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CT 20 2
Frequency Domain Methods & Linear State Feedback

EEEN40010 Control Theory

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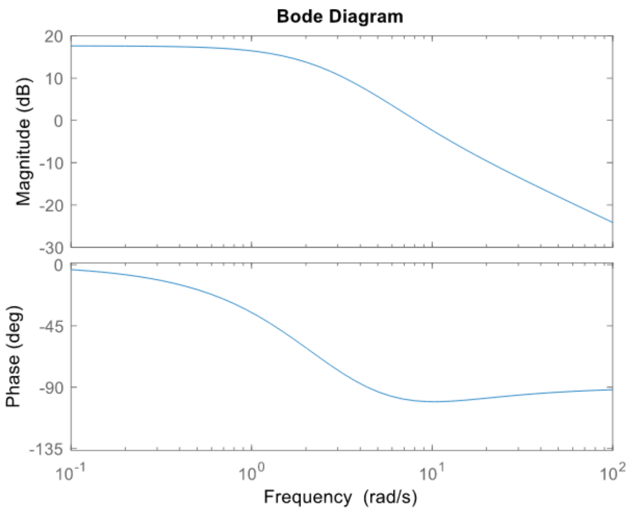

Figure 1.1: Bode plot of plant.

Problem 1

From fig. 1.1, the DC gain of the plant appears to be about 17 dB. This results in the following approximate steady state error:

$$\begin{aligned} e &\simeq \frac{1}{1+G_p(0)} \\ &\simeq \frac{1}{1+10^{\frac{17}{20}}} \\ &\simeq 12.4\% \end{aligned}$$

1|1 Margins

The gain margin is infinite as the phase is never less than 180°.

The phase at $\omega_g c \simeq 8\,\mathrm{rad\,s^{-1}}$ is $\sim -95°$, therefore the phase margin is $\sim -180° + 95° = 85°$.

1|2 Specifications

Closed loop system must be stable

For the closed loop system to be stable, both gain and phase margins must be positive. As the gain margin is infinite, any increase in gain will not destabilise this system. The phase margin is positive by 85°, this is a good margin to use during controller design.

Steady state error $< 5\%$

The steady state error of the closed loop system to a step input must be less than 5%. The minimum DC gain of the open loop system to accomplish this is calculated as follows:

$$\begin{aligned} G_o(0) &> \frac{1}{e} - 1 \\ &= 19 \simeq 25.6\,\mathrm{dB} \end{aligned}$$

The required controller gain is calculated from the following expression for the open loop gain:

$$\begin{aligned} & G_o(s) = G_c(s) + G_p(s) \\ \iff & G_c(0) = G_o(0) - G_p(0) \\ & \qquad = 25.6\,\mathrm{dB} - 17\,\mathrm{dB} \\ & \qquad = 8.6\,\mathrm{dB} \end{aligned}$$

I took 10 dB as the gain to give a margin.

With this controller gain, the open loop dc gain becomes 27 dB, shifting the gain plot in fig. 1.1 up 10 dB. The gain crossover frequency of the open loop system is about $20\,\mathrm{rad\,s^{-1}}$ (rising from about $8\,\mathrm{rad\,s^{-1}}$ in the closed loop system).

Percentage overshoot less than 5%

Approximating the system as a second order system, the relationship between the percentage overshoot and the damping ratio is given below:

$$PO100e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}$$

Solving this in MATLAB gives a minimum damping ratio of 0.69 for a percentage overshoot of 5%. To satisfy the specification with a decent margin I chose $\zeta = 0.72$.

Settling time $t_{s,2\%} \leq 1\,\mathrm{s}$

The settling time in relation to the natural frequency and the damping ratio is given by (expression for a second order system):

$$\begin{aligned} & t_{s,2\%} \simeq \frac{4}{\zeta\omega_n} \\ \iff & \omega_n \geq \frac{4}{\zeta t_{s,2\%}} \end{aligned}$$

Which translates to the following minimum gain crossover frequency:

$$\begin{aligned} \omega_g c &\geq \omega_n \sqrt{\sqrt{1+4\zeta^4} - 2\zeta^2} \\ &\geq 3.53\,\mathrm{rad\,s^{-1}} \end{aligned}$$

1|3 Controller

The controller that satisfies all of these conditions is a lead-lag controller with $\alpha = 0$, $\tau = 0$ and $k = 10\,\mathrm{dB} = 3.16$. The gain crossover frequency introduced by the gain satisfies the settling time and percentage overshoot specifications on its own..

$$G_c(s) = k, \qquad k = 3.16$$

Problem 2

$$
\begin{aligned}
A &= \begin{bmatrix} 5.91 & -8.5914 \\ 1.00 & 0 \end{bmatrix} \\
B &= \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\
C &= \begin{bmatrix} 64.8 & 425.088 \end{bmatrix} \\
D &= \begin{bmatrix} 0 \end{bmatrix}
\end{aligned} \tag{1}
$$

Equation 1: State space model of plant.

Problem 3

3|1 Specifications

Stability

All poles and zeroes should be in the left half plane. For the state space matrices, eigenvalues of $A-BK < 0$ and eigenvalues of $A - LC < 0$.

Settling time

The 2% settling time must be no greater than 40% that of the plant:

$$t_{s.2\%} \leq 0.41.8333\,\text{s} = 0.7333\,\text{s}$$

I will aim for a 2% settling time of 0.7 s. If the system is to behave as a first-order one with a dominant pole, that dominant pole must be to the left of $-\frac{4}{t_{s,2\%}} = -5.71$.

The plant has a zero at -6.806. To reduce the retarding effect of this zero I will introduce a pole near to it, at $p_1 = -6.7$.

Percentage overshoot

The plant's step response exhibits no overshoot. This specification then becomes that the closed loop step response must have a maximum of 25% overshoot. This translates to a damping ratio $\zeta = 0.4037$, I chose $\zeta = 0.5$ (PO $\sim$ 16%) to give a margin. From the roots of the characteristic polynomial (below) we get two suggested poles of the system:

$$s^2 + 2\zeta\omega_n s + \omega_n^2, \qquad \omega_n = \frac{4}{\zeta t_{s,2\%}}$$
$$s = -5.7143 \pm 9.8975\text{i}$$

To choose the above complex pair as poles would not allow me to satisfy the settling time specification with any margin—the real part of the pair is equal to the minimum magnitude requirement.

I will choose a pole at $p_2 = -6$ to satisfy both the PO and t_s requirements. The real pole should suppress oscillations and it is sufficiently negative that, were it dominant, the plant would be fast enough.

Steady state error

The closed loop system must have zero steady state error to a step input. To this end a pre-amplifier is introduced to scale the reference input by a constant amount (K is the gain matrix defined in the next section):

$$\bar{N} = \frac{-1}{C(A-(BK))^{-1}B} \qquad = 0.0955$$

3|2 Introduced poles

The poles of the plant are at −3.378 and −2.838. Both introduced poles are sufficiently negative to dominate these plant poles. The gains such that the closed loop system has the introduced poles $\{-6.7, -6\}$ were found using `place`. The gains below are the gains thus produced rounded up to the next integer gain:

$$K = [7, 32]$$

3|3 Observer

Observer gain

To implement the observer, first the gain of the difference between the system output and the observer output must be chosen. The aim is that the error reduce rapidly. As unnecessarily high gains should be avoided, I chose gains such as would introduce poles three times the greatest pole of the system as this appeared to produce acceptable results while not requiring very large gains. The simplified gains used are given below:

$$L = \begin{bmatrix} 2 \\ -0.3 \end{bmatrix}$$

These were aimed to correspond to poles at −21 and −22.

State space model

The matrices of the state space model of the closed loop system without the observer is given below:

$$A = \begin{bmatrix} -12.91 & -40.5914 \\ 1 & 0 \end{bmatrix}$$
$$B = \begin{bmatrix} 0.0955 \\ 0 \end{bmatrix}$$
$$C = \begin{bmatrix} 64.8 & 425.088 \end{bmatrix}$$
$$D = \begin{bmatrix} 0 \end{bmatrix}$$

The matrices of the state space model of the system with the observer are given below:

$$A = \begin{bmatrix} -12.91 & -40.5914 & 7 & 32 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -135.51 & -858.7674 \\ 0 & 0 & 20.44 & 127.5264 \end{bmatrix}$$
$$B = \begin{bmatrix} 0.0955 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$
$$C = \begin{bmatrix} 64.8 & 425.088 & 0 & 0 \end{bmatrix}$$
$$D = \begin{bmatrix} 0 \end{bmatrix}$$

3|4 Results

Characteristic	Value
Steady State Error	0.07%
$t_{s,2\%}$	0.6622 s
Overshoot	0%

Table 1: Step characteristics of closed loop system

Figure 3.1 gives the step response of the final closed loop system and table 1 some characteristics of the response.

All specifications have been met. The steady state error over the default MATLAB time range is in the fourth decimal place and when the step response is extended goes to zero. All introduced poles have real magnitudes less than 0 so the system is stable.

Sensitivity to model parameters

?? gives the magnitude of the error in the steady state value for deviations of ±10% in all parameters and fig. 3.3 gives the relative error in the 2% settling time for the same deviations.

The range of the settling time was from 5.52% below the specified value to 14.59% below. The settling time specification was always met.

The steady state error ranged from 06.87%

The overshoot was 0 for all deviations, satisfying the specification.

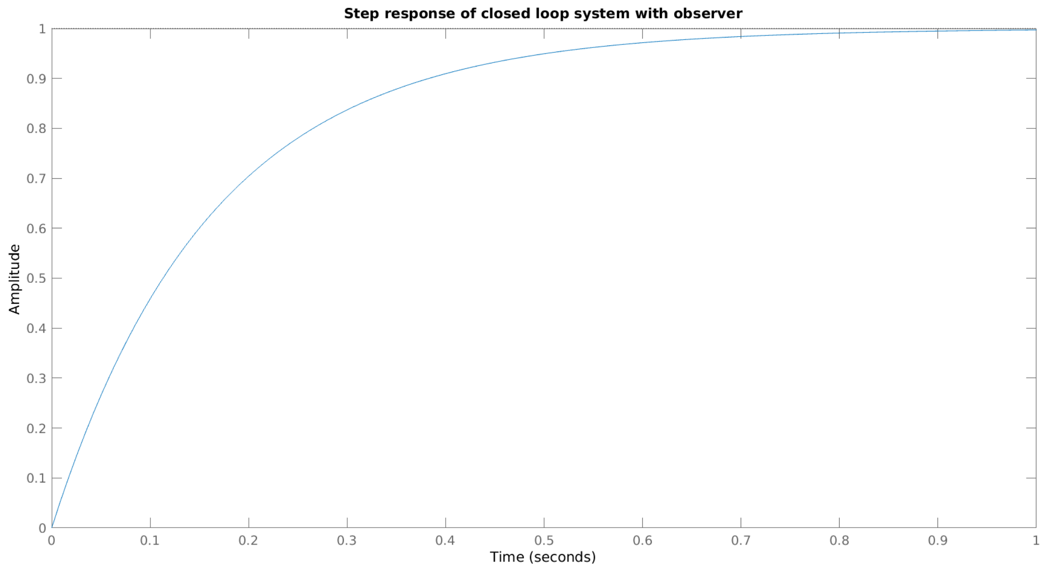

Figure 3.1: Step response of closed loop system with observer

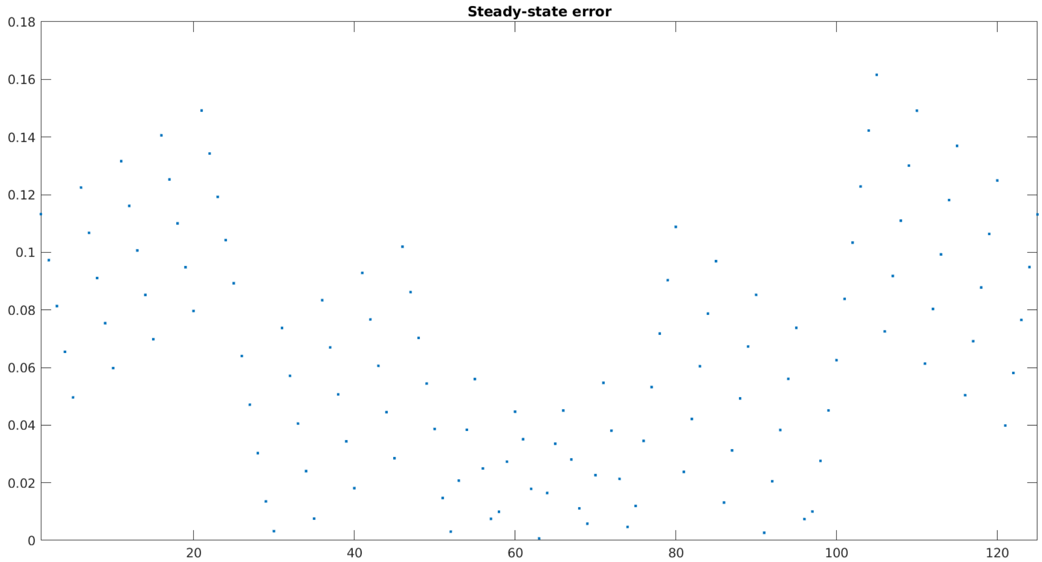

Figure 3.2: Steady state error.

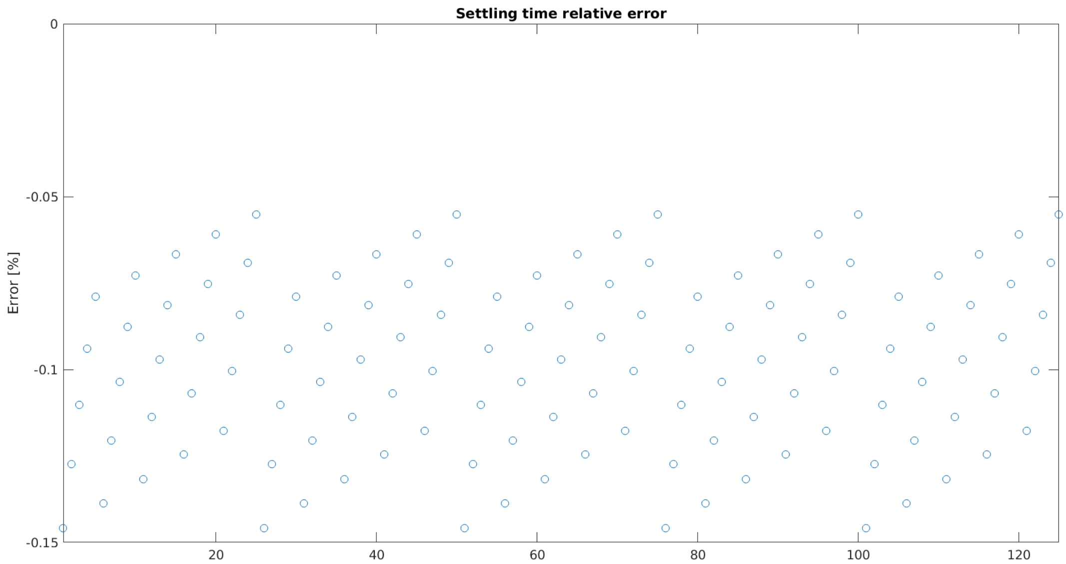

Figure 3.3: Relative error in the 2% settling time. Note that this is $\frac{t_s - t_{s,specified}}{t_{s,specified}}$.