$t\bar{t}$ + jet factorization checks

define $B^X_{P\to D} := \dfrac{\Gamma^X_{P\to D}}{\Gamma^{LO}_{tot,P}}$ with $X = LO, NLO$; $P = t, W$

$$\Gamma^{LO}_{tot,t} = \Gamma^{LO}_{t\to Wb} = 1.46533\,\text{GeV}$$

$$\Gamma^{LO}_{tot,W} = \Gamma_{exp,W} = 2.14\,\text{GeV}$$

$$B^{LO}_{t\to bW} = 1\,, \qquad B^{NLO}_{t\to bW} \overset{\text{hep-ph/0408158, 2-loop } \alpha_s(m_t)}{=} \frac{1.33745\,\text{GeV}}{1.46533\,\text{GeV}} = 0.912729$$

$$B^{LO}_{W\to l\nu} = \frac{1}{\Gamma_{exp,W}} \cdot \frac{G_F \cdot M_W^3}{6\pi\sqrt{2}} = \frac{227.6\,\text{MeV}}{2.14\,\text{GeV}} = 10.64\,\%$$

$$B^{LO}_{W\to jj} = 6 \cdot B^{LO}_{W\to l\nu} = 63.84\,\%$$

$$B^{NLO}_{W\to l\nu} = B^{LO}_{W\to l\nu}$$

$$B^{NLO}_{W\to jj} = B^{LO}_{W\to jj}\left(1 + \frac{\alpha_s^{2\text{-loop}}}{\pi}\right) = 66.07\,\%$$

$m_{top} = 172\,\text{GeV}$, $m_b = 0\,\text{GeV}$, $M_W = 80.419\,\text{GeV}$

$\alpha_s^{1\text{-loop}} = 0.1258113$ (MSTW08, $\mu = m_t$)

$\alpha_s^{2\text{-loop}} = 0.1095170$

1-loop α_s: $B^{LO}_{t\to bWj} \overset{\text{MadGraph}}{=} \frac{0.682813\,\text{GeV}}{1.46533\,\text{GeV}} = 46,60\%$, $s_{gb} \geq 10^{-3} m_t^2$ ($(p_g \cdot p_b)$)

1-loop α_s: $B^{LO}_{W\to jjj} \overset{\text{set flavor}}{=} \frac{1.05596\,\text{GeV}}{2.14\,\text{GeV}} \times 2 = 98,69\%$, $s_{gq} \geq 10^{-3} M_W^2$

2-loop α_s: $B^{LO}_{t\to bWj} = \frac{0.59438\,\text{GeV}}{1.46533\,\text{GeV}} = 40,56\%$, as above

2-loop α_s: $B^{LO}_{W\to jjj} = \frac{0.9192\,\text{GeV}}{2.14\,\text{GeV}} \times 2 = 85.91\%$, as above

check A

$$d\sigma^{LO}_{t\bar{t}j} \cdot dB^{LO}_{\bar{t}} \cdot dB^{LO}_{t} = d\sigma^{LO}_{t\bar{t}j} \cdot \left(dB^{LO}_{\bar{t}\to \bar{b}W}\, dB^{LO}_{W\to jj} \right) \cdot \left(dB^{LO}_{t\to bW}\, dB^{LO}_{W\to e\nu} \right)$$

$$\xrightarrow{\int dPS} \sigma^{LO}_{t\bar{t}j} \cdot B^{LO}_{W\to jj}\, B^{LO}_{W\to e\nu}$$

check B

$$d\sigma^{LO}_{t\bar{t}} \left(dB^{LO}_{\bar{t}j}\, dB^{LO}_{t} + dB^{LO}_{\bar{t}}\, dB^{LO}_{tj} \right)$$

$$\xrightarrow{\int dPS} \sigma^{LO}_{\bar{t}t} \Big[\left(B^{LO}_{t\to bWj}\, B^{LO}_{W\to jj} + B^{LO}_{W\to jjj} \right) \cdot B^{LO}_{W\to e\nu} + B^{LO}_{W\to jj} \cdot B^{LO}_{t\to bWj}\, B_{W\to e\nu} \Big]$$

Top D $k=4$, LHC (14 TeV), MSTW08,

check **E-F1a,b**

F1a: F, corr=2 without DK dipoles

F1b: F, corr=3

$$d\sigma^{X}_{\bar{t}t} \left(dB^{LO}_{\bar{t}j}\, dB^{LO}_{t} + dB^{LO}_{\bar{t}}\, dB^{LO}_{tj} \right)$$

$$\xrightarrow{\int dPS} \sigma^{X}_{\bar{t}t} \Big[\left(B^{LO}_{t\to bWj}\, B^{LO}_{W\to jj} + B^{LO}_{W\to jjj} \right) \cdot B^{LO}_{W\to e\nu} + B^{LO}_{W\to jj} \cdot B^{LO}_{t\to bWj}\, B^{LO}_{W\to e\nu} \Big]$$

with $X = \begin{cases} VI & \boxed{E} \\ RE & \boxed{F1a} \\ ID & \boxed{F1b} \end{cases}$

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check **F1c-G**

F1c: F, corr=2 without PR dipoles

$$d\sigma^{LO}_{\bar{t}tj} \Big[\left(dB^{X}_{t\to bW(j)} \cdot dB^{LO}_{W\to jj} + dB^{LO}_{t\to bW} \cdot dB^{X}_{W\to jj(j)} \right) B^{LO}_{W\to e\nu} + B^{LO}_{W\to jj} \cdot B^{X}_{t\to bW(j)}\, B^{LO}_{W\to e\nu} \Big]$$

with $X = \begin{cases} VI & \boxed{G} \\ RE & \boxed{F1c} \end{cases}$

$$\xrightarrow[VI+RE]{\int dPS} \sigma^{LO}_{\bar{t}tj} \cdot \Big[\left(B^{NLO}_{t\to bW}\, B^{LO}_{W\to jj} + B^{NLO}_{W\to jj} \right) B^{LO}_{W\to e\nu} + B^{LO}_{W\to jj}\, B^{NLO}_{t\to bW}\, B^{LO}_{W\to e\nu} \Big]$$

check H-I

NOTE: in the following I factor out $d\sigma^{LO}_{t\bar{t}}$ everywhere

- **V1 + R1 + R11 + R12** , **V9 + R8 + R13 + R14**

$$dB^{virt}_{\bar{t}\to \bar{b}Wj} \cdot dB^{LO}_{W\to jj}\, dB^{LO}_{W\to e\nu}$$

$$+\; dB^{real}_{\bar{t}\to \bar{b}Wjj} \cdot dB^{LO}_{W\to jj} \cdot dB^{LO}_{W\to e\nu}$$

$\xrightarrow{\text{fdPS}}$ $B^{\delta NLO}_{\bar{t}\to \bar{b}Wj}\; B^{LO}_{W\to jj}\; B^{LO}_{W\to e\nu}$ A

- **V3 + R2 + R15 + R16 + R17**

$$dB^{LO}_{\bar{t}\to \bar{b}W} \cdot dB^{virt}_{W\to jjj} \cdot dB^{LO}_{W\to e\nu}$$

$$+\; dB^{LO}_{t\to bW} \cdot dB^{real}_{W\to jjjj} \cdot dB^{LO}_{W\to e\nu}$$

$\xrightarrow{\text{fdPS}}$ $B^{\delta NLO}_{W\to jjj} \cdot B^{LO}_{W\to e\nu}$ A

- **V2 + V4 + R3**

$$d\mathcal{B}^{LO}_{t\to \bar{b}Wj} \cdot d\mathcal{B}^{virt}_{W\to jj} \cdot d\mathcal{B}^{LO}_{W\to e\nu}$$

$$+ \; d\mathcal{B}^{virt}_{t\to \bar{b}W} \cdot d\mathcal{B}^{LO}_{W\to jjj} \cdot d\mathcal{B}^{LO}_{W\to e\nu}$$

$$+ \; d\mathcal{B}^{real}_{t\to \bar{b}Wj} \cdot d\mathcal{B}^{real}_{W\to jjj} \cdot d\mathcal{B}^{LO}_{W\to e\nu}$$

$\xrightarrow{\int dPS}$ $\mathcal{B}^{LO}_{t\to bWj} \; \mathcal{B}^{\delta NLO}_{W\to jj} \; \mathcal{B}^{LO}_{W\to e\nu}$

$$+ \; \mathcal{B}^{\delta NLO}_{t\to bW} \cdot \mathcal{B}^{LO}_{W\to jjj} \; \mathcal{B}^{LO}_{W\to e\nu}$$

- **V5 + V13 + R4**

$$d\mathcal{B}^{virt}_{t\to \bar{b}W} \cdot d\mathcal{B}^{LO}_{W\to jj} \cdot d\mathcal{B}^{LO}_{t\to bWj} \cdot d\mathcal{B}^{LO}_{W\to e\nu}$$

$$+ \; d\mathcal{B}^{LO}_{t\to \bar{b}Wj} \; d\mathcal{B}^{LO}_{W\to jj} \cdot d\mathcal{B}^{virt}_{t\to bW} \cdot d\mathcal{B}^{LO}_{W\to e\nu}$$

$$+ \; d\mathcal{B}^{real}_{t\to \bar{b}Wj} \cdot d\mathcal{B}^{LO}_{W\to jj} \cdot d\mathcal{B}^{real}_{t\to bWj} \cdot d\mathcal{B}^{LO}_{W\to e\nu}$$

$\xrightarrow{\int dPS}$ $2 \cdot \mathcal{B}^{\delta NLO}_{t\to bW} \; \mathcal{B}^{LO}_{W\to jj} \cdot \mathcal{B}^{LO}_{t\to bWj} \cdot \mathcal{B}^{LO}_{W\to e\nu}$

• **V6 + V15 + R6**

$$d\mathcal{B}^{virt}_{W\to jj} \cdot d\mathcal{B}^{LO}_{t\to bWj} \cdot d\mathcal{B}^{LO}_{W\to \ell\nu}$$

$$+\; d\mathcal{B}^{LO}_{W\to jjj} \cdot d\mathcal{B}^{virt}_{t\to bW} \cdot d\mathcal{B}^{LO}_{W\to \ell\nu}$$

$$+\; d\mathcal{B}^{real}_{W\to jjj} \cdot d\mathcal{B}^{real}_{t\to bWj} \cdot d\mathcal{B}^{LO}_{W\to \ell\nu}$$

$\int d\mathrm{PS}$ →

$$\mathcal{B}^{\delta NLO}_{W\to jj} \cdot \mathcal{B}^{LO}_{t\to bWj} \cdot \mathcal{B}^{LO}_{W\to \ell\nu}$$

$$+\; \mathcal{B}^{LO}_{W\to jjj} \cdot \mathcal{B}^{\delta NLO}_{t\to bW} \cdot \mathcal{B}^{LO}_{W\to \ell\nu}$$

check C-D

$$d\sigma^{virt}_{t\bar{t}j}\; d\mathcal{B}^{LO}_{W\to jj} \cdot d\mathcal{B}^{LO}_{W\to \ell\nu}$$

$$+\; d\sigma^{real}_{t\bar{t}jj}\; d\mathcal{B}^{LO}_{W\to jj} \cdot d\mathcal{B}^{LO}_{W\to \ell\nu}$$

$\int d\mathrm{PS}$ →

$$\sigma^{\delta NLO}_{t\bar{t}j} \cdot \mathcal{B}^{LO}_{W\to jj} \cdot \mathcal{B}^{LO}_{W\to \ell\nu}$$