DATA.ML.300 Computer Vision

Exercise Round 2

Answered to questions 1-2

1)

a)

Hidden unit output:

$$\sigma(i_h) = \frac{1}{1 + exp(-i_h)} = \frac{1}{1 + exp(-(1 * (-2) + 2))} = 0.5$$

Output unit output:

$$y = 4 * 0.5 + 0 = 2$$

b)

$$\frac{1}{2}(1 - 2)^2 = 0.5$$

c)

$$\frac{\partial E}{\partial w_2} = \frac{\partial E}{\partial y}\frac{\partial y}{\partial w_2} = \frac{1}{2}(-2(t - y)) * x_2 = (y - t)x_2$$

Numerical value: $(2 - 1) * 0.5 = 0.5$

d)

$$E = \frac{1}{2}(t - y)^2$$

$$y = w_2 x_2 + b_2$$

$$x_2 = \sigma(i_h) = \frac{1}{1 + exp(-i_h)}$$

$$i_h = w_1 x_1 + b_1$$

$$\begin{aligned}\frac{\partial E}{\partial w_1} &= \frac{\partial E}{\partial y}\frac{\partial y}{\partial \sigma(i_h)}\frac{\partial \sigma(i_h)}{\partial i_h}\frac{\partial i_h}{\partial w_1} \\ &= [y - t]\,[w_2]\,[\sigma(i_h)(1 - \sigma(i_h))]\,[x_1] \\ &= [y - t]\,[w_2]\left[\frac{1}{1 + exp(-i_h)}(1 - \frac{1}{1 + exp(-i_h)})\right][x_1]\end{aligned}$$

Numerical value: $(2 - 1)(4)\frac{1}{1 + exp(0)}(1 - \frac{1}{1 + exp(0)})(1) = 1$

2.

a)

$$dist(Q, A) = \sqrt{(2-1)^2 + (1-2)^2 + (6-3)^2 + (4-4)^2 + (2-1)^2} = \sqrt{12}$$

$$dist(B, A) = \sqrt{(1-3)^2 + (2-1)^2 + (3-4)^2 + (4-1)^2 + (1-5)^2} = \sqrt{31}$$

$$cos(\theta)_{QA} = \frac{Q * A}{|Q|\,|A|} = \frac{40}{\sqrt{61}\sqrt{31}}$$

$$cos(\theta)_{BA} = \frac{B * A}{|B|\,|A|} = \frac{26}{\sqrt{52}\sqrt{31}}$$

b)

By euclidean distance, the vector A is closer to Q (smaller distance). By cosine similarity, the vector A is closer to Q (cosine of the angle is closer to 1 $\Rightarrow$ angle between vectors is closer to 0 $\Rightarrow$ vectors are more parallel).