DATA.ML.300 Computer Vision, Exercise Round 1

Answered to all questions (1-4).

1.

a) Homogenous coordinates for:

$$\mathbf{x}_1 = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \Rightarrow \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

$$\mathbf{x}_2 = \begin{pmatrix} 1 \\ -2 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

$$\mathbf{x}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\mathbf{x}_4 = \begin{pmatrix} -1 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

b)

$$\mathbf{l}_1 = \mathbf{x_1} \times \mathbf{x_2} = \begin{pmatrix} x_{11}x_{22} - x_{12}x_{21} \\ x_{12}x_{20} - x_{10}x_{22} \\ x_{10}x_{21} - x_{11}x_{20} \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ -3 \end{pmatrix}$$

Equation for $\mathbf{l_1}$: $x - y - 3 = 0$

$$\mathbf{l}_2 = \mathbf{x_3} \times \mathbf{x_4} = \begin{pmatrix} x_{31}x_{42} - x_{32}x_{41} \\ x_{32}x_{40} - x_{30}x_{42} \\ x_{30}x_{41} - x_{31}x_{40} \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

Equation for $\mathbf{l_2}$: $x - 2y + 1 = 0$

c)

Homogenous intersection point:

$$\mathbf{p_h} = \mathbf{l_1} \times \mathbf{l_2} = \begin{pmatrix} l_{11}l_{22} - l_{12}l_{21} \\ l_{12}l_{20} - l_{10}l_{22} \\ l_{10}l_{21} - l_{11}l_{20} \end{pmatrix} = \begin{pmatrix} -7 \\ -4 \\ -1 \end{pmatrix}$$

Cartesian intersection point:

$$\mathbf{p}_c = \begin{pmatrix} \frac{-7}{-1} \\ \frac{-4}{-1} \end{pmatrix} = \begin{pmatrix} 7 \\ 4 \end{pmatrix}$$

2.

a) & b)

Translation matrix, 2 dof:

$$\begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix}$$

Rotation + translation matrix, 3 dof:

$$\begin{pmatrix} cos(\theta) & -sin(\theta) & t_x \\ sin(\theta) & cos(\theta) & t_y \\ 0 & 0 & 1 \end{pmatrix}$$

Similarity transformation matrix, 4 dof:

$$\begin{pmatrix} s * cos(\theta) & -s * sin(\theta) & t_x \\ s * sin(\theta) & s * cos(\theta) & t_y \\ 0 & 0 & 1 \end{pmatrix}$$

Affine transformation matrix, 6 dof:

$$\begin{pmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ 0 & 0 & 1 \end{pmatrix}$$

Projective transformation matrix, 8 dof:

$$\begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix}$$

c)

Due to fundamental properties of homogenous coordinate systems, $\begin{pmatrix} \mathbf{x} \\ 1 \end{pmatrix}$ and $c \begin{pmatrix} \mathbf{x} \\ 1 \end{pmatrix}$ represents the same point after conversion. Therefore it is possible to express projective transformation as

$$\begin{pmatrix} \mathbf{x}' \\ 1 \end{pmatrix} = H_{3\times3} \begin{pmatrix} \mathbf{x} \\ 1 \end{pmatrix} \Leftrightarrow \begin{pmatrix} \mathbf{x}' \\ 1 \end{pmatrix} = \mathrm{H}_{3\times3} c \begin{pmatrix} \mathbf{x} \\ 1 \end{pmatrix}.$$

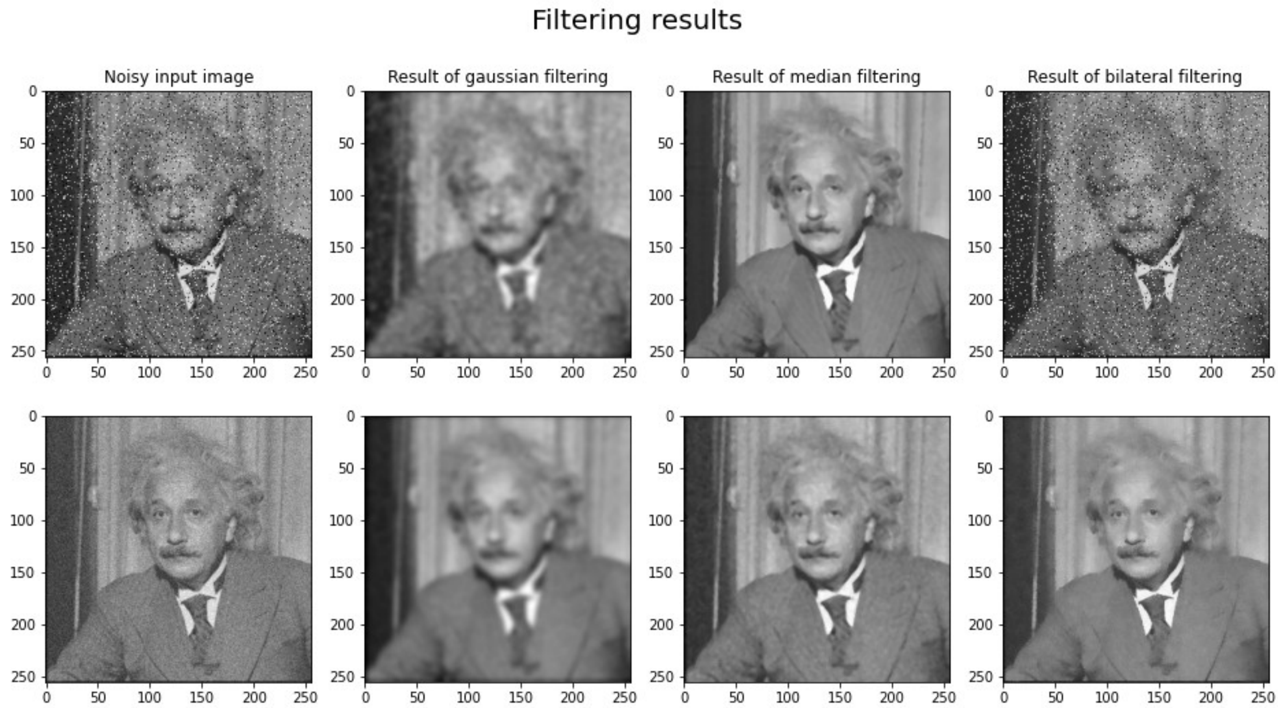
Because c is arbitary, it can be chosen to be, for example, equal to the inverse of the arbitary element of $H_{3\times3}$, such as $c = \dfrac{1}{h_{11}}$. Therefore the transformation can be stated as

$$\begin{pmatrix} \mathbf{x}' \\ 1 \end{pmatrix} = H_{3\times3} \frac{1}{h_{11}} \begin{pmatrix} \mathbf{x} \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & h_{12}/h_{11} & h_{13}/h_{11} \\ h_{21}/h_{11} & h_{22}/h_{11} & h_{23}/h_{11} \\ h_{31}/h_{11} & h_{32}/h_{11} & h_{33}/h_{11} \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ 1 \end{pmatrix}.$$

Now projective transformation matrix consist of an eight independent ratios and thus it has eight degrees of freedom.

3.

Output image of the program *image_denoising.py*



4.

Output images of the program *hybrid_images.py*