Bayesian Analysis II, Midterm EXAM (Dec. 2, 2020)

INSTRUCTIONS: Show all your work to receive full credit. If you can't solve a problem completely, give the steps you would take to solve the problem to receive partial credit!! No help from others are allowed. In this exam, two small extra-credit problems are included.

- Problems from 1-4 should be done by using the statistical package R.

- Problem 5 should be done by Winbugs or rjags.

1. a) Using a random variable from a uniform distribution, estimate the integral

$$\int_0^{10} \exp(-2|x-5|)dx.$$

In R, abs( ) is the function for computing an absolute value.

b) Using the N(5,1) random variable, estimate the above integral.

c) Compare these estimates in a) and b) and give a brief comment on your findings.

2. a) We are to simulate from a distribution with density $f(x) = rx^{r-1}$ for $x \geq 1$, $r \geq 2$ and c.d.f,

$$F(x) = \int_1^x ry^{r-1}dy = 1 - \frac{1}{x^r}.$$

Starting from independent uniform random variables $U \sim \text{unif}(0,1)$, generate samples from the above distribution with $r = 2$. Make a histogram with the samples.

b) Consider a posterior density for $0 < \theta < \pi/2$,

$$p(\theta) = \sin(\theta)$$

Give an accept-reject sampling algorithm for obtaining independent draws of θ using the instrumental function, g that is uniform density on the interval $(0, \pi/2)$.

3. Let $x \sim \text{Binom}(n, \theta)$ be a single observation from the Binomial distribution. Assume independent priors for the unknown parameters $n \sim \text{Poisson}(\lambda)$ and $\theta \sim \text{Beta}(\alpha, \beta)$.

a) Derive the full conditional distributions: $p(\theta|n, x)$ and $p(n|\theta, x)$.

Note that $p(n|\theta, x)$ is proportional to the kernel of a Poisson distribution for $n-x$, not for n. (Multiply a constant term λ^x to give the complete form for a Poisson kernel.)

The transformed variable $W = n - x$ can be used first for drawing from the full conditional derived above. Thus, to sample from $n|(\theta, x)$, one can sample from the (2nd) derived full conditional for W and set $n = W + x$.

b) Implement the full conditional distributions to a Gibbs sampler.

Assume $x = 20, \lambda = 30, \alpha = 2$ and $\beta = 2$. Draw at least 2000 posterior samples and use 500 samples for burn-in. Make the histograms of samples from the marginal posterior distribution of each variable obtained by a Gibbs sampler.

4. Suppose that we wish to draw samples from the posterior density proportional to $\exp(-\theta), \theta > 0$. Write a function for the Metropolis-Hastings algorithm with the random walk Normal(0,1) (centered at the current $\theta^{(t)}$) proposal distribution. Make a histogram with the samples.

5. A study was done to find some relationship between the smoking habits and the age for some population.

   For each age-group, the total number of people and the number of deaths were recorded. These age groups are divided into 4 types according to the smoking habits: non-smokers, Cigar and pipe, Cigarette and other, Cigarette only.

   Let Y_i be the number of deaths in age/smoking group i from a population of n_i: $Y_i \sim$Binom(n_i, p_i). It is expected that the probability of death varies systematically as a linear function of the age depending on the group: set up a random intercept and a random slope for each age. Use 'age-60' to scale the covariate 'age'.

   a) Analyze this data using the noninformative normal priors. Give the posterior inference summaries and make a short comment on the results.

   b) Extend the model to a 3-stage hierarchical model for which the random coefficients have some common means. (Fix the scale parameters using some numbers.)

   age: 40 45 50 55 60 65 70 75 80 40 45 50 55 60 65 70 75 80 40 45 50 55 60 65 70 75 80 40 45 50 55 60 65 70 75 80

   deaths: 18 22 19 55 117 170 179 120 120 2 4 3 38 113 173 212 243 253 149 169 193 576 1001 901 613 337 189 124 140 187 514 778 689 432 214 63

   n: 656 359 249 632 1067 897 668 361 274 145 104 98 372 846 949 824 667 537 4531 3030 2267 4682 6052 3880 2033 871 345 3410 2239 1851 3270 3791 2421 1195 436 113

   g : 1 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3 3 4 4 4 4 4 4 4 4 4