

We need to solve $\underline{\underline{M}} \cdot \underline{u}^{n+1} = \underline{u}^n$, where

$$\underline{\underline{M}} = \begin{bmatrix} 1+2\alpha & -\alpha & 0 & 0 & 0 & \dots \\ -\alpha & 1+2\alpha & -\alpha & 0 & 0 & \dots \\ 0 & -\alpha & 1+2\alpha & -\alpha & 0 & \dots \\ 0 & 0 & -\alpha & 1+2\alpha & \alpha & \dots \\ & 0 & & & & \ddots \\ & & 0 & & & & \ddots \\ & & & 0 & & & & \ddots \end{bmatrix}, \quad \underline{u}^n = \begin{bmatrix} u_0^n \\ u_1^n \\ \vdots \\ u_{N-1}^n \end{bmatrix}$$

The goal of **forward substitution** is to manipulate the system into the form

$$\begin{bmatrix} d_0 & -\alpha & 0 & 0 & 0 & \dots \\ 0 & d_1 & -\alpha & 0 & 0 & \dots \\ 0 & 0 & d_2 & -\alpha & 0 & \dots \\ 0 & 0 & 0 & d_3 & -\alpha & \dots \\ & & & & & \ddots \end{bmatrix} \cdot \underline{u}^{n+1} = \underline{\tilde{u}}^n.$$

1. What is d_0 ?

2. What is d_1 ?

3. What is \tilde{a}_1^n ?

4. What is d_j ? What is \tilde{a}_j^n ?

Once all d_j and \tilde{a}_j^n are known, we find
via backsubstitution u_{N-1}^{n+1} , u_{N-2}^{n+1} , ...

What is u_{N-1}^{n+1} ?

What is u_{N-2}^{n+1} ?

What is u_j^{n+1} ?