**#1** (10 Points)

Is the following function a proper distance function? Why? Explain your answer.

$$d(\mathbf{x}, \mathbf{y}) = \left(\sum_{i} |x_{i} - y_{i}|\right)^{3}$$

Hint: Measure the distance between (0,0), (0,1) and (1,1)

#### **Solution:**

Let us assume that,X=(0,0),Y=(0,1) and Z=(1,1)

For any distance function to work the following conditions must be satisfied:

$$d(x,y) \ge 0$$
 1. Non-negativity or separation axiom

$$d(x,y)=0 \Leftrightarrow x=y \hspace{1cm} \text{2.} \hspace{1cm} \text{Identity of indiscernible}$$

$$d(x,y) = d(y,x) \qquad \qquad \text{3.} \qquad \qquad \text{Symmetry} \\ d(x,z) \leq d(x,y) + d(y,z) \quad \text{4.} \qquad \qquad \text{Subadditiv}$$

$$d(x,z) \le d(x,y) + d(y,z)$$
 4. Subadditivity or triangle inequality

Using given distance function,

The distance between X (0,0) & Y (0,1) => d (x, y) 
$$= (|0-0| + |0-1|)^3$$
$$= (0+1)^3$$
$$= (1)^3$$
$$= 1$$

The distance between Y (0,1) & X (0,0) => d (y, x) = 
$$(|0-0| + |1-0|)^3$$
  
=  $(0+1)^3$   
=  $(1)^3$   
= 1

The distance between Y (0,1) & Z (1,1) => d (y, z) 
$$= (|0-1| + |1-1|)^3$$
$$= (1+0)^3$$
$$= (1)^3$$
$$= 1$$

The distance between Z (1,1) & Y (0,1) => d (z, y) 
$$= (|1-0| + |1-1|)^3$$

$$= (1+0)^3$$

$$= (1)^3$$

$$= 1$$
The distance between Z (1,1) & X (0,0) => d (z, x) 
$$= (|1-0| + |1-0|)^3$$

$$= (1+1)^3$$

$$= (2)^3$$

$$= (1+1)^3$$

$$= (2)^3$$

$$= (2)^3$$

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$$= (2)^3$$

$$= (2)^3$$

Checking validity of the distance function properties on the distance values calculated using given distance function.

1. 
$$d(x, y) \ge 0$$
,  $d(y, x) \ge 0$ ,  $d(y, z) \ge 0$ ,  $d(z, y) \ge 0$ ,  $d(z, x) \ge 0$ ,  $d(x, z) \ge 0$ . Clearly  $d(x, y) \ge 0$  and  $d(x, y) = 0 \Leftrightarrow x = y$  are satisfied.

2. 
$$d(x, y) = d(y, x)$$
,  $d(y, z) = d(z, y)$ ,  $d(z, x) = d(x, z)$   
 $Clearly d(x, y) = d(y, x)$  is satisfied.

3. 
$$d(x, z) = 8$$
,  $d(x, y) = 1$ ,  $d(y, z) = 1$   
 $d(x, z) \le d(x, y) + d(y, z)$   
 $8 \le 1 + 1$   
 $8 \le 2$  which is false. So, condition 4 failed d  
 $(z, x) = 8$ ,  $d(z, y) = 1$ ,  $d(y, x) = 1$ .  
 $d(z, x) \le d(z, y) + d(y, x)$   
 $8 \le 1 + 1$   
 $8 \le 2$  which is false. So, condition 4 failed here as well.

As per above calculations and observations, given distance function satisfies the first 3 conditions but fails to meet the last condition (Triangle inequality). Therefore, given function is not a proper distance function.

### # 2 (15 Points)

There are three major manufacturing companies that make a product: Manufacturers A, B, and C. Manufacturer A has a 60% market share, and Manufacturer B has a 30% market share. 5% of A's products are defective, 6% of B's products are defective, and 8% of C's products are defective.

- a) What is the probability that a randomly selected product is defective? P(Defective)?
- **b)** What is the probability that a randomly selected product is defective and that it came from A? P(A and Defective)?
- c) What is the probability that a defective product came from B? P(B/Defective)?
- **d)** Are these events (being defective and coming from B) independent? Why?

#### **Solution:**

Let's assume there are 1000 items of the product in the market => N = 1000 Based on Market Share,

A has 50% of market share.  $\Rightarrow$  N(A) = 60% of 1000 = 600

B has 30% of market share.  $\Rightarrow$  N(B) = 30% of 1000 = 300

Remaining are from C => N(c) = 1000-600-300 = 100

Number of defective pieces by manufacturer are as follows:

A's defective products = N (Defective  $\mid$  A) = 5% of 600 items = 30

B's defective products = N (Defective | B) = 6% of 300 items = 18

C's defective products = N (Defective | C) = 8% of 100 items = 8

**b)** P (A  $\cap$  Defective) = N (Defective | A) / N = 30 / 1000 = 0.030 = 3%

**d)** 
$$P(B) = 300 / 1000 = 0.3$$

$$P(Defective) = 56 / 1000 = 0.056$$

For events to be independent =>  $P(B \cap Defective) = P(B) * P(Defective)$ 

P (B 
$$\cap$$
 Defective) = 18 / 1000 = 0.018

Since,  $P(B \cap Defective) \neq P(B) * P(Defective)$ 

Therefore, the events are **not independent** of each other.

# **#3** (20 Points)

The following training dataset (table) is a subset of "census data" for workers with the following characteristics:

• Age: between 15 and 65

• Education: between 0 and 16 years

• Average hours worked per week: between 0 and 80 per week

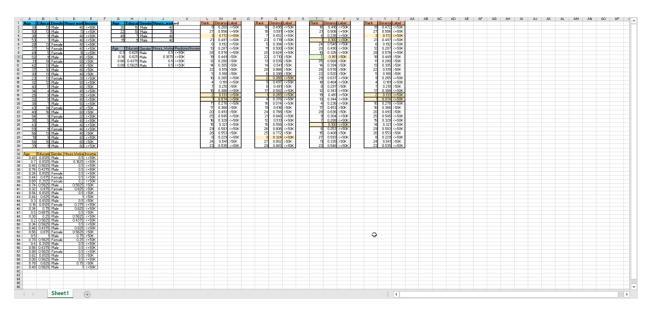
			Hours	
Age	Education	Gender	worked	Income
39	13	Male	40	<=50K
50	13	Male	13	<=50K
38	9	Male	40	<=50K
53	7	Male	40	<=50K
28	13	Female	40	<=50K
37	14	Female	40	<=50K
49	5	Female	16	<=50K
52	9	Male	45	>50K
31	14	Female	50	>50K
42	13	Male	40	>50K
37	10	Male	80	>50K
30	13	Male	40	>50K
23	13	Female	30	<=50K
32	12	Male	50	<=50K
40	11	Male	40	>50K
34	4	Male	45	<=50K
25	9	Male	35	<=50K
32	9	Male	40	<=50K
38	7	Male	50	<=50K
43	14	Female	45	>50K
40	16	Male	60	>50K
54	9	Female	20	<=50K
35	5	Male	40	<=50K
43	7	Male	40	<=50K
59	9	Female	40	<=50K
56	13	Male	40	>50K
19	9	Male	40	<=50K
54	10	Male	60	>50K
39	9	Male	80	<=50K

Use **EXCEL**, weighted knn (k=3) and the above training dataset to predict the income level of the following people (test dataset).

Age	Education_Years	Gender	Hours_worked
30	10	Male	40
22	10	Male	15
48	7	Male	40
19	9	Male	40

Remember, you can copy and paste data into Excel.

**Solution:** file: Q3\_KNN.xlsx



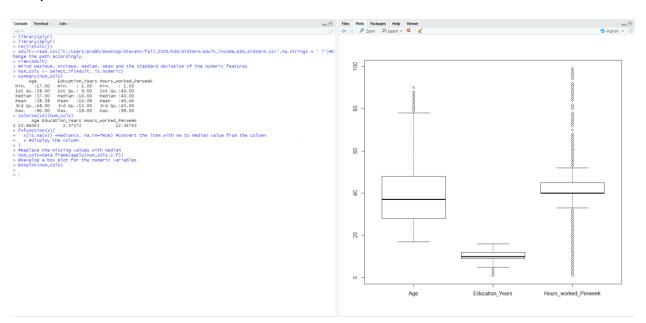
The following questions (#4 through #6) refer to various subsets of the "Census Data". The original dataset was produced by the "Census Bureau". Use the dataset and methods mentioned below to predict salary level.

#4 (15 Points)

Load the "Adult\_income\_EDA.csv" and perform the following exploratory data analysis:

- Find maximum, minimum, median, mean and the standard deviation of the numeric features
- Replace the missing value with the median of the numbers
- Develop a box plot for the numeric variables

**Solution:** file: Q4\_EDA.R



#5 (20 Points): Naïve Bayes:

Load the "Adult\_Income\_Bayes.csv"

- a) Remove any row with missing values
- b) Select every fourth record, starting with the first record, as the test dataset and the remaining records as the training dataset
- c) Perform Naïve Bayes
- d) Score the test dataset
- e) Measure the error rate.

**Solution:** file: Q5\_NB.R

#6 (20 Points): CART Analysis:

Load the "Adult\_Income\_Dtree.csv"

- a) Select every fourth record, starting with the first record, as the test dataset and the remaining records as the training dataset
- b) Perform CART analysis
- c) Score the test dataset
- d) Measure the error rate.

Solution: file: Q6\_CART.R

