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CT 20 1
Matlab for control

EEEN40010 Control Theory

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Problems

Problem 1

$$\mathcal{G}_p = \frac{24(s+26)}{(s+50.8)(s+16.4)} \tag{1}$$

To achieve a closed loop system with a controller in series with the plant given in eq. (1) with the following specifications I implemented a PI controller. The step response of the plant is given in fig. 1.1

i. **Stable closed loop system.** This translates to all poles and zeroes of the closed loop system being in the left half plane.

ii. **Zero steady-state error to a step input.** This translates to an open-loop pole at zero. Since the plant does not already have a pole at zero, it must be introduced as an I term in the controller. The plant's steady-state error is 0.2516.

iii. **Closed loop system 2% settling time** $\leq 80\%$ **that of the plant.** Approximating the closed loop system as a first-order system (assuming it has a single dominant real pole) allows an approximate equation between the dominant pole and the 2% settling time:

$$\sigma > \frac{4}{t_{s,2\%}} = \frac{4}{0.8t_{s,2\%,plant}} = \frac{4}{0.1612\,\mathrm{s}} = 24.8$$

iv. As the percentage overshoot of the plant is zero, the **percentage overshoot must be less than 25%**. To meet this specification, I will minimise the complex part of any poles introduced as the plant has no complex poles.

Using the P and I terms of the PID controller, the following controller transfer function is used:

$$\mathcal{G}_c(s) = \frac{k(s+z)}{s} \tag{2}$$

I chose a value of $\sigma = -28 < -24.8$ for z to satisfy the 2% settling time requirement. Despite this, the resulting system did not satisfy the settling time requirement with $t_s = 0.1747\,\mathrm{s} > 0.1612\,\mathrm{s}$. This was due to the zero at $\sigma = -26$, close to the introduced zero and to the minimum pole location for the settling time requirement. This zero was slowing down the response. Using the `rlocfind` command on the open loop system, I calculated the gain necessary for a pole near enough the plant zero to offset its slowing effects. This pole was at $\sigma - 26.6$ and the necessary gain was 18.

The resulting controller equation is given below.

$$\mathcal{G}_c(s) = \frac{18(s+28)}{s}$$

The step response of the closed loop system of this controller in series with the plant in a unity feedback loop is given in fig. 1.2.

i. The closed loop system is stable, all poles $(-446.69, -26.26 \pm 3.83\mathrm{i})$ are negative.

ii. The steady-state value of the system is 1, so the steady-state error is zero.

iii. The 2% settling time ($0.0147\,\mathrm{s}$) is $\sim 7\%$ that of the plant ($0.2016\,\mathrm{s}$). This is much faster than the necessary 80% of the plant's settling time and may be unrealistically fast, depending on the implementation of this system.

iv. The percentage overshoot is nil, satisfying the requirement.

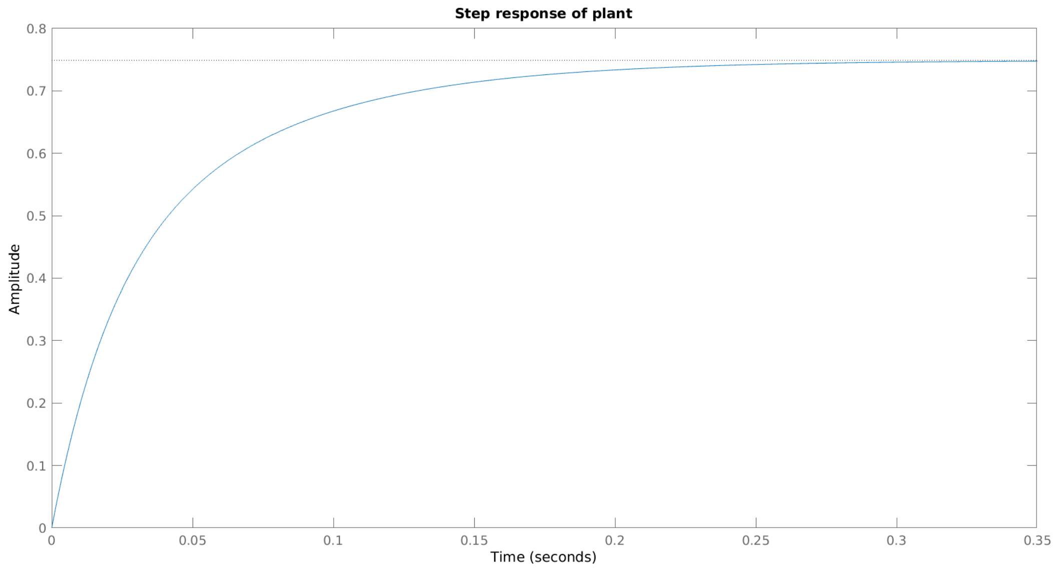

Figure 1.1: Step response of plant

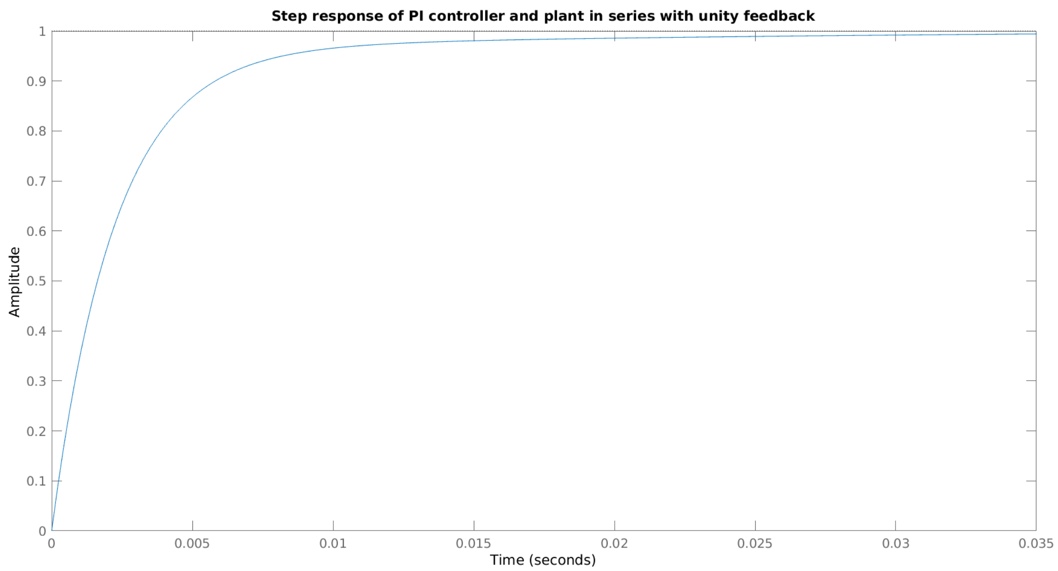

Figure 1.2: Step response of closed loop system.

Problem 2

$$\begin{aligned} F(t) &= m\frac{d^2}{dt^2}y + \delta\frac{d}{dt}y + ky + \epsilon y^3 \quad (3)\\ m &= a+b+c = 16\\ \delta &= 5.3c = 31.8\\ k &= 0.5(a+2c) = 8\\ \epsilon &: \text{ nonlinearity parameter} \end{aligned}$$

Equation 3: Global model of spring-mass-damper system with $F(t)$ as input and $y(t)$ as output.

2|1 Linearised system model

$$\begin{aligned} y &= 0\\ \frac{d}{dt}y &= 0 \end{aligned} \quad (4)$$

To linearise eq. (3) about operating point eq. (4) I first replaced the output $y(t)$ with its operating point value (denoted Y_0) and a quantity representing a small deviation about its operating point (denoted $\tilde{y}$) :

$$y(t) = Y_0 + \tilde{y}(t)$$

Subbing this expression into eq. (3) we arrive at the following equation:

$$m\frac{d^2}{dt^2}(Y_0+\tilde{y})+\delta\frac{d}{dt}(Y_0+\tilde{y})+k(Y_0+\tilde{y})+\epsilon(Y_0+\tilde{y})^3 = F(t)$$

Which—neglecting powers of $\tilde{y}$ as these would be very small for small deviations from the operating point, neglecting derivatives of Y_0 as Y_0 is constant, and neglecting Y_0 as $Y_0 = 0$—becomes the following:

$$m\frac{d^2}{dt^2}\tilde{y} + \delta\frac{d}{dt}\tilde{y} + k\tilde{y} = F(t) \quad (5)$$

Let the elements of the state vector $\boldsymbol{x}$ be the deviation of the deflection $\tilde{y}$ and its first derivative $\frac{d}{dt}\tilde{y}$:

$$\boldsymbol{x} = \begin{bmatrix} \tilde{y} \\ \frac{d}{dt}\tilde{y} \end{bmatrix} \quad (6)$$

To find an expression for the derivative of the state vector $\dot{\boldsymbol{x}}$, equations are derived as follows:

$$\begin{aligned} \frac{d}{dt}x_1 &= x_2\\ \frac{d}{dt}x_2 &= \frac{1}{m}\left(F(t) - \delta x_2 - k\tilde{y}\right) \end{aligned}$$

Representing the input $F(t)$ by its operating point value of $F_0 = 0$ and its small deviation from this value by $\tilde{F}$ we arrive at the following representation of $\dot{\boldsymbol{x}}$:

$$\begin{aligned} \dot{\boldsymbol{x}} &= \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{\delta}{m} \end{bmatrix}\boldsymbol{x} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}\tilde{F} \quad (7)\\ &= A\boldsymbol{x} + B\boldsymbol{u}\\ \boldsymbol{u} &= \begin{bmatrix}\tilde{F}\end{bmatrix} \text{ : input vector} \end{aligned}$$

As the output y in this model is just the deviation $\tilde{y}$ the output is given by the following equation:

$$y = \begin{bmatrix}10\end{bmatrix}\boldsymbol{x} + \begin{bmatrix}0\end{bmatrix}\boldsymbol{u}$$

The transfer function of this local LTI model was calculated in MATLAB using the `ss` and `tf` commands:

$$\mathcal{G}_{p,lin} = \frac{0.0625}{s^2 + 1.987s + 0.5} \quad (8)$$

The characteristics of the system's response to a step input are given in table 1.

Characteristic	Value
Steady-state error	0.8752
$t_{s,2\%}$	13.8878 s
PO%	0

Table 1: Step characteristics of the local LTI model of the plant eq. (8).

2|2 PID Controller Design

The implications of the design specifications are given below:

i. **Closed loop system must be stable.** All poles and zeroes of the closed loop system must have negative real part.

ii. **Zero steady-state error to a step input.** The open-loop system must have a pole at zero. This implies the presence of an I term in the controller as the plant does not have a pole at zero.

iii. $t_{s,2\%} \leq 60\% t_{s,2\%,plant}$. The settling time must be less than 8.3327 s. Approximating as a first-order system, this requirement implies a pole at $\sigma < -\frac{4}{t_s} = -0.48$.

iv. **Percentage overshoot less than 50% that of the plant or 25%, whichever is greater.** The plant's overshoot is 0, so the controller must ensure that the closed loop system exhibits less than 25% overshoot.

First I tried a PI controller, as these were thus far the only necessary terms. The equation for this controller is given below:

$$\mathcal{G}_c = \frac{k(s+z)}{s} \quad (9)$$

I chose z to be 0.5 to satisfy the settling time requirement. Using `rlocfind` I calculated the gain for there to be a dominant pole at $\sigma = -0.7$ in the open-loop system. I chose this value as it seemed reasonably greater in magnitude than the required $\sigma = -0.48$ to give some margin for errors in the assumption governing the settling time requirement. I chose a real pole as introducing a complex pair would have added oscillations unnecessarily into the system. The value of gain returned was ~ 23.

The final controller equation is the following:

$$\mathcal{G}_c(s) = \frac{23(s+0.5)}{s} \tag{10}$$

The step response of the closed loop system is given in fig. 2.1 and the associated characteristics in table 2. As the table shows, the system is stable (the real part of all poles is negative), the steady state error is zero (final value is one), the 2% settling time is less than the requisite 60% of the plant's settling time given in table 1, and the percentage overshoot is less than the requisite 25%.

Characteristic	Value
Poles	$-0.3941 \pm 0.4038\text{i}, -1.1994$
Zeroes	-0.5
Steady-state error	0
$t_{s,2\%}$	6.1386 s
PO%	18.3072%

Table 2: Step characteristics of the closed loop system of G_c in series with $G_{p,lin}$ with ideal feedback.

2|3 Evaluation of Failure Mechanisms

For the case where the gain fails to 1% of the nominal, the step response is shown in fig. 2.2. As the plot shows, the system is extremely slow ($t_s = 266\,\text{s}$), but exhibits no overshoot. This lack of overshoot implies that the system is failsafe, unless a failure condition is extremely sluggish response.

For the case where the gain fails to 500% of its nominal value, the step response is shown in fig. 2.3. Contrary to the previous failure case, the 2% settling time of the step response of this failure mode is not too dissimilar to the nominal value. Its overshoot is 49% however, nearly twice the specified value. This is most likely unsafe.

Judged by these two failure modes, the closed-loop system is not failsafe due to the magnitude of the overshoot exhibited by the fail-high mechanism and possibly also the extreme slowness exhibited by the fail-low mechanism.

2|4 Equations of motion

The PI controller is described by the following equation where E is the error signal and OP is the output of the controller:

$$OP = K_p E + K_i \int E$$

But $OP = F$ where F is the input to the plant. Letting them equal and differentiating to remove the integral we arrive at the following equation:

$$K_p \dot{E} + K_i E = m\dddot{y} + \delta\ddot{y} + (k + 3\epsilon y^2)\dot{y}$$

If the input to the controller, $E(t)$, is represented as the difference of the reference input x and the output y as in a unity negative feedback system, the above equation becomes the following:

$$K_p\dot{x} + K_i x = m\dddot{y} + \delta\ddot{y} + (k + 3\epsilon y^2 K_p)\dot{y} + K_i y$$

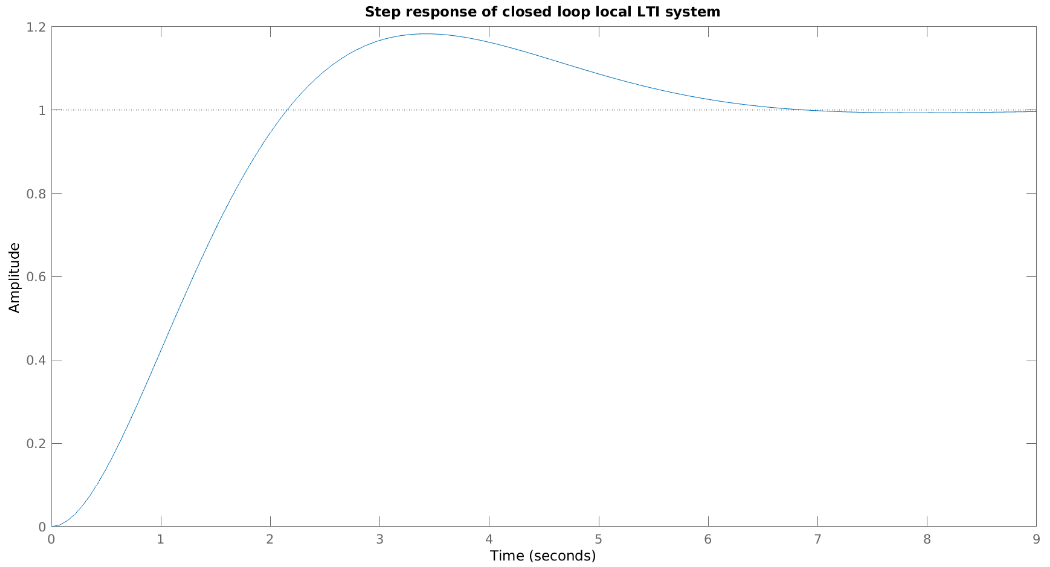

Figure 2.1: Step response of closed loop system of G_c in series with $G_{p,lin}$ with ideal feedback.

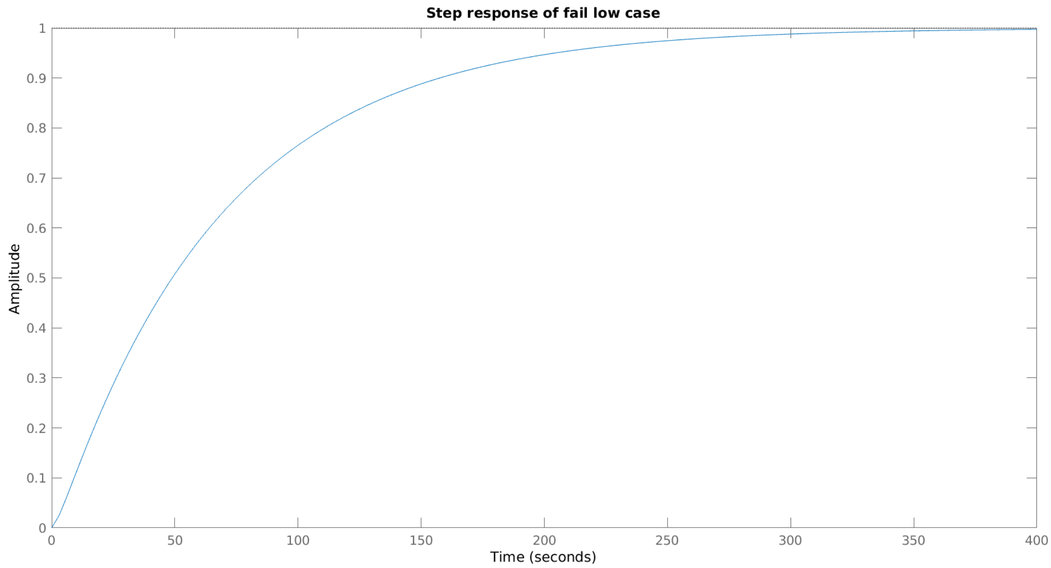

Figure 2.2: Step response of fail low scenario.

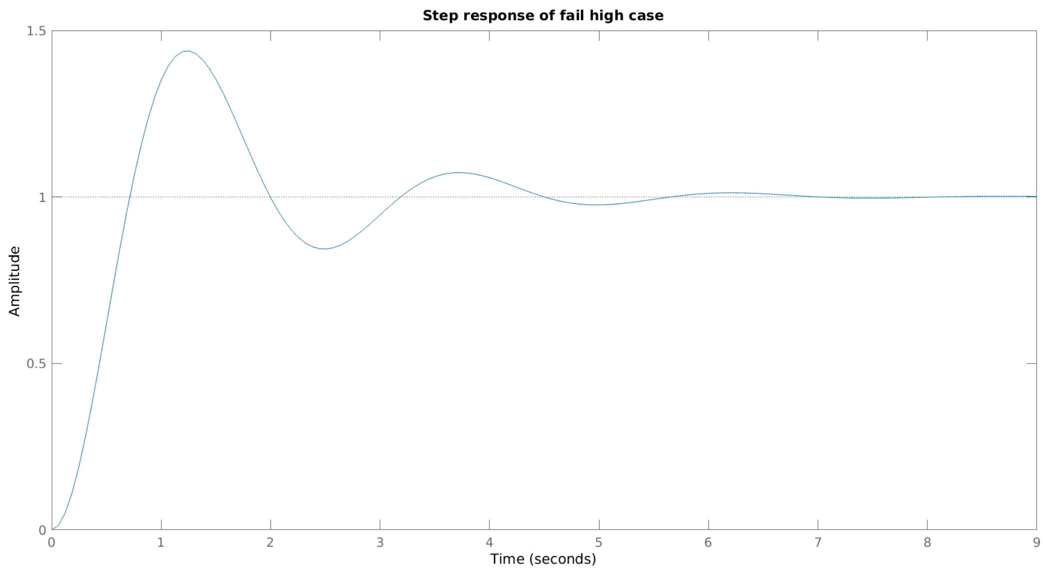

Figure 2.3: Step response of fail high scenario.