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CT 20 0 Matlab for control

EEEN40010 Control Theory

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Problems

Problem 1

The plant is given in eq. (1).

$$\mathcal{H}(s) = \frac{s + 3.6}{s^2 + 20s + 64} \tag{1}$$

The poles of this plant are $\{-16, -4\}$ and it has a zero at -3.6.

The DC gain of the plant is 0.05625. This was found by evaluating eq. (1) at $s = 0$.

The steady-state value of the response to a unit step was calculated using the `residue` command in MATLAB to get the partial fraction expansion of the product of the laplace transform of the unit step and the transfer function. The denominator corresponding to the pole at 0 is the steady-state value, 0.05625.

There is a potentially dominant pole at -4. This should result in a 2% settling time of $\frac{4}{4} = 1\,\text{s}$ and, if the pole truly is dominant, no overshoot.

The step response of this plant is given in fig. 1.1. Characteristics visible on the plot are reproduced in table 1.

Table 1: Characteristics visible in step response plot fig. 1.1

	Observed value	Predicted Value
Steady-state value	0.0563	0.05625
10% to 90% rise time	0.105 s	-
2% settling time	0.496 s	1 s
Percentage overshoot	3.54%	0%

The predicted and observed steady state values are identical to the level of precision given in the plot. The 2% settling time and the percentage overshoot values differ however. This is caused by the supposed dominant pole at -4 not being sufficiently dominant for the plant's response to approximate that of a first order system. The step response in the time domain is given in eq. (2). This shows that the non-dominant pole at -16 is 7× greater than the pole at -4; since this pole is four times more negative and has a greater magnitude than the supposed dominant pole, this will decrease the settling time. The dominant pole's diminished magnitude relative to the non-dominant pole will also cause overshoot to occur, as the time taken for the dominant pole to assert its dominance is increased.

$$h(t) = 0.0563 + 0.0083e^{-4t} - 0.0646e^{-16t} \tag{2}$$

As fig. 1.1 shows, the step response exhibits overshoot but no ringing.

Problem 2

The partial fraction expansion of the plant (given in eq. (1)) when the unit step is the input, as obtained using the `residue` command, is as follows:

$$\mathcal{H}(s) = \frac{s + 3.6}{s^2 + 20s + 64} \cdot \frac{1}{s} = -\frac{0.0646}{s + 16} + \frac{0.0083}{s + 4} + \frac{0.0563}{s}$$

Taking the inverse Laplace transform of the above, we arrive at an expression for the step response of the system.

$$h(t) = 0.0563 + 0.0083e^{-4t} - 0.0646e^{-16t}$$

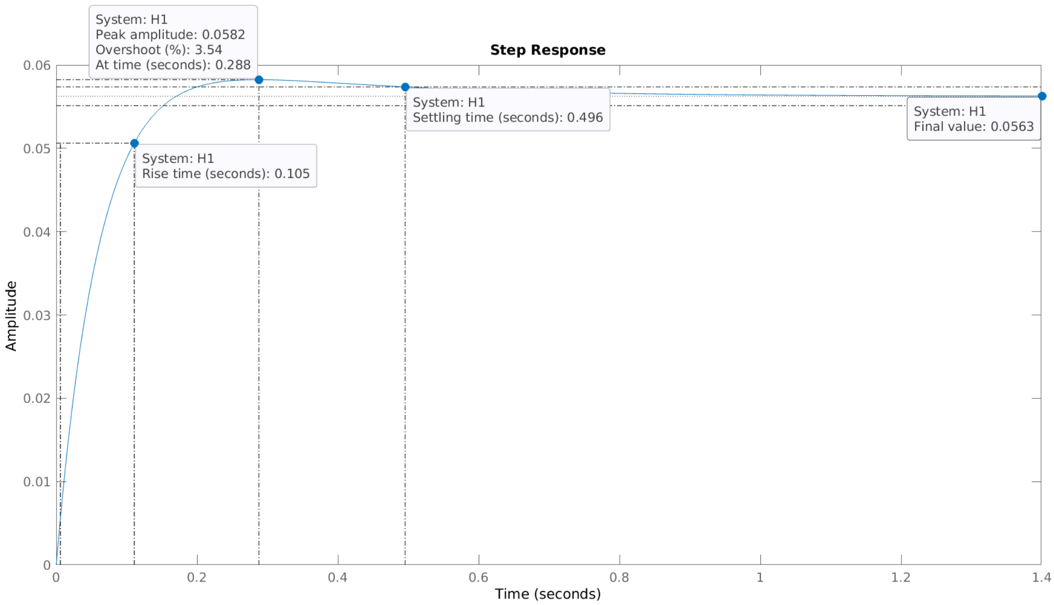

Figure 1.1: Step response for eq. (1)

Problem 3

$$\mathcal{G}_1 = \frac{1}{s^2 + 50s + 184} \tag{3}$$

$$\mathcal{G}_2 = \frac{s + 16}{s^2 + 50s + 184} \tag{4}$$

Predicted values

Evaluating the plant transfer functions at $s = 0$ gives dc gains of $\mathcal{G}_{1,0} = 5.43 \times 10^{-3}$ and $\mathcal{G}_{2,0} = 86.96 \times 10^{-3}$.

The formula $t_s \simeq 4\tau = \frac{4}{\sigma}$ gives approximate 2% settling times for the plants—assuming they can be approximated as first-order systems with a pole at $-\sigma$—of $t_{s,1} = t_{s,2} = 1\,\text{s}$.

As the plants are being assumed to be first order systems (i.e. they have a dominant pole), there should be 0% overshoot for either.

Matlab calculations

Figure 3.1 gives the values of the steady-state response, the settling time, and the percentage overshoot for both plants.

The steady state values for booth plants are identical to the predicted values to three significant digits.

The 2% settling time of the first plant is identical to the predicted value. The settling time of the second plant differs by 0.071 (7.1% error) which is quite close.

Neither plant exhibits any overshoot.

Using the `residue` command, the poles of both systems are at -4 and -46. This gives a partial fraction expansion for both systems (with A and B differing) of

$$G(s) = \frac{A}{s+4} + \frac{B}{s+46}$$

Via the inverse laplace transform, we get the time domain representation:

$$g(t) = Ae^{-4t} + Be^{-46t}$$

Filling in values for A and B for each plant:

$$g_1(t) \simeq 0.0238e^{-4t} - 0.0238e^{-46t}$$
$$g_2(t) \simeq 0.2857e^{-4t} + 0.7143e^{-46t}$$

The term with the time constant $\tau = \frac{1}{46}$ will decay much faster than the term with $\tau = \frac{1}{4}$, thus both plants will exhibit behaviour very similar to a first order system i.e. no overshoot.

The step response of both plants is given by the product of their transfer function with the laplace transform of the step response, $\frac{1}{s}$. The inverse laplace transform of the equation formed from the partial fraction expansion of

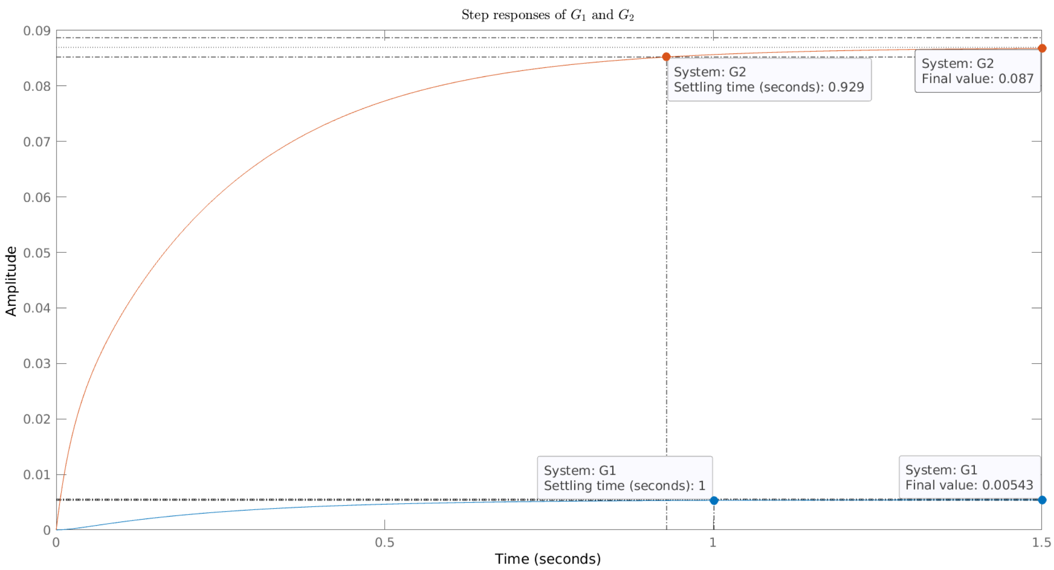

Figure 3.1: Step responses of eqs. (3) and (4)

the result is the time-domain step response.

$$\mathcal{G}_1 = \frac{0.0005}{s+46} - \frac{0.0060}{s+4} + \frac{0.0054}{s}$$

$$\mathcal{G}_2 = -\frac{0.0155}{s+46} - \frac{0.0714}{s+4} + \frac{0.0870}{s}$$

Applying inverse Laplace

$$g_1 = 0.0054 - 0.0060e^{-4t} + 0.0005e^{-46t}$$

$$g_2 = 0.0870 - 0.0714e^{-4t} - 0.0155e^{-46t}$$

Problem 4

Figure 4.1 gives the step responses and settling times for $k \in \{0.1, 0.5, 2, 10\}$ and $z \in \{0.1, 0.5, 2\}$. The steady state error is zero for every combination of positive k and z. There is zero percentage overshoot for any choice of k and z. The closed loop poles are given in table 2.

$$\mathcal{G}_c = \frac{k(s+z)}{s} \tag{5}$$

k	z	p_2	p_1	p_0	t_s
0.100	0.100	−16.103	−3.997	−0.001	6980
0.100	0.500	−16.100	−3.997	−0.003	1400
0.100	2.000	−16.090	−3.998	−0.011	349
0.500	0.100	−16.513	−3.984	−0.003	1420
0.500	0.500	−16.500	−3.986	−0.014	284
0.500	2.000	−16.453	−3.992	−0.055	70.9
2.000	0.100	−18.046	−3.944	−0.010	376
2.000	0.500	−18.000	−3.949	−0.051	75.2
2.000	2.000	−17.827	−3.970	−0.203	18.7
10.000	0.100	−26.142	−3.822	−0.036	96.2
10.000	0.500	−25.986	−3.833	−0.181	19.2
10.000	2.000	−25.384	−3.886	−0.730	4.75

Table 2: Closed loop poles p_0, p_1, p_2 of plant eq. (1) in a unity negative feedback loop with controller eq. (5). p_0 is the dominant pole in each case.

As the table shows, the dominant poles p_0 come to dominate more as both k and z increase. For lower values of k, the system is

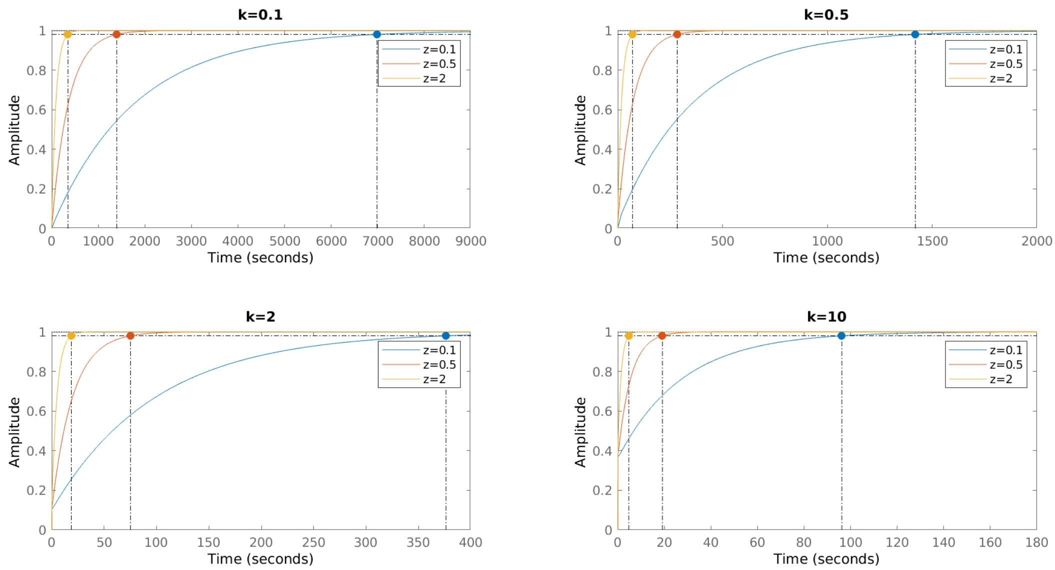

Figure 4.1: Step responses of plant eq. (1) in a unity negative feedback loop with controller eq. (5) for various values of k—the proportional gain—and z— the controller zero.

very sluggish, with settling times being up to several thousand seconds, contrasting with the much quicker times for higher values of k.