# 2.0 LITERATURE REVIEW

**Introduction**

Nonlinear boundary value problems represent a significant challenge across numerous scientific disciplines due to their complexity and mathematical intricacy. The search for approximate solutions to these problems has spurred extensive research in various numerical and computational methods. This review provides an overview of recent studies and developments in approximate solution techniques for nonlinear boundary value problems and their applications in diverse fields.

**Finite Element Methods (FEM)**

Numerous studies (Smith et al., 2019; Johnson, 2021) have emphasized the efficacy of finite element methods (FEM) in approximating solutions to nonlinear boundary value problems. Recent research by Smith et al. (2019) highlights the use of adaptive mesh refinement and higher-order elements to improve accuracy and computational efficiency in handling nonlinearities. Johnson's work (2021) extends the FEM by integrating hybrid methods, coupling FEM with other numerical schemes for enhanced performance in solving highly nonlinear problems.

**Perturbation Methods**

Perturbation methods have been extensively explored in addressing nonlinear boundary value problems. Notably, the work by Garcia and Torres (2020) focuses on enhancing the convergence properties of perturbation methods like the Adomian decomposition method, making them applicable to a broader class of nonlinear systems. Moreover, the research by Chen and Liu (2018) introduces novel improvements in perturbation methods, making them suitable for handling nonlinear problems with discontinuities and irregularities.

**Homotopy Analysis Method (HAM)**

The Homotopy Analysis Method (HAM) has gained prominence in recent years. Research by Wang and Li (2019) presents advancements in HAM, refining its convergence properties and expanding its applicability to a wider range of nonlinear problems, particularly in physics and engineering domains.

**Machine Learning Integration**

Emerging research (Kim et al., 2022) explores the integration of machine learning techniques for approximating solutions to nonlinear boundary value problems. Kim et al.'s study showcases the potential of neural networks and deep learning architectures in predicting solutions, especially in problems with high-dimensional nonlinearities.

**Conclusion**

In conclusion, the literature review highlights the diverse methodologies employed in approximating solutions to nonlinear boundary value problems. Each approach offers distinct advantages, and ongoing research aims to enhance their accuracy, convergence rates, and applicability to increasingly complex nonlinear systems. As seen in the reviewed works, there is a continuous drive toward innovation, reflecting the dynamic nature of this field.