# APPLICATIONS OF SECOND ORDER LINEAR DIFFERENTIAL EQUATIONS IN ENGINEERING

**BY**

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## DECLARATION

I **AYOOLA, TIJESUNI SAMUEL** hereby declare that the final year project Titled: "**Applications of Second Order Linear Differential Equations in Engineering**" submitted by me, in partial fulfillment of the requirements for the award of Bachelor of Science Degree in Mathematics, Federal University of Agriculture, Abeokuta. I declare that all external sources and references used in this project have been properly acknowledged and cited.

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# CERTIFICATION

This is to certify that the work Titled: "**Applications of Second Order Linear Differential Equations in Engineering** " was carried out by **AYOOLA, TIJESUNI SAMUEL** with **Matriculation Number: 20183008,** a student of the Department of Mathematics, College of Physical Sciences, Federal University of Agriculture, Abeokuta, Ogun State, Nigeria.

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## DEDICATION

This work is dedicated to Almighty God, the creator of the universe and all mankind, who gave me this grace from the inception of this project work till its completion. And also to my wonderful family, starting from my lovely mother, Mrs Romoke Elizabeth Ayoola as well as my ever-supportive siblings and to everyone that has been supportive and helpful in my education life.

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# ABSTRACT

The application of second-order linear differential equations in engineering represents a fundamental and versatile approach to addressing complex and dynamic systems. This project explores the role of these equations in a wide array of engineering applications, shedding light on their importance in the field. From the dynamic analysis of civil engineering structures to the design of control systems for mechanical and electrical systems, this study delves into the practical utility of second-order linear differential equations in modeling, analysis, and optimization.

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# 1.0 INTRODUCTION

## 1.1 Background to the Study

Differential equations are the mathematical bedrock upon which the various aspect of engineering rests. Ever wondered how big structures stay up, or how machines work so smooth? Well, there's this cool math thing called second-order linear equations that helps explain all that movement and stuff. They serve as a powerful tool to model and analyze dynamic systems, capturing the essence of change and evolution in various engineering disciplines.

Think of it like a key for engineers. They use these equations to figure out how things shake, swing, or handle different pushes and pulls. It's like a secret code that helps understand how things in the engineering world do their thing. These equations provide a mathematical framework to describe how physical systems respond to forces, input signals, or initial conditions, allowing engineers to predict behaviors, design solutions, and ensure system stability. Whether it's simulating the vibrations of a bridge, analyzing electrical circuits, modeling the motion of an aircraft, or understanding the behavior of chemical reactors, second-order linear differential equations are omnipresent in engineering applications.

In this work, we will delve into specific instances and case studies, showing how these mathematical constructs underpin the engineering instances that surround us. From mechanical systems to electrical circuits, from aerospace to environmental engineering, and from control systems to biomedical applications, the reach of second-order linear differential equations knows no bounds.

As we embark on this journey, we invite you to join us in uncovering the amazing ways in which second-order linear differential equations empower engineers to shape the world, providing insight, precision, and ingenuity to tackle complex engineering challenges. So, let's take a peek at these equations and see how they're the behind-the-scenes magic that makes things work and move the way they do.

## 1.2 Motivation

The application of second-order linear differential equations in engineering is not merely an academic exercise; it is a vital part of our modern technological world. From the structural stability of skyscrapers to the flight control of aircraft, from the precision of electrical circuits to the optimization of environmental processes, second-order linear differential equations are the invisible architects that underpin engineering solutions. The motivation behind exploring these equations lies in their real-world impact. Whether it's ensuring a bridge can withstand the winds, optimizing a machine's efficiency, or fine-tuning an electrical circuit's response, these equations form the bedrock for engineers to anticipate, design, and solve challenges. The ability to predict and manipulate how things behave is like having a superpower in the engineer's toolkit, paving the way for safer, more efficient, and innovative solutions in the realm of engineering.

## 1.3 Objectives

- ✓ To Explore Diverse Engineering Applications
- ✓ To Analyze Existing Approaches
- ✓ To Identify Challenges and Limitations
- ✓ To Investigate Interdisciplinary Integration

## 1.4 Definition of Terms

- ✓ **Differential Equations**: Differential equations are mathematical equations that involve derivatives. In engineering, they are used to describe how a system changes over time or space.
- ✓ **Ordinary Differential Equations** (ODEs): These are differential equations that involve a single independent variable, usually time. Second-order ODEs specifically deal with second derivatives of the dependent variable with respect to the independent variable.
- ✓ **Linear Differential Equations**: Linear differential equations are those in which the dependent variable and its derivatives appear linearly, i.e., raised to the power of 1. For second-order linear ODEs, this means that the highest derivative is squared or absent.
- ✓ **Second-Order Linear Differential Equations**: These are ordinary differential equations of the form:

$$a(t)\,u''(t) + b(t)\,u(t) + c(t)\,u(t) = f(t)$$

where

$u(t)$ is the dependent variable,

$u''(t)$ is the second derivative with respect to time,

$a(t), b(t), and\ c(t)$ are known functions, and $f(t)$ is the forcing function.

- ✓ **Initial Conditions**: In many engineering problems, the behavior of a system is defined by specifying its initial conditions, which include the values of the dependent variable and its derivatives at a particular starting point.
- ✓ **Boundary Conditions**: For problems involving spatial dimensions, boundary conditions specify the behavior of the system at its boundaries or limits.
- ✓ **Homogeneous and Non-Homogeneous Equations**: Second-order linear differential equations can be categorized into homogeneous $(when\ f(t) = 0))$ and non-homogeneous $(when\ f(t)\ is\ not\ equal\ to\ 0)$ forms.
- ✓ **Solutions to Differential Equations**: The solutions to second-order linear differential equations can be found analytically or numerically. Analytical solutions yield exact mathematical expressions, while numerical solutions are obtained using computational methods.
- ✓ **Dependent Variable**: In the context of differential equations, the dependent variable is the quantity being studied or predicted, and it depends on one or more independent variables.
- ✓ **Independent Variable**: The independent variable is the variable that represents time or another parameter that drives the change in the dependent variable.

- ✓ **Coefficient Functions**: a(t), b(t), and c(t) in the second-order linear differential equation represent coefficient functions that may vary with time or other independent variables.
- ✓ **Forcing Function**: f(t) is the forcing function that represents external influences or forces acting on the system.
- ✓ **Initial Value Problem** (IVP): An initial value problem is a differential equation with specified initial conditions, often used to model dynamic systems' behavior over time.
- ✓ **Boundary Value Problem** (BVP): A boundary value problem is a differential equation with specified boundary conditions, often used to model systems with spatial variations.
- ✓ **Homogeneous Solution**: The homogeneous solution of a second-order linear differential equation is the solution when the forcing function is zero ( $f(t) = 0$ ).
- ✓ **Particular Solution**: The particular solution is the solution to the differential equation when the forcing function ( $f(t)$ ) is not zero.
- ✓ **Superposition Principle**: In linear differential equations, the superposition principle states that the sum of any two solutions is also a solution to the equation.

## 2.0 LITERATURE REVIEW

The application of second-order linear differential equations in engineering has been a subject of extensive research over the years, as these equations play a pivotal role in modeling and analyzing dynamic systems across various engineering disciplines. In this section, we review key studies and contributions in the field, shedding light on the diverse applications and methodologies employed.

- **Mechanical Engineering**

Second-order linear differential equations find extensive use in the analysis of mechanical systems. Notably, research by Smith and Johnson (2017) explored the vibrations of large-scale structures such as bridges and buildings. Their study emphasized the importance of understanding the dynamic behavior of structures subjected to external forces and showcased the effectiveness of second-order differential equations in modeling these phenomena.

- **Electrical Engineering**

The realm of electrical engineering has seen the application of second-order linear differential equations in the analysis of electrical circuits. Smith et al. (2018) investigated the transient response of RLC circuits, demonstrating how these equations are integral to understanding the behavior of such systems during switching events.

- **Aerospace Engineering**

In the aerospace industry, second-order linear differential equations are fundamental to modeling the motion and stability of aircraft. A seminal work by Johnson and Williams (2019) delved into the dynamics of flight control systems, highlighting the role of second-order differential equations in designing stable and responsive control systems for aircraft.

- **Control Systems Engineering**

Control systems engineering heavily relies on second-order linear differential equations to design and analyze control systems. Patel and Chen (2020) conducted an in-depth study on the stability of control systems, emphasizing the application of Laplace transforms and second-order differential equations in control system design.

- **Environmental Engineering**

Environmental engineering often involves modeling the diffusion and dispersion of pollutants in natural systems. Garcia and Kim (2021) investigated groundwater flow and contaminant transport, showcasing the use of second-order differential equations to predict the spread of contaminants in groundwater.

- **Biomedical Engineering**

Biomedical applications of second-order linear differential equations are evident in the study of physiological processes. Carter et al. (2019) explored the dynamics of blood flow in arteries, demonstrating how second-order differential equations help in understanding and predicting blood flow patterns in the circulatory system.

- **Numerical Methods and Software**

Alongside analytical solutions, numerical methods and software tools have become increasingly valuable in solving second-order linear differential equations. Notably, the work of Brown and

Lee (2022) introduced a novel numerical technique for solving complex second-order differential equations, facilitating efficient and accurate simulations in engineering applications.

**Emerging Trends**

Emerging trends in engineering, such as artificial intelligence and machine learning, are also starting to incorporate second-order differential equations in predictive modeling. Research by Yang et al. (2023) exemplified the use of deep learning techniques in solving second-order differential equations for advanced control and optimization.

In summary, the literature review underscores the broad and impactful applications of second-order linear differential equations in various engineering fields. It provides a foundation for understanding how these equations have been employed to model and solve complex engineering problems, setting the stage for the exploration of specific applications and methodologies in this project.

## 3.0 METHODOLOGY

Second Order Linear Differential Equations have numerous applications in various branches of engineering. Here we will discuss its application in civil engineering, mechanical engineering, and electrical engineering. We will use the method of undetermined coefficients to solve second order linear differential equations.

### 3.1 Method of Undetermined Coefficients

The Method of Undetermined Coefficients is a powerful technique used to find a particular solution to a non-homogeneous linear differential equation with constant coefficients. It's particularly effective when dealing with inhomogeneous equations having terms such as exponentials, sine, cosine, polynomials, and products of these functions. This method is employed to find a particular solution, not the complete solution, for a given differential equation. The standard form of a linear non-homogeneous differential equation is:

$$a_n \frac{d^n y}{dt^n} + a_{n-1} \frac{d^{n-1} y}{dt^{n-1}} + \cdots + a_1 \frac{dy}{dt} + a_0 y = g(t)$$

Where:

y is the dependent variable (usually a function of t).

$a_n$, $a_{n-1}$,…, $a_0$ are constants.

g(t) is the non-homogeneous function.

The general approach of the Method of Undetermined Coefficients involves the following steps:

**Homogeneous Solution**: Find the solution to the associated homogeneous equation

$$a_n y^{(n)} + a_{n-1} y^{(n-1)} + \cdots + a_1 y' + a_0 y = 0$$

**Particular Solution**: Assume a form for the particular solution based on the form of the non-homogeneous term $g(t)$. The assumed particular solution should not be a solution of the associated homogeneous equation.

- ✓ For polynomial functions: Assume a polynomial with undetermined coefficients for each term.
- ✓ For exponential functions: Assume a general form for the exponential term.
- ✓ For trigonometric functions: Assume trigonometric functions with undetermined coefficients.
- ✓ For other functions: Choose the appropriate form that represents the non-homogeneous term.

**Substitution and Determination of Coefficients**: Substitute the assumed particular solution into the original differential equation and solve for the undetermined coefficients. This step often involves equating coefficients of similar terms on both sides of the equation.

**Complete Solution**: Combine the particular solution with the solution to the associated homogeneous equation to obtain the complete solution to the non-homogeneous equation.

This method simplifies the process of finding a particular solution by assuming a form that matches the non-homogeneous term and solving for the undetermined coefficients. However, this method has limitations, especially when the form of the non-homogeneous term is not straightforward or when the assumed form overlaps with the homogeneous solution. For more complex non-homogeneous terms, other methods like variation of parameters or Laplace transforms may be more appropriate.

The Method of Undetermined Coefficients remains a valuable tool for solving a wide range of linear non-homogeneous differential equations in engineering, physics, and other scientific fields.

# 4.0 APPLICATIONS

## 4.1 Illustrative Examples

### 4.1.1 Vibrations in Civil Engineering - Dynamic Analysis of Bridges

**Problem Statement**

Consider a bridge segment with a $m = 1000kg$, a $stiffness\ k = 5000N/m$, and a $damping\ coefficient\ c = 200Ns/m$. An external force $F(t) = 200sin(2\pi t)$ is applied to the bridge. Determine the vertical displacement $x(t)$ of the bridge as a function of time t.

**Solution**

The equation of motion for the bridge is given by:

$$1000\ \frac{d^2x(t)}{dt^2} + 200\ \frac{dx(t)}{dt}\ + 500\ x(t)\ =\ 200\ sin(2\pi t)$$

This is a second-order linear differential equation. To solve it, we can use the method of undetermined coefficients.

Assume a solution of the form

$$x(t)\ =\ A\sin(2\pi t) +\ B\cos(2\pi t)\quad ------\ (*)$$

where A and B are constants to be determined.

Taking the derivatives:

$$\frac{dx(t)}{dt} = 2\pi\ (\ A\ cos(2\pi t) -\ B\ sin(2\pi t)\ )$$

$$\frac{d^2x(t)}{dt^2} = -4\pi^2 \left( A\, sin(2\pi t) + B\, cos(2\pi t) \right)$$

Substituting these into the equation of motion:

$$1000\left[ -4\pi^2 ( A\sin(2\pi t) + B\cos(2\pi t))\right] + 200[\, 2\pi\, ( A\cos(2\pi t) - B\sin(2\pi t))]$$

$$+\ 5000( A\sin(2\pi t) + B\cos(2\pi t))$$

$$= 200\, sin(2\pi t)$$

Now, simplify and equate coefficients:

$$-4000\pi^2 A - 400\pi\, B + 5000\, A = 200\ (coefficients\ of\ sin(2\pi t))$$

$$4000\pi^2 B + 400\pi A + 5000\, B = 0\ (coefficients\ of\ cos(2\pi t))$$

Solve these equations for A and B, and then substitute them back into $(*)$ to get the solution $x(t)$.

To solve this second-order linear differential equation, we'll determine the constants A and B by applying initial conditions. Let's assume that at t = 0, the initial displacement $x(0) = 0$ and the initial velocity $v\,(0) = 0$.

Solving these equations for A and B, we get:

$$A = -\frac{10}{\pi}$$

$$B = -\frac{500}{\pi^2}$$

Now, we can express the solution $---\,(*)$ with these constants:

$$x(t) \ = \ -\frac{10}{\pi}\ sin(2\pi t) - \frac{500}{\pi^{\ 2}}\ cos(2\pi t)$$

**Discussion**

This solution represents the vertical displacement of the bridge as a function of $time\ t$. It demonstrates how second-order linear differential equations can be used to model real-world vibrations in civil engineering applications, providing insights into the bridge's response to external forces.

#### 4.1.2 Mechanical Engineering - Forced Vibration Analysis

**Problem Statement**

Consider a mechanical system with a mass (m) of 4 kg, a spring constant (k) of 100 N/m, and a damping coefficient (c) of 4 Ns/m. The system is subjected to an external harmonic force

F(t)=20cos(2t) N. The objective is to analyze the forced vibration response of the system and determine the steady-state solution.

**Solution:**

The equation of motion for the spring-mass-damper system can be represented as a second-order linear differential equation based on Newton's second law:

$$m\,\frac{d^2x(t)}{dt^2} + c\,\frac{dx(t)}{dt} + k\,x(t) = F(t)$$

Given the system parameters:

Mass $(m)$: 4 $kg$

Spring constant $(k)$: 100 $N/m$

Damping coefficient $(c)$: 4 $Ns/m$

External force $(F(t))$: 20 $cos(2t)$ $N$

We'll find the steady-state solution, which describes the response of the system after transient effects have decayed. To do this, we assume a solution of the form:

$$x(t) = Xcos(2t - \phi)$$

Where:

$X$ is the amplitude of the steady-state response.

$\phi$ is the phase angle.

Taking the derivatives of $x(t)$:

$$\frac{dx(t)}{dt} = -2Xsin(2t - \phi)$$

$$\frac{d^2x}{dt^2} = -4Xcos(2t - \phi)$$

Now, we can substitute these into the equation of motion:

$$4(-4Xcos(2t-\phi))+4(-2Xsin(2t-\phi))+100Xcos(2t-\phi)=20cos(2t)$$

Simplify the equation:

$$-16Xcos(2t-\phi)-8Xsin(2t-\phi)+100Xcos(2t-\phi)=20cos(2t)$$

Now, we'll match the coefficients of the cosine and sine terms on both sides of the equation:

For the cosine term:

$$-16Xcos(2t-\phi)+100Xcos(2t-\phi)=20cos(2t)$$

$$84Xcos(2t-\phi)=20cos(2t)$$

For the sine term:

$$-8Xsin(2t-\phi)=0$$

From the sine term equation, we have $8Xsin(2t-\phi)=0$, which means $sin(2t-\phi)=0$.

This implies that $\phi=2t$.

Now, solving for X in the cosine term equation:

$$84Xcos(2t-2t)=20cos(2t)$$

$$84Xcos(0)=20cos(2t)$$

$$84X=20cos(2t)$$

$$X=\frac{20}{84}\ cos(2t)$$

Simplify $X$:

$$X = \frac{5}{21}\, cos(2t)$$

So, the steady-state response of the system is:

$$x(t) = \frac{5}{21}\, cos(2t)$$

This is the amplitude of the steady-state response, and it shows how the system responds to the external harmonic force $F(t) = 20cos(2t)$.

### 4.1.3 Electrical Engineering – RLC Circuit

**Problem Statement**

An RLC circuit consists of a resistor (R = 3Ω), an inductor (L = 0.5H), and a capacitor (C = 0.2F) connected in series. The circuit is initially uncharged. At t = 0, a voltage source E(t) = 5cos(2πt) volts is suddenly applied.

**Objective**

Determine the current i(t) flowing through the circuit at any given time t after the voltage source is applied

**Solution**

The behavior of the circuit is described by the second order linear differential equation:

$$L\frac{d^2 i(t)}{dt^2} + \ R\frac{di(t)}{dt} + \frac{1}{C} i(t) = E(t)$$

Where:

- i(t) is the current flowing through the circuit (in amperes).
- L is the inductance of the inductor (in henries),
- R is the resistance of the resistor (in ohms)
- C is the capacitance of the capacitor (in farads),
- E(t) is the electromotive force (EMF) or voltage source (in volts), and
- t is time (in seconds).

$$0.5\frac{d^2i(t)}{dt^2} + \ 3\frac{di(t)}{dt} + 5i(t) = 5\cos(2\pi t)$$

Assume the particular solution has the form i(t) = $A\cos(2\pi t) + B\sin(2\pi t)$

1. **Differentiate i(t) to find $\frac{di(t)}{dt}$ and $\frac{d^2i(t)}{dt^2}$**

$$\frac{\mathrm{di(t)}}{\mathrm{dt}} = -2\pi A\ \sin(2\pi t) + 2\pi B\ \cos(2\pi t)$$

$$\frac{\mathrm{d^2i(t)}}{\mathrm{dt^2}} = -4\pi^2 A\ \cos(2\pi t) - 4\pi^2 B\ \sin(2\pi t)$$

2. **Substitute i(t) and its derivatives into the given differential equation**

$$0.5(-4\pi^2 A\ \cos(2\pi t) - 4\pi^2 B\ \sin(2\pi t)) + 3(-2\pi A\ \sin(2\pi t) + 2\pi B\ \cos(2\pi t)) +$$

$$5(A\cos(2\pi t) + B\sin(2\pi t)) = 5\cos(2\pi t)$$

3. **Equate coefficients of like terms on both sides of the equation**

Solving the resulting system of equations for A and B will yield the particular solution components.

$$0.5(-4\pi^2 A\ \cos(2\pi t) - 4\pi^2 B\ \sin(2\pi t)) + 3(-2\pi A\ \sin(2\pi t) + 2\pi B\ \cos(2\pi t)) +$$

$$5(A\cos(2\pi t) + B\sin(2\pi t)) = 5\cos(2\pi t)$$

*Now, let's equate coefficients of $\cos(2\pi t)$ terms on both sides:*

$$-2\pi^2\mathrm{A} + 6\mathrm{B} + 5\mathrm{A} = 5 \qquad (1)$$

*And let's equate coefficients of* $sin(2\pi t)$ *terms on both sides:*

$$-2\pi^2\mathrm{B} - 6\mathrm{A} + 5\mathrm{B} = 0 \qquad (2)$$

We have a system of two equations with two unknowns (A and B). Let's solve this system to find the values of A and B.

*From equation (1):*

$5\mathrm{A} - 2\pi^2\mathrm{A} = 5 - 6\mathrm{B}$

$\mathrm{A}(5 - 2\pi^2) = 5 - 6\mathrm{B}$

$A = \dfrac{5 - 6B}{5 - 2\pi^2}$

*Substitute this expression for A into equation (2):*

$-2\pi^2\mathrm{B} - 6(\frac{5-6B}{5-2\pi^2}) + 5\mathrm{B} = 0$

Solving this equation for B will give:

$-2\pi^2\mathrm{B} - \frac{30-36B}{5-2\pi^2} + 5\mathrm{B} = 0$

*To solve for B, first, find a common denominator for the fractions*

$-2\pi^2(5 - 2\pi^2) - (30 - 36\mathrm{B}) + 5\mathrm{B}(5 - 2\pi^2) = 0$

*Expand and collect like terms*

$-10\pi^2 B + 4\pi^4 B - 30 + 36\mathrm{B} + 25\mathrm{B} - 10\pi^2\mathrm{B} = 0$

$(4\pi^4 - 20\pi^2 + 61)\mathrm{B} = 30$

*Now Solve for B*

$B = \dfrac{25}{4\pi^4 - 20\pi^2 + 61}$

Therefore, the particular solution has the form

$$i(t) = \frac{5-6B}{5-2\pi^2}\cos(2\pi t) + \frac{25}{4\pi^4-20\pi^2+61}\sin(2\pi t)$$

# 5.0 CONCLUSION AND RECOMMENDATIONS

## 5.1 Conclusion

The application of second-order linear differential equations stands as a cornerstone in the realms of civil, mechanical, and electrical engineering.

In civil engineering, the use of second-order linear differential equations is instrumental in structural analysis, allowing engineers to comprehend the behavior of buildings, bridges, and infrastructure subjected to dynamic forces. It provides a predictive framework to ensure structural stability, assess vibration effects, and design earthquake-resistant structures.

In mechanical engineering, second-order differential equations are fundamental in understanding mechanical systems' motion, such as vibrations, pendulum movements, and harmonic oscillations. These equations aid in optimizing machine design, analyzing control systems, and enhancing the efficiency of mechanical components.

Within electrical engineering, second-order linear differential equations find extensive application in circuit analysis. They allow engineers to comprehend the dynamic response of electrical systems to varying inputs, providing insights into the behavior of circuits, filters, and control systems. Such equations facilitate the design and analysis of electrical networks, ensuring efficient and robust system performance.

In summary, the broad applications of second-order linear differential equations play an important role in civil, mechanical, and electrical engineering. Their utilization in modeling and analyzing dynamic systems underpins the design, stability, and efficiency of a wide array of engineering applications, paving the way for innovation and advancements in these fields.

## 5.2 Recommendation

I recommend that other researchers should further their research into advanced methods and techniques for solving second-order linear differential equations in engineering applications. This could include exploring numerical algorithms and mathematical models that enhance the accuracy and efficiency of solutions. Also, I advocate for the development and adoption of innovative educational materials and approaches to teach second-order linear differential equations in engineering programs. Emphasize real-world applications and hands-on experiences to enhance students' understanding and practical skills.

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