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CT 20 0 Matlab for control

EEEN40010 Control Theory

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Problems

Problem 1

The plant is given in eq. (1).

$$\mathcal{H}(s) = \frac{s+3.6}{s^2+20s+64} \tag{1}$$

The poles of this plant are $\{-16, -4\}$ and it has a zero at -3.6.

The DC gain of the plant is 0.05625. This was found by evaluating eq. (1) at $s = 0$.

The steady-state value of the response to a unit step was calculated using the `residue` command in MATLAB to get the partial fraction expansion of the product of the laplace transform of the unit step and the transfer function. The denominator corresponding to the pole at 0 is the steady-state value, 0.05625.

There is a potentially dominant pole at -4. This should result in a 2% settling time of $\frac{4}{4} = 1\,\mathrm{s}$ and, if the pole truly is dominant, no overshoot.

The step response of this plant is given in fig. 1.1. Characteristics visible on the plot are reproduced in table 1.

Table 1: Characteristics visible in step response plot fig. 1.1

	Observed value	Predicted Value
Steady-state value	0.0563	0.05625
10% to 90% rise time	0.105 s	-
2% settling time	0.496 s	1 s
Percentage overshoot	3.54%	0%

The predicted and observed steady state values are identical to the level of precision given in the plot. The 2% settling time and the percentage overshoot values differ however. This is caused by the supposed dominant pole at -4 not being sufficiently dominant for the plant's response to approximate that of a first order system. The step response in the time domain is given in eq. (2). This shows that the non-dominant pole at -16 is 7× greater than the pole at -4; since this pole is four times more negative and has a greater magnitude than the supposed dominant pole, this will decrease the settling time. The dominant pole's diminished magnitude relative to the non-dominant pole will also cause overshoot to occur, as the time taken for the dominant pole to assert its dominance is increased.

$$h(t) = 0.0563 + 0.0083e^{-4t} - 0.0646e^{-16t} \tag{2}$$

As fig. 1.1 shows, the step response exhibits overshoot but no ringing.

Problem 2

The partial fraction expansion of the plant (given in eq. (1)) when the unit step is the input, as obtained using the `residue` command, is as follows:

$$\mathcal{H}(s) = \frac{s+3.6}{s^2+20s+64} \cdot \frac{1}{s} = -\frac{0.0646}{s+16} + \frac{0.0083}{s+4} + \frac{0.0563}{s}$$

Taking the inverse Laplace transform of the above, we arrive at an expression for the step response of the system.

$$h(t) = 0.0563 + 0.0083e^{-4t} - 0.0646e^{-16t}$$

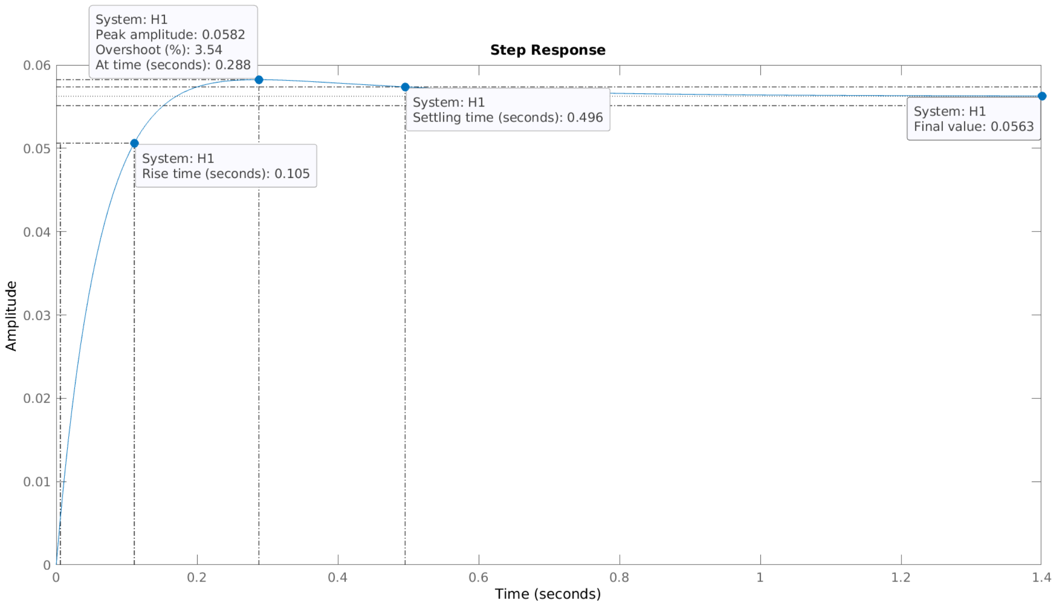

Figure 1.1: Step response for eq. (1)

Problem 3

$$\mathcal{H}_1 = \frac{1}{s^2 + 50s + 184} \tag{3}$$

$$\mathcal{H}_2 = \frac{s + 16}{s^2 + 50s + 184} \tag{4}$$

Problem 4

Figure 4.1 gives the step responses and settling times for $k \in \{0.1, 0.5, 2\}$ and $z \in \{0.1, 0.5, 2\}$. The steady state error is zero for every combination of positive k and z. There is zero percentage overshoot for any choice of k and z. The closed loop poles are given in ??

$$\mathcal{G}_c = \frac{k(s + z)}{s} \tag{5}$$

k	z	pole 1	pole 2	pole 3
0.100	0.100	−16.103	−3.997	−0.001
0.100	0.500	−16.100	−3.997	−0.003
0.100	2.000	−16.090	−3.998	−0.011
0.500	0.100	−16.513	−3.984	−0.003
0.500	0.500	−16.500	−3.986	−0.014
0.500	2.000	−16.453	−3.992	−0.055
2.000	0.100	−18.046	−3.944	−0.010
2.000	0.500	−18.000	−3.949	−0.051
2.000	2.000	−17.827	−3.970	−0.203

Table 2: Closed loop poles of plant eq. (1) in a unity negative feedback loop with controller eq. (5).

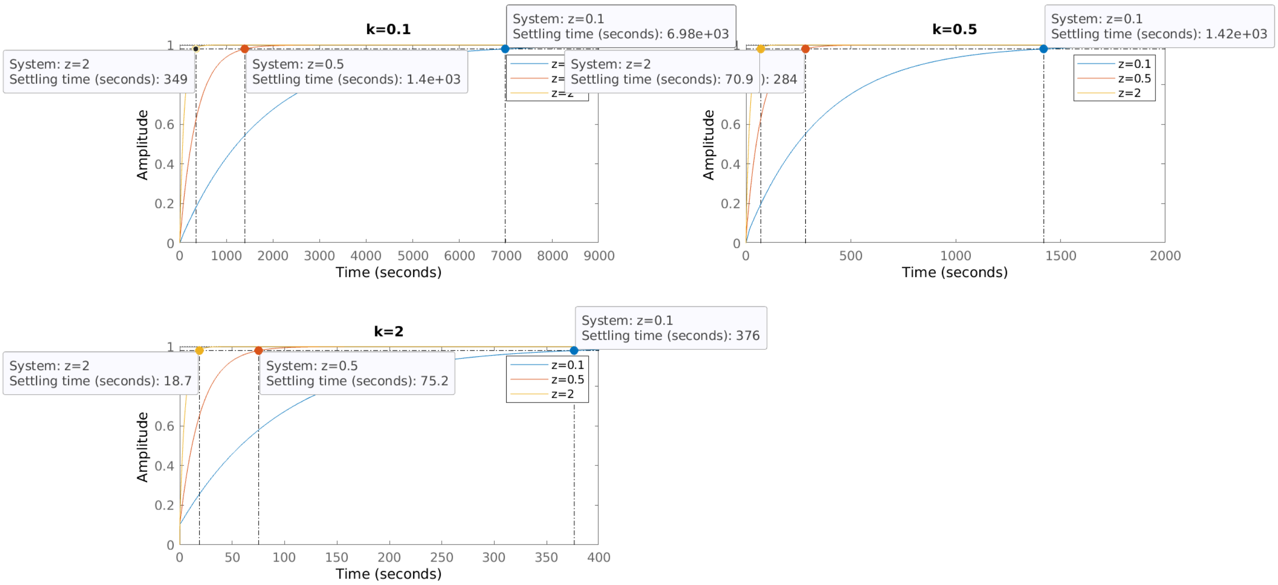

Figure 4.1: Step responses for various values of k—the proportional gain—and z— the controller zero.