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Submit until: 11.12.2018, 16:30

Numerical Simulations II
Homework assignment no. 7

These are analytic problems, no programming is required. We expect you to submit paper-work.

Problem 1 (1,5 p) :
The advection equation

$$\frac{\partial u}{\partial t} + V\frac{\partial u}{\partial x} = 0 \tag{1}$$

has, since it is a linear PDE with constant coefficients, plane wave solutions of the form

$$u(x,t) = \exp(\mathrm{i}(\omega t - kx))\,.$$

a) The frequency ω and the wavenumber k are related via the dispersion relation $\omega(k)$. What is the analytic dispersion relation for Eq. (1)?

b) What are the dispersion relations when we semi-discetize Eq. (1) by approximating the spatial derivative in the following three ways? If $\omega(k)$ becomes complex, please seperate real and complex parts explicitly in the final expression.

i)
$$\frac{\partial u(x,t)}{\partial x} \to \frac{u(x,t) - u(x-\Delta x,t)}{\Delta x} \tag{2}$$

ii)
$$\frac{\partial u(x,t)}{\partial x} \to \frac{u(x+\Delta x,t) - u(x,t)}{\Delta x} \tag{3}$$

iii)
$$\frac{\partial u(x,t)}{\partial x} \to \frac{u(x+\Delta x,t) - u(x-\Delta x,t)}{2\Delta x} \tag{4}$$

c) Produce two plots, one showing $\mathrm{Re}(\omega)$ vs. k, and the second one showing $|\omega|$ vs. k. For this choose $V = 1$ and $\Delta x = 0.5$. The largest k value that is of interest is $\pi/\Delta x$.