remapping of x in integr. dipoles

pdfs — part. cross sect.

$$\sigma_{h_1 h_2} = \int_0^1 d\eta_1 \cdot \int_0^1 d\eta_2 \cdot f_a(\eta_1) \cdot f_b(\eta_2) \cdot \int_0^1 dx \; \sigma_{ab}(x\eta_1 P_1, \eta_2 P_2) \times I_a(x, x\eta_1 P_1, \eta_2 P_2)$$

integr. dipoles

$$\Rightarrow \boxed{\eta' = x\eta_1}$$

$$= \int_0^1 dx \cdot \int_0^x \frac{d\eta'}{x} \int_0^1 d\eta_2 \; f_a\!\left(\frac{\eta'}{x}\right) \cdot f_b(\eta_2) \cdot \sigma_{ab}(\eta' P_1, \eta_2 P_2) \times I_a(x, \eta' P_1, \eta_2 P_2)$$

$\int_0^x d\eta'$ can be promoted to $\int_0^1 d\eta'$
if we require $f_a\!\left(\frac{\eta'}{x}\right) = 0$ for $\frac{\eta'}{x} > 1$.

Now, do the same for η_2 and add up:

$$\Rightarrow \boxed{\begin{aligned} \sigma_{h_1 h_2} = \int_0^1 dx \int_0^1 d\eta_1 \int_0^1 d\eta_2 \; \sigma_{ab}(\eta_1 P_1, \eta_2 P_2) \cdot \Big\{ & \frac{1}{x} f_a\!\left(\frac{\eta_1}{x}\right) \cdot f_b(\eta_2) \cdot I_a(x, \eta_1 P_1, \eta_2 P_2) \\ & + f_a(\eta_1) \cdot \frac{1}{x} f_b\!\left(\frac{\eta_2}{x}\right) \cdot I_b(x, \eta_1 P_1, \eta_2 P_2) \Big\} \end{aligned}}$$

The integr. dipoles I_p decompose into a delta-fct. part (soft) and into a regular part (fini) and into a plus-distr. part (plus):

$$I_p = \overset{\text{„soft"}}{c_p \cdot \delta(x-1)} + \overset{\text{„fini"}}{g_p(x)} + \overset{\text{„plus"}}{h_p(x)_+}$$

pdfs + part. cross sect.

$$\Rightarrow \quad s_p(x) \cdot I_p = s_p(1) \cdot c_p + s_p(x)\, g_p(x) + \big(s_p(x) - s_p(1)\big)\, h_p(x)$$

$$= s_p(1) \cdot \big[c_p - h_p(x) \big] + s_p(x) \cdot \big[g_p(x) + h_p(x) \big]$$

$$\Rightarrow \quad \boxed{\begin{aligned} \sigma_{h_1 h_2} = \int_0^1 dx \int_0^1 d\eta_1 \int_0^1 d\eta_2 \cdot \sigma_{ab}(\eta_1 P_1, \eta_2 P_2) \cdot \Big\{ & \\ & f_a(\eta_1) \cdot f_b(\eta_2) \cdot \Big[\big(c_a - h_a(x)\big) + \big(c_b - h_b(x)\big) \Big] \\ & + \frac{1}{x} f_a\!\left(\frac{\eta_1}{x}\right) \cdot f_b(\eta_2) \cdot \big[g_a(x) + h_a(x) \big] \\ & + f_a(\eta_1) \cdot \frac{1}{x} f_b\!\left(\frac{\eta_2}{x}\right) \cdot \big[g_b(x) + h_b(x) \big] \quad \Big\} \end{aligned}}$$