UCD / School of Engineering and Architecture / Electronic Engineering

CT 20 1
Matlab for control

EEEN40010 Control Theory

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Problems

Problem 1

Problem 2

$$F(t) = m\frac{d^2}{dt^2}y + \delta\frac{d}{dt}y + ky + \epsilon y^3 \tag{1}$$
$$m = a + b + c = 16$$
$$\delta = 5.3c = 31.8$$
$$k = 0.5(a + 2c) = 8$$
$$\epsilon : \text{nonlinearity parameter}$$

Equation 1: Global model of spring-mass-damper system with $F(t)$ as input and $y(t)$ as output.

2|1 Linearised system model

$$\begin{aligned} y &= 0 \\ \frac{d}{dt}y &= 0 \end{aligned} \tag{2}$$

To linearise eq. (1) about operating point eq. (2) I first replaced the output $y(t)$ with its operating point value (denoted Y_0) and a quantity representing a small deviation about its operating point (denoted $\tilde{y}$) :

$$y(t) = Y_0 + \tilde{y}(t)$$

Subbing this expression into eq. (1) we arrive at the following equation:

$$m\frac{d^2}{dt^2}(Y_0+\tilde{y}) + \delta\frac{d}{dt}(Y_0+\tilde{y}) + k(Y_0+\tilde{y}) + \epsilon(Y_0+\tilde{y})^3 = F(t)$$

Which—neglecting powers of $\tilde{y}$ as these would be very small for small deviations from the operating point, neglecting derivatives of Y_0 as Y_0 is constant, and neglecting Y_0 as $Y_0 = 0$—becomes the following:

$$m\frac{d^2}{dt^2}\tilde{y} + \delta\frac{d}{dt}\tilde{y} + k\tilde{y} = F(t) \tag{3}$$

Let the elements of the state vector $\boldsymbol{x}$ be the deviation of the deflection $\tilde{y}$ and its first derivative $\frac{d}{dt}\tilde{y}$:

$$\boldsymbol{x} = \begin{bmatrix} \tilde{y} \\ \frac{d}{dt}\tilde{y} \end{bmatrix} \tag{4}$$

To find an expression for the derivative of the state vector $\dot{\boldsymbol{x}}$, equations are derived as follows:

$$\begin{aligned} \frac{d}{dt}x_1 &= x_2 \\ \frac{d}{dt}x_2 &= \frac{1}{m}\left(F(t) - \delta x_2 - k\tilde{y}\right) \end{aligned}$$

Representing the input $F(t)$ by its operating point value of $F_0 = 0$ and its small deviation from this value by $\tilde{F}$ we arrive at the following representation of $\dot{\boldsymbol{x}}$:

$$\begin{aligned} \dot{\boldsymbol{x}} &= \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{\delta}{m} \end{bmatrix}\boldsymbol{x} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}\tilde{F} \\ &= A\boldsymbol{x} + B\boldsymbol{u} \\ \boldsymbol{u} &= \begin{bmatrix}\tilde{F}\end{bmatrix} : \text{input vector} \end{aligned} \tag{5}$$

As the output y in this model is just the deviation $\tilde{y}$ the output is given by the following equation:

$$y = \begin{bmatrix}10\end{bmatrix}\boldsymbol{x} + \begin{bmatrix}0\end{bmatrix}\boldsymbol{u}$$

The transfer function of this local LTI model was calculated in MATLAB using the `ss` and `tf` commands:

$$\mathcal{G}_{p,lin} = \frac{0.0625}{s^2 + 1.987s + 0.5} \tag{6}$$

The characteristics of the system's response to a step input are given in table 1.

Characteristic	Value
Steady-state error	0.8752
$t_{s,2\%}$	13.8878 s
PO%	0

Table 1: Step characteristics of the local LTI model of the plant eq. (6).

2|2 PID Controller Design

The implications of the design specifications are given below:

i. **Closed loop system must be stable.** All poles and zeroes of the closed loop system must have negative real part.

ii. **Zero steady-state error to a step input.** The open-loop system must have a pole at zero. This implies the presence of an I term in the controller as the plant does not have a pole at zero.

iii. $t_{s,2\%} \leq 60\% t_{s,2\%,plant}$. The settling time must be less than 8.3327 s. Approximating as a first-order system, this requirement implies a pole at $\sigma < -\frac{4}{t_s} = -0.48$.

iv. **Percentage overshoot less than 50% that of the plant or 25%, whichever is greater.** The plant's overshoot is 0, so the controller must ensure that the closed loop system exhibits less than 25% overshoot.

First I tried a PI controller, as these were thus far the only necessary terms. The equation for this controller is given below:

$$\mathcal{G}_c = \frac{k(s+z)}{s} \tag{7}$$

I chose z to be 0.5 to satisfy the settling time requirement. Using `rlocfind` I calculated the gain for there to be a dominant pole at $\sigma = -0.7$ in the open-loop system. I chose this value as it seemed reasonably greater in magnitude than the required $\sigma = -0.48$ to give some margin for errors in the assumption governing the settling time requirement. I chose a real pole as introducing a complex pair would have added oscillations unnecessarily into the system. The value of gain returned was ~ 23.

The final controller equation is the following:

$$\mathcal{G}_c(s) = \frac{23(s + 0.5)}{s} \tag{8}$$

The step response of the closed loop system is given in fig. 2.1 and the associated characteristics in table 2. As the table shows, the system is stable (the real part of all poles is negative), the steady state error is zero (final value is one), the 2% settling time is less than the requisite 60% of the plant's settling time given in table 1, and the percentage overshoot is less than the requisite 25%.

Characteristic	Value
Poles	$-0.3941 \pm 0.4038\mathrm{i}, -1.1994$
Zeroes	-0.5
Steady-state error	0
$t_{s,2\%}$	6.1386 s
PO%	18.3072%

Table 2: Step characteristics of the closed loop system of G_c in series with $G_{p,lin}$ with ideal feedback.

2|3 Evaluation of Failure Mechanisms

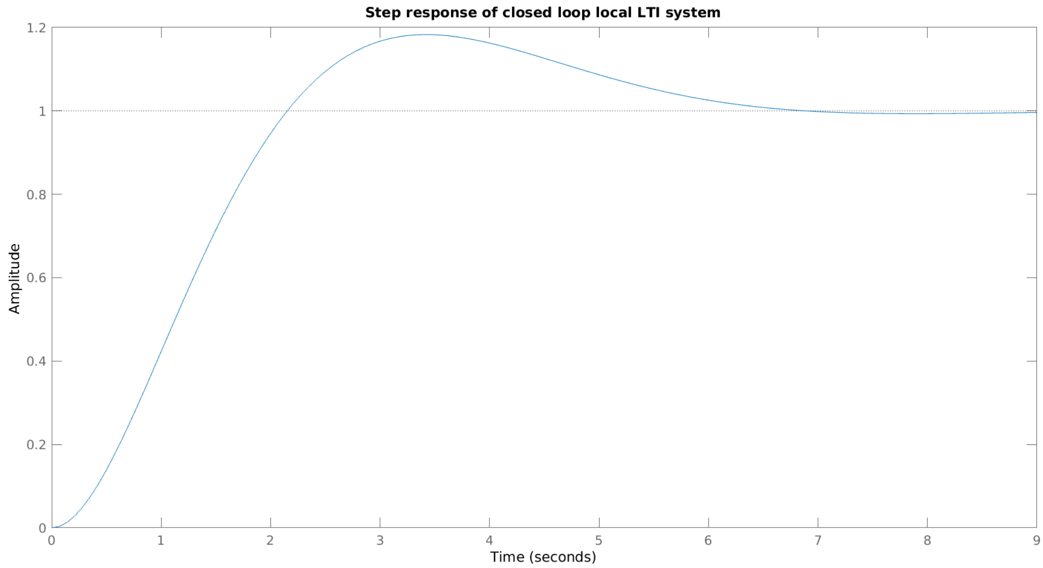

Figure 2.1: Step response of closed loop system of G_c in series with $G_{p,lin}$ with ideal feedback.