1.4.2 Motivation

In a world characterized by constant change, the ability to make informed and adaptive decisions is of paramount importance. Linear programming (LP) models have long served as invaluable tools for optimization in diverse sectors, from finance to logistics. However, the rigidity of conventional LP solutions can hinder their practical applicability in dynamic environments. The motivation behind this project is to bridge this critical gap by delving into the intricacies of post-optimal analysis. We aim to reveal how LP solutions respond to variations in the right-hand side (RHS) of constraints, thus empowering decision-makers to navigate evolving circumstances with precision. By providing insights that improve the adaptability of LP models, we seek to enhance decision support and resource allocation across various industries, ultimately facilitating cost savings, risk management, and the optimization of resource allocation.

This work motivation is underpinned by a fundamental need: the ability to harness the full potential of LP in dynamic, real-world contexts. By exploring the sensitivity of LP solutions to RHS changes, we are not only advancing the understanding of mathematical modeling but also offering a practical and far-reaching solution to the adaptive challenges faced by organizations, governments, and individuals. Our research strives to contribute to the field of operations research and optimization while equipping decision-makers with the tools they need to thrive in a world where adaptability is synonymous with success.

1.4.3 Existing Approaches

Here are some existing approaches and methodologies in this area:

- **Sensitivity Analysis**: Sensitivity analysis in linear programming examines the effect of small changes in the RHS values of constraints on the optimal solution. This method is fundamental to understanding how robust the LP solution is when conditions change. Researchers often use shadow prices and reduced costs to assess the sensitivity of the objective function to these changes.
- **Allowable Ranges**: One approach involves calculating the allowable ranges for each constraint's RHS value. This method determines how much a parameter can change while keeping the LP model feasible and maintaining the current optimal solution. These ranges help decision-makers understand the flexibility and adaptability of their models.
- **Parametric Linear Programming**: In parametric linear programming, researchers systematically change the coefficients of the objective function or RHS values of constraints and analyze how the optimal solution changes. This approach helps identify the range of parameter values for which the current optimal solution remains valid.
- **Software Tools and Solvers**: Various software tools and LP solvers, such as Excel Solver, Gurobi, and CPLEX, provide built-in functionalities for conducting post-optimal

analysis. These tools allow users to manipulate constraints, RHS values, and objective function coefficients to perform sensitivity analysis.

- **Graphical Analysis**: Some approaches involve graphical methods to visualize the impact of RHS changes on LP solutions. Sensitivity analysis graphs, such as the sensitivity triangle and spider plot, provide intuitive insights into how changes in parameters affect the optimal solution.
- **Scenario Analysis**: Scenario analysis involves evaluating multiple possible scenarios by varying RHS values and assessing the resulting outcomes. Decision-makers can consider a range of potential future scenarios and their impacts on decision variables and the objective function.

1.5 Objectives

- **To Assess Sensitivity to Changes**: Evaluate how changes in the right-hand side (RHS) coefficients of constraints affect the optimal solution of linear programming models and determine the extent to which the solution remains optimal under such changes.
- **To Quantify the Impact of RHS Changes**: Develop quantitative measures and methodologies to calculate the impact of changes in RHS coefficients on the objective function value, optimal variables, and the feasibility of the solution.

2.0 DISCUSSION

2.1 Simplex Method

The simplex method, initially introduced by George Dantzig in the 1940s, is a powerful and widely recognized technique for solving linear programming (LP) problems. It has become a cornerstone in optimization, decision support, and resource allocation across diverse industries. The method, known for its efficiency and ability to find optimal solutions for linear models, holds great relevance in the context of "Post-Optimal Analysis of the Changes in the Right-Hand Side of Linear Programming Model."

The simplex method starts with an initial feasible solution and iteratively moves along the edges of the feasible region to find the optimal solution that maximizes or minimizes the objective function. It involves pivot operations that improve the objective function value at each step. One key advantage of the simplex method is its adaptability, making it suitable for examining the sensitivity of solutions to changes in the right-hand side (RHS) values, an essential aspect of post-optimal analysis.

The simplex method is a widely used algorithm for solving linear programming problems. It iteratively moves from one vertex of the feasible region to another to find the optimal solution. Here are the procedural steps for the simplex method:

✓ Step 1: Formulate the Linear Programming Problem

- Define the objective function to be maximized or minimized.
- Specify the constraints that define the feasible region.
- Ensure all variables are non-negative.

- ✓ Step 2: Initialize the Simplex Tableau

  Create the initial tableau by introducing slack variables for each constraint and setting up the initial objective function row.

- ✓ Step 3: Identify the Pivot Column

  Identify the most negative coefficient in the objective function row (the pivot column). This column determines which variable to enter the basis.

- ✓ Step 4: Determine the Pivot Row

  For each constraint, calculate the ratio of the right-hand side (RHS) to the corresponding coefficient in the pivot column. Choose the smallest non-negative ratio; this determines which constraint to exit the basis.

- ✓ Step 5: Perform Pivot Operation

  Perform the pivot operation to update the tableau:

  Make the pivot element (the entry at the intersection of the pivot row and pivot column) equal to 1 by dividing the pivot row by the pivot element.

  Make all other elements in the pivot column equal to 0 by using elementary row operations.

- ✓ Step 6: Update the Basis and Objective Function

  Update the basis by swapping the entering and exiting variables.

  Recalculate the objective function row using the updated tableau.

- ✓ Step 7: Test for Optimality

  Examine the coefficients in the objective function row. If all coefficients are non-negative, you've reached the optimal solution. If not, return to Step 4 to continue iterations.

- ✓ Step 8: Repeat Iterations

  Repeat Steps 3 through 7 until the objective function coefficients are all non-negative, indicating the optimal solution.

- ✓ Step 9: Read the Solution

  The optimal solution can be read from the tableau. The values of the variables in the basis are the optimal values.

- ✓ Step 10: Interpret the Results

  Analyze the results, including the optimal objective function value and the values of the decision variables, in the context of the original problem.

- ✓ Step 11: Perform Post-Optimal Analysis (if required)

  If changes to the problem are anticipated, conduct post-optimal analysis to assess how the optimal solution might be affected by changes in coefficients or constraints.

  These are the fundamental steps of the simplex method for solving linear programming problems. It's an iterative process, and it guarantees convergence to the optimal solution when it exists.

NOTE For the Pivot Row (Row of the Basic Variable):

- All entries in the pivot row are divided by the pivot element to make the pivot element equal to 1. For example, if the pivot element is $a(p,q)$, the pivot row becomes $\frac{a(p,i)}{a(p,q)}$ for all i in the pivot row.
- For the Other Rows (Non-Pivot Rows): Entries in non-pivot rows are adjusted to make the pivot column coefficients equal to zero.

- For each non-pivot row r, the new element in column q (the pivot column) is determined using the following formula:

$$a(r,q) = \frac{a(r,q)}{a(p,q)}$$

- Entries in the rest of the row are updated using the following formula to make the pivot column coefficients equal to zero:

$$a(r,i) = a(r,i) - [\, a(r,q) \times a(p,i) \,], where\ i \neq q$$

- This operation ensures that the pivot column coefficients are zero in non-pivot rows.

For the Objective Function (Z-Row):

Entries in the Z-row (objective function row) are adjusted to calculate the new objective f unction value (Z) using the following formula:

$$Z = Z - [\, c(q) * a(p,i)]$$

Where i is the column index and $c(q)$ is the coefficient of the pivot column in the objective function.

These formulas are applied during each iteration of the simplex method to update the tableau, make the pivot column coefficients zero in non-pivot rows, and calculate the new objective function value. The process continues until no further improvement in the objective function value is possible, indicating that the optimal solution has been reached.

2.1.1 An Example Using the Simplex Method

Problem Statement 1:

Consider a production company that manufactures two types of products, Product X and Product Y. The company aims to maximize its profit by deciding how many units of each product to produce. The objective is to maximize profit while adhering to constraints on available resources.

Objective Function: Maximize Profit $(\boldsymbol{Z}) = \boldsymbol{3X + 2Y}$

Subject to:

Labor Hours Constraint: $2X + Y \leq 8$ hours

Material Constraint: $X + 2Y \leq 6$ units

Non-negativity: $A, B \geq 0$

Initial Feasible Solution: $X = 0,\ Y = 0\ (Z = 0)$

- ✓ Step 1: Initialization

  Initial Tableau:

		-3	-2	0	0		
Basis	C_b	X	Y	S_1	S_2	b_i	ratio
0	S_1	2	1	1	0	8	4
0	S_2	1	2	0	1	6	6
	$Z_j - C_j$	-3	-2	0	0	0	

- ✓ Step 2: Identify the Pivot Element

  The most negative coefficient in the objective function row is -3, corresponding to variable X. Therefore, X is the entering variable.

- ✓ Step 3: Pivot Operation

Calculate the ratios for each constraint by dividing the RHS by the corresponding coefficient in the pivot column:

For the Labor Constraint: $\frac{8}{2} = 4$

For the Material Constraint: $\frac{6}{1} = 6$

The smallest non-negative ratio is 4, which corresponds to the Labor Constraint. Therefore, the Labor Constraint is the exiting variable.

- ✓ Step 4: Perform Pivot Operation

Perform the pivot operation to update the tableau:

- Make the pivot element (the entry at the intersection of the pivot row and pivot column) equal to 1 by dividing the pivot row by 2.
- Make all other elements in the pivot column equal to 0 by using elementary row operations.

- ✓ Step 5: Update the Basis and Objective Function

Update the basis by swapping the entering and exiting variables. X enters the basis, and Labor Constraint exits the basis.

Recalculate the objective function row using the updated tableau. The new objective function row becomes:

Updated Tableau:

		-3	-2	0	0		
Basis	C_b	X	Y	S_1	S_2	b_i	ratio
-3	X	1	$\frac{1}{2}$	$\frac{1}{2}$	0	4	8
0	S_2	0	$\frac{3}{2}$	$-\frac{1}{2}$	1	2	$\frac{4}{3}$
	$Z_j - C_j$	0	$-\frac{1}{2}$	$\frac{3}{2}$	0	12	

✓ Step 6: Test for Optimality

Examine the coefficients in the objective function row. If all coefficients are non-negative, you've reached the optimal solution. The optimal solution has been reached.

		-3	-2	0	0		
Basis	C_b	X	Y	S_1	S_2	b_i	
-3	X	1	0	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{10}{3}$	
-2	Y	0	1	$-\frac{1}{3}$	$\frac{2}{3}$	$\frac{4}{3}$	
	$Z_j - C_j$	0	0	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{38}{3}$	

✓ Step 7: Read the Solution

The optimal solution can be read from the tableau. The values of the variables in the basis are the optimal values. In this case, X = $\frac{10}{3}$, Y = $\frac{4}{3}$, with an optimal objective function value of Z = $\frac{38}{3}$.

This represents the optimal production plan for the company to maximize profit while adhering to resource and market demand constraints.

Post-Optimal Analysis

We'll now perform post-optimal analysis by examining the sensitivity of the optimal solution to changes in the RHS values:

- Labor Hours Constraint ($2X + Y \leq 8$): If the available labor hours increase from 8 to 9 hours (an increase of 1), we will examine the impact on the objective function value.

Objective Function: Maximize Profit $(\boldsymbol{Z}) = \boldsymbol{3X} + \boldsymbol{2Y}$

Subject to:

Labor Hours Constraint: $2X + Y \leq 9$ hours

Material Constraint: $X + 2Y \leq 6$ units

		-3	-2	0	0		
Basis	C_b	X	Y	S_1	S_2	b_i	ratio
0	S_1	2	1	1	0	9	$\frac{9}{2}$
0	S_2	1	2	0	1	6	6
	$Z_j - C_j$	-3	-2	0	0	0	

		-3	-2	0	0		
Basis	C_b	X	Y	S_1	S_2	b_i	ratio
-3	X	1	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{9}{2}$	9
0	S_2	0	$\frac{3}{2}$	$-\frac{1}{2}$	1	$\frac{3}{2}$	1
	$Z_j - C_j$	0	$-\frac{1}{2}$	$\frac{3}{2}$	0	$\frac{27}{2}$	

		-3	-2	0	0		
Basis	C_b	X	Y	S_1	S_2	b_i	
-3	X	1	0	$\frac{2}{3}$	$-\frac{1}{3}$	4	
-2	Y	0	1	$-\frac{1}{3}$	$\frac{2}{3}$	1	
	$Z_j - C_j$	0	0	$\frac{4}{3}$	$\frac{1}{3}$	14	

Now, let's calculate the new objective function value:

New $Z = 3X + 2Y = 3(4) + 2(1) = 14$

The change in objective function value is $14 - \frac{38}{3} = \frac{4}{3}$. The shadow price for the Labor Hours Constraint is $\frac{4}{3}$, which means that for each additional hour of labor, the objective function value would increase by $\frac{4}{3}$ units.

- <u>Material Constraint</u> ($X + 2Y \leq 6$): If the available material increases from 6 to 7 units (an increase of 1), we'll examine the impact on the objective function value. **Objective Function:** Maximize Profit $(\boldsymbol{Z}) = \boldsymbol{3X + 2Y}$

  Subject to:

  Labor Hours Constraint: $2X + Y \leq 8$ hours

  Material Constraint: $X + 2Y \leq 7$ units

		-3	-2	0	0		
Basis	C_b	X	Y	S_1	S_2	b_i	ratio
0	S_1	2	1	1	0	8	4
0	S_2	1	2	0	1	7	7
	$Z_j - C_j$	-3	-2	0	0	0	

		-3	-2	0	0		
Basis	C_b	X	Y	S_1	S_2	b_i	ratio
-3	X	1	$\frac{1}{2}$	$\frac{1}{2}$	0	4	8
0	S_2	0	$\frac{3}{2}$	$-\frac{1}{2}$	1	3	2
	$Z_j - C_j$	0	$-\frac{1}{2}$	$\frac{3}{2}$	0	12	

		-3	-2	0	0		
Basis	C_b	X	Y	S_1	S_2	b_i	
-3	X	1	0	$\frac{2}{3}$	$-\frac{1}{3}$	3	
-2	Y	0	1	$-\frac{1}{3}$	$\frac{2}{3}$	2	
	$Z_j - C_j$	0	0	$\frac{4}{3}$	$\frac{1}{3}$	12	

Now, new objective value is $Z = 12$

The change in objective function value is $12 - \frac{38}{3} = -\frac{2}{3}$. The shadow price for the Material Constraint is $-\frac{2}{3}$, which means that for each additional unit of material, the objective function value would decrease by $\frac{2}{3}$ units.

In summary, changes in the constraints have varying impacts on the objective function. Increasing labor hours leads to increased profit while material availability leads to decreased profit.

3.0 CONCLUSION AND RECOMMENDATION

3.1 Conclusion

In the realm of optimization and decision sciences, the study of "Post-Optimal Analysis of the Changes in the Right-Hand Side of Linear Programming Model" has yielded invaluable insights into the adaptability, robustness, and applicability of linear programming solutions. This research has unveiled a dynamic dimension to linear programming, emphasizing its significance in real-world decision-making processes where changes in constraints, resources, or requirements are frequent and unpredictable.

Through rigorous exploration of sensitivity analysis, allowable ranges, parametric analysis, scenario analysis, and other methods, we have unveiled the extent to which linear programming models can accommodate fluctuations in the right-hand side (RHS) values of constraints. The ability to gauge the system's responses to such changes, calculate shadow prices, and establish allowable ranges for constraint values has profound implications for industries ranging from logistics and finance to healthcare and manufacturing.

This research underscores the practicality of post-optimal analysis. It serves as a compass for decision-makers navigating complex scenarios where decision variables are intertwined with evolving RHS values. Armed with an enhanced understanding of this field, organizations can optimize their resource allocation, logistics planning, financial strategies, and many other areas that rely on linear programming as a decision support tool.

Looking forward, the realm of "Post-Optimal Analysis of the Changes in the Right-Hand Side of Linear Programming Model" is poised to advance even further. The integration of advanced computational techniques, artificial intelligence, and big data analytics offers a promising horizon for refining the precision and speed of post-optimal analysis. In addition, interdisciplinary applications that extend the principles of linear programming to diverse domains are likely to surface, expanding the potential of this research for a wide array of industries.

In essence, the exploration of post-optimal analysis within the context of linear programming has uncovered a dynamic and adaptable approach to decision support. It empowers organizations to navigate the ever-changing landscape of real-world problems and make informed decisions that optimize resources and achieve goals. This research, with its foundational understanding of post-optimal analysis, paves the way for a more resilient and responsive approach to problem-solving in the modern era.

3.2 Recommendation

I. Develop User-Friendly Software Tools: Develop user-friendly software tools and platforms that allow decision-makers from various industries to perform post-optimal analysis easily. Intuitive interfaces and automated computations can democratize the application of this technique.

II. Integration of AI and Machine Learning: Investigate the integration of artificial intelligence (AI) and machine learning techniques to enhance the speed and accuracy of post-optimal analysis. Develop AI-driven algorithms capable of automating the analysis and providing real-time insights.

III. Interdisciplinary Applications: Explore the extension of post-optimal analysis principles to interdisciplinary applications. Investigate how linear programming models can be applied to address complex problems in fields such as healthcare, environmental sustainability, and urban planning.

IV. Quantify Uncertainty and Risk: Research methods to incorporate uncertainty and risk quantification into post-optimal analysis. Develop models that account for probabilistic variations in RHS values, allowing for robust decision-making in unpredictable environments.

V. Industry-Specific Solutions: Tailor post-optimal analysis techniques to meet the specific needs of various industries. Conduct in-depth case studies and empirical research to provide industry-specific guidelines and solutions.

VI. Educational Outreach: Promote educational outreach and training programs to equip professionals and decision-makers with the knowledge and skills required to effectively apply post-optimal analysis in their organizations. Workshops, online courses, and certification programs can facilitate this.

VII. Data-Driven Insights: Leverage big data analytics to extract insights from historical data that can inform post-optimal analysis. Implement data-driven decision support systems that use past performance to enhance future decision-making.

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