DATA.ML.100

My derivations:

$$SSE = \sum (y_i - \hat{y}_i)^2, \quad \hat{y}_i = ax_i + b$$

$$\frac{d}{da} SSE = \sum \frac{d}{da}(y_i - ax_i - b)^2 = \sum -2x_i(-ax_i - b + y_i)$$
$$= -2[-a\sum x_i^2 - b\sum x_i + \sum y_i x_i]$$

$$-a\sum x_i^2 - b\sum x_i + \sum y_i x_i = 0$$
$$\therefore a = \frac{\sum y_i x_i - b\sum x_i}{\sum x_i^2}$$

$$\frac{d}{db} SSE = \sum \frac{d}{db}(y_i - ax_i - b)^2 = \sum -2(-ax_i - b + y_i) = 2[-a\sum x_i - \sum b + \sum y_i]$$
$$= -2[-a\sum x_i - Nb + \sum y_i]$$

$$-2[-a\sum x_i - Nb + \sum y_i] = 0$$
$$\therefore b = \frac{\sum y_i - a\sum x_i}{N} = \overset{\text{Mean}}{\overline{y}} - a\overline{x}$$

$$a = \frac{\sum y_i x_i}{\sum x_i^2} - b\frac{\sum x_i}{\sum x_i^2}$$

$$a = \frac{\sum y_i x_i}{\sum x_i^2} - \left(\frac{(\sum y_i)(\sum x_i)}{N\sum x_i^2} - a\frac{(\sum x_i)^2}{N\sum x_i^2}\right)$$

$$a - a\left(\frac{(\sum x_i)^2}{N\sum x_i^2}\right) = \frac{\sum y_i x_i}{\sum x_i^2} - \frac{(\sum y_i)(\sum x_i)}{N\sum x_i^2}$$

$$a\left(1 - \frac{(\sum x_i)^2}{N\sum x_i^2}\right) = --''--$$

$$a\left(\frac{N\sum x_i^2 - (\sum x_i)^2}{N\sum x_i^2}\right) = --''--$$

$$a = \frac{N\sum y_i x_i - (\sum y_i)(\sum x_i)}{N\sum x_i^2} \cdot \frac{N\sum x_i^2}{N\sum x_i^2 - (\sum x_i)^2}$$

$$a = \frac{N\sum y_i x_i - (\sum x_i)(\sum y_i)}{N\sum x_i^2 - (\sum x_i)^2} = \frac{\sum y_i x_i - \frac{1}{N}(\sum x_i)(\sum y_i)}{\sum x_i^2 - \frac{1}{N}(\sum x_i)^2}$$

$$= \frac{\sum y_i x_i - \overline{x} N \overline{y}}{\sum x_i^2 - \frac{1}{N}(\sum x_i)^2}$$

$$\boxed{a = \frac{\sum y_i x_i - N\overline{x}\overline{y}}{\sum x_i^2 - \frac{1}{N}(\sum x_i)^2}}$$

$$\boxed{b = \overline{y} - a\overline{x}}$$

Screenshots from the results:

LEFT CLICK: Add point, RIGHT CLICK: Draw a fitted linear model.

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