Bayesian Analysis II, FALL 2020

Lab Exercise 2: on **Wednesday Oct. 28, 14-16**

1. Starting from independent uniform random variables $U \sim \text{unif}(0,1)$, we are to simulate independent draws from a Logistic distribution, having density

$$f(x) = \frac{e^{-x}}{(1+e^{-x})^2}, \qquad \text{for } -\infty < x < \infty$$

a) Give an algorithm (in words) to simulate draws from a Logistic distribution.

b) Write a function in R to generate samples from the Logistic based on your algorithm. Make a histogram with the samples.

c) Use the samples to estimate $P(X \in (2,3))$.

2. Consider a random variable X having the density

$$f(x) = e^{-(x+1)} + (e-1)e^{-ex}, \qquad \text{for } 0 < x < \infty$$

a) Give an accept-reject sampling algorithm for obtaining independent draws of X using the instrumental function $g(x) = e^{-x}$.

b) Implement the algorithm in R and make a histogram with the samples drawn to esimate the density of X.

3. For the following functions below, plot the integrands, and use Monte Carlo integration based on a Cauchy simulation to calculate the integrals.

a)

$$\int_{-\infty}^{\infty} \frac{\theta}{1+\theta^2} \exp(-(x-\theta)^2/2)d\theta, \text{ where } x = 2.$$

b)

$$\int_{-\infty}^{\infty} \frac{\theta}{1+\theta^2} \exp(-(x-\theta)^2/2)d\theta \Big/ \int_{-\infty}^{\infty} \frac{1}{1+\theta^2} \exp(-(x-\theta)^2/2)d\theta,$$

where $x = 2$.

4. Determine if Markov chains given below are recurrent and ergodic.

i) $P_1 = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$ ii) $P_2 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

iii) $P_3 = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

5. A particle moves on a circle through points which have been marked 0,1,2,3,4 (in a clockwise order). At each step it has a probability p moving to the right (clockwise) and $1-p$ to the left (counterclockwise). Let X_n denote its location on the circle after nth step. The process is a Markov chain.

i) Find the transition probability matrix.

ii) Give the equations to solve for limiting distributions and find the limiting probabilities.