Final state dipoles

Here we discuss what to do with final state dipoles, associated with initial spectators; in particular in case when final state is $g \rightsquigarrow g+g$ splitting & top is the spectator.

The requirements:

1) must subtract soft singularity
2) must subtract collinear singularity
3) must survive in the sense of Ellis/Campbell maping

To begin, define the collinear limit

$$|M|^2 \mapsto M_\mu P_{gg}^{\mu\nu} M_\nu \quad \text{where}$$

$$P_{gg}^{\mu\nu} \sim \left[-g^{\mu\nu}\left(\frac{\xi}{1-\xi} + \frac{1-\xi}{\xi}\right) - 2(1-\varepsilon)\xi(1-\xi)\frac{k_\perp^\mu k_\perp^\nu}{k_\perp^2} \right]$$

Here ξ & $k_\perp^\mu$ are defined as:

$$p_i^\mu = (1-\xi)p^\mu + k_\perp^\mu - \frac{k_\perp^2 n^\mu}{(1-\xi)(2pn)}$$

$$p_j^\mu = \xi p^\mu - k_\perp^\mu - \frac{k_\perp^2 n^\mu}{\xi\, 2pn}$$

Vector p^μ with $p^2=0$ defines collinear direction.

We now use the fact that $\xi p_i^\mu - (1-\xi)p_j^\mu = k_\perp^\mu$ and $2p_ip_j = -k_\perp^2/\xi(1-\xi)$, to write an extension of the splitting function:

$$P^{\mu\nu}_{g_i g_j} \longmapsto \left[-g^{\mu\nu}\left(\frac{\xi}{1-\xi} + \frac{1-\xi}{\xi}\right) + (1-\varepsilon)\frac{a^\mu a^\nu}{p_i p_j} \right]$$

where $a^\mu = \xi p_i^\mu - (1-\xi) p_j^\mu$.

Now, to describe the dipoles, we will split this into two pieces

$$P^{\mu\nu}_{g_i g_j} \frac{1}{2p_i p_j} \longrightarrow D^{\mu\nu}_{ij} + D^{\mu\nu}_{ji} \quad (\text{both for } gg)$$

where

$$\underbrace{D^{\mu\nu}_{ij}}_{\text{soft}} = \frac{1}{2p_i p_j}\left\{ -g^{\mu\nu}\frac{\xi_i}{1-\xi_i} + \frac{1-\epsilon}{2}\frac{a^\mu a^\nu}{p_i p_j} \right\}$$

$$= \frac{1}{2p_i p_j}\left\{ -g^{\mu\nu}\left(\frac{1}{1-\xi_i} - 1\right) + \frac{1-\varepsilon}{2}\frac{a^\mu a^\nu}{p_i p_j} \right\}$$

Now, we need to write all this staff using Campbell - Ellis parametrization.

We have $\dfrac{t p_j}{t p_i} \approx \dfrac{\xi_i}{1-\xi_i} \Rightarrow$

we can <u>identify ξ with z</u> defined throug

$$\boxed{t p_i = \frac{m_t^2}{2}(1-r^2)(1-z)}$$

$$\boxed{p_i p_j = \frac{m_t^2}{2}(1-r)^2 y}$$

$$\Rightarrow D^{\mu\nu}_{ij,j} = \frac{1}{2p_ip_j}\left\{-g^{\mu\nu}\left(\frac{1}{1-z} - \cancel{1}\right) + \frac{1-\varepsilon}{2}\frac{a^\mu a^\nu}{p_ip_j}\right\}$$

Now, we need to do something about spin-correlated part which is only relevant in the collinear regime. We have, originally

$$a^\mu = \xi p_i^\mu - (1-\xi) p_j^\mu \equiv z p_i^\mu - (1-z) p_j^\mu$$

$$\approx \left[(tp_j) p_i^\mu - (tp_i)^\mu p_j^\mu\right] \frac{1}{\frac{m_t^2}{2}(1-r^2)} =$$

$$\equiv \frac{2}{m_t^2(1-r^2)}\left[(tp_j) p_i^\mu - (tp_i)^\mu p_j^\mu\right]$$

Now, we make this vector transverse to $\tilde{p}_{ij}$ by multiplying it with the transverse projection

$$Pr^{\mu\nu} = \frac{(t\tilde{p}_{ij}) g^{\mu\nu} - (t^\mu \tilde{p}_{ij}^\nu + t^\nu \tilde{p}_{ij}^\mu)}{t\tilde{p}_{ij}} \Rightarrow$$

$$a^\mu \to \pi^\mu = Pr^{\mu\nu} a^\nu = \frac{1}{t\tilde{p}_{ij}}\left((t\tilde{p}_{ij}) a^\mu - t^\mu (\tilde{p}_{ij}\cdot a)\right),$$

since $t\cdot a = 0$. $\Rightarrow$ The dipole (soft i, arbitrary j):

$$\left\{\begin{array}{l} D^{\mu\nu}_{ij} = \frac{1}{2p_ip_j}\left(-g^{\mu\nu}\left(\frac{1}{1-z} - 1\right) + \frac{1-\varepsilon}{2p_ip_j}\pi^\mu\pi^\nu\right), \text{ with} \\ tp_i = \frac{m_t^2}{2}(1-r^2)(1-z) \;;\; \tilde{p}_{ij}^\mu = (t-\tilde{W})^\mu \quad \Rightarrow \end{array}\right.$$

The general result that we can write down is

$$\boxed{D^{\mu\nu}_{i,j} = \frac{1}{2p_i p_j}\left(-g^{\mu\nu}\left(\frac{1}{1-z}-1\right) + \frac{1-\varepsilon}{2p_i p_j}\,\pi^{\mu}\pi^{\nu}\cdot f(y)\right),}$$

$\Rightarrow$ See page 7 for full definition

where $f(y)$ is an arbitrary function of y such that $f(\phi)=1$ and it is free from soft singularities & momenta are given by

$$\pi^{\mu} = \frac{1}{t\hat{p}_{ij}}\left((t\hat{p}_{ij})a^{\mu} - t^{\mu}(\hat{p}_{ij}\cdot a)\right)$$

$$a^{\mu} = \frac{\alpha}{m_t^2(1-r^2)}\left[(tp_j)p_i^{\mu} - (tp_i)^{*}p_j^{\mu}\right]$$

$$tp_i = m_t^2/2\,(1-r^2)(1-z) \qquad p_i p_j = \frac{m_t^2}{2}(1-r)^2 y$$

$$t\hat{p}_{ij} = m_t^2(1-r^2)/2$$

The function $f(y)$ can be anything but, it appears that for ~~introducing~~ calculating ~~a dip~~ dependence of the ~~tadp~~ dipole on α-parameter in a reasonable fashion, we need to choose it as

$$\boxed{f(y) = \frac{4}{m_t^4}\,\frac{\left[(tw)^2 - r^2 m_t^4\right]}{(1-z^2)^2}}$$

The phase space factorization is

$$d\phi^{(3)} = d\phi^{(2)} \times \frac{(1-z)^2 (m_t^2)^{1-\varepsilon} (4\pi)^{\varepsilon}}{16\pi^2 \Gamma(1-\varepsilon)} \times$$

$$\times \left(\frac{1+r}{1-r}\right)^{2\varepsilon} \int_0^1 dz \left[r^2 + z(1-r^2)\right]^{-\varepsilon} \int_0^{y_{max}} dy\, y^{-\varepsilon} (y_{max} - y)^{-\varepsilon},$$

where $\boxed{y_{max} \equiv \frac{(1+r)^2 z(1-z)}{\left(z + r^2(1-z)\right]}}$

Define $\langle g(y,z) \rangle = \left(\frac{1+r}{1-r}\right)^{2\varepsilon} \int_0^1 dz \left[\ldots\right]^{-\varepsilon} \int_0^{y_{max}} dy\, y^{-\varepsilon} (\ldots)^{-\varepsilon} \times$
$\times g(y,z)$.

Campbell / Ellis table gives results for

$\left\langle \frac{2}{y(1-z)} \right\rangle$ & $\left\langle \frac{1}{y} \right\rangle$.

For spin-correlation piece, we need

$(1-\varepsilon) \left\langle \frac{\pi^\mu \pi^\nu}{(2 p_i p_j)^2} f(y) \right\rangle$ Because $\tilde{p}_{ij}^{\mu} \cdot \pi_\mu = 0$,

this tensor is transverse $\Rightarrow$

$$(1-\varepsilon) \left\langle F(y) \frac{\pi^\mu \pi^\nu}{(2 p_i p_j)^2} \right\rangle = A_1 \left(-g_{\mu\nu} + \frac{t_\mu \tilde{p}_{ij\nu} + \tilde{p}_{ij\mu} t_\nu}{(t \cdot \tilde{p}_{ij})} \right)$$

$$+ A_2\, \tilde{p}_{ij}^{\mu}\, \tilde{p}_{ij}^{\nu};$$

Because of transversality of on-shell matrix elements, we only need A_1. We get A_1, by contracting both sides with $g_{\mu\nu}$ $\Rightarrow$

$$A_1 = \frac{(1-\varepsilon)}{(2-d)} \left\langle f(y) \frac{\pi^2}{(2p_ip_j)^2} \right\rangle = -\frac{1}{2} \left\langle f(y) \frac{\pi^2}{(2p_ip_j)^2} \right\rangle$$

When I calculate ~~A_4~~ π^2 in the form program, I use <u>un-normalized</u> expressions for π^μ & a^μ.

This gives me the following factor <u>per</u> <u>1 power</u> of <u>π^μ</u> to account for in the Maple program:

$$\frac{1}{t\hat{p}_{ij}} \frac{2}{m_t^2(1-r^2)} = \left[\frac{2}{m_t^2(1-r^2)}\right]^2$$

After that all calculations are done with Maple. To deal with α-dependence, we introduce, as before

$$\int dy\,dz \to \int dy\,dz \left[1 - \theta(1-\alpha-z)\,\theta(y-\alpha y_{max})\right]$$

$$\equiv \int dy\,dz - \int_0^{1-\alpha} dz \int_{\alpha y_{max}}^{y_{max}} dy\,.$$

Finally, to account for soft emissions from the top quark, we need to add a term. The full $D^{\mu\nu}_{g_i, g_j}$ splitting becomes:

$$D^{\mu\nu}_{g_i, g_j} = \frac{1}{2p_i p_j}\left[-g^{\mu\nu}\left(\frac{1}{1-z} - 1 - \frac{m_t^2}{4} \frac{2p_i p_j}{(t \cdot p_i)^2} \right) + \frac{1-\varepsilon}{2p_i p_j} \pi^{\mu}\pi^{\nu} \cdot f(y) \right]$$

where most of the definitions are found on page 4.