Dipoles: momentum mappings & logic

① Final - final dipoles. For our purposes - they should be all massless. Then, we require that those final state spectators do not change directions. Hence:

we require:

$$p_i + p_j + p_k \equiv \tilde{p}_{ij} + \tilde{p}_k \quad \text{and} \quad \boxed{\tilde{p}_k = (1-\lambda) p_k}$$

Hence $\tilde{p}_{ij} = p_i + p_j + p_k - \tilde{p}_k = p_i + p_j + p_k - (1-\lambda) p_k$

$$= \underbrace{p_i + p_j}_{\pi_{ij}} + \lambda p_k \Rightarrow$$

$$\boxed{\tilde{p}_{ij} = \pi_{ij} + \lambda p_k}$$

We choose $\tilde{p}_{ij}^2 = 0$, eventually. For this to happen; λ should be such that

$$0 = \pi_{ij}^2 + 2\lambda_0 \pi_{ij} p_k \Rightarrow \lambda_0 = -\pi_{ij}^2 / (2\pi_{ij} \cdot p_k) \Rightarrow$$

We will also consider constraining variable

$$z = \frac{p_i p_k}{\pi_{ij} \cdot p_k}.$$

Hence, we have the phase-space element to be:

$$d\Phi_{ijk} \equiv [dp_i][dp_j][dp_k](2\pi)^d \delta(\dots - p_i - p_j - p_k)$$

$$\equiv [dp_i][dp_j] \frac{[d\tilde{p}_k]}{(1-\lambda)^{d-2}} (2\pi)^d \delta\left(\dots - p_i - p_j - \frac{\tilde{p}_k}{1-\lambda}\right),$$

where $[dp] = \dfrac{d^{D-1} p}{(2\pi)^{D-1} 2p_0}$;

Now, we put in λ-constraints & z constraints

Then $d\phi_{ijk} \longmapsto d\Phi_q$

$$d\phi_{ijk} \equiv [dp_i][dp_j]\frac{[d\tilde{p}_k]}{(1-\lambda)^{d-2}}(2\pi)^d\,\delta\left(\dots - p_i - p_j - \frac{\tilde{p}_k}{1-\lambda}\right)$$

$$\times \frac{d^d\hat{p}_{ij}}{(2\pi)^d}(2\pi)^d\,\delta\left(\hat{p}_{ij} - \pi_{ij} - \lambda p_k\right)$$

$$\times d\lambda \quad \delta\left(\lambda + \pi_{ij}^2/(2\pi_{ij}\cdot p_k)\right)$$

$$\times dz \quad \delta\left(z - \frac{p_i p_k}{\pi_{ij}\cdot p_k}\right).$$

Now, write $\dfrac{d^d\hat{p}_{ij}}{(2\pi)^d} = \dfrac{d\hat{p}_{ij}^2}{2\pi}\,[d\hat{p}_{ij}]$, use

$$-p_i - p_j - \tilde{p}_k/1-\lambda = -\left(\hat{p}_{ij} - \frac{\lambda\tilde{p}_k}{1-\lambda}\right) - \tilde{p}_k/1-\lambda = -\hat{p}_{ij} - \tilde{p}_k \Rightarrow$$

$$d\phi_{ijk} = \cancel{[dp_i][dp_j]}\,\frac{[d\tilde{p}_k]}{(1-\lambda)^{d-2}}(2\pi)^d\,\delta\left(\dots - \hat{p}_{ij} - \tilde{p}_k\right)[d\hat{p}_{ij}]$$

$$\times \frac{d\hat{p}_{ij}^2}{2\pi}\,\cancel{\delta}(2\pi)^d\,\delta\left(\hat{p}_{ij} - \pi_{ij} - \frac{\lambda\tilde{p}_k}{1-\lambda}\right)$$

$$\times d\lambda\ \delta\left(\lambda + (1-\lambda)\pi_{ij}^2/2\pi_{ij}\cdot\tilde{p}_k\right)$$

$$\times dz\ \delta\left(z - \frac{p_i\tilde{p}_k}{\pi_{ij}\cdot\tilde{p}_k}\right)[dp_i][dp_j] \Rightarrow$$

Fine, so now we need to integrate over the unresolved structures.

First: $\pi_{ij}\tilde{p}_k = \tilde{p}_{ij}\cdot\tilde{p}_k$. Then

$$\lambda + (1-\lambda)\,\pi_{ij}^2 \big/ 2\pi_{ij}\tilde{p}_k = \lambda + (1-\lambda)\frac{(\tilde{p}_{ij} - \lambda\tilde{p}_k/1-\lambda)^2}{2\tilde{p}_{ij}\cdot\tilde{p}_k} \equiv$$

$$\equiv (1-\lambda)\frac{\tilde{p}_{ij}^2}{2\tilde{p}_{ij}\cdot\tilde{p}_k} + \lambda - \lambda = (1-\lambda)\frac{\tilde{p}_{ij}^2}{2\tilde{p}_{ij}\cdot\tilde{p}_k} \Rightarrow$$

$$X = \frac{d\,\tilde{p}_{ij}^2}{2\pi}(2\pi)^d\,\delta\left(\tilde{p}_{ij} - \pi_{ij} - \frac{\lambda\tilde{p}_k}{1-\lambda}\right)\times$$

$$\times\, d\lambda\;\delta\left((1-\lambda)\frac{\tilde{p}_{ij}^2}{2\tilde{p}_{ij}\cdot\tilde{p}_k}\right)\,dz\,\delta\left(z - \frac{p_i\cdot\tilde{p}_k}{\tilde{p}_{ij}\cdot\tilde{p}_k}\right)[dp_i][dp_j]$$

$$\equiv \frac{d\lambda\,(2\tilde{p}_{ij}\cdot\tilde{p}_k)}{2\pi(1-\lambda)}\times(2\pi)^d\,\delta\left(\tilde{p}_{ij} - \pi_{ij} - \frac{\lambda\tilde{p}_k}{1-\lambda}\right)$$

$$(dz)\times\delta\left(z - p_i\tilde{p}_k\big/\tilde{p}_{ij}\cdot\tilde{p}_k\right)[dp_i][dp_j]$$

Now: $[dp_j] = \dfrac{d^dp_j}{(2\pi)^d}\,\delta(p_j^2)\cdot 2\pi \Rightarrow$

$$X = \frac{d\lambda\,dz\,(2\tilde{p}_{ij}\cdot\tilde{p}_k)}{(2\pi)(1-\lambda)}\,2\pi\,\delta\left(\left(\tilde{p}_{ij} - p_{ij} - \frac{\lambda\tilde{p}_k}{1-\lambda}\right)^2\right)$$

$$\times\,\delta\left(z - \frac{p_i\tilde{p}_k}{\tilde{p}_{ij}\cdot\tilde{p}_k}\right)[dp_i]$$

$$[dp_i] \Rightarrow \quad \boxed{p_i = a\,\hat{p}_{ij} + B\,\hat{p}_k + p_{i,\perp}} \Rightarrow$$

$$[dp_i] = \frac{d^d p_i}{(2\pi)^d}\, 2\pi\, \delta(p_i^2) = \frac{d^d p_i}{(2\pi)^d}\, 2\pi\, \delta\left(2aB\,\tilde{p}_{ij}\hat{p}_k + p_{i\perp}^2\right)$$

$$= \frac{d^d p_i}{(2\pi)^d}\,(2\pi)\,\delta\left(2aB\,\hat{p}_{ij}\cdot\hat{p}_k - \vec{p}_{i\perp}^{\,2}\right)$$

$$d^d p_i \equiv (\hat{p}_{ij}\cdot\hat{p}_k)\,da\,db\,d^{d-2}\vec{p}_{i,\perp} \equiv [\hat{p}_{ij}\cdot\hat{p}_k]\,da\,db\,d\vec{p}_{i,\perp} \Rightarrow$$

$$[dp_i] \equiv \frac{(\hat{p}_{ij}\cdot\hat{p}_k)\,da\,dB\,d\vec{p}_{i,\perp}^{\,d-2}\,\delta\left(2aB\,\hat{p}_{ij}\cdot\hat{p}_k - \vec{p}_{i\perp}^{\,2}\right)}{(2\pi)^{d-1}} \Rightarrow$$

$$X = \frac{d\lambda\,dz\,(2\tilde{p}_{ij}\cdot\tilde{p}_k)}{(1-\lambda)\,(2\pi)^{d-1}}\,(\hat{p}_{ij}\cdot\tilde{p}_k)\quad da\,db\,d^{d-2}\vec{p}_{i,\perp}$$

$$\times\,\delta\left(2ab(\hat{p}_{ij}\cdot\hat{p}_k) - \vec{p}_{i\perp}^{\,2}\right)\delta(z-a)$$

$$\times\,\delta\left(\frac{-2\lambda\,\hat{p}_k\hat{p}_{ij}}{1-\lambda} - 2p_i\left(\tilde{p}_{ij} - \frac{\lambda\hat{p}_k}{1-\lambda}\right)\right) =$$

$$= \frac{d\lambda\,dz\,(2\tilde{p}_{ij}\cdot\tilde{p}_k)(\hat{p}_{ij}\cdot\hat{p}_k)}{(1-\lambda)\,(2\pi)^{d-1}}\quad da\,db\,\frac{p_\perp^{d-3}}{2p_\perp}\,dp_\perp^2\,d\Omega_{d-2}$$

$$\delta\left(2aB\,(\hat{p}_{ij}\cdot\hat{p}_k) - \vec{p}_{i\perp}^{\,2}\right)\delta(z-a)$$

$$\delta\left(\frac{-2\lambda\,\tilde{p}_k\tilde{p}_{ij}}{1-\lambda} - 2p_i\hat{p}_{ij} + \frac{\lambda}{1-\lambda}\,2p_i\tilde{p}_k\right)$$

$$p_i\cdot\hat{p}_{ij} = B\,\tilde{p}_{ij}\hat{p}_k \qquad p_i\hat{p}_k = a\,\hat{p}_{ij}\cdot\hat{p}_k \Rightarrow$$

$$X = \frac{d\lambda\, dz}{(1-\lambda)(2\pi)^{d-1}} \cancel{\left(2\hat{p}_{ij}\hat{p}_k\right)} \left(\hat{p}_{ij}\cdot\hat{p}_k\right) \cdot \cancel{da}\, \cancel{dB} \left(p_\perp^2\right)^{\frac{d-4}{2}}$$

$$d\Omega_{d-2}\ \cancel{d\vec{p}_{i\perp}^{\,2}}\ \cancel{\delta\left(2ab\left(\hat{p}_{ij}\cdot\hat{p}_k\right) - \vec{p}_{i\perp}^{\,2}\right)}\ \cancel{\delta(z-a)}\ \cancel{da}$$

$$\cancel{\delta\left(-\frac{\lambda}{1-\lambda}\hat{p}_k\hat{p}_{ij} - B + \frac{\lambda}{1-\lambda} z\right)}\ \cancel{\frac{1}{2\hat{p}_{ij}\hat{p}_k}}$$

$$\Rightarrow \quad B = -\frac{\lambda}{1-\lambda}(1-z) \qquad \vec{p}_{i\perp}^{\,2} = 2ab\left(\hat{p}_{ij}\cdot\hat{p}_k\right) \equiv$$

$$\equiv 2\left(\hat{p}_{ij}\cdot\hat{p}_k\right) z(1-z)\left(\frac{-\lambda}{1-\lambda}\right)$$

$$\Rightarrow \quad X = \frac{d\lambda\, dz}{(1-\lambda)(2\pi)^{d-1}} \left(\hat{p}_{ij}\cdot\hat{p}_k\right)\left[2\hat{p}_{ij}\cdot\hat{p}_k\ z(1-z)\left(-\frac{\lambda}{1-\lambda}\right)\right]^{-\varepsilon}$$

$d\Omega_{d-2}$ $\Rightarrow$ <u>The phase-space is:</u>

$$d\phi_{ijk} = d\phi_{[ij],k} \times \frac{d\lambda\, dz\, d\Omega_{d-2}}{(1-\lambda)^{d-1}\, 2(2\pi)^{d-1}} \left(2\hat{p}_{ij}\cdot\hat{p}_k\right)^{1-\varepsilon}\left[z(1-z)\left(\frac{-\lambda}{1-\lambda}\right)\right]^{-\varepsilon}$$

It is conventional to redefine

$$1-\lambda \equiv \frac{1}{1-y} \Rightarrow \frac{1}{1-\lambda} = 1-y \Rightarrow y = 1 - \frac{1}{1-\lambda} = \frac{-\lambda}{1-y}$$

$$\Rightarrow \quad y = \frac{-\lambda}{1-\lambda} \Rightarrow \quad dy = \frac{d\lambda}{(1-\lambda)^2} \Rightarrow$$

$$d\phi_{ijk} = d\phi_{[ij,k]} \times \frac{dy\, dz\, d\Omega_{d-2}}{2(2\pi)^{d-1}} \frac{1}{(1-y)^2}(1-y)^{d-1}$$

$$\times \left(2\hat{p}_{ij}\cdot\hat{p}_k\right)^{1-\varepsilon}\left[z(1-z)\cdot y\right]^{-\varepsilon} =$$

$$\Rightarrow \quad d\Phi_{ijk} = d\Phi_{[ij,k]} \times \frac{dy\, dz\, d\Omega_{d-2}}{2(2\pi)^{d-1}} (1-y)^{1-2\varepsilon} [z(1-z)y]^{-\varepsilon}$$
$$\times (2\hat{p}_{ij}\cdot\hat{p}_k)^{1-\varepsilon} \quad \blacksquare$$

The momenta mapping then:

$$p_i = z\hat{p}_{ij}^{\mu} + y(1-z)\hat{p}_k^{\mu} + p_{i\perp}^{\mu}$$
$$p_j = (1-z)\hat{p}_{ij}^{\mu} + yz\hat{p}_k^{\mu} \mp p_{i\perp}^{\mu}$$

And integration over restricted phase-space is straightforward, for this case

② ~~Final emitter / initial spectator:~~

~~We will study the case of interest: massless final state emitter & massive initial state spectator~~

~~We have the following mapping → for the massless case~~

$$\cancel{\tilde{p}_j = p_j + p_i - (1-x)\tilde{p}_a}$$
$$\cancel{\tilde{p}_a = x\, p_a}$$

~~$\Rightarrow \quad p_a^2 = \tilde{p}_a^2 = 0,$~~

~~but for massive initial state it doesn't work.~~

$$\cancel{\tilde{p}_j \equiv \tilde{p}_a = \Pi_{ji} - p_a}$$

~~Hence, what can be done?~~

~~$p_i + p_j - p_a \equiv \Pi_{ija} \leftarrow$ this is fixed.~~

Final emitter / initial spectator

Here, the idea is to avoid ~~b~~oosting the initial momentum; so that recoil must be absorbed by final state particles. Hence:

$$t = K + b + g \Rightarrow t = \tilde{K} + \tilde{b}$$

We then compute $\tilde{K}$, requiring that

$$\underline{K^2 = \tilde{K}^2} \quad \text{and} \quad \underline{\tilde{b}^2 = b^2} \Rightarrow$$

$$\tilde{K}^\mu = x_1 \left(K^\mu - \frac{(Kt)}{t^2} t^\mu \right) + x_2 t^\mu \Rightarrow$$

$$\frac{t \cdot \tilde{K}}{t^2} = x_2 \quad \text{and} \quad (t - \tilde{K})^2 = \tilde{b}^2 = \phi \Rightarrow$$

$$\frac{m_t^2 + m_K^2}{2} = t\tilde{K} \Rightarrow$$

$$\tilde{K} = x_1 \left(K^\mu - \frac{(Kt)}{t^2} t^\mu \right) + \cancel{tk} \frac{(m_t^2 + m_K^2)}{2t^2} t^\mu$$

To find x_1, we square this equation.

$$K^2 = x_1^2 \left(K^2 - \frac{(Kt)^2}{t^2} \right) + \left(\frac{m_t^2 + m_K^2}{2m_t^2} \right)^2 m_t^2$$

$$\Rightarrow x_1^2 \left(K^2 - (Kt)^2/m_t^2 \right) = K^2 - \frac{(m_t^2 + K^2)^2}{4m_t^2} \equiv$$

$$\equiv - \frac{(m_t^2 - K^2)^2}{4m_t^2} \Rightarrow$$

$$x_1 = \frac{1}{2} \sqrt{\frac{(m_t^2 - K^2)^2}{(Kt)^2 - m_t^2 K^2}} =$$

$$\boxed{x_1 = \frac{1}{2} \frac{(m_t^2 - K^2)}{\sqrt{(Kt)^2 - m_t^2 K^2}}}$$

$$\boxed{\hat{K} = \frac{(m_t^2 - k^2)}{2\sqrt{(kt)^2 - m_t^2 k^2}} \left(k^\mu - \frac{(kt)}{m_t^2} t^\mu\right) + \frac{(m_t^2 + m_K^2)}{2 m_t^2} t^\mu}$$

To get further, we need to understand decays $t \to b + W$ & $t \to b + W + g$

$$[dp] = \frac{d^{d-1}\vec{p}}{2p_0 (2\pi)^{d-1}}$$

$\boxed{t \to b + W}$

$$d\phi_2(t; W, b) = \int [dW][db](2\pi)^d \delta(t - W - b)$$

$$= \int [dW] \frac{d^d b}{(2\pi)^{d-1}} (2\pi)^d \delta(t - W - b)\, \delta(b^2) =$$

$$= \int [dW]\ 2\pi\, \delta((t-W)^2) = \int [dW]\ 2\pi\, \delta(m_t^2 + m_W^2 - 2tW)$$

$$\equiv \frac{d\Omega_W^{d-1}}{(2\pi)^{d-1}\, 2E_W}\, p_W^{D-2} \frac{dp_W}{dE_W}\, dE_W\ 2\pi\, \delta(m_t^2 + m_W^2 - 2m_t E_W)$$

$$\equiv \frac{d\Omega_W^{d-1}}{(2\pi)^{d-2}\, 2E_W} \frac{p_W^{D-2}}{\frac{p_W}{E_W}} \frac{1}{2m_t} \Bigg|_{E_W = \frac{m_t^2 + m_W^2}{2m_t}}$$

$$= \frac{d\Omega_W^{d-1}}{(2\pi)^{d-2}\, 4} \frac{p_W^{D-3}}{m_t}; \qquad p_W^2 = E_W^2 - m_W^2 = \frac{(m_t^2 - m_W^2)^2}{4m_t^2} \Rightarrow$$

$$p_W = \frac{m_t^2 - m_W^2}{2m_t};$$

Take $m_W^2 = m_t^2 r^2 \Rightarrow$

$$d\phi_2 \equiv \frac{d\Omega_W^{d-1}}{(2\pi)^{d-2}} \left[\frac{m_t}{2}(1 - r^2)\right]^{1-2\varepsilon} \frac{1}{4m_t} =$$

$$\boxed{d\phi_2 = \frac{d\Omega_W^{d-1}}{(2\pi)^{d-2}} \cdot \frac{1}{2^{3-2\varepsilon}}\, m_t^{-2\varepsilon} (1 - r^2)^{1-2\varepsilon}}$$

$\boxed{t \to b + W + g}$ We introduce constraints

$$\begin{cases} b \cdot g = \dfrac{m_t^2}{2}(1-r)^2 \cdot y \\ t \cdot g = \dfrac{m_t^2}{2}(1-r^2)(1-z) \end{cases}$$

Hence, we compute

$$d\Phi_3(t; W, b, g) = \int [dW][db][dg]\,(2\pi)^d \delta(t-W-b-g)$$
$$\times\, dy\ \delta\left(y - \frac{2bg}{m_t^2(1-r)^2}\right)$$
$$\times\, dz\ \delta\left(1-z - \frac{2tg}{m_t^2(1-r^2)}\right).$$

We "integrate out" the b-quark:

$$[db]\,(2\pi)^d\,\delta(t-W-b-g) \equiv 2\pi\,\delta((t-W-g)^2) \Rightarrow$$

$$(t-W-g)^2 \equiv (t-W)^2 - 2(t-W)g = (t-W)^2 - 2(b+g)\cdot g =$$
$$= (t-W)^2 - 2bg = m_t^2(1-r)^2 y.$$

$$2\pi\,\delta\left((t-W)^2 - 2(t-W)g\right) = 2\pi\,\delta\left(m_t^2[(1+r^2)-(1-r)^2 y] - 2tW\right)$$

Write $2bg \equiv 2E_b\,\omega_g\,(1-\cos\theta_{bg})$

with $E_b = (m_t - E_W - \omega_g) = m_t - \dfrac{m_t}{2}(1-z)(1-r^2)$

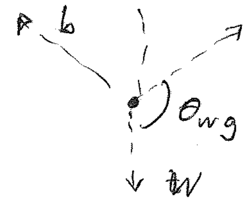
$$- \frac{m_t}{2}\left(1+r^2 - (1-r)^2 y\right) \Rightarrow$$

$$\boxed{E_b^* = \frac{m_t}{2}\left(1-r^2+(1-r)^2 y - (1-z)(1-r^2)\right)},$$

where we used

$$\omega_g^* = \frac{m_t}{2}(1-z)(1-r^2), \quad \& \quad E_W^* = \frac{m_t}{2}\left(1+r^2-(1-r)^2 y\right)$$

We have $\cos\theta_{wg}$ as a parameter:



$$d\Phi_3(t; W, b, g) = \int [dW][dg]\, 2\pi\, \delta\left(2m_t E_W^* - 2m_t E_W\right)$$
$$\times\, dy\, dz\, \delta\left(y - \cancel{2E_b}\, \frac{2bg}{m_t^2(1-r)^2}\right) \times$$
$$\times\, \delta\left(1 - z - \frac{2m_t \omega_g}{m_t^2(1-r^2)}\right)$$

We choose direction of the $\vec{W}$ momentum as d free parameter(s) and use relative angle between $\vec{W}$ & $\vec{g}$ to ~~move~~ parametrize the gluon direction.

$$d\Phi_3(t; W, b, g) \equiv \frac{d\Omega_W^{(d-1)}}{(2\pi)^{d-1}\, 2E_W}\, p_W^{D-2}\, \frac{E_W}{p_W}\, dE_W$$
$$\times \frac{d\Omega_g^{(d-2)}}{(2\pi)^{d-1}\, 2}\, d\cos\theta_g \left(1-\cos^2\theta_g\right)^{\frac{D-4}{2}}$$
$$\times\, \omega_g^{D-3}\, d\omega_g\, 2\pi\, \delta\left(2m_t E_W^* - 2m_t E_W\right)$$
$$dy\, dz\, \delta\left(1 - z - \frac{2m_t \cdot \omega_g}{m_t^2(1-r^2)}\right)$$
$$\delta\left(y - \frac{2bg}{m_t^2(1-r)^2}\right).$$

Now $\quad 2bg = 2(t-W-g)g = 2tg - 2Wg \Rightarrow$

$$y - \frac{2bg}{m_t^2(1-r)^2} \equiv y - \frac{2tg}{m_t^2(1-r)^2} + \frac{2E_W\omega_g(1-\beta_W\cos\theta_g)}{m_t^2(1-r)^2}$$

$$\equiv y - \frac{m_t^2(1-r^2)(1-z)}{m_t^2(1-r)^2} + \frac{2E_W\omega_g(1-\beta_W\cos\theta_g)}{m_t^2(1-r)^2}$$

$$\frac{m_t^2\left[y(1-r)^2-(1-r^2)(1-z)\right]+2E_W\omega_g(1-\beta_W\cos\theta_g)}{m_t^2(1-r)^2}$$

We therefore : integrate over $\cos\theta_g$ to remove the δ- function, & get a factor $\frac{m_t^2(1-r)^2}{2p_W\omega_g}$ & get a solution

for the angle $\quad \cos\theta_g^* \equiv \frac{m_t^2\left[y(1-r)^2-(1-r^2)(1-z)\right]+2E_W\omega_g}{2E_W\omega_g\beta_W}$

$$\Rightarrow 1-\cos\theta_g^{*2} \equiv \frac{4p_W^2\omega_g^2-\left[m_t^2\left(y(1-r)^2-(1-r^2)(1-z)\right)+2E_W\omega_g\right]^2}{4p_W^2\omega_g^2}$$

$$= \frac{4p_W^2\omega_g^2-\left[m_t^2\left(y(1-z)^2-(1-r^2)(1-z)\right)+2E_W\omega_g\right]^2}{4p_W^2\omega_g^2}$$

$$1-\cos\theta_g^{*2} \equiv \frac{y(1-z)^4\left[z(1-z)(1+r)^2-y\left(z+r^2(1-z)\right)\right]m_t^4}{4p_W^2\omega_g^2} =$$

$$= \frac{y(1-z)^4\left(z+r^2(1-z)\right)\left[y_{max}-y\right]m_t^4}{4p_W^2\omega_g^2}$$

$$y_{max} = z(1-z)(1+z)^2 / \left(z+r^2(1-z)\right).$$

Hence:

$$d\Phi_3(t,W,b,g) = \frac{d\vec{\Omega}_W^{(d-1)}}{(2\pi)^{d-1}\,2}\;\frac{d\vec{\Omega}_g^{(d-2)}}{(2\pi)^{d-1}\,2} \times \cancel{p_W^{*\,D-3}}\;\cancel{\omega_g^{*\,D-3}}$$

$$\times\;\frac{2\pi}{2m_t}\;\frac{m_t^2(1-r^2)}{2m_t}\;\frac{m_t^2(1-r)^2}{2\cancel{p_W^*}\cancel{\omega_g^*}}\;\frac{m_t^{-4\varepsilon}\,y^{-\varepsilon}(y_{max}-y)^{-\varepsilon}\left(z+r^2(1-z)\right)^{-\varepsilon}}{2^{-2\varepsilon}\,\cancel{p_W^{-2\varepsilon}}\,\cancel{\omega_g^{-2\varepsilon}}}$$

$$\times\,(1-r)^{-4\varepsilon}\;dy\,dz \;=$$

$$d\Phi_3 = \frac{d\vec{\Omega}_W^{(d-1)}\;d\vec{\Omega}_g^{(d-2)}}{(2\pi)^{2d-3}\;2^{5-2\varepsilon}}\;m_t^{2-4\varepsilon}(1-r^2)(1-r)^{2-4\varepsilon}\;\times$$

$$\times\;y^{-\varepsilon}(y_{max}-y)^{-\varepsilon}\left(z+r^2(1-z)\right)^{-\varepsilon}\;dy\,dz$$

$$d\Phi_3 = \frac{d\Phi_2}{\cancel{(2\pi)^d}}\cdot\frac{d\vec{\Omega}_g^{(d-2)}}{(2\pi)^{d-1}\cdot 4}\;m_t^{2-2\varepsilon}\;\frac{(1-r)^{2-4\varepsilon}}{(1-r^2)^{-2\varepsilon}}$$

$$\times\;y^{-\varepsilon}(y_{max}-y)^{-\varepsilon}\left(z+r^2(1-z)\right)^{-\varepsilon}\;dy\,dz.$$