(6.) $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i \quad , i = 1, \dots, n \quad , \varepsilon_i \sim N(0, \tau)$

$$p(\beta_0, \beta_1, \tau) \propto \frac{1}{\tau} \qquad n = 15$$

$$\therefore y_i \mid \beta_0, \beta_1, \tau, \underline{x} \sim N\left(\beta_0 + \beta_1 x_i , \frac{1}{\tau}\right)$$

Lkhd:

$$p(y \mid \beta_0, \beta_1, \tau, \underline{X}) = \prod^{n} p(y_i \mid \beta_0, \beta_1, \tau, x_i) = \prod^{n} \left(\sqrt{2\pi\tau^{-1}}\right)^{-\frac{1}{2}} \exp\left(\frac{1}{2\tau^{-1}}\left(y_i - (\beta_0 + \beta_1 x_i)\right)^2\right)$$

$$\propto \left(\sqrt{\tau^{-1}}\right)^{-\frac{n}{2}} \exp\left(-\frac{1}{2\tau^{-1}} \sum (y_i - \beta_0 - \beta_1 x_i)^2\right)$$

$$= \tau^{\frac{n}{4}} \exp\left(-\frac{\tau}{2} \sum (y_i - \beta_0 - \beta_1 x_i)^2\right)$$

Joint

$$p(y, \beta_0, \beta_1, \tau, \underline{X}) = p(y \mid \beta_0, \beta_1, \tau, \underline{X})\, p(\beta_0, \beta_1, \tau) = p(\beta_0, \beta_1, \tau \mid y, \underline{x})$$

$$\propto \tau^{\frac{n}{4}} \tau^{-1} \exp\left(-\tau \frac{1}{2} \sum (y_i - \beta_0 - \beta_1 x_i)^2\right) = \tau^{\frac{n}{4} - 1} \exp\left(-\tau \frac{1}{2} \sum (y_i - \beta_0 - \beta_1 x_1)^2\right)$$

Full-cond for τ

$$\tau \mid \beta_0, \beta_1, y, \underline{x} \sim Gamma\left(\frac{n}{4} , \left(\frac{1}{2} \sum (y_i - \beta_0 - \beta_1 x_i)^2\right)^{-1}\right)$$

Full-cond for β_0:

$$p(\beta_0 \mid \beta_1, \tau, y, \underline{x}) \propto \exp\left(-\tau \frac{1}{2} \sum (y_i - \beta_0 - \beta_1 x_i)^2\right)$$

$$\propto \exp\left(-\tau \frac{1}{2} \sum \beta_0^2 - 2y_i \beta_0 + 2\beta_0 \beta_1 x_i\right) = \exp\left(-\tau \frac{1}{2} \sum \beta_0^2 - 2(y_i - \beta_1 x_i)\beta_0\right)$$

$$= \exp\left(-\tau \frac{1}{2} \left[n\beta_0^2 - 2\left[\sum (y_i - \beta_1 x_i)\right]\beta_0\right]\right)$$

$$= \exp\left(-\frac{1}{2} \tau n \left(\beta_0 - \frac{1}{n} \sum (y_i - \beta_1 x_i)\right)^2\right)$$

$$\therefore \beta_0 \mid \beta_1, \tau, y, \underline{x} \sim N\left(\frac{1}{n} \sum (y_i - \beta_1 x_i) , \frac{1}{\tau n}\right)$$

Full-cond for β_1:

$$p(\beta_1 \mid \beta_0, \tau, y, \underline{x}) \propto \exp\left(-\tau \frac{1}{2} \sum (y_i - \beta_0 - \beta_1 x_i)^2\right)$$

$$\propto \exp\left(-\tau \frac{1}{2} \sum \beta_1^2 x_i^2 - 2y_i x_i \beta_1 + 2\beta_0 \beta_1 x_i\right) = \exp\left(-\tau \frac{1}{2} \sum \beta_1^2 x_i^2 - 2(y_i x_i - \beta_0 x_i)\beta_1\right)$$

$$= \exp\left(-\tau \frac{1}{2} \left[\left(\sum x_i^2\right)\beta_1^2 - 2\left[\sum (y_i x_i - \beta_0 x_i)\right]\beta_1\right]\right)$$

$$= \exp\left(-\frac{1}{2} \tau \left(\sum x_i^2\right)\left(\beta_1^2 - \frac{\sum (y_i x_i - \beta_0 x_i)}{\sum x_i^2}\right)^2\right)$$

$$\therefore \beta_1 \mid \beta_0, \tau, y, \underline{x} \sim N\left(\frac{\sum (y_i x_i - \beta_0 x_i)}{\sum x_i^2} , \frac{1}{\tau \sum x_i^2}\right)$$